Stringy inspired solution to Strong CP violation problem

Gerardo Aldazabal

Instituto Balseiro-Bariloche

Argentina

Durham , 2003

In collaboration with L.E. Ibañez and A.M. Uranga

THE STRONG CP VIOLATION PROBLEM

One of the oldest fine-tuning problems in particle physics.

Non-trivial structure of QCD vacuum (instantons) allows for

$$q\bar{\theta} = \frac{\bar{\theta}}{32\pi^2} \int F_{\mu\nu}^{QCD} \tilde{F}^{\mu\nu}_{QCD}$$

where q is the instanton number.

$$\int F_{\mu\nu}^{QCD} \tilde{F}^{\mu\nu}_{QCD} \propto \mathbf{E}^{QCD}.\mathbf{B}^{QCD}$$

$\Longrightarrow$ CP violating effects in strong interactions

i.e. electric dipole moment of neutron measurements require

$$\bar{\theta} \leq 10^{-10}$$

Why ?

- *A massless quark*.

Global chiral (classical) symmetry

$$\Psi \longrightarrow e^{i\pi\alpha\gamma_5}\Psi$$

leads to

$$\partial_\mu J_5^\mu = -\frac{\delta\mathcal{L}}{\delta\alpha} = 2\alpha\frac{1}{32\pi^2}\int F_{\mu\nu}^{QCD}\tilde{F}_{QCD}^{\mu\nu}$$

$$\downarrow$$

$$\bar{\theta} \longrightarrow \bar{\theta} + 2\alpha$$

Thus, $\bar{\theta}$ is unphysical, it can be shifted to zero by field redefinition

Problem: quarks are massive

- *The axion solution*

  Introduce a dynamical field $\eta^0(x)$ with an axial coupling to the QCD field strength

$$\frac{\eta^0}{f_a} F^{QCD}_{\mu\nu} \tilde{F}^{\mu\nu}_{QCD}$$

a periodic scalar potential is generated. Minimized by

$$\eta = \eta^0 + \bar{\theta} = 0$$

$\Longrightarrow$ Zero effective θ-parameter. There is no strong CP violation

Problem: Experiments rule out most of the parameter space for this model. Only a small window with $f_a \propto 10^{10}$ GeV seems to be allowed.

PROPOSAL

G. A., L.E. Ibañez, A.M. Uranga

Introduce a:

- *pseudoscalar state* η with axionic couplings to the QCD field strength, very similar to axion solution.
- $U(1)_X$ *gauge interaction* whose gauge boson gets a Stuckelberg mass M by combining with the axion introduced above.
- *WZ term*.

$$\begin{aligned}\mathcal{L} \quad = \quad & \mathcal{L}_{QCD} + \eta F^{QCD}_{\mu\nu} \tilde{F}^{\mu\nu}_{QCD} \\ & -\frac{1}{4g_X^2} F_X^{\mu\nu} F^X_{\mu\nu} - \frac{M^2}{2}(A_X^\mu + \partial^\mu \eta)^2 + S_{WZ}\end{aligned}$$

$\mathcal{L}$ is gauge invariant under the transformation

$$A_X^\mu \to A_X^\mu - \partial^\mu \Lambda(x) \; ; \; \eta(x) \to \eta(x) + \Lambda(x)$$

provided

$$\delta S_{WZ} = - \int_{M_4} \Lambda \, \mathrm{tr} \, F^2_{QCD}$$

In dual variables $*dB = d\eta$

$$\mathcal{L} = \mathcal{L}_{QCD} - \frac{1}{4g_X^2} F_X^{\mu\nu} F^X_{\mu\nu} - \frac{1}{12} H^{\mu\nu\rho} H_{\mu\nu\rho} + \frac{M}{4} \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} F^X_{\rho\sigma} + \ldots$$

where

$$H^{\mu\nu\rho} = \partial^\mu B^{\nu\rho} + \partial^\rho B^{\mu\nu} + \partial^\nu B^{\rho\mu}$$

$$\eta F^{QCD} \wedge F_{QCD}$$

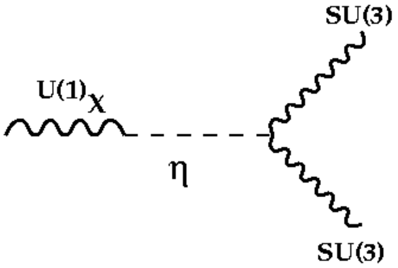
and

$$B \wedge F^X$$

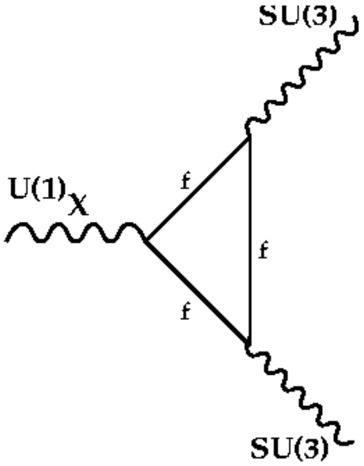
lead to potential mixed

$$U(1)_X - SU(3)^2_{QCD}$$

anomaly generated by axion transformation

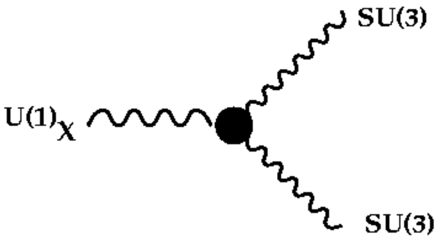


Cancelled if quarks are $U(1)_X$ charged



Requires a massless chiral quark

THE WESS-ZUMINO TERM



Introduce a fake dimension

$$\mathbf{X}_5 \qquad \text{with} \qquad \partial \mathbf{X}_5 \equiv M_4$$

$$S_{WZ} = \int_{\mathbf{X}_5} A_X \,\mathrm{tr}\, F^2_{QCD}$$

$$A_X \to A_X + d\Lambda$$

$$\begin{aligned} \delta S_{WZ} &= \int_{\mathbf{X}_5} d\Lambda \,\mathrm{tr}\, F^2_{QCD} = \int_{\mathbf{X}_5} d\left(\Lambda \,\mathrm{tr}\, F^2_{QCD}\right) \\ &= \int_{M_4} \Lambda \,\mathrm{tr}\, F^2_{QCD} \end{aligned}$$

gauge variation axion shift is cancelled

Removing the strong CP problem

When extending

$$M_4 \to \mathbf{X}_5$$
$$A^\mu(x) \to A^M(x,y)$$

with

$$A^M(x,y)|_{\partial\mathbf{X}_5 \equiv M_4} = A^\mu(x)$$

Extra global symmetry due to freedom to choose $A^4(x,y)$

In particular $A^4 \to A^4 + \Phi(y)$ field redefinition leads to

1. No changes in $D = 4$ action
2. $S_{WZ} \to S_{WZ} + \int_{\mathbf{X}_5} (\,\Phi \,\mathrm{tr}\, F^2_{QCD}\,) \wedge dy =$

   $S_{WZ} + \phi \int_{M_4} (\,\mathrm{tr}\, F^2_{QCD}\,)$

$$\phi = \int (\,\Phi\, dy)$$

Thus,

$$A^4 \longrightarrow A^4 + \Phi(y)$$

and choosing

$$\phi = -\bar{\theta}$$

removes CP violating term ($\bar{\theta}$ is unphysical).

Similar to chiral fermion redefinition

$$\Psi \longrightarrow e^{i\pi\alpha\gamma_5}\Psi$$

$$\alpha = -\bar{\theta}$$

A *pseudoscalar state* with axionic couplings to the QCD field strength.

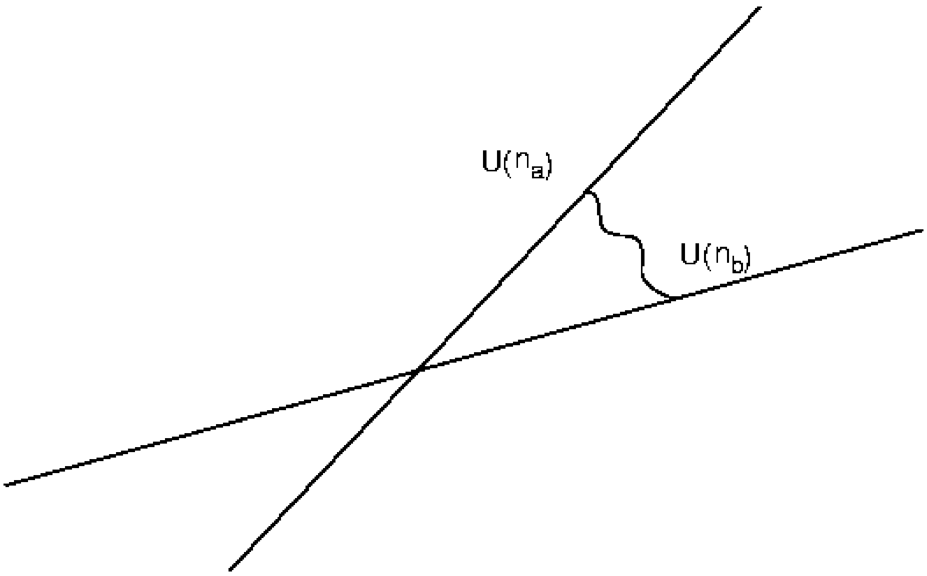
A *WZ term* to cancel θ

A $U(1)_X$ to ensure gauge invariance .

WZ term *mimics* chiral fermion

Non-local $D = 4$ WZ arise from local higher dimensional theories.

WZ term from branes at angles



$$\prod_{a=1}^{K} U(N_a)$$

$$\sum_{a<b} I_{ab}\,(N_a, \overline{N}_b)$$

where I_{ab}

$$I_{ab} = [\Pi_a] \cdot [\Pi_b] = \prod_i (n_a^i m_b^i - m_a^i n_b^i)$$

Closed sector RR p-forms C_1, C_3, C_5, C_7 can lead to A_X 1-form upon compactification. i.e $C_1 = A_X$.

Gauge fields on the D6-brane couple to the closed string RR modes via Chern-Simons couplings.

$$\int_{D6_a} C_3 \, \mathrm{tr}\, F_a^2 \longrightarrow \int_{M_4} \eta_a \, \mathrm{tr}\, F_a^2 \quad \longrightarrow \text{axion}$$

$$\int_{M_4 \times Y_6} C_5 \, H_{NS} F_2 \longrightarrow k_{\text{flux}} \int_{M_4} B_2^{a_0} F_2 \quad \longrightarrow \text{Stuckelberg}$$

Here

$$\begin{aligned} F_2 &= dC_1 \\ \eta_a &= \int_{\Pi_a} C_3 \\ B_2^a &= \int_{\tilde{\Pi}_a} C_5 \\ k_{\text{flux}} &= \int_{\Pi_{a_0}} H_{NS} \end{aligned}$$

Chern-Simon interactions provide WZ term

$$\int_{D6} (C_1 + C_3 + ...)\, e^{F+B_{NS}}$$

B_{NS} is NS-NS 2-form field. $\Longrightarrow$ WZ like term

$$S_{CS} = \int_{D6_{a_0}} B_{NS}\, C_1 \,\mathrm{tr}\, F_{a_0}^2 \;=\; \int_{D6_{a_0}} B_{NS} \,[\, F_2 \,\mathrm{tr}\, F_{a_0}^2 \,]^{(0)}$$

Hence,

$$\begin{aligned} \delta S_{CS} &= \int_{D6_{a_0}} H_{NS} \,[\, F_2 \,\mathrm{tr}\, F_{a_0}^2 \,]^{(1)} \\ &= k_{\text{flux}} \int_{M_4} [\, F_2 \,\mathrm{tr}\, F_{a_0}^2 \,]^{(1)} = k_{\text{flux}} \int_{M_4} [\, \Lambda \,\mathrm{tr}\, F_{a_0}^2 \,] \end{aligned}$$

provides the required term to cancel Green-Schwarz

Notice dangerous terms

$$\int_{D6_a} C_5\, F_a \longrightarrow B_2^{a_0} \wedge F_2^{brane}$$

might lead to brane-bulk $U(1)$ mixings

Explicit D-brane example

$N_1 = 3$	(1,0)	(1,-1)	(1,-1)
$N_2 = 2$	(2,1)	(1,2)	(1,0)
$N_3 = 1$	(2,1)	(-1,-2)	(1,0)
$N_4 = 1$	(1,0)	(1,-1)	(-2,2)
$N_5 = 1$	(1,0)	(1,-1)	(-1,1)
$N_6 = 1$	(2,1)	(-1,-2)	(1,0)

$$\begin{aligned} I_{12} &= 3 = I_{56} = I_{25} \\ I_{13} &= I_{16} = I_{35} = -3 \\ I_{24} &= I_{46} = 6 = -I_{34} \end{aligned}$$

$$\Longrightarrow \quad SU(3) \times SU(2) \times U(1)_Y (\times U(1)'s)$$

chiral fermion spectrum

$$3(3,2,1/6)_{(1,-1,0,0,0,0)} + 3(\bar{3},1,-2/3)_{(-1,0,1,0,0,0)} +$$
$$3(\bar{3},1,1/3)_{(-1,0,0,0,0,1)}$$
$$+3(1,2,1/2)_{(0,1,0,0,-1,0)} + 3(1,2,-1/2)_{(0,1,0,\underline{-1,0},0,0)} +$$
$$3(1,1,1)_{(0,0,0,\underline{1,0},0,-1)} + 3(1,1,-1)_{(0,0,-1,0,1,0)} +$$
$$3(1,1,0)_{(0,0,0,0,1,-1)} + 3(1,1,0)_{(0,0,-1,\underline{1,0},0,0)}$$

$$Y = -(\frac{Q_1}{3} + \frac{Q_2}{2} + Q_5 + Q_6)$$

where Q_a is the $U(1)$ generator in $U(N_a)$.

Closed 0-forms:

$$\begin{aligned} \eta_{123} &= \int_{\mathbf{e_1}^1 \otimes \mathbf{e_1}^2 \otimes \mathbf{e_1}^3} C_3 \\ \eta_I &= \int_{\mathbf{e_1}^I \otimes \mathbf{e_2}^J \otimes \mathbf{e_2}^K} C_3 \\ \eta_{IJ} &= \int_{\mathbf{e_1}^I \otimes \mathbf{e_1}^J \otimes \mathbf{e_2}^K} C_3 \\ \eta &= \int_{\mathbf{e_2}^1 \otimes \mathbf{e_2}^2 \otimes \mathbf{e_2}^3} C_3 \end{aligned}$$

where $I \neq J \neq K \neq I$ in the second and third rows. Also, 2-forms, i.e.

$$B^{(2)}_{123} = \int_{\mathbf{e_2}^1 \otimes \mathbf{e_2}^2 \otimes \mathbf{e_2}^3} C_5$$

$\Longrightarrow$D6_a brane gauge bosons- RR fields couplings

$$\begin{array}{ccc} N_a\, m_a^1\, m_a^2\, m_a^3 \int_{M_4} B^{(2)}_{123} \wedge F_a & ; & n_b^1\, n_b^2\, n_b^3 \int_{M_4} \eta_{123} \wedge F_b \wedge F_b \\ N_a\, n_a^J\, n_a^K\, m_a^I \int_{M_4} B^{(2)}_I \wedge F_a & ; & n_b^I\, m_b^J\, m_b^K \int_{M_4} \eta_I \wedge F_b \wedge F_b \\ N_a\, n_a^K\, m_a^I\, m_a^J \int_{M_4} B^{(2)}_{IJ} \wedge F_a & ; & n_b^I\, n_b^J\, m_b^K \int_{M_4} \eta_{IJ} \wedge F_b \wedge F_b \\ N_a\, n_a^1\, n_a^2\, n_a^3 \int_{M_4} B^{(2)} \wedge F_a & ; & m_b^1\, m_b^2\, m_b^3 \int_{M_4} \eta \wedge F_b \wedge F_b \end{array}$$

From wrapping numbers above:

η_{123} has an axion coupling to QCD through

$$\int_{M_4} \eta_{123} \wedge \mathrm{tr}\,(F_{QCD} \wedge F_{QCD})$$

BF Stuckelberg coupling

$$\int_{M_4 \times T^6} C_5 F_2 H_{NS} = k_{\text{flux}} \int_{M_4} B^{(2)}_{123} F_2$$

Dual 2-form $B^{(2)}_{123}$ has no BF coupling to brane gauge fields, since $m^1_a\, m^2_a\, m^3_a = 0$ for all a, ($a = 1, \ldots, 6$).

CONCLUSIONS

QCD $\bar{\theta}$ parameter is *gauged away* if theory contains

- *pseudoscalar state* η with axionic couplings to the QCD field strength.
- $U(1)_X$ *gauge interaction* with Stuckelberg coupling
- *WZ term*.

Available in String theory

M-theory lift:

If

$$A_X \equiv C_1 \qquad \text{and} \qquad C_\mu = G_{11\mu}$$

$\bar{\theta}$ is removed by M-theory Diff invariance

References

[1] C. Callan, R. Dashen and D. Gross, *"The structure of the gauge theory vacuum"*, Phys. Lett. B61(1976)334 ;
R. Jackiw and C. Rebbi, *"Vacuum periodicity in a Yang-Mills quantum theory"*, Phys.Rev.Lett.37(1976)172.

[2] G. 't Hooft, *"Computation of the quantum effects due to a four-dimensional pseudoparticle"*, Phys.Rev.D14(1976)3432.

[3] R. Peccei and H. Quinn, *"CP conservation in the presence of instantons"*, Phys.Rev.Lett.38(1977)1440; *"Constraints imposed by CP conservation in the presence of instantons"*, Phys.Rev.D16(1977)1791;
S. Weinberg, *"A new light boson?"*, Phys.Rev.Lett.40(1978)223;
F. Wilczek, *" Problem of strong P and T invariance in the presence of instantons"*, Phys.Rev.Lett.40(1978)279;
J. E. Kim, *"Weak interaction singlet and strong CP invariance"*, Phys.Rev.Lett.43(1979)103;
M. Shifman, A. Vahinstein and V. Zakharov, Nucl.Phys.B166(1980)493;
M. Dine, W. Fischler and M. Srednicki, Phys.Lett.B104(1981)104.

[4] A. M. Uranga, 'D-brane, fluxes and chirality', JHEP 0204 (2002) 016, hep-th/0201221.