Electroweak two-loop corrections in the Standard Model

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Durham, 04/2002

1. Introduction
2. Evaluation of electroweak two-loop contributions to M_W
3. Numerical results and estimate of remaining theory uncertainty
4. Prospects for future precision tests
5. Conclusions

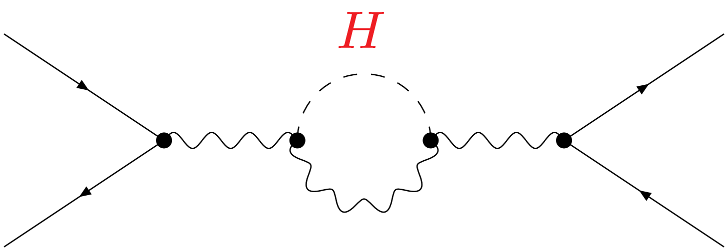
1. Introduction

Comparison of ew precision data with theory:

EW precision data: $M_{\mathrm{Z}}, M_{\mathrm{W}}, \sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}, \ldots$	Theory: SM, MSSM, ...

$$\Downarrow$$

Test of theory at quantum level:



$$\Downarrow$$

Indirect constraints on M_{H}, ...

Leading corrections to precision observables:

$$\sim m_{\mathrm{t}}^2, \ \sim \ln M_{\mathrm{H}}$$

$\Rightarrow$ Very high accuracy of measurements and theoretical predictions needed

Theoretical uncertainties:

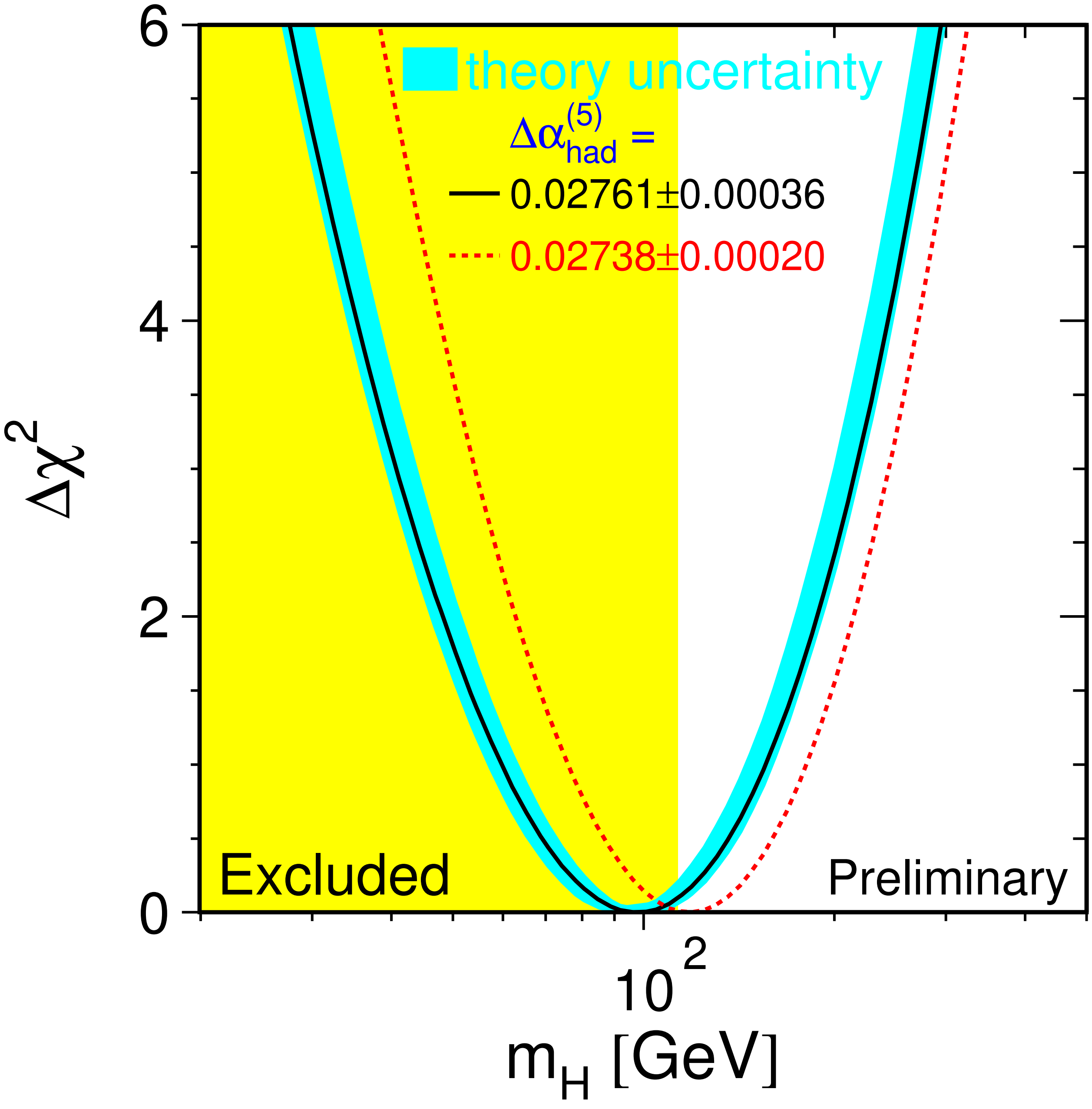
– unknown higher-order corrections

– exp. error of input parameters: m_{t}, $\Delta\alpha_{\mathrm{had}}$, ...

Global fit to all data in the SM:

[*LEPEWWG '01*]

[*Moriond '01*]



$\Rightarrow M_\mathrm{H} = 98^{+58}_{-38}$ GeV

$M_\mathrm{H} < 212$ GeV, 95% C.L.

“Blue band” :

Estimate of unknown higher-order corrections by varying different options in

ZFITTER [*D. Bardin et al.*]

TOPAZ0 [*G. Passarino*]

Different renormalization schemes, different resummations of non-leading terms

$\Rightarrow$ differ by higher-order corrections

Error estimate for blue band plot up to Moriond ’01:

$$\begin{aligned} \Delta M_{\mathrm{W}}^{\mathrm{theo}} &\approx \pm 3\ \mathrm{MeV} \\ \Delta \sin^2\theta_{\mathrm{eff}}^{\mathrm{theo}} &\approx \pm 3 \times 10^{-5} \end{aligned}$$

Too optimistic!

Comparison: estimate of higher-order uncertainties

[*G. Degrassi et al. ’00*]

[*U. Baur et al. ’01*]

$$\Delta \sin^2\theta_{\mathrm{eff}}^{\mathrm{theo}} \approx \pm 6 - 7 \times 10^{-5}$$

Theoretical prediction for M_{W} in the SM:

Definition of Fermi constant G_μ via muon lifetime:

$$\tau_\mu^{-1} = \frac{G_\mu^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right) \left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right) (1 + \Delta q)$$

Δq: QED corrections in Fermi Model, included in definition

[*R. Behrends, R. Finkelstein, A. Sirlin '56*]

[*T. van Ritbergen, R. Stuart '99*]

Comparison with SM prediction for muon decay:

$$\Rightarrow \quad M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2} G_\mu} (1 + \underbrace{\Delta r}_{\Updownarrow\ \text{loop corrections}}),$$

$\Rightarrow$ Theoretical prediction for M_{W} in terms of M_{Z}, α, G_μ

Current experimental values:

$M_{\mathrm{W}}[\,\mathrm{GeV}]$	$=$	80.451 ± 0.033	0.04%
$M_{\mathrm{Z}}[\,\mathrm{GeV}]$	$=$	91.1875 ± 0.0021	0.002%
$G_\mu[\,\mathrm{GeV}^{-2}]$	$=$	$1.16637(1)\,10^{-5}$	0.0009%
$\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$	$=$	0.23149 ± 0.00017	0.07%

SM prediction for Δr, one-loop:
[*A. Sirlin '80*], [*W. Marciano, A. Sirlin '80*]

$$\Delta r_{1-\mathrm{loop}} = \Delta\alpha - \frac{c_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\Delta\rho + \Delta r_{\mathrm{rem}}(M_{\mathrm{H}})$$

$$\sim \log\frac{M_{\mathrm{Z}}}{m_f} \qquad \sim m_{\mathrm{t}}^2$$

$$\sim 6\% \qquad \sim 3.3\% \qquad \sim 1\%$$

$\Delta\alpha$, $\Delta\rho$: fermion-loop contributions from
charge and mixing-angle renormalization

Higher-order results for M_W in the SM:

Resummation of leading 1-loop contributions:

$$1 + \Delta r \to \frac{1}{(1 - \Delta\alpha)\left(1 + \frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}\Delta\rho\right) - \Delta r_\mathrm{rem}}$$

$\Rightarrow$ $(\Delta\alpha)^2$, $(\Delta\rho)^2$, $(\Delta\alpha\Delta\rho)$, $(\Delta\alpha\Delta r_\mathrm{rem})$ terms correct at 2-loop order

[*W. Marciano '79*], [*A. Sirlin '84*]

[*M. Consoli, W. Hollik, F. Jegerlehner '89*]

QCD corrections:

Results in $\mathcal{O}(\alpha\alpha_\mathrm{s})$ and $\mathcal{O}(\alpha\alpha_\mathrm{s}^2)$ known

[*A. Djouadi, C. Verzegnassi '87*], [*A. Djouadi '89*]

[*B.A. Kniehl '90*], [*F. Halzen, B.A. Kniehl '91*]

[*L. Avdeev, J. Fleischer, S.M. Mikhailov, O. Tarasov '94*]

[*K. Chetyrkin, J. Kühn, M. Steinhauser '95*]

EW 2-loop corrections:

Expansion for asymptotically large m_t:

Leading $m_\mathrm{t}^4/M_\mathrm{W}^4$ terms:

[*J. van der Bij, F. Hoogeveen '87*]

[*R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci, A. Vicere '92*]

[*J. Fleischer, O.V. Tarasov, F. Jegerlehner '93*]

Next-to-leading $m_\mathrm{t}^2/M_\mathrm{W}^2$ terms:

expansions:

$$M_\mathrm{W}, M_\mathrm{Z}, M_\mathrm{H} \ll m_\mathrm{t}$$

$$M_\mathrm{W}, M_\mathrm{Z} \ll m_\mathrm{t}, M_\mathrm{H}$$

$+$ interpolation

[*G. Degrassi, P. Gambino, A. Vicini '96*]

[*G. Degrassi, P. Gambino, A. Sirlin '97*]

$\rightarrow$ Same order of magnitude as $m_\mathrm{t}^4/M_\mathrm{W}^4$ terms, same sign

$\Rightarrow$ Exact evaluation of EW 2-loop contributions desirable (no expansion in masses)

Expansion for asymptotically large M_{H}:

Leading $M_{\mathrm{H}}^2/M_{\mathrm{W}}^2$ terms:

expansion: $M_{\mathrm{W}}, M_{\mathrm{Z}}, m_{\mathrm{t}} \ll M_{\mathrm{H}}$

[*J. van der Bij, M. Veltman '84*], [*J. van der Bij '84*]

Exact result for Higgs-mass dependence of fermionic contributions:

$M_{\mathrm{W,subtr}}(M_{\mathrm{H}}) = M_{\mathrm{W}}(M_{\mathrm{H}}) - M_{\mathrm{W}}(M_{\mathrm{H}} = 65\,\mathrm{GeV})$

[*S. Bauberger, G.W. '98*]

Pure bosonic two-loop contributions to on-shell counterterms for M_{W} and M_{Z}:

expansion in $\sin^2\theta_{\mathrm{W}}$, $M_{\mathrm{Z}}^2/M_{\mathrm{H}}^2$

[*F. Jegerlehner, M. Kalmykov, O. Veretin '01*]

Leading EW corrections beyond 2-loop:

Pure fermion-loop contributions up to 4-loop order

[*A. Stremplat '98*], [*G.W. '98*]

Leading 3-loop corrections to $\Delta\rho$:

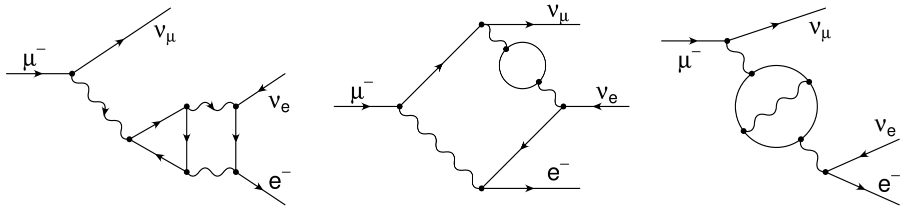
$\mathcal{O}(G_\mu^3 m_{\mathrm{t}}^6)$, $\mathcal{O}(G_\mu^2 \alpha_{\mathrm{s}} m_{\mathrm{t}}^4)$ terms

limit: $M_{\mathrm{H}} = 0$

[*J. van der Bij, K. Chetyrkin, M. Faisst, G. Jikia, T. Seidensticker '01*]

2. Evaluation of electroweak two-loop contributions to M_{W}

Two-loop fermionic contributions to muon decay:
[*A. Freitas, W. Hollik, W. Walter, G.W. '00, '02*]



⇒ full dependence on m_{t}, complete light fermion contributions at two-loop order

Method of calculation:

– Algebraic evaluation:

FeynArts [*J. Küblbeck, A. Denner, M. Böhm '90*] [*H. Eck '95*], [*T. Hahn '99*]

⇒ Feynman amplitudes and counterterms

TwoCalc [*G. W. '92*], [*G. W., R. Scharf, M. Böhm '94*]

⇒ tensor integral reduction, etc.
standard scalar integrals

– General R_ξ gauge

– Regularisation:

DREG [*G. 't Hooft, M. Veltman '72*]

Careful treatment of γ_5 necessary

[*P. Breitenlohner, D. Maison '77*]

– Two-loop renormalization:

On-shell scheme (physical parameters)

Renormalization in gauge-fixing and ghost sector

Mass renormalization of unstable particles

$\Rightarrow$ determines physical meaning of M_{W}, M_{Z}

– Numerical evaluation:

One-dimensional integral representations with elementary functions

[*S. Bauberger, F. Berends, M. Böhm, M. Buza '95*]

[*S. Bauberger, M. Böhm '95*]

$\Rightarrow$ Exact evaluation of 2-loop 2-point functions

Mass renormalization of unstable particles

From 2-loop order on:

real part of complex pole		pole of real part
$\mathcal{M}^2 = \overline{M}^2 - i\overline{M}\,\overline{\Gamma}$	$\neq$	$\widetilde{M}^2$

$$\delta\overline{M}^2_{(2)} = \delta\widetilde{M}^2_{(2)} + \underbrace{\mathrm{Im}\left\{\Sigma'_{\mathrm{T},(1)}(M^2)\right\}\,\mathrm{Im}\left\{\Sigma_{\mathrm{T},(1)}(M^2)\right\}}_{\text{gauge-parameter dependent!}}$$

S-matrix theory

[*A. Sirlin '91*], [*R.G. Stuart '91*], [*H. Veltman '94*]

[*P. Gambino, P.A. Grassi '99*]

Explicit 2-loop calculation in general R_ξ-gauge:

[*A. Freitas, W. Hollik, W. Walter, G.W. '00*]

$\Rightarrow$ Only complex pole gauge-invariant

Comparison theory prediction – experimental result:

Definition via real part of complex pole:

$$\Rightarrow \quad \sigma(s) \sim \frac{s}{(s-\overline{M}^2)^2 + \overline{M}^2\overline{\Gamma}^2}$$

$\Rightarrow$ Breit–Wigner shape with constant width

Experimental determinations at LEP/Tevatron use:

$$\sigma(s) \sim \frac{s}{(s-M^2)^2 + s^2\Gamma^2/M^2}$$

$\Rightarrow$ Breit–Wigner shape with running width

Relation:

$$M^2 = \overline{M}^2 + \overline{\Gamma}^2 \qquad \Rightarrow \quad \underbrace{\Delta M_W \approx 27\,\text{MeV}}_{\text{formally a 2-loop effect!}}$$

$\Rightarrow$ For theoretical prediction of M_W:
need prediction for $\overline{\Gamma}$

$\Rightarrow$ (Slightly) larger theoretical uncertainty for M_W than for $\overline{M}_W$

$\Rightarrow$ Using $\overline{M}_W$ instead of M_W as experimental value would be preferable

3. Numerical results and estimate of remaining theory uncertainty

Checks of the calculation:

UV-, IR-finiteness, general R_ξ-gauge

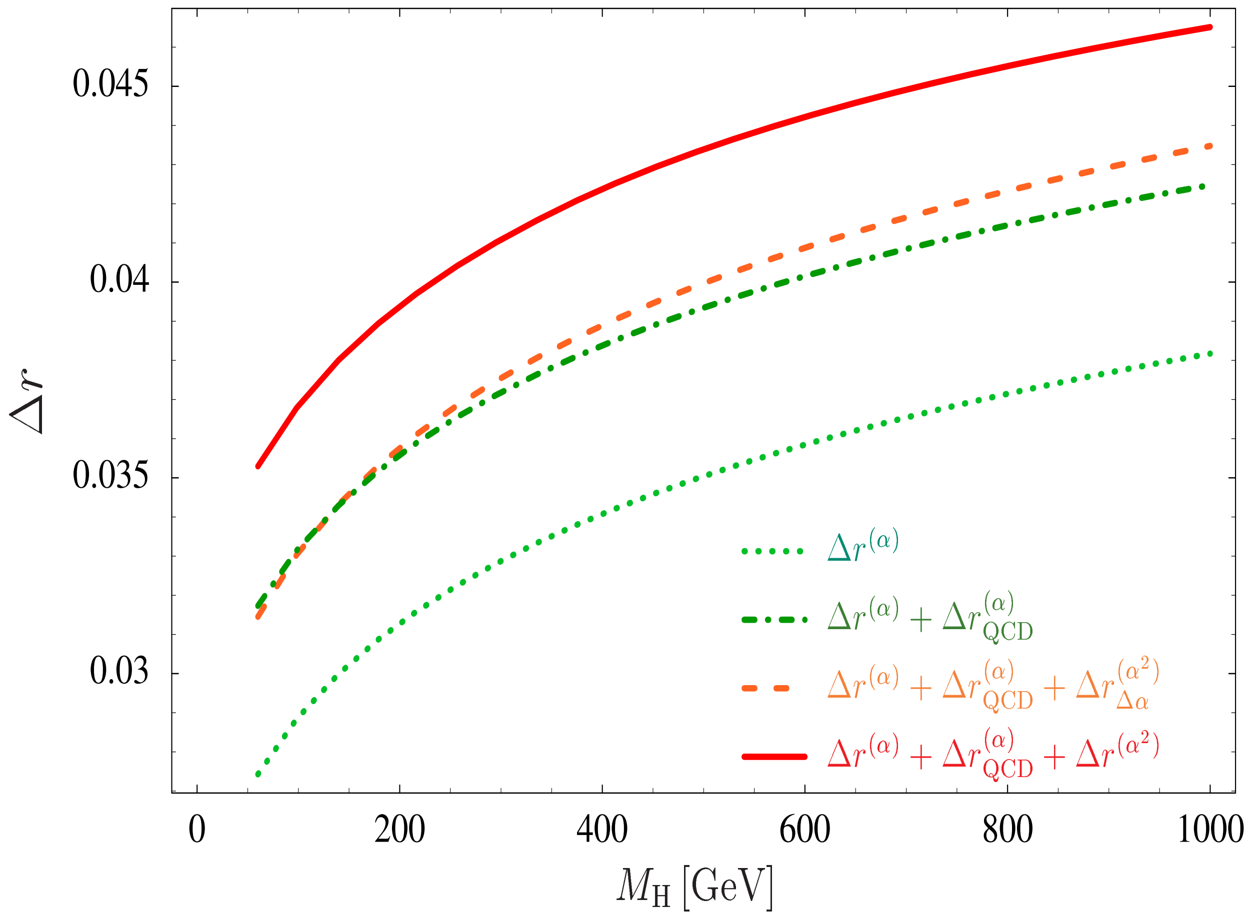
$\Rightarrow$ gauge-parameter independence at algebraic level

Higgs-mass dependence agrees with existing result
[*S. Bauberger, G.W. '98*]

1-loop, QCD and ew 2-loop contributions to Δr:

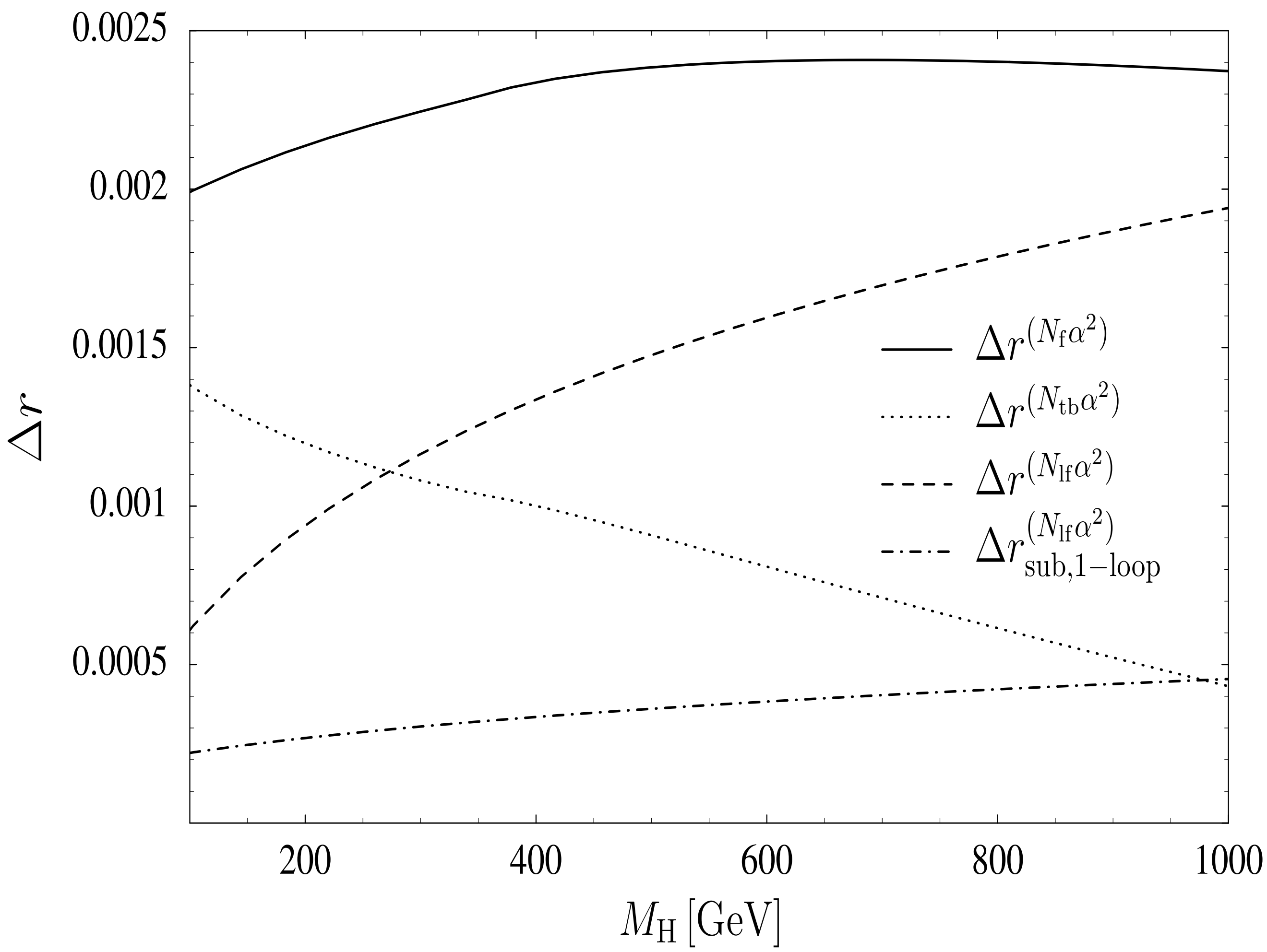
$$\Delta r = \Delta r^{(\alpha)} + \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)} + \Delta r^{(N_f\alpha^2)} + \Delta r^{(N_f^2\alpha^2)}$$

fully expanded, no resummation needed



Comparison of contributions from top/bottom doublet and from light fermions:

[A. Freitas, W. Hollik, W. Walter, G.W. '02]



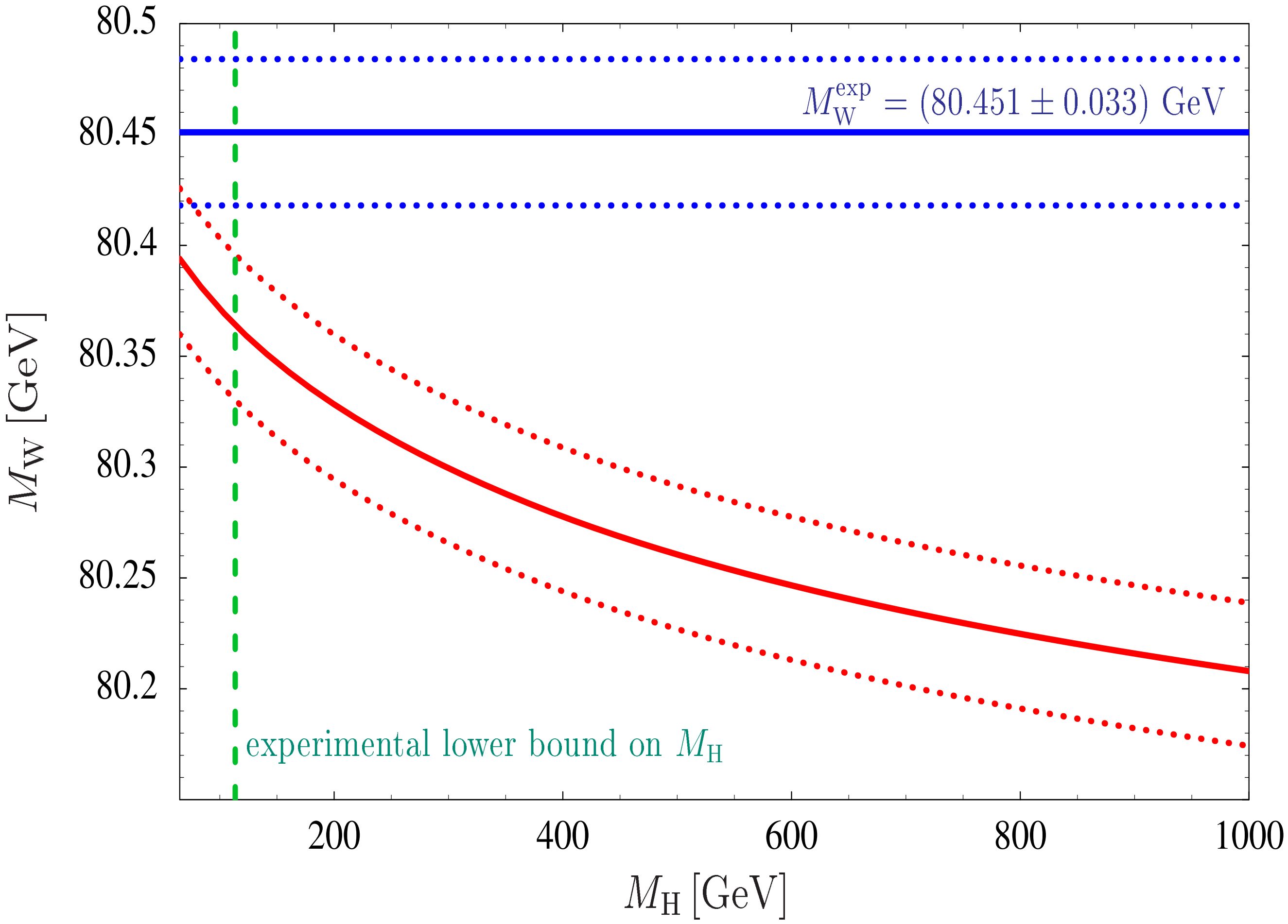
$$\Delta r^{(N_{\mathrm{lf}}\alpha^2)}_{\mathrm{sub,1-loop}} = \Delta r^{(N_{\mathrm{lf}}\alpha^2)} - 2\Delta\alpha\Delta r^{(\alpha)}_{\mathrm{bos}}$$

$\Rightarrow$ genuine two-loop light fermion contributions lead to shift in M_{W} of up to 5 MeV

smaller than top/bottom corrections for $M_{\mathrm{H}} \lesssim 1$ TeV

SM prediction for M_W vs. experimental result:

[*A. Freitas, W. Hollik, W. Walter, G.W. '00, '02*]



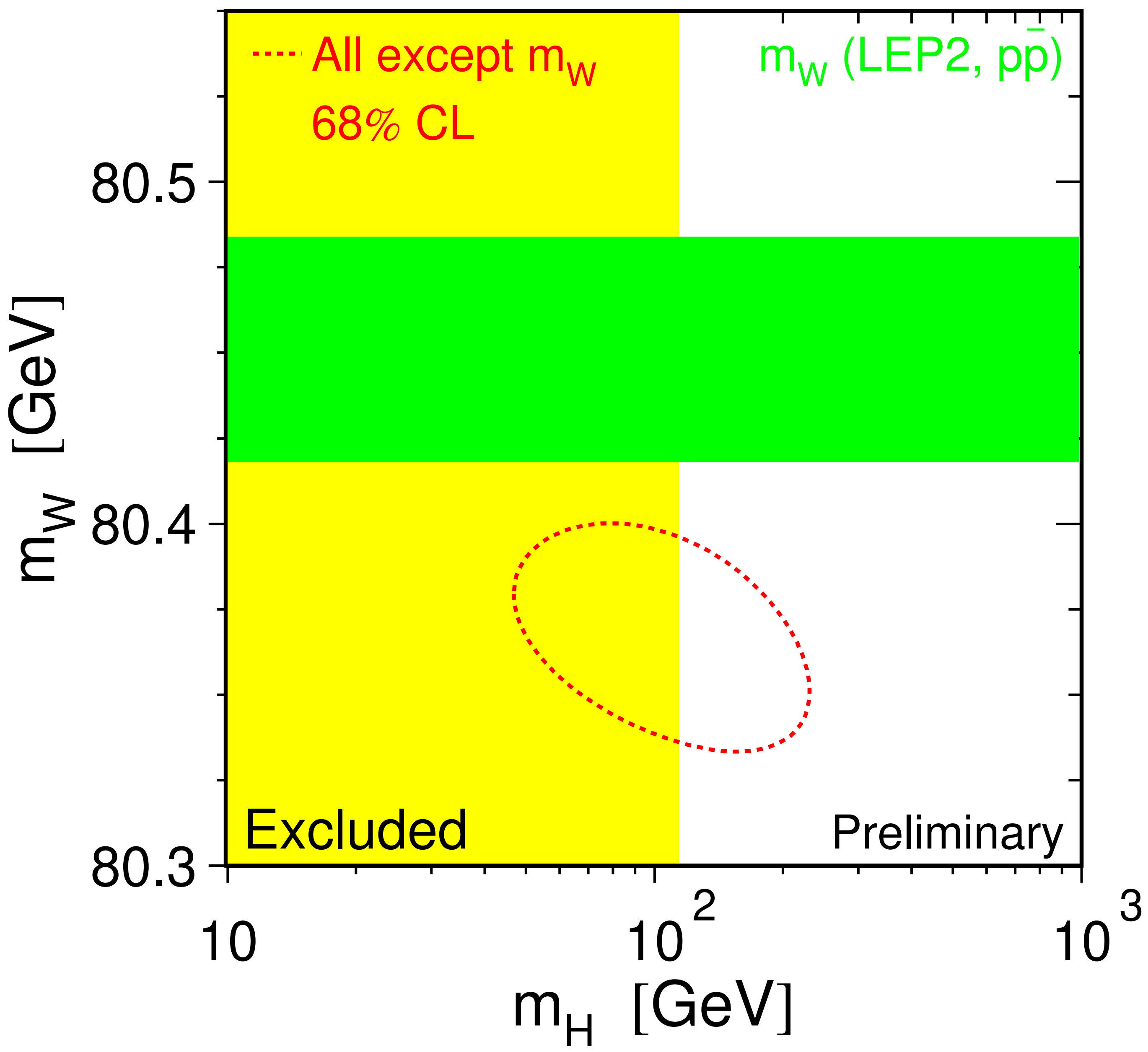
$\Rightarrow$ Light Higgs preferred

$M_W^{exp} = 80.451 \pm 0.033$ GeV, $m_t^{exp} = 174.3 \pm 5.1$ GeV

$M_H > 114.1$ GeV, 95% C.L.

$\Rightarrow$ 1σ regions do not overlap

Comparison of indirect and direct information in the (M_H, M_W) plane:
[*LEPEWWG '02*]

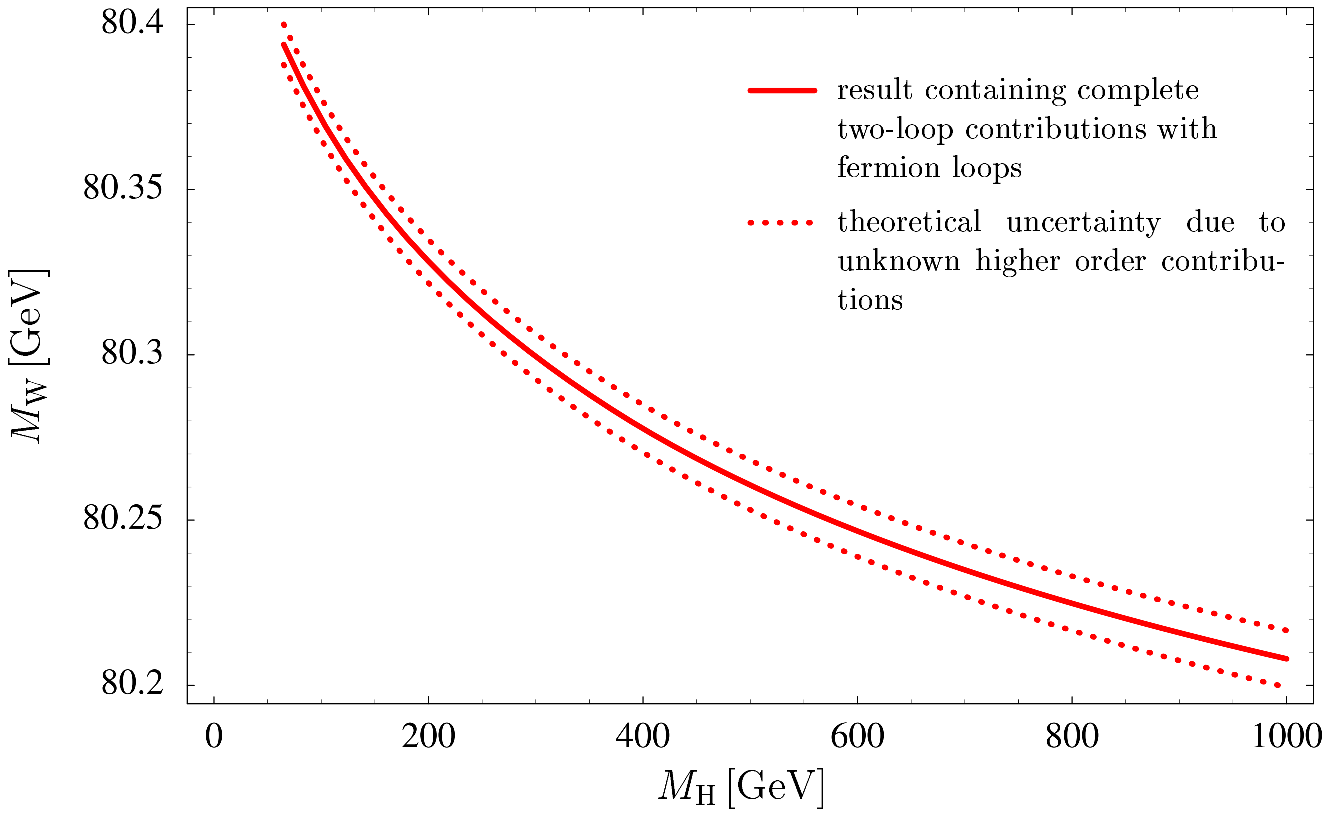


$\Rightarrow$ SM prediction for M_W below experimental value

Uncertainty from experimental error of input parameters: $\delta m_{\mathrm{t}} = \pm 5.1$ GeV $\Rightarrow$ $\delta M_{\mathrm{W}} \approx \pm 31$ MeV

Estimate of theoretical uncertainty from unknown higher-order corrections:

- Purely bosonic $\mathcal{O}(\alpha^2)$ electroweak corrections
- Higher-order electroweak corrections: $\mathcal{O}(\alpha^3)$
- Higher-order QCD corrections: $\mathcal{O}(\alpha\alpha_{\mathrm{s}}^3)$, $\mathcal{O}(\alpha^2\alpha_{\mathrm{s}})$



For light Higgs masses:

$$\Delta M_{\mathrm{W}}^{\mathrm{theo}} \approx \pm 6\,\mathrm{MeV}$$

Simple parameterization of full result:

$$
\begin{aligned}
M_{\mathrm{W}} \quad = \quad & M_{\mathrm{W}}^{0} - c_1\,\mathrm{dH} - c_2\,\mathrm{dH}^2 + c_3\,\mathrm{dH}^4 \\
& - c_4\,\mathrm{d}\alpha + c_5\,\mathrm{dt} - c_6\,\mathrm{dt}^2 \\
& - c_7\,\mathrm{dH}\,\mathrm{dt} - c_8\,\mathrm{d}\alpha_{\mathrm{s}} + c_9\,\mathrm{dz}
\end{aligned}
$$

where

$$
\mathrm{dH} = \ln\left(\frac{M_{\mathrm{H}}}{100\ \mathrm{GeV}}\right), \qquad \mathrm{dt} = \left(\frac{m_{\mathrm{t}}}{174.3\ \mathrm{GeV}}\right)^2 - 1,
$$

$$
\mathrm{d}\alpha = \frac{\Delta\alpha}{0.05924} - 1, \qquad \mathrm{d}\alpha_{\mathrm{s}} = \frac{\alpha_{\mathrm{s}}(M_{\mathrm{Z}})}{0.119} - 1,
$$

$$
\mathrm{dz} = M_{\mathrm{Z}} - 91.1875\ \mathrm{GeV}
$$

$\Rightarrow$ approximates full result within 0.3 MeV
for 65 GeV $\leq M_{\mathrm{H}} \leq$ 1 TeV, 2σ variations
[*A. Freitas, W. Hollik, W. Walter, G.W. '02*]

Result now implemented in *ZFITTER*
[*D. Bardin '01*]

Comparison with previous result for M_{W}
(for $M_{\mathrm{H}} = 100$ GeV):

⇒ Downward shift of $M_{\mathrm{W}}^{\mathrm{theo}}$ by $\delta M_{\mathrm{W}}^{\mathrm{theo}} \approx 4.3$ MeV

⇒ Tendency towards light Higgs sharpened

Theoretical uncertainty of $\sin^2\theta_{\mathrm{eff}}$:

$$\sin^2\theta_{\mathrm{eff}} = \left(1 - \frac{M_{\mathrm{W}}^2}{M_{\mathrm{Z}}^2}\right)\kappa(M_{\mathrm{W}}^2)$$

New contribution (complete fermionic two-loop result) for κ not yet known

⇒ Effect of $\delta M_{\mathrm{W}}^{\mathrm{theo}}$ (non-leading electroweak two-loop corrections) in $\sin^2\theta_{\mathrm{eff}}$ yields estimate of uncertainty

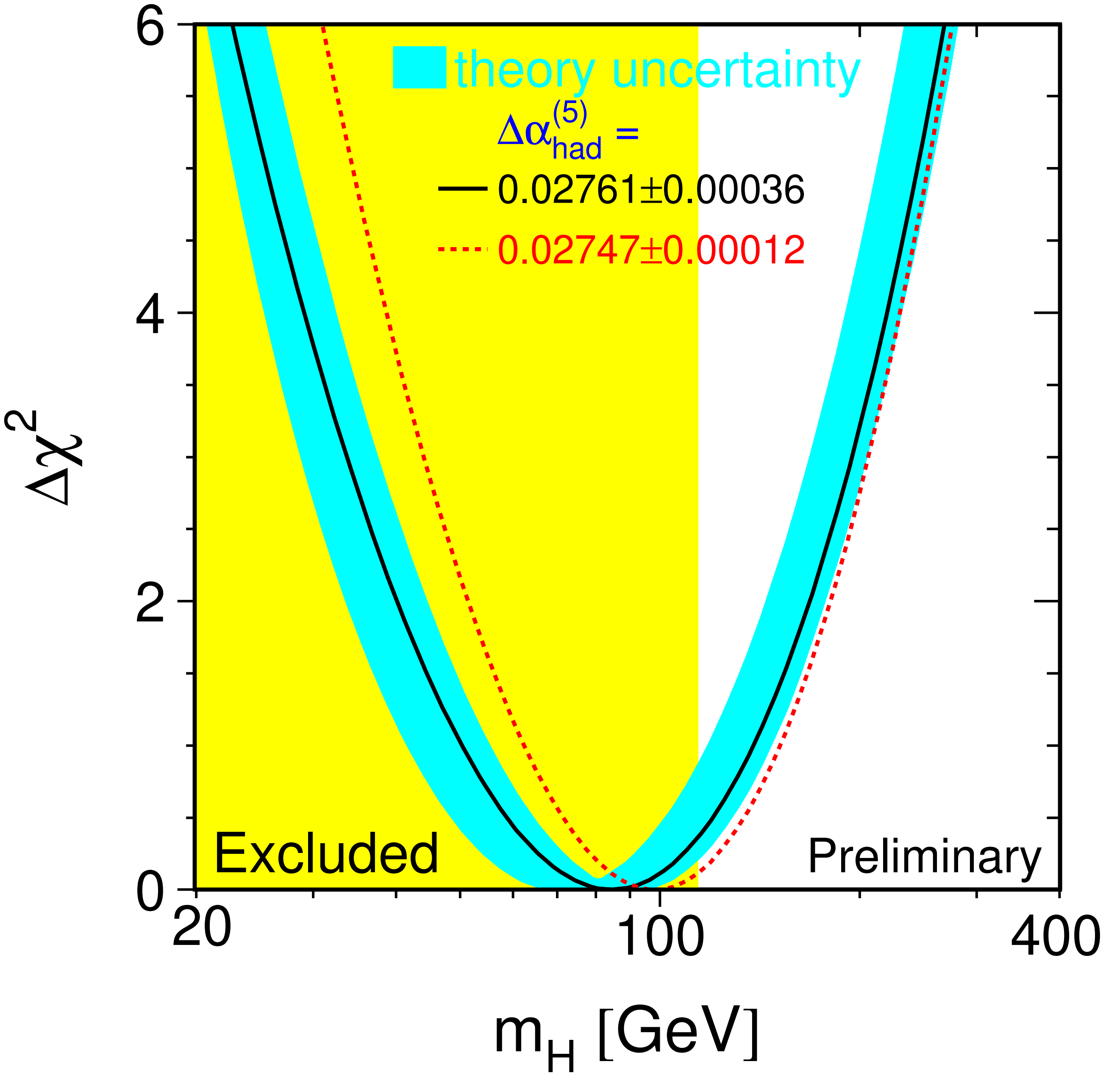
Upward shift in $\sin^2\theta_{\mathrm{eff}}$ caused by $\delta M_{\mathrm{W}}^{\mathrm{theo}}$:

$$\begin{aligned} \delta\sin^2\theta_{\mathrm{eff}}^{\mathrm{theo}} \approx\; & 8.2\times 10^{-5}, \quad M_{\mathrm{H}} = 100 \text{ GeV} \\ & 6.3\times 10^{-5}, \quad M_{\mathrm{H}} = 200 \text{ GeV} \\ & 5.4\times 10^{-5}, \quad M_{\mathrm{H}} = 300 \text{ GeV} \end{aligned}$$

Global fit to all data in the SM:
[*LEPEWWG '02*]

[*Moriond '02*]

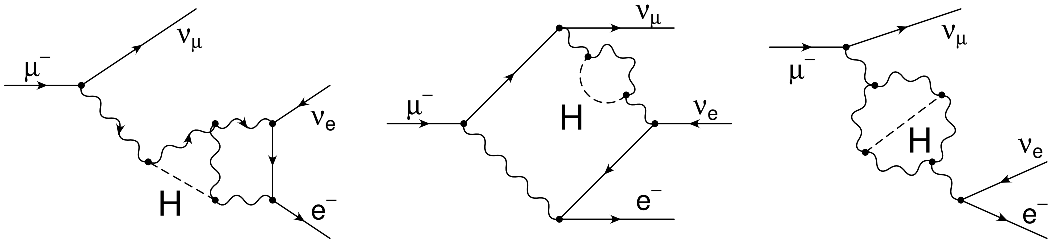
New result for M_W, new error estimate for "blue band":



$$\Rightarrow M_H = 85^{+54}_{-34} \text{ GeV}$$

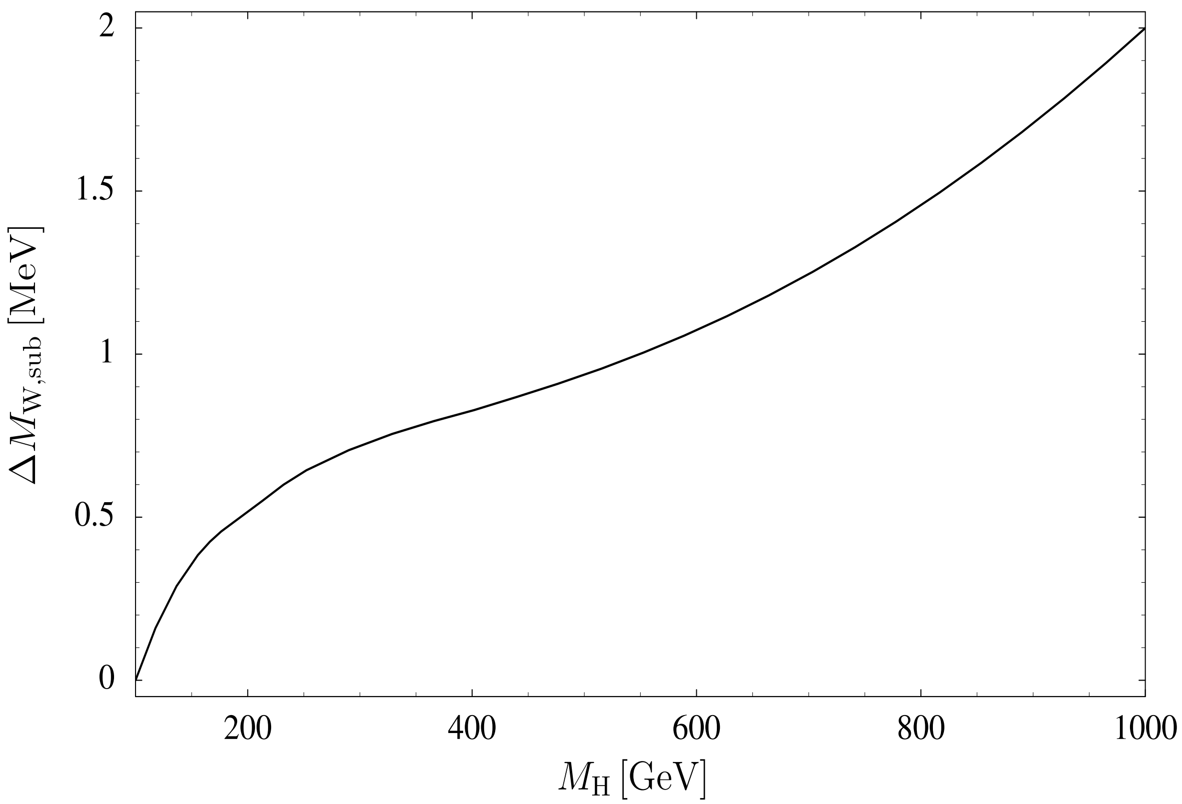
$$M_H < 196 \text{ GeV, } 95\% \text{ C.L.}$$

Higgs-mass dependence of purely bosonic two-loop contributions:



Shift in M_W:

[*A. Freitas, W. Hollik, W. Walter, G.W. '02*]

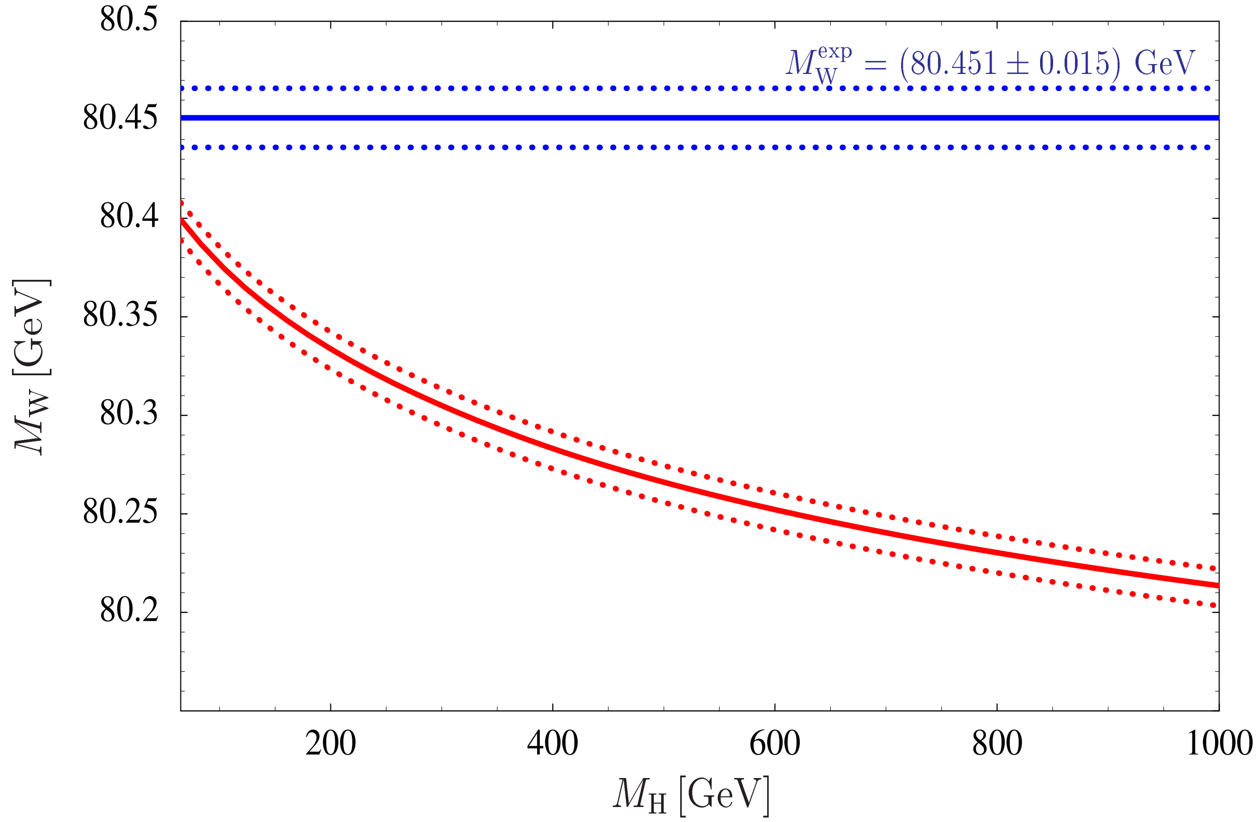


$$\Rightarrow \Delta M_W \leq 2 \text{ MeV}$$

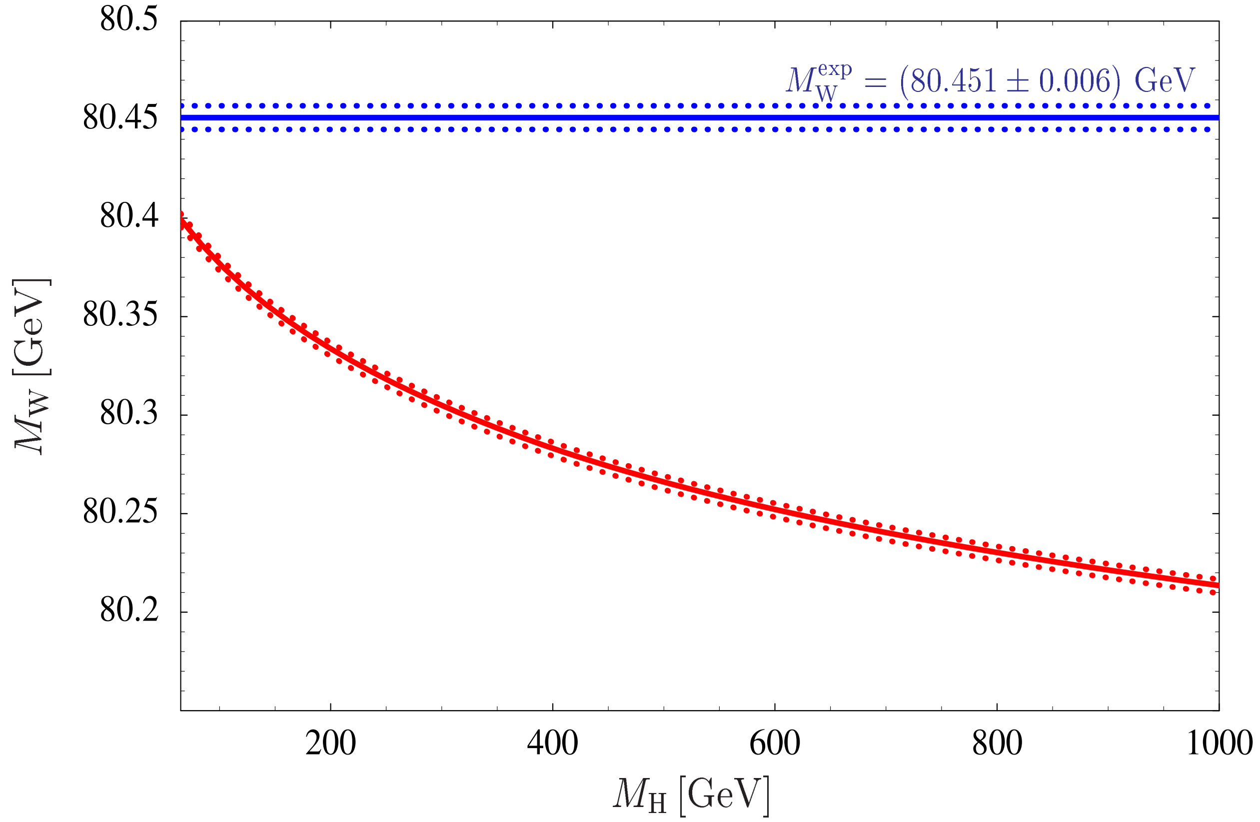
$\Rightarrow$ Only missing piece of complete two-loop result for M_W in the SM: M_H-independent constant

4. Prospects for future precision tests

Tevatron Run II, LHC:

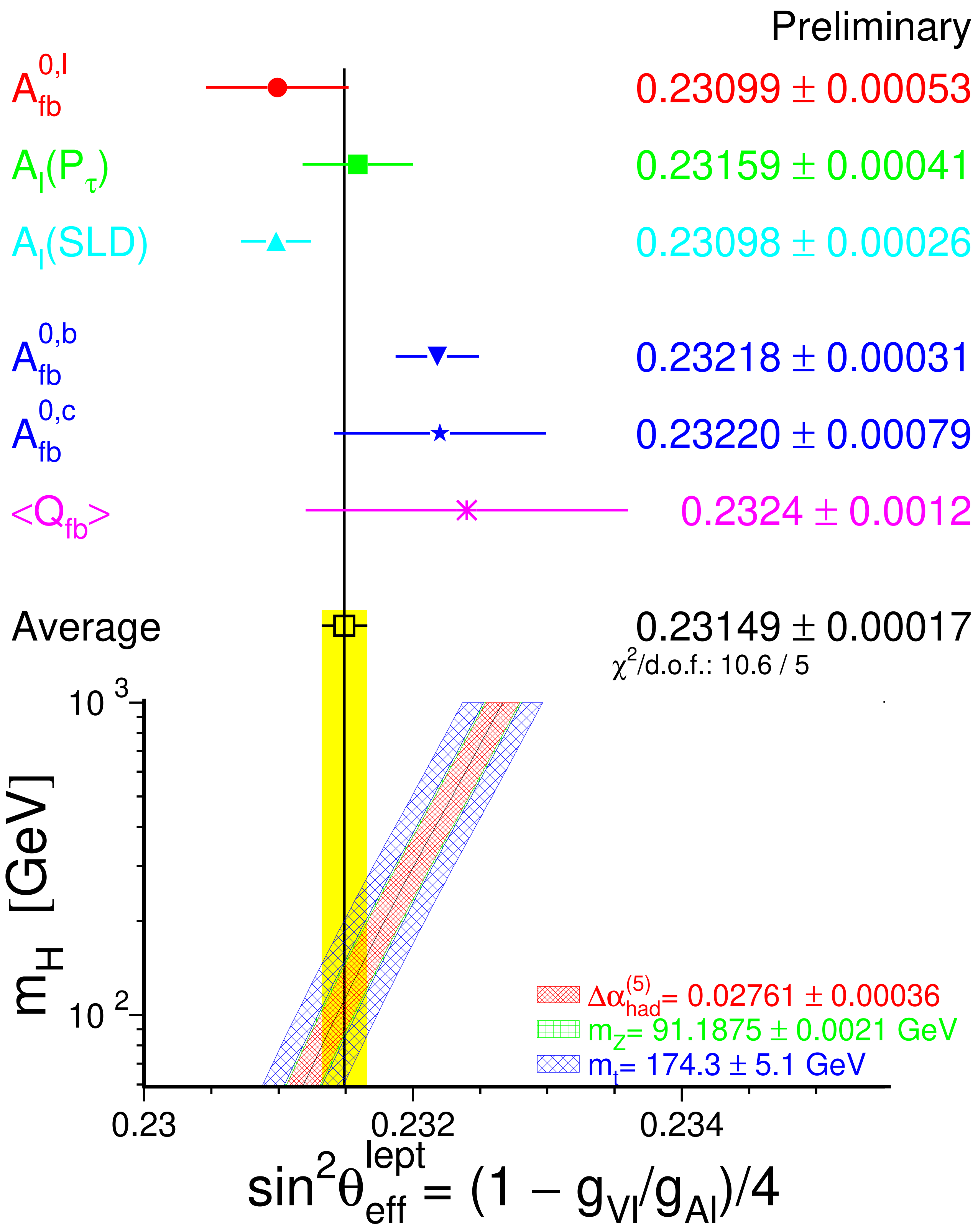


LC with high luminosity in low-energy mode: GigaZ



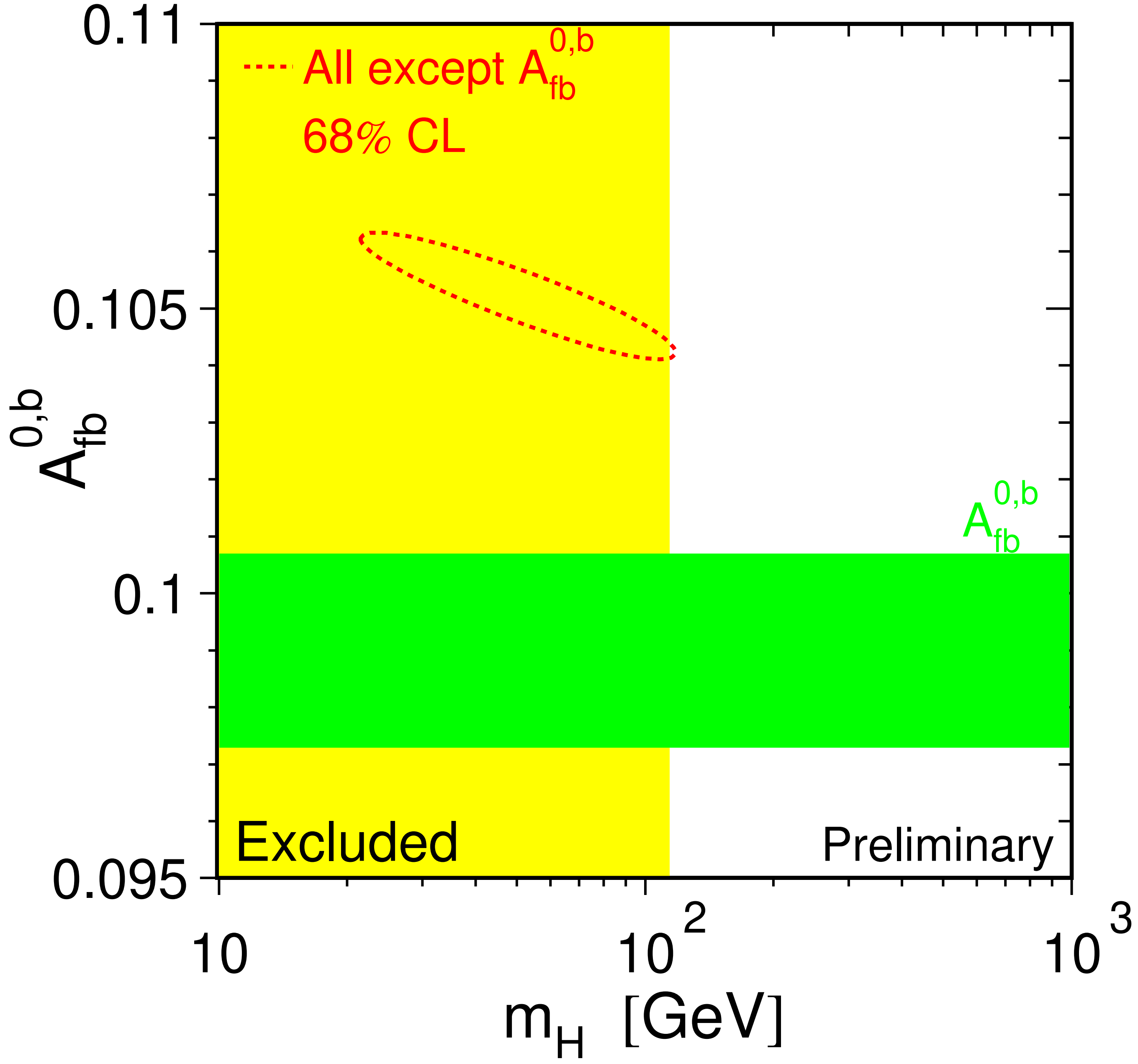
The effective weak mixing angle $\sin^2\theta_{\mathrm{eff}}$:

[LEPEWWG '02]



Forward–backward asymmetry of b quarks:

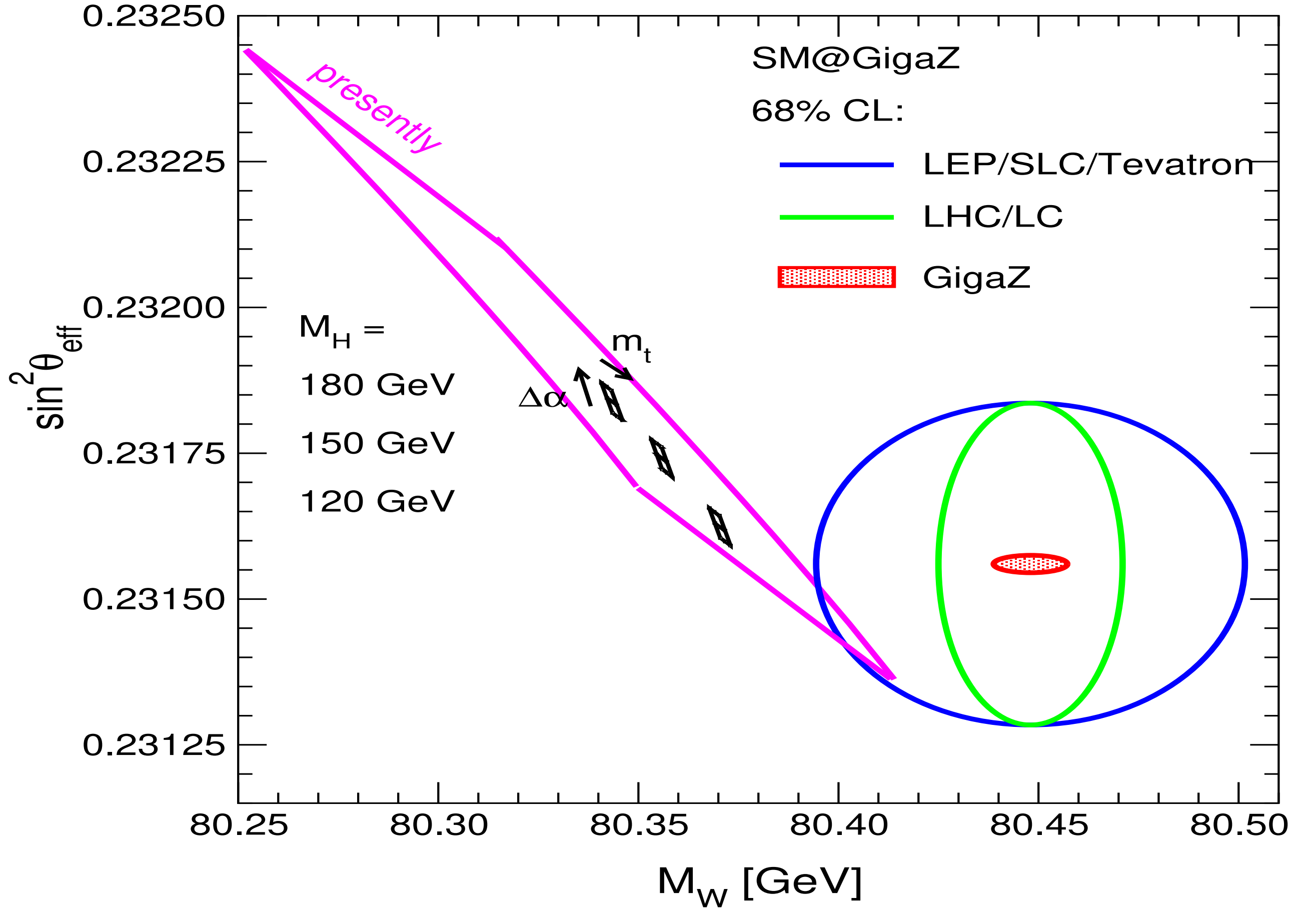
[*LEPEWWG '01*]



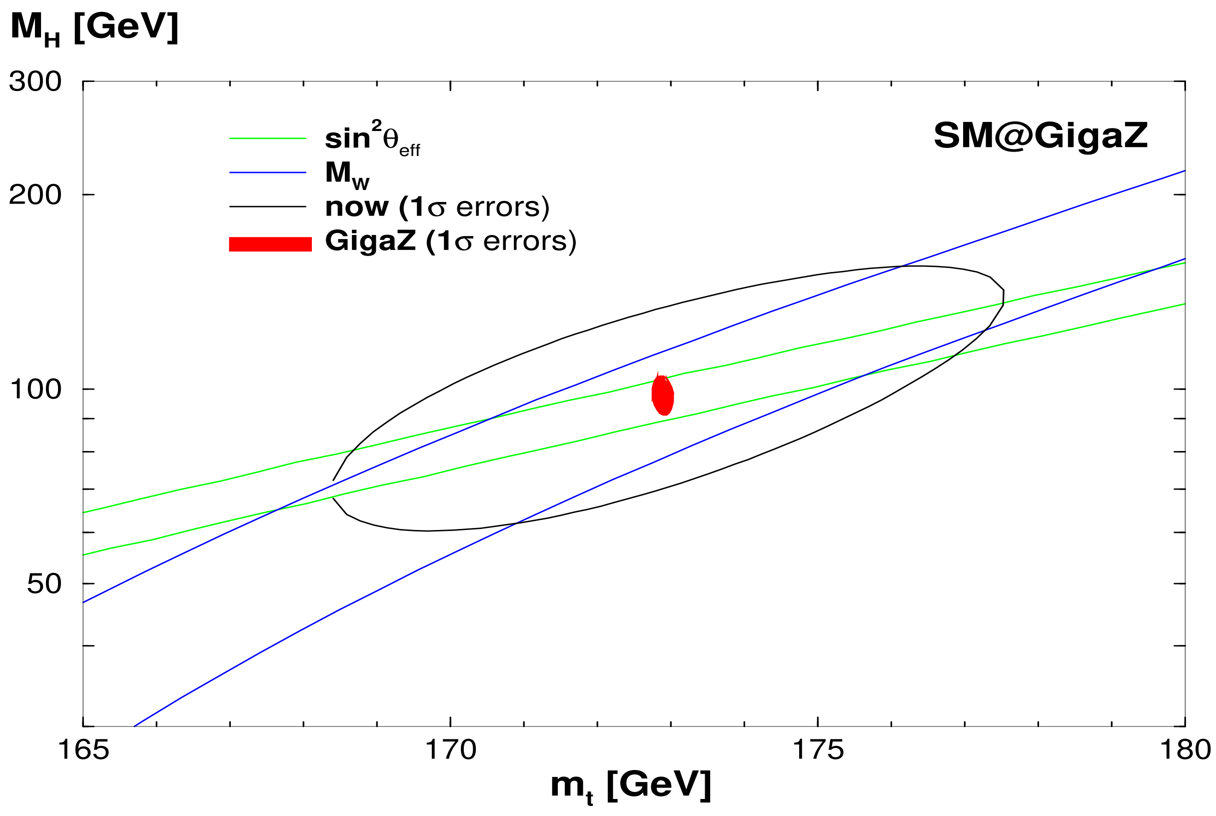
⇒ the "$A_{\text{fb}}^{0,b}$ crisis"

SM prediction for M_W and $\sin^2\theta_{\rm eff}$ vs. prospective accuracies at LEP2/Tevatron, Tev./LHC/LC, GigaZ:

[*J. Erler, S. Heinemeyer, W. Hollik, G.W., P. Zerwas '00*]



Indirect determination of m_t, M_H at GigaZ:



Precision observables in the MSSM:

In order to treat MSSM at same level of accuracy as SM

⇒ Include higher-order corrections

Prediction for M_W, $\sin^2\theta_{\text{eff}}$ in the MSSM:

- Complete 1-loop results
  [*P. Chankowski, A. Dabelstein, W. Hollik, W. Mösle, S. Pokorski, J. Rosiek '94*]
  [*D. Garcia, J. Solà '94*]
  [*D. Garcia, R. Jiménez, J. Solà '95*]
  [*A. Dabelstein, W. Hollik, W. Mösle '95*]
  [*P. Chankowski, S. Pokorski '96*]

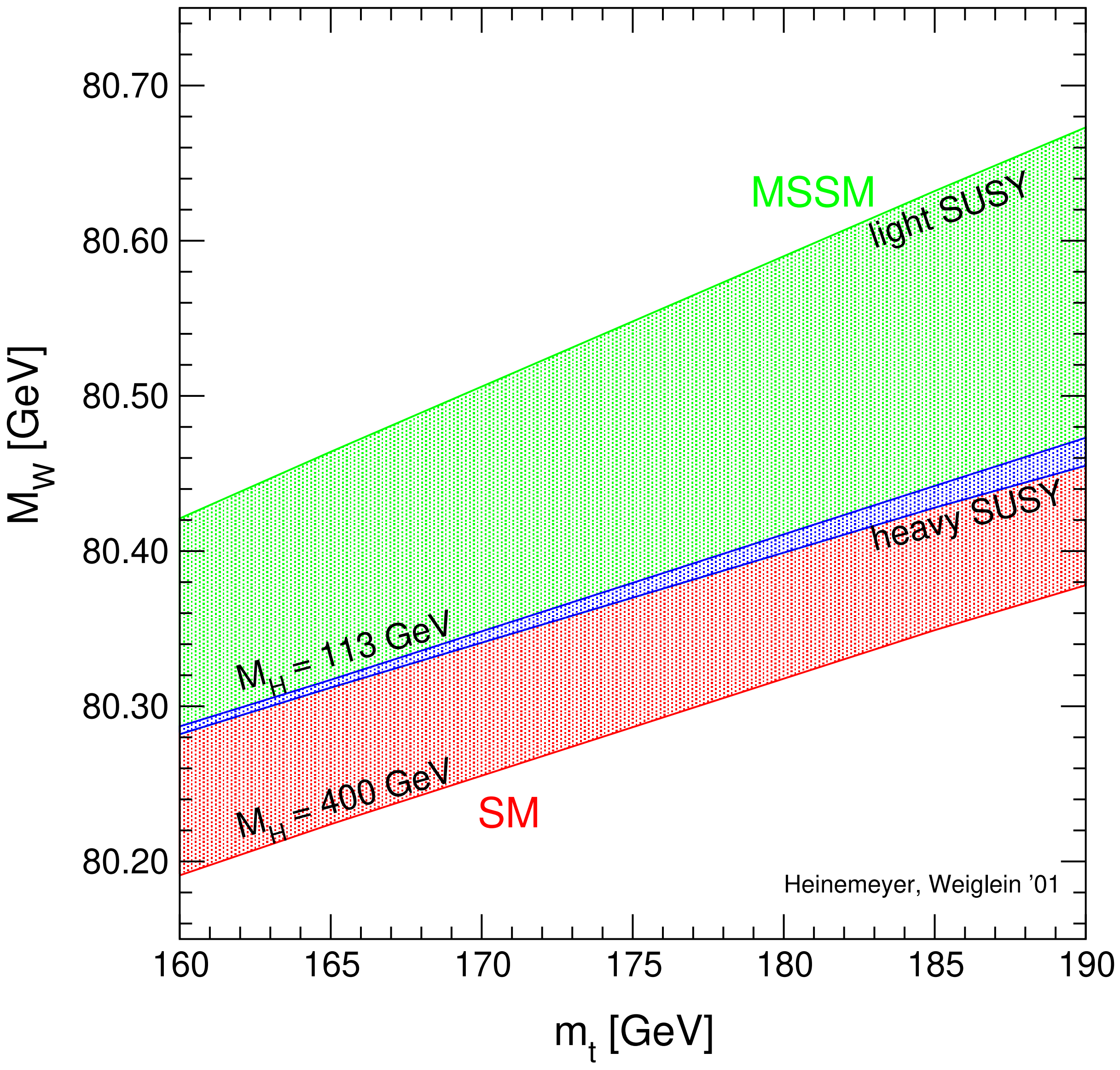
- Leading 2-loop $\mathcal{O}(\alpha\alpha_s)$ corrections
  [*A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger, G. W. '97*]
  [*S. Heinemeyer, W. Hollik, G. W. '98*]

- Leading 2-loop $\mathcal{O}(\alpha^2)$ corrections
  [*S. Heinemeyer, G. W. '01*]

Prediction for M_{W} in the SM and the MSSM:

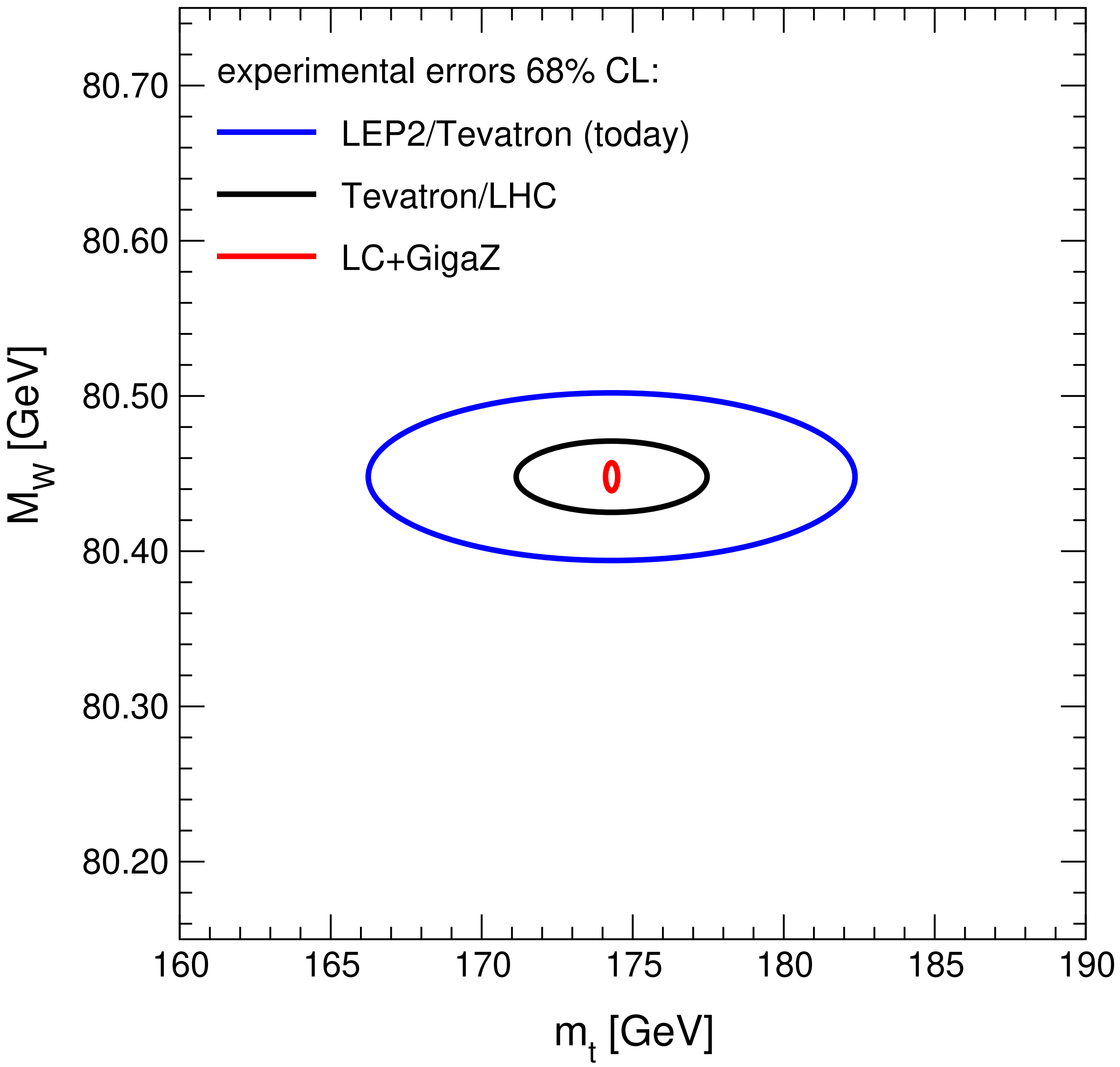
[A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger, G. W. '97]

[S. Heinemeyer, W. Hollik, G. W. '98]



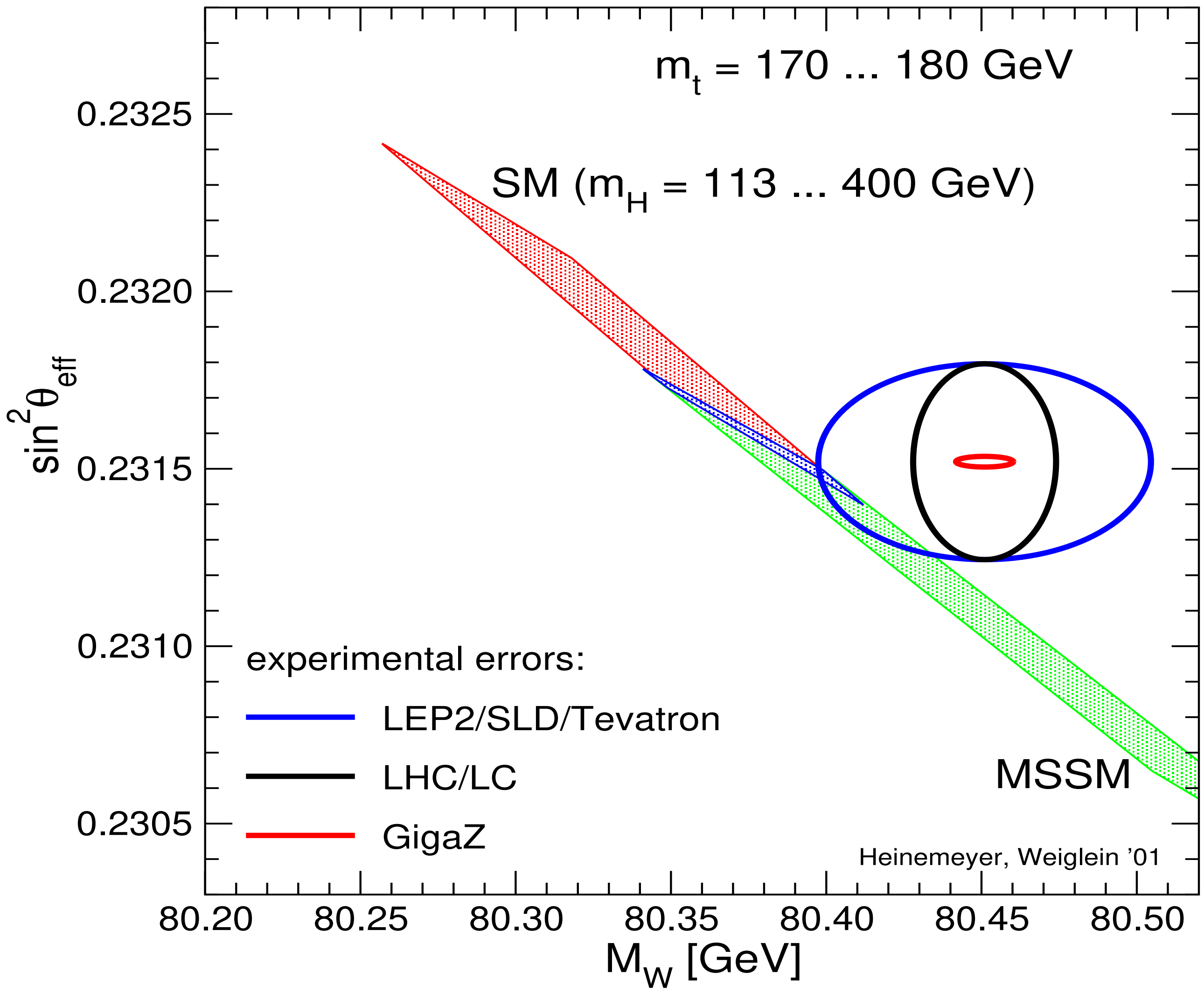
SM: M_{H} varied

MSSM: SUSY parameters varied



experimental errors 68% CL:
LEP2/Tevatron (today)
Tevatron/LHC
LC+GigaZ
80.70
80.60
80.50
80.40
80.30
80.20
M_W [GeV]
160
165
170
175
180
185
190
m_t [GeV]

Prediction for M_{W}, $\sin^2\theta_{\mathrm{eff}}$ in SM and MSSM:



$\Rightarrow$ Large improvement in experimental accuracy LEP/TevI $\rightarrow$ TevII/LHC/LC $\rightarrow$ GigaZ:

$\Rightarrow$ Very sensitive test of theory

5. Conclusions

- New results for M_W in the electroweak SM:
  - Complete fermionic two-loop corrections
  - Higgs dependence of bosonic corrections

- Estimate of theoretical uncertainties from unknown higher orders:

  $\delta M_W \approx \pm 6$ MeV

- Implementation of new result into *ZFITTER*

  $\Rightarrow$ Downward shift of $M_W^{\rm theo}$ by ≈ 4 MeV

  $\Rightarrow$ Tendency towards light Higgs sharpened

- New estimate of theoretical uncertainties

  $\Rightarrow$ "Blue band" has widened

- Comparison SM/MSSM predictions for M_W:

  Preference for non-zero SUSY contribution

- At TevII/LHC/LC: improved accuracy of precision observables M_W, $\sin^2\theta_{\rm eff}$, m_h, . . . and input parameters m_t, $m_{\tilde{t}}$, . . .

  $\Rightarrow$ Highly sensitive test of electroweak theory