Gauge Unification in Supersymmetric Intersecting Brane Worlds

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based on:

R. Blumenhagen, V. Braun, B. Körs and D. Lüst, *Orientifolds of K3 and Calabi-Yau Manifolds with Intersecting D-branes*, JHEP 0207 (2002) 026, hep-th/0206038.

R. Blumenhagen, D. Lüst and S. Stieberger, *Gauge Unification in Supersymmetric Intersecting Brane Worlds*, hep-th/0305146.

I. Introduction to Gauge Unification

The three Standard Model gauge couplings g_s, g_w and g_y have different values at the weak scale and are all in the perturbative regime $g_i < 1$.

Extrapolating these couplings due to the one-loop running

$$\frac{4\pi}{g_a^2(\mu)} = k_a \frac{4\pi}{g_X^2} + \frac{b_a}{2\pi} \log\left(\frac{\mu}{M_X}\right) + \Delta_a$$

to higher scales, one finds that they all meet at

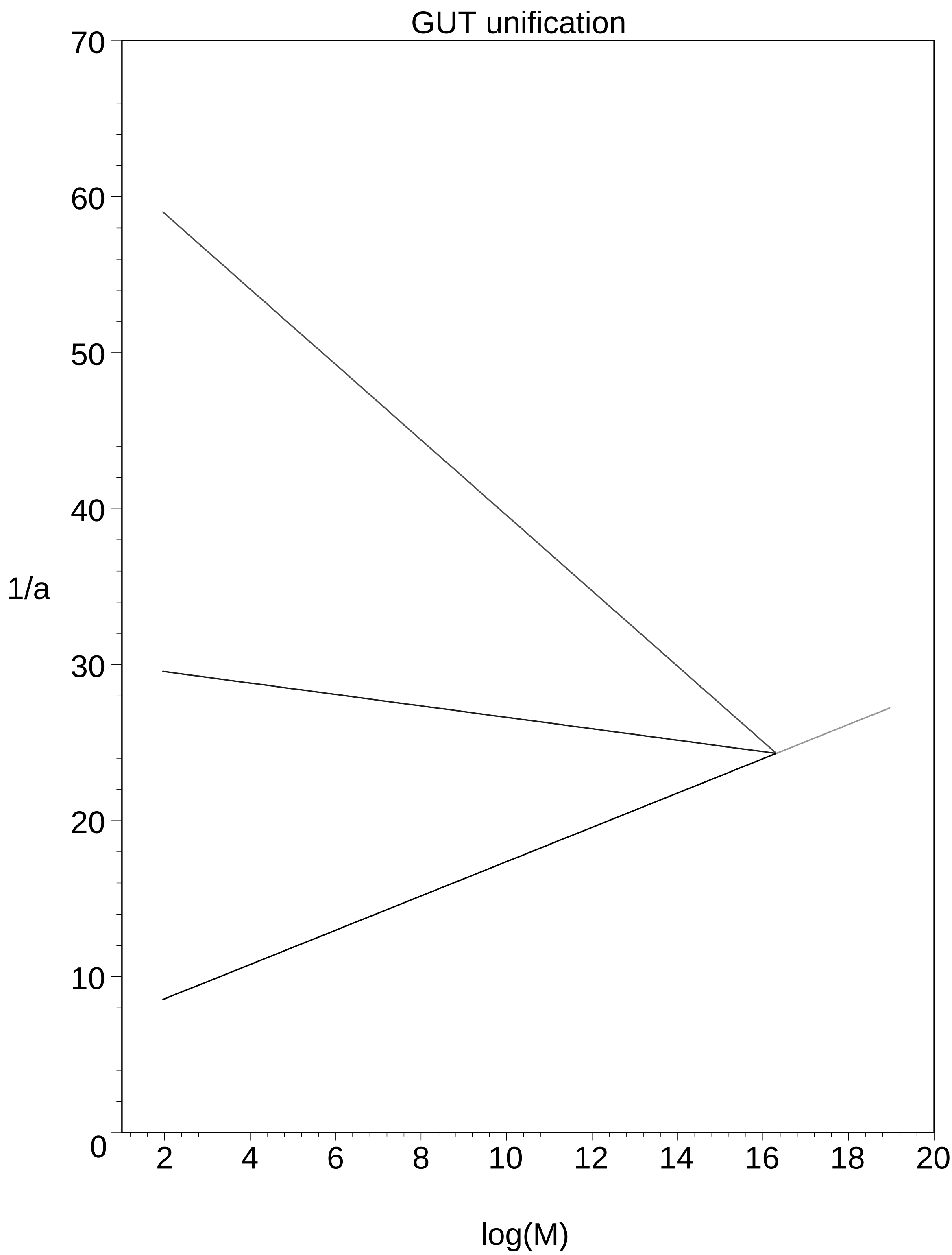
$$M_X \simeq 2 \cdot 10^{16}\ \text{GeV}, \qquad \alpha_s = \alpha_w = \frac{3}{5}\alpha_Y = \alpha_X \simeq \frac{1}{24},$$

if the light spectrum contains just the MSSM particles.

This is in accord with for instance an $SU(5)$ Grand Unified gauge group at the GUT scale.

<u>Shortcomings:</u>

- Inherent problems with GUT models, like doublet-triplet splitting problem
- Since $M_X < M_{pl}$ no unification with gravity.


GUT unification
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String model building

- 1986-1994: weakly coupled $E_8 \times E_8$ heterotic string on Calabi-Yau threefolds, E_6 GUT model with $\chi/2$ generations of chiral fermions, $\mathcal{N} = 1$ supersymmetry, string scale $M_s \simeq 10^{18} GeV$, concrete models have a lot of exotic matter.

review:(K. Dienes, hep-th/9602045)

- $>$1995: open string model building using D-branes, orientifolds, supersymmetric models with branes on singularities and intersecting D-branes, string scale in principle a free parameter, extreme case: $M_s \sim 1$ TeV allows for non-supersymmetric models, exotic matter, stability of non-susy configurations.

reviews:(C. Angelantonj, A. Sagnotti, hep-th/0204089)
(A.M. Uranga, hep-th/0301032)

Gauge Unification for Heterotic Strings

In string theory one has a new scale M_s, so that it is natural to identify M_X with M_s. In the heterotic string one finds

$$k_a = \text{level of SU(N}_\text{a}\text{) Kac} - \text{Moody algebra.}$$

At one loop level the relation between the string and the Planck scale was found to be

$$M_s \simeq g_{st} \cdot 0.058 \cdot M_{pl},$$

which, using $g_{st} \simeq 0.7$, led to $M_s \simeq 5 \cdot 10^{17}$ GeV.

(Kaplunovsky, Nucl. Phys. B307 (1988) 145, hep-th/9205068)

The discrepancy between M_X and M_s needs to be explained by moduli-dependent string threshold corrections Δ_a (or alternatively by heterotic M-theory).

For adjoint scalars one needs $k_a > 1$, which makes the string construction fairly messy. Better to have direct string unification of the Standard model couplings without an enhanced grand unified gauge symmetry (no doublet-triplet splitting problem).

II. Introduction to Intersecting Brane Worlds

Intersecting Brane Worlds (IBW) have been proven to be a new branch in the M-theory moduli space, which exhibit nice phenomenological properties and where computations can be performed quite explictly.

The general set-up:

Consider an orientifold background

$$\mathcal{X} = \mathbb{R}^{3,1} \times \frac{\mathcal{M}^6}{\Omega\overline{\sigma}}$$

with $\overline{\sigma}$ an anti-holomorphic involution.

The fixed point locus of $\overline{\sigma}$ gives rise to an orientifold $O6$ plane, whose RR charge must be canceled by introducing D-branes in the background.

Giving up the restriction of placing these D-branes parallel to the O-planes, they will intersect each other non-trivially leading to chirality and supersymmetry breaking.
(Berkooz, Douglas, Leigh, hep-th/9606139)

One gets the following massless modes in the effective four-dimensional space-time.

- Supergravity in the 10D bulk
- Gauge fields localized on the 7-dimensional world-volume of the D-branes
- Chiral matter localized on the 4-dimensional intersection locus of two D-branes

The number of chiral fermions is given by the topological intersection numbers between two D-branes.

The mass of the bosonic partners depend on the intersection angles between two-branes. Therefore, the breaking of supersymmetry depends on the conformal structure of the manifold $\mathcal{M}$.

Issues studied so far:

- String model building with intersecting D-branes, construction of both non-supersymmetric and non-trivial supersymmetric models.

(Blumenhagen, Görlich, Körs, Lüst, hep-th/0007024)
(Angelantonj, Antoniadis, Dudas, Sagnotti, hep-th/0007090)
(Aldazabel, Franco, Ibáñez, Rabadán, Uranga, hep-th/0011073, hep-ph/0011132)

(Blumenhagen, Körs, Lüst, hep-th/0012156)
(Ibanez, Marchesano, Rabadan, hep-th/0105155)
(Cvetic, Shiu, Uranga, hep-th/0107143, hep-th/0107166)

- Generic structure of intersecting brane worlds on general K3 and Calabi-Yau manifolds

  (Blumenhagen, Braun, Körs, Lüst, hep-th/0206038, hep-th/0210083)

- Computation of Yukawa and four-point couplings and discussion of their phenomenological implications, like FCNC and proton decay in GUT-like models

  (Cvetiv, Papadimitriou, hep-th/0303197)

  (Abel, Owen, hep-th/0303124)

  (Klebanov, Witten, hep-th/0304079)

- Computation of gauge threshold corrections

  (Friedmann, Witten, hep-th/0211269)

  (Lüst, Stieberger, hep-th/0302221)

Tadpole conditions

(Blumenhagen, Braun, Körs, Lüst, hep-th/0206038)

Choose the anti-holomorphic involution in local coordinates to be $\overline{\sigma} : z_i \to \overline{z}_i$.

The fixed locus, Fix($\overline{\sigma}$), is a sLag 3-cycle, implying

$$\mathrm{Vol}(\mathrm{Fix}(\overline{\sigma})) = \int_{\mathrm{Fix}(\overline{\sigma})} \Re(\Omega_3).$$

The RR tadpole cancellation conditions can be deduced from the Chern-Simons terms

$$\mathcal{S}_{\mathrm{CS}}^{(\mathrm{D6})} = \mu_6 \int_{\mathrm{D6}} \mathrm{ch}(\mathcal{F}) \wedge \sqrt{\frac{\hat{\mathcal{A}}(\mathcal{R}_T)}{\hat{\mathcal{A}}(\mathcal{R}_N)}} \wedge \sum_p C_p$$

for the D-branes and the O-planes

$$\mathcal{S}_{\mathrm{CS}}^{(\mathrm{O6})} = Q_6 \mu_6 \int_{\mathrm{O6}} \sqrt{\frac{\hat{\mathcal{L}}(\mathcal{R}_T/4)}{\hat{\mathcal{L}}(\mathcal{R}_N/4)}} \wedge \sum_p C_p.$$

Introduce general D6 branes wrapped on the homology cycles π_a and their $\Omega\overline{\sigma}$ images π'_a.

For A-type D-branes wrapping sLag cycles with $\mathcal{F} = 0$ the RR 7-form charge on the compact manifold $\mathcal{M}^6$ vanishes if

$$\sum_a N_a \, (\pi_a + \pi'_a) + Q_6 \, \pi_{O6} = 0.$$

Tadpole conditions

The disc level scalar potential (tension of the branes) can be deduced from the DBI action for the D-branes. For sLag branes it can be written as

$$\mathcal{V} = T_6\, e^{-\phi_4} \left(\sum_a N_a \int_{\pi_a + \pi'_a} \Re(e^{i\phi_a} \widehat{\Omega}_3) + Q_6 \int_{\pi_{\mathrm{O6}}} \Re(\widehat{\Omega}_3) \right),$$

where the D-branes are calibrated with respect to the 3-form $\Re(e^{i\phi_a}\Omega_3)$.

In a supersymmetric configuration all branes are calibrated with respect to the same 3-form as the orientifold plane, so that

$$\mathcal{V} = T_6\, e^{-\phi_4} \left(\sum_a N_a \int_{\pi_a + \pi'_a} \Re(\widehat{\Omega}_3) + Q_6 \int_{\pi_{\mathrm{O6}}} \Re(\widehat{\Omega}_3) \right),$$

which vanishes due to the RR-tadpole condition.

Note, $\mathcal{V}$ only depends on the complex structure (conformal structure) of $\mathcal{M}$.

The Chiral Massless Spectrum

Since the chiral spectrum has to satisfy some anomaly constraints, we expect that it is given by purely topological data (Atiyah-Singer index theorem).

The chiral massless spectrum indeed is completely fixed by the topological intersection numbers of the 3-cycles of the configuration.

Sector	**Rep.**	**Number**
$a'\,a$	A_a	$\frac{1}{2}\left(\pi'_a \circ \pi_a + \pi_{O6} \circ \pi_a\right)$
$a'\,a$	S_a	$\frac{1}{2}\left(\pi'_a \circ \pi_a - \pi_{O6} \circ \pi_a\right)$
$a\,b$	$(\overline{N}_a, N_b)$	$\pi_a \circ \pi_b$
$a'\,b$	(N_a, N_b)	$\pi'_a \circ \pi_b$

The non-abelian gauge anomalies cancel automatically and mixed $U(1)_a - SU(N)_b^2$ anomalies are canceled by a generalized Green-Schwarz mechanism involving dimensionally reduced RR-forms.

III. Gauge Couplings at Tree Level

In contrast to the heterotic string, here each gauge factor comes with its own gauge coupling, which at string tree-level can be deduced from the Dirac-Born-Infeld action

$$\frac{4\pi}{g_a^2} = \frac{M_s^3\, V_a}{(2\pi)^3\, g_{st}\, \kappa_a}$$

with $\kappa_a = 1$ for $U(N_a)$ and $\kappa_a = 2$ for $SP(2N_a)/SO(2N_a)$.

By dimensionally reducing the type IIA gravitational action one can similarly express the Planck mass in terms of stringy parameters ($M_{pl} = (G_N)^{-\frac{1}{2}}$)

$$M_{pl}^2 = \frac{8\, M_s^8\, V_6}{(2\pi)^6\, g_{st}^2}.$$

Eliminating the unknown string coupling g_{st} gives

$$\frac{1}{\alpha_a} = \frac{M_{pl}}{2\sqrt{2}\,\kappa_a\, M_s} \frac{V_a}{\sqrt{V_6}}.$$

Due to

$$\frac{V_a}{\sqrt{V_6}} = \int_{\pi_a} \Re(e^{i\phi_a}\widehat{\Omega}_3)$$

the gauge coupling only depends on the complex structure moduli.

MSSM-like models

There are so far two simple ways to embed the standard model gauge group into products of unitary and symplectic gauge groups.

Both of them use four stacks of $D6$-branes, which give rise to the initial gauge symmetries

$$\begin{aligned} A &: \quad U(3) \times SP(2) \times U(1) \times U(1) \\ B &: \quad U(3) \times U(2) \times U(1) \times U(1). \end{aligned}$$

The chiral spectrum of the intersecting brane world model should be identical to the chiral spectrum of the standard model particles.

This fixes uniquely the intersection numbers of the four 3-cycles, $(\pi_a, \pi_b, \pi_c, \pi_d)$.

field	**sector**	**I**	$SU(3) \times SU(2) \times U(1)^3$
q_L	(ab)	3	$(3, 2; 1, 0, 0)$
u_R	(ac)	3	$(\overline{3}, 1; -1, 1, 0)$
d_R	(ac')	3	$(\overline{3}, 1; -1, -1, 0)$
e_L	(db)	3	$(1, 2; 0, 0, 1)$
e_R	(dc')	3	$(1, 1; 0, -1, -1)$
ν_R	(dc)	3	$(1, 1; 0, 1, -1)$

The Hypercharge

The hypercharge Q_Y is given as the following linear combination of the three $U(1)$s

$$Q_Y = \frac{1}{3}Q_a - Q_c - Q_d.$$

In general some of the stringy $U(1)$s are anomalous and get a mass via some generalized Green-Schwarz mechanism.

However, for intersecting brane worlds it can also happen that via axionic couplings some anomaly-free abelian gauge groups become massive.

The condition that a linear combination $U(1)_Y = \sum_i c_i U(1)_i$ remains massless reads

$$\sum_i c_i\, N_i\, \left(\pi_i - \pi_i'\right) = 0.$$

In general, if the hypercharge is such a linear combination of $U(1)$s, $Q_Y = \sum_i c_i Q_i$, then the gauge coupling is given by

$$\frac{1}{\alpha_Y} = \sum_i \frac{N_i\, c_i^2}{2} \frac{1}{\alpha_i}.$$

In our case

$$\frac{1}{\alpha_Y} = \frac{1}{6}\frac{1}{\alpha_a} + \frac{1}{2}\frac{1}{\alpha_c} + \frac{1}{2}\frac{1}{\alpha_d}.$$

Realizing the MSSM in IBWs

There exists a most natural and economical way of realizing the Standard Model intersection numbers.

Say one finds two supersymmetric 3-cycles π_a and π_b with the intersection numbers $\pi_a \circ \pi_b = 3$, then homologically choosing $\pi_d = \pi_a$ gives the right intersection numbers for π_d.

Therefore

$$V_a = V_d.$$

The condition that $U(1)_Y$ remains massless simply implies $\pi_c' = \pi_c$.

Therefore, at the bottom of this simple realization there lies an extended Pati-Salam like model

$$U(4) \times SU(2) \times SU(2).$$

From the field theory point of view it is very natural to assume, that the two gauge couplings of the two $SU(2)$ factors are the same, i.e.

$$V_c = V_b.$$

From the stringy point of view, even though we cannot rigorously prove it in the general case, the constraints from supersymmetry and the intersection numbers $\pi_a \circ \pi_b = -\pi_a \circ \pi_c = 3$ do not seem to leave very much room to evade that the internal volumes for the cycles π_b and π_c agree.

Therefore

$$\alpha_d = \alpha_a = \alpha, \quad \alpha_c = \frac{1}{2}\alpha_b = \frac{1}{2}\alpha_w,$$

which implies the Pati-Salam like tree-level relation

$$\frac{1}{\alpha_Y} = \frac{2}{3}\frac{1}{\alpha_s} + \frac{1}{\alpha_w}.$$

IV. Running of the gauge couplings

Using the string prediction of the relation among the gauge couplings at the string scale, we can now use the one-loop running of the gauge couplings down to the weak scale.

In the absence of threshold corrections, the one-loop running of the three gauge couplings is described by the well known formulas

$$\begin{aligned}\frac{1}{\alpha_s(\mu)} &= \frac{1}{\alpha_s} + \frac{b_3}{2\pi}\ln\left(\frac{\mu}{M_s}\right)\\ \frac{\sin^2\theta_w(\mu)}{\alpha(\mu)} &= \frac{1}{\alpha_w} + \frac{b_2}{2\pi}\ln\left(\frac{\mu}{M_s}\right)\\ \frac{\cos^2\theta_w(\mu)}{\alpha(\mu)} &= \frac{1}{\alpha_Y} + \frac{b_1}{2\pi}\ln\left(\frac{\mu}{M_s}\right),\end{aligned}$$

where (b_3, b_2, b_1) are the one-loop beta-function coefficients for $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$.

Using tree level relation at the string scale yields

$$\frac{2}{3}\frac{1}{\alpha_s(\mu)} + \frac{2\sin^2\theta_w(\mu) - 1}{\alpha(\mu)} = \frac{B}{2\pi}\ln\left(\frac{\mu}{M_s}\right)$$

with

$$B = \frac{2}{3}b_3 + b_2 - b_1.$$

Running of the Gauge Couplings

Employing the measured Standard Model parameters

$$
\begin{aligned}
M_Z &= 91.1876\ \text{GeV}, \quad \alpha_s(M_Z) = 0.1172, \\
\alpha(M_Z) &= \frac{1}{127.934}, \quad \sin^2\theta_w(M_Z) = 0.23113
\end{aligned}
$$

the resulting value of the unification scale only depends on the combination B of the beta-function coefficients.

In general besides the chiral matter string theory contains also additional vector-like matter.

This is also localized on higher dimensional intersection loci of the $D6$ branes and also comes with multiplicity n_{ij} with $i, j \in \{a, b, c, d\}$.

One finds the following contribution to B

$$
\begin{aligned}
B &= 12 - 2\, n_{aa} - 4\, n_{ab} + 2\, n_{a'c} + 2\, n_{a'd} - 2\, n_{bb} + 2\, n_{c'c} \\
&\quad + 2\, n_{c'd} + 2\, n_{d'd}.
\end{aligned}
$$

B does not depend on the number of weak Higgs multiplets n_{bc}.

Running of the Gauge Couplings

For the unification scale one finds

B	$\mathrm{M_s}$
18	$3.36 \cdot 10^{11}$
16	$5.28 \cdot 10^{12}$
14	$1.82 \cdot 10^{14}$
12	$2.04 \cdot 10^{16}$
10	$1.51 \cdot 10^{19}$
8	$3.06 \cdot 10^{23}$

For the MSSM one has $(b_3, b_2, b_1) = (3, -1, -11)$, i.e $B = 12$ and the unification scale is the usual GUT scale

$$M_X = 2.04 \cdot 10^{16}\ \text{GeV}.$$

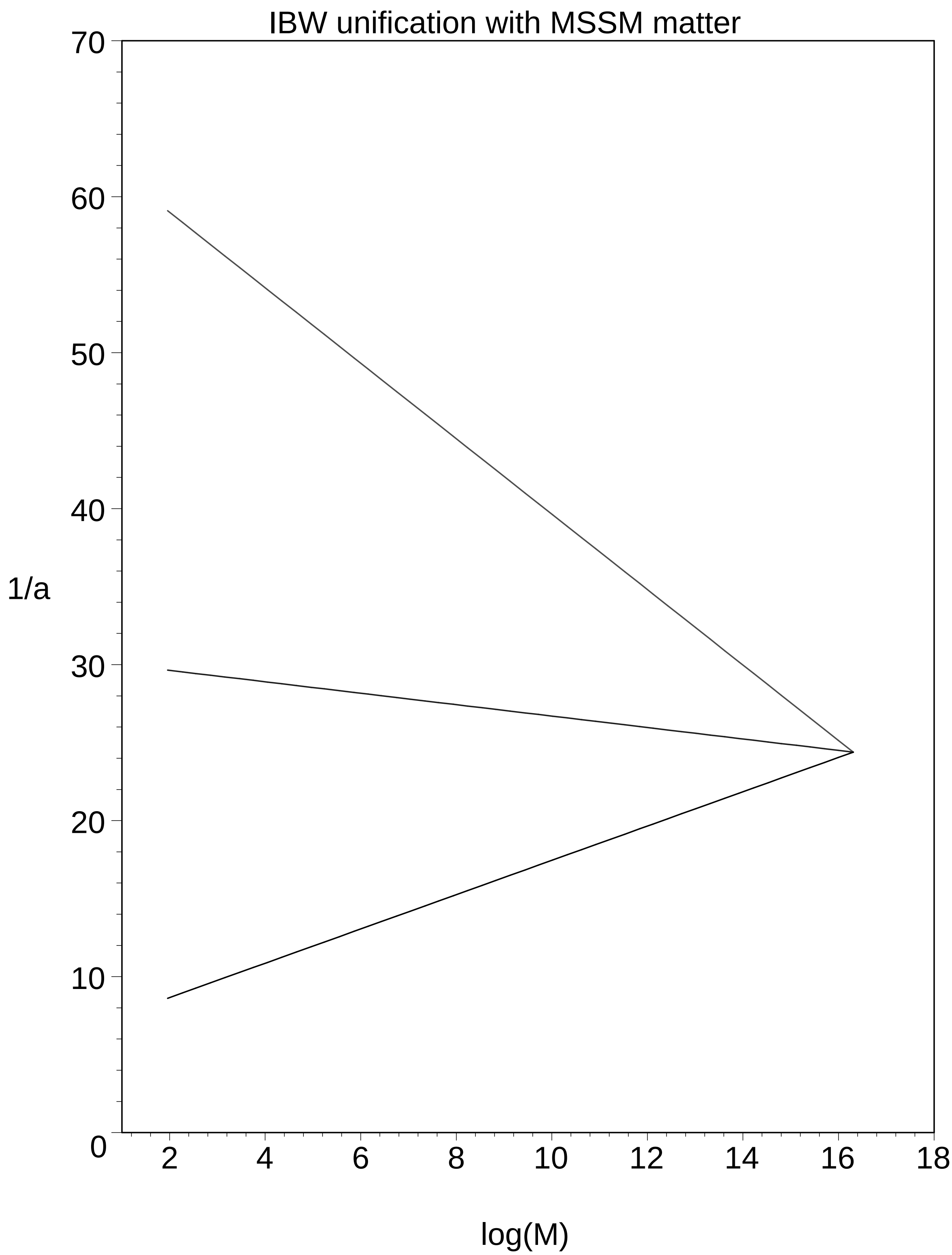
Of course, for the individual gauge couplings at the string scale we get

$$\alpha_s(M_s) = \alpha_w(M_s) = \frac{5}{3}\alpha_Y(M_s) = 0.041,$$

which are just the supersymmetric GUT scale values with the Weinberg angle being $\sin^2\theta_w(M_s) = 3/8$.

Assuming $g_{st} = g_X$, for the internal radii one obtains

$$M_s R = 5.32, \quad M_s R_s = 2.6, \quad M_s R_w = 3.3.$$


IBW unification with MSSM matter
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log(M)

Comparison to Heterotic String

Comparison to Heterotic String:

- Only one relation among the three gauge couplings at the string scale.
- No discrepancy between M_X and M_s.
- Difference for the string scale couplings achieved for appropriate choice of radii (string tree level) as opposed to one-loop threshold corrections for the heterotic string.
- Concerning $SU(5)$ GUT: Adjoint scalars are easy to get (in most cases one even has to many of them), but chiral matter in anti-symmetric representations poses some problems with Yukawa couplings.

V. More examples

Example A:

If we have a model with a second weak Higgs field, i.e. $n_{bc} = 1$, we still get $B = 12$ but with

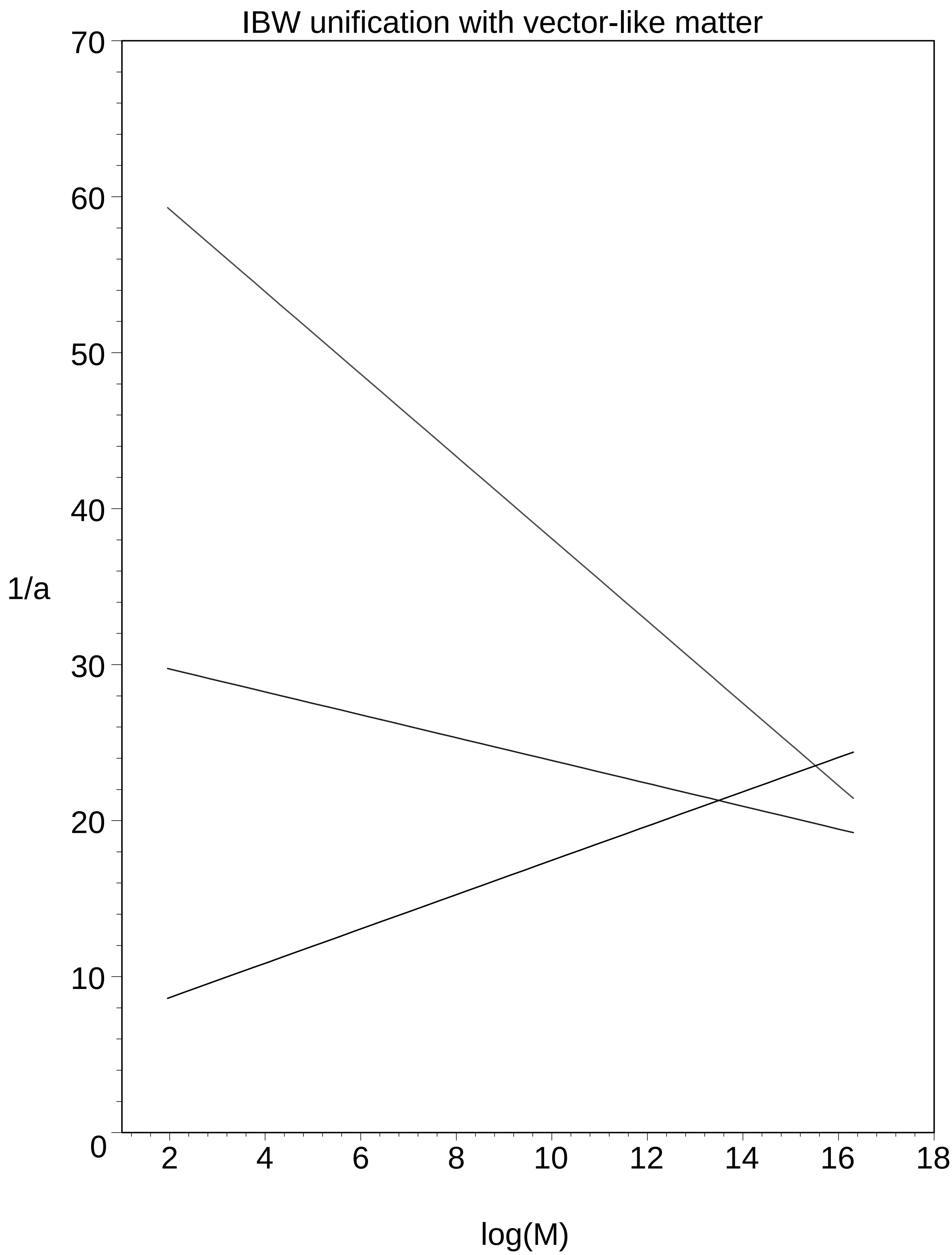
$$(b_3, b_2, b_1) = (3, -2, -12).$$

The gauge couplings "unify" at the scale

$$M_s = 2.02 \cdot 10^{16} \text{GeV}.$$

However they are not all equal at that scale

$$\alpha_s(M_s) = 0.041, \quad \alpha_w(M_s) = 0.052, \quad \alpha_Y(M_s) = 0.028.$$


IBW unification with vector-like matter
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Intermediate Scale Model

Example B:

For models with gravity mediated supersymmetry breaking (hidden anti-branes) the string scale is naturally in the intermediate regime $M_s \simeq 10^{11}$GeV.

Choosing vector-like matter

$$n_{a'a} = n_{a'd} = n_{d'd} = 2, \quad n_{bb} = 1$$

leads to $B = 18$.

The string scale turns out to be

$$M_s = 3.36 \cdot 10^{11}\text{GeV}.$$

The running of the couplings with

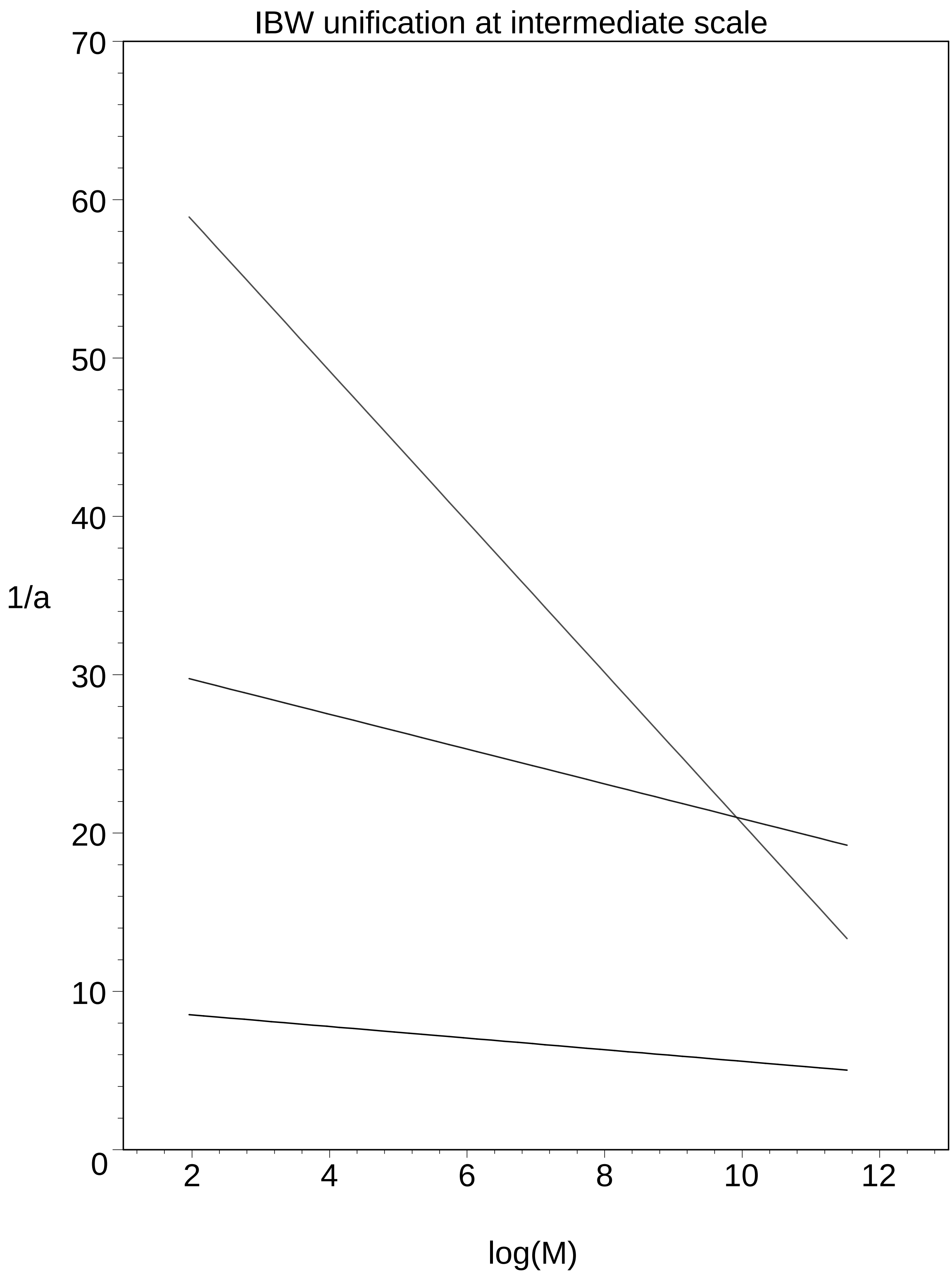
$$(b_3, b_2, b_1) = (-1, -3, -65/3)$$

leads to the values of the gauge couplings at the string scale

$$\alpha_s(M_s) = 0.199, \quad \alpha_w(M_s) = 0.052, \quad \alpha_Y(M_s) = 0.045.$$

Assuming $g_{st} \simeq 1$, for the internal radii one obtains

$$M_s R = 230, \quad M_s R_s = 1.7, \quad M_s R_w = 3.3.$$



IBW unification at intermediate scale
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Example C:

Interestingly for $B = 10$ one gets

$$\frac{M_s}{M_{pl}} = 1.24 \sim \sqrt{\frac{\pi}{2}}.$$

Choosing vector-like matter

$$n_{aa} = 1,$$

the beta-function coefficients read

$$(b_3, b_2, b_1) = (0, -1, -11).$$

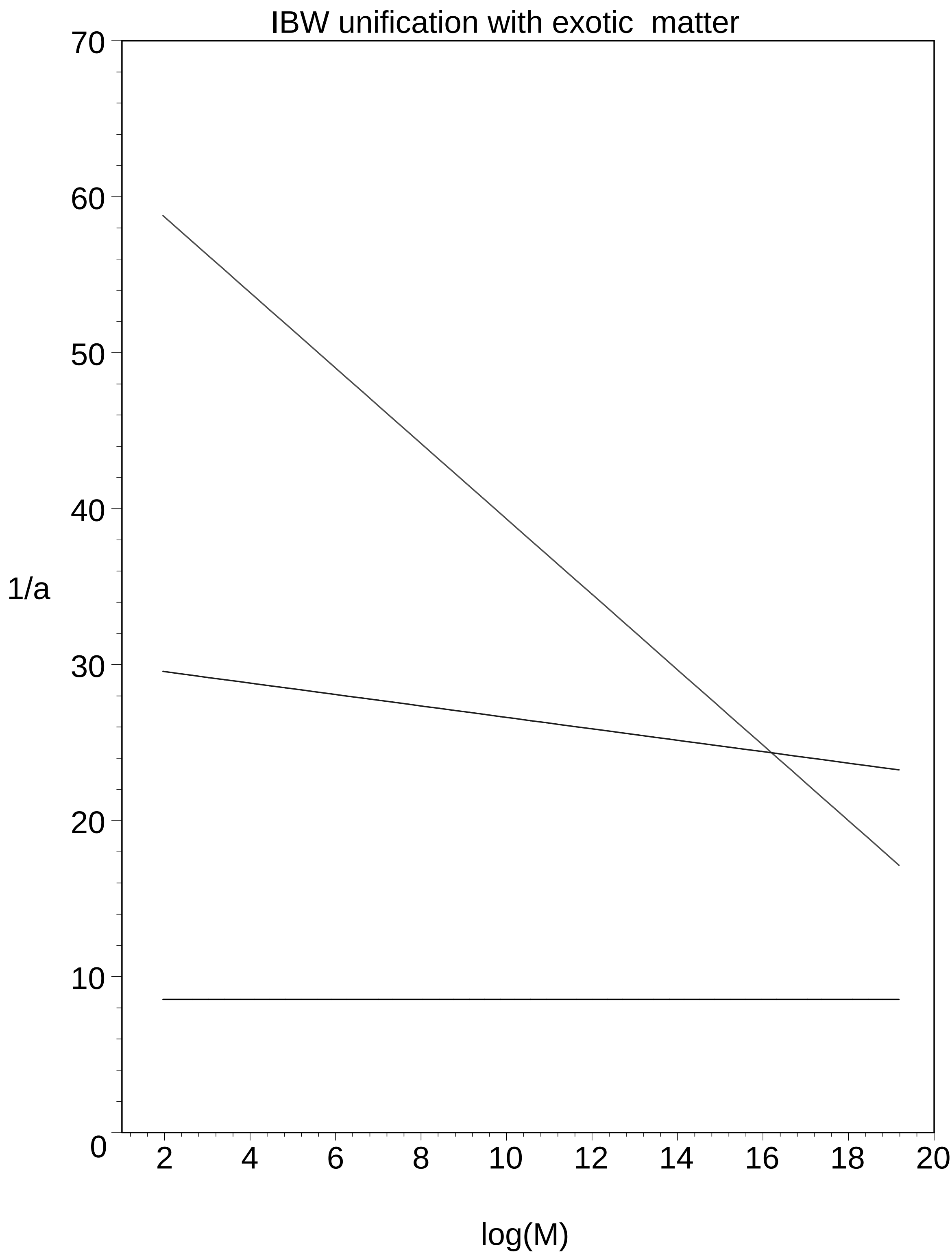
The couplings at the string scale turn out to be

$$\alpha_s(M_s) = 0.117, \quad \alpha_w(M_s) = 0.043, \quad \alpha_Y(M_s) = 0.035$$

leading to $\sin^2\theta_w(M_s) = 0.445$.

For the scales of the overall Calabi-Yau volume and the 3-cycles we obtain

$$M_s R = 0.6, \quad M_s R_s = 1.9, \quad M_s R_w = 3.3.$$


IBW unification with exotic matter
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log(M)

VI. Conclusions

- Under a few natural assumptions supersymmetric Intersecting Brane World Models can make interesting predictions about gauge coupling unification.

- The challenge remains to construct realistic supersymmetric IBW models with the chiral spectrum of the MSSM and only a mild amount of vector-like matter.

- For a concrete model it would be interesting to compute the string threshold corrections.