One-loop electroweak corrections at high energies

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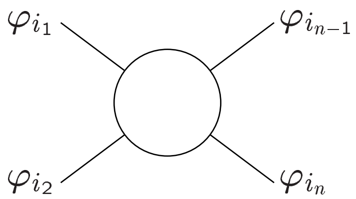
- Leading logarithmic approximation
- Universality of leading logarithmic corrections
- Derivation and results
- WZ and Wγ production at the LHC (introduction)

High-energy limit, logarithmic approximation

On-shell matrix elements

$$\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}}(p_1,\dots,p_n) =$$



with arbitrary external particles

$$\varphi_{i_k} = f^{\mathrm{L}}, f^{\mathrm{R}}, \quad \gamma_{\mathrm{T}}, \mathrm{Z}_{\mathrm{T}}, \mathrm{W}^{\pm}_{\mathrm{T}}, \quad H, \chi, \phi^{\pm}$$

High-energy limit

$$r_{kl} = (p_k + p_l)^2 \gg m_{\mathrm{t}}^2, M_{\mathrm{H}}^2 > M_{\mathrm{W,Z}}^2 \gg m_{f\neq\mathrm{t}}^2 \gg \lambda^2$$

Virtual corrections: logarithmic approximation

$$\frac{\delta\mathcal{M}}{\mathcal{M}_0} \sim \alpha \log^m \frac{|r_{kl}|}{M^2} + \mathcal{O}\left(\alpha,\, \alpha\frac{M^2}{|r_{kl}|}\log^m \frac{|r_{kl}|}{M^2}\right)$$

with $m = 1, 2$.

Motivation

Applicable at LHC and LC energies

- predict leading EW corrections $\sim \mathcal{O}(10\%)$
- reduce theoretical error $\sim \mathcal{O}(1\%)$

Easy to implement

- simple analytical form
- numerical stability

Theoretical understanding

- high-energy limit of $\mathcal{L}_{\mathrm{SB}}$ $(\sqrt{s} \gg v)$
- universal corrections as expected from $\mathcal{L}_{\mathrm{symm}}$?

at $\sqrt{s} \simeq 0.5\,\mathrm{TeV}(1\,\mathrm{TeV})$

$$\log \frac{s}{M_{\mathrm{W}}^2} \simeq 3.7(5.0), \qquad \log^2 \frac{s}{M_{\mathrm{W}}^2} \simeq 13.4(25.4)$$

$$\boxed{\text{Typical size of } \delta\sigma/\sigma}$$

per external leg

$$\text{W,Z:} \qquad -\frac{\alpha}{4\pi s_{\mathrm{W}}^2} \log^2 \frac{s}{M_{\mathrm{W}}^2} \simeq -3.5\%(-6.6\%)$$

$$\text{W,Z:} \qquad +\frac{3\alpha}{4\pi s_{\mathrm{W}}^2} \log \frac{s}{M_{\mathrm{W}}^2} \simeq +3.0\%(+3.9\%)$$

$$\text{t:} \qquad -\frac{\alpha}{4\pi s_{\mathrm{W}}^2} \frac{m_{\mathrm{t}}^2}{M_{\mathrm{W}}^2} \log \frac{s}{m_{\mathrm{t}}^2} \simeq -2.6\%(-4.3\%)$$

double- and single- logarithms have

- similar size
- different sign

$\Rightarrow$ important cancellations

Effects of spontaneous symmetry breaking

Mixing effects

gauge sector:

$$\begin{pmatrix} A \\ Z \\ W^+ \\ W^- \end{pmatrix} = \underbrace{\begin{pmatrix} c_W & -s_W & 0 & 0 \\ s_W & c_W & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix}}_{U(\theta_W)} \begin{pmatrix} B \\ W^3 \\ W^1 \\ W^2 \end{pmatrix}$$

tree-level $U(\theta_W)$, renormalization $\delta U(\theta_W)$

Longitudinal gauge bosons

- $\mathcal{L}_{\mathrm{symm}}$: WI's $\Rightarrow$ decoupling
- $\mathcal{L}_{\mathrm{SB}}$: WI's $\Rightarrow$ GBET:

$$\mathcal{M}^{\varphi_{i_1}\dots Z_L \dots \varphi_{i_n}} \simeq i(1+\delta A^{\chi})\mathcal{M}^{\varphi_{i_1}\dots \chi \dots \varphi_{i_n}}$$
$$\mathcal{M}^{\varphi_{i_1}\dots W_L^{\pm} \dots \varphi_{i_n}} \simeq \pm(1+\delta A^{\phi})\mathcal{M}^{\varphi_{i_1}\dots \phi^{\pm} \dots \varphi_{i_n}}$$

- **corrections δA to LO GBET have to be derived from $\mathcal{L}_{\mathrm{SB}}$ and lead to large logarithms!**

$\boxed{\text{General form of results}}$

In 1-loop logarithmic approximation

- $\mathcal{M}_0$ **not mass-suppressed** $\Rightarrow \delta\mathcal{M}$ **universal**

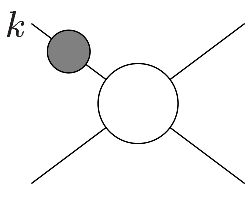
- $\mathcal{L}_{\mathrm{SB}} \longrightarrow \mathcal{L}_{\mathrm{symm}}, \qquad \mathrm{Z_L}, \mathrm{W}^{\pm}_{\mathrm{L}} \longrightarrow \chi, \phi^{\pm}$

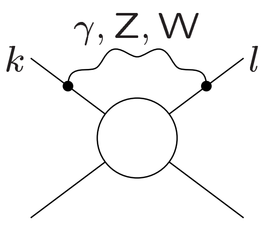
Running Couplings

$$\delta\mathcal{M} = \sum_{\lambda_i} \delta\lambda_i \frac{\partial\mathcal{M}_0}{\partial\lambda_i}$$

Mass singularities $\Rightarrow$ external-leg factorization

$$\delta\mathcal{M} = \underbrace{\sum_{k=1}^{n} \delta(k)\,\mathcal{M}_0}_{\text{ang-indep}} \quad + \quad \underbrace{\sum_{k=1}^{n}\sum_{l<k} \delta(k,l)\,\mathcal{M}_0}_{\text{ang-dep}}$$





A. Denner and S. P., Eur. Phys. J. **C 18** (2001)
S. P., hep-ph/0201077

Strategy of calculation

Dimensional regularization: scale $\mu^2 = s$

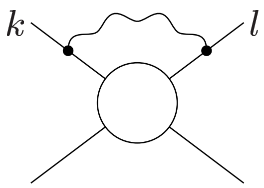
$$\underbrace{\log\frac{s}{\mu^2}}_{\text{UV-div loops}} + \underbrace{\log\frac{\mu^2}{M^2}}_{\text{CT's \& MS loops}} = \underbrace{\log\frac{s}{M^2}}_{\mu\text{-indep}}$$

$\Rightarrow$ Large logarithms from:

A) Running couplings in $\delta\lambda$

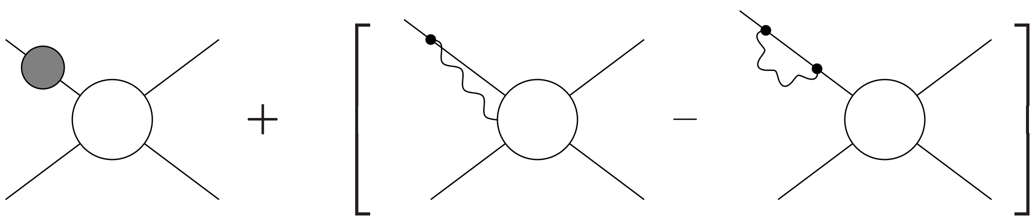
B) Mass singularities in δZ and **loops** with **soft** and/or **collinear** virtual gauge bosons coupling to external legs. In the **Feynman gauge**:

- Double logarithms



from soft-collinear region ($q^\mu \to xp^\mu$, $x \to 0$)

- Single logarithms (non-trivial)



from collinear region ($q^\mu \to xp^\mu$, $x \neq 0$)

Running couplings

$$\delta\mathcal{M} = \sum_{\lambda_i} \frac{\partial \mathcal{M}_0}{\partial \lambda_i} \delta\lambda_i \Big|_{\mu^2=s}$$

independent $\{\lambda_i\}$ for OS renormalization

$$e, \quad c_{\mathrm{w}} = \frac{M_{\mathrm{W}}}{M_{\mathrm{Z}}}, \quad m_{\mathrm{t}}, \quad M_{\mathrm{H}}$$

convenient $\{\lambda_i\}$ at high energies

$$g_1 = \frac{e}{c_{\mathrm{w}}}, \quad g_2 = \frac{e}{s_{\mathrm{w}}}, \quad \lambda_{\mathrm{t}} = \frac{e}{\sqrt{2}s_{\mathrm{w}}}\frac{m_{\mathrm{t}}}{M_{\mathrm{W}}}, \quad \lambda_{\mathrm{H}} = \frac{e^2}{2s_{\mathrm{w}}^2}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}^2}$$

Counterterms for gauge interactions

$$\frac{\delta g_1^2}{g_1^2} = \frac{\alpha}{4\pi}\left[-b_1 \log\frac{\mu^2}{M_{\mathrm{W}}^2} - \frac{5}{6c_{\mathrm{w}}^2}\log\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}^2} + \frac{9+6s_{\mathrm{w}}^2-32s_{\mathrm{w}}^4}{18s_{\mathrm{w}}^2c_{\mathrm{w}}^2}\log\frac{m_{\mathrm{t}}^2}{M_{\mathrm{W}}^2}\right] + \Delta\alpha(M_{\mathrm{W}}^2),$$

$$\frac{\delta g_2^2}{g_2^2} = \frac{\alpha}{4\pi}\left[-b_2 \log\frac{\mu^2}{M_{\mathrm{W}}^2} + \frac{5}{6s_{\mathrm{w}}^2}\log\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}^2} - \frac{9+6s_{\mathrm{w}}^2-32s_{\mathrm{w}}^4}{18s_{\mathrm{w}}^4}\log\frac{m_{\mathrm{t}}^2}{M_{\mathrm{W}}^2}\right] + \Delta\alpha(M_{\mathrm{W}}^2)$$

Running from M_{W}^2 to s

$$b_1 = -\frac{41}{6c_{\mathrm{w}}^2}, \qquad b_2 = \frac{19}{6s_{\mathrm{w}}^2}$$

Running from 0 to M_{W}^2

$$\Delta\alpha(M_{\mathrm{W}}^2) = \frac{\alpha}{3\pi}\sum_{f\neq \mathrm{t}} N_{\mathrm{C}}^f Q_f^2 \log\frac{M_{\mathrm{W}}^2}{m_f^2}$$

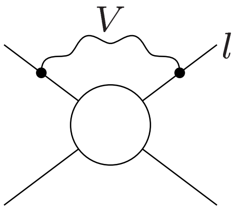
Counterterms for Yukawa interactions

$$\frac{\delta\lambda_{\mathrm{t}}}{\lambda_{\mathrm{t}}} = \frac{\alpha}{4\pi}\left\{\left[-\frac{3}{2}(C^{\mathrm{ew}}_{\mathrm{t^L}} + C^{\mathrm{ew}}_{\mathrm{t^R}}) + \frac{3+2N^t_{\mathrm{C}}}{8s^2_{\mathrm{w}}}\frac{m^2_{\mathrm{t}}}{M^2_{\mathrm{W}}}\right]\log\frac{\mu^2}{m^2_{\mathrm{t}}}\right.$$
$$-\left(\frac{9-12s^2_{\mathrm{w}}-32s^4_{\mathrm{w}}}{36s^4_{\mathrm{w}}} + 2C^{\mathrm{ew}}_{\Phi}\right)\log\frac{m^2_{\mathrm{t}}}{M^2_{\mathrm{W}}}$$
$$\left.-\frac{3}{8s^2_{\mathrm{w}}}\frac{m^2_{\mathrm{t}}}{M^2_{\mathrm{W}}}\log\frac{M^2_{\mathrm{H,t}}}{m^2_{\mathrm{t}}} + \frac{5}{6s^2_{\mathrm{w}}}\log\frac{M^2_{\mathrm{H}}}{M^2_{\mathrm{W}}}\right\} + \frac{1}{2}\Delta\alpha(M^2_{\mathrm{W}})$$

Counterterms for scalar interactions

$$\frac{\delta\lambda_{\mathrm{H}}}{\lambda_{\mathrm{H}}} = \frac{\alpha}{4\pi}\left\{\left[\frac{3}{s^2_{\mathrm{w}}}\frac{M^2_{\mathrm{W}}}{M^2_{\mathrm{H}}}\left(1+\frac{1}{2c^4_{\mathrm{w}}}\right) - 6C^{\mathrm{ew}}_{\Phi}\right]\log\frac{\mu^2}{M^2_{\mathrm{W}}}\right.$$
$$+\frac{3}{2s^2_{\mathrm{w}}}\frac{M^2_{\mathrm{H}}}{M^2_{\mathrm{W}}}\log\frac{\mu^2}{M^2_{\mathrm{H}}} + \frac{N^t_{\mathrm{C}}}{s^2_{\mathrm{w}}}\frac{m^2_{\mathrm{t}}}{M^2_{\mathrm{W}}}\left(1-2\frac{m^2_{\mathrm{t}}}{M^2_{\mathrm{H}}}\right)\log\frac{\mu^2}{m^2_{\mathrm{t}}}$$
$$+\left[-\frac{9}{2s^2_{\mathrm{w}}}\frac{M^2_{\mathrm{W}}}{M^2_{\mathrm{H}}}\left(1+\frac{1}{2c^4_{\mathrm{w}}}\right) + \frac{3}{2}C^{\mathrm{ew}}_{\Phi} + \frac{5}{3s^2_{\mathrm{w}}}\right]\log\frac{M^2_{\mathrm{H}}}{M^2_{\mathrm{W}}}$$
$$-\frac{9-12s^2_{\mathrm{w}}-32s^4_{\mathrm{w}}}{18s^4_{\mathrm{w}}}\log\frac{m^2_{\mathrm{t}}}{M^2_{\mathrm{W}}}$$
$$\left.-\frac{N^t_{\mathrm{C}}}{2s^2_{\mathrm{w}}}\frac{m^2_{\mathrm{t}}}{M^2_{\mathrm{W}}}\left(1-4\frac{m^2_{\mathrm{t}}}{M^2_{\mathrm{H}}}\right)\log\frac{M^2_{\mathrm{H,t}}}{m^2_{\mathrm{t}}}\right\} + \Delta\alpha(M^2_{\mathrm{W}})$$

Mass singularities I : double logarithms

$$\delta\mathcal{M} = \sum_{k=1}^{n}\sum_{l<k}\sum_{V=\gamma,Z,W^\pm}$$ 

$$= \ldots \text{eikonal approximation} \ldots$$

$$= \frac{\alpha}{8\pi}\sum_{k=1}^{n}\sum_{l\neq k}\sum_{V=\gamma,Z,W^\pm} I^{\bar{V}}(k)I^{V}(l)\mathcal{M}_0 \left[\log^2\frac{|r_{kl}|}{M_V^2} - \delta_{V\gamma}\log^2\frac{m_k^2}{\lambda^2}\right]$$

- **Angular-dependent** part: $\log\frac{|r_{kl}|}{s} \sim 1$

$$\log^2\frac{|r_{kl}|}{M^2} \simeq \log^2\frac{s}{M^2} + 2\log\frac{s}{M^2}\log\frac{|r_{kl}|}{s}$$

- **Angular-independent** part: global SU(2)×U(1) invariance

$$\sum_{l=1}^{n} I^V(l)\mathcal{M}_0 \simeq 0, \qquad V = A, Z, W^\pm$$

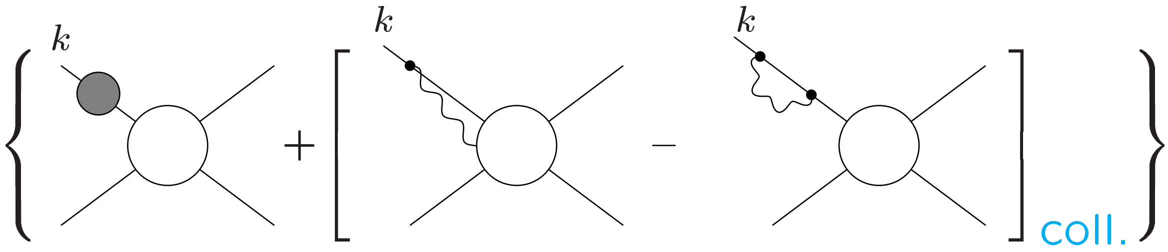
$$\delta\mathcal{M} = \frac{\alpha}{2\pi}\sum_{k=1}^{n}\sum_{l<k}\sum_{V=\gamma,Z,W^\pm} I^{\bar{V}}(k)I^{V}(l)\mathcal{M}_0 \log\frac{s}{M_V^2}\log\frac{|r_{kl}|}{s}$$
$$-\frac{\alpha}{8\pi}\sum_{k=1}^{n}\left[C^{\mathrm{ew}}(k)\log^2\frac{s}{M_\mathrm{W}^2} + Q^2(k)\left(2\log\frac{s}{m_k^2}\log\frac{M_\mathrm{W}^2}{\lambda^2} - \log^2\frac{M_\mathrm{W}^2}{m_k^2}\right)\right]\mathcal{M}_0$$

Electroweak Casimir operator

$$C^{\mathrm{ew}} = \sum_{V=\gamma,Z,W^\pm} I^V I^{\bar{V}} = \frac{1}{c_\mathrm{w}^2}\left(\frac{Y}{2}\right)^2 + \frac{1}{s_\mathrm{w}^2}C^{\mathrm{SU(2)}}$$

Mass singularities II : single logarithms

$$\delta\mathcal{M} = \sum_{k=1}^{n}$$



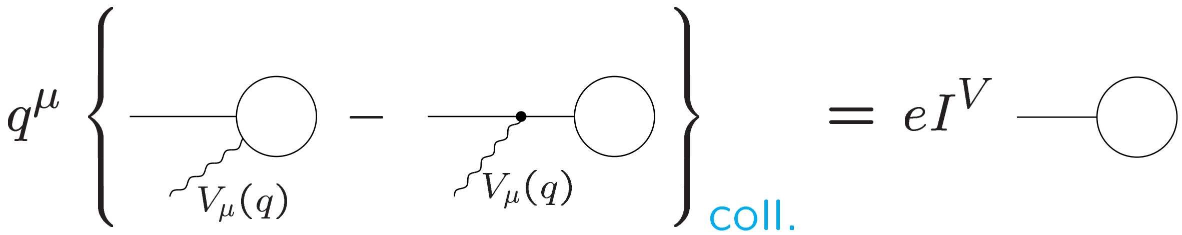
Factorization of collinear singularities

$$\delta\mathcal{M} = \sum_{k=1}^{n} \left\{ \frac{1}{2}\delta Z(k) + \delta^{\mathrm{coll}}(k) \right\}\Big|_{\mu^2=s} \mathcal{M}_0$$

with

$$\delta^{\mathrm{coll}}(k) = K\frac{\alpha}{4\pi}C^{\mathrm{ew}}(k)\log\frac{\mu^2}{M_\mathrm{W}^2}, \qquad K = \begin{cases} 1 & \text{bosons} \\ 2 & \text{fermions} \end{cases}$$

obtained from collinear WI's



- for arbitrary LO matrix elements
- up to mass-supp terms
- for BRS inv. spont. broken gauge theory

A. Denner and S. P., Eur. Phys. J. **C 21** (2001)

$$\boxed{\delta^{\mathrm{C}} = \frac{1}{2}\delta Z + \delta^{\mathrm{coll}}}$$

Light fermions: $f^{\mathrm{R,L}}$ with $f = \mathrm{l}, \nu_{\mathrm{l}}, \mathrm{u}, \mathrm{d}, \mathrm{c}, \mathrm{s}$

$$\delta^{\mathrm{C}}_{f^\lambda} = \frac{\alpha}{4\pi}\left[\frac{3}{2}C^{\mathrm{ew}}_{f^\lambda}\log\frac{s}{M_{\mathrm{W}}^2} + Q_f^2\left(\frac{1}{2}\log\frac{M_{\mathrm{W}}^2}{m_f^2} + \log\frac{M_{\mathrm{W}}^2}{\lambda^2}\right)\right]$$

- diagonal in isospin and flavour indices
- depend on chirality

Heavy fermions: $\mathrm{t^L}, \mathrm{t^R}, \mathrm{b^L}$

$\Rightarrow$ additional Yukawa contributions

$$\delta^{\mathrm{C,Yuk}}_{\mathrm{t^L}} = \frac{\alpha}{4\pi}\left\{-\frac{m_{\mathrm{t}}^2}{8s_{\mathrm{w}}^2M_{\mathrm{W}}^2}\left[\log\frac{s}{m_{\mathrm{t}}^2} + \frac{1}{2}\log\frac{m_{\mathrm{t}}^2}{M_{\mathrm{H,t}}^2}\right] + \frac{16c_{\mathrm{w}}^4 + 10c_{\mathrm{w}}^2 + 1}{24s_{\mathrm{w}}^2c_{\mathrm{w}}^2}\log\frac{M_{\mathrm{W}}^2}{m_{\mathrm{t}}^2}\right\}$$

$$\delta^{\mathrm{C,Yuk}}_{\mathrm{t^R}} = \frac{\alpha}{4\pi}\left\{-\frac{m_{\mathrm{t}}^2}{8s_{\mathrm{w}}^2M_{\mathrm{W}}^2}\left[2\log\frac{s}{m_{\mathrm{t}}^2} + \frac{1}{2}\log\frac{m_{\mathrm{t}}^2}{M_{\mathrm{H,t}}^2}\right] + \frac{2s_{\mathrm{w}}^2}{3c_{\mathrm{w}}^2}\log\frac{M_{\mathrm{W}}^2}{m_{\mathrm{t}}^2}\right\}$$

$$\delta^{\mathrm{C,Yuk}}_{\mathrm{b^L}} = \frac{\alpha}{4\pi}\left\{-\frac{m_{\mathrm{t}}^2}{8s_{\mathrm{w}}^2M_{\mathrm{W}}^2}\log\frac{s}{m_{\mathrm{t}}^2} + \frac{3}{4s_{\mathrm{w}}^2}\log\frac{M_{\mathrm{W}}^2}{m_{\mathrm{t}}^2}\right\}$$

$$\boxed{\delta^{\mathrm{C}} = \tfrac{1}{2}\delta Z + \delta^{\mathrm{coll}}}$$

Transverse gauge bosons: $V_{\mathrm{T}}, V'_{\mathrm{T}} = \gamma_{\mathrm{T}}, \mathrm{Z}_{\mathrm{T}}, \mathrm{W}^{\pm}_{\mathrm{T}}$

$$\delta^{\mathrm{C}}_{V_{\mathrm{T}}V'_{\mathrm{T}}} = \frac{\alpha}{4\pi}\left\{\frac{1}{2}K_{VV'}\log\frac{s}{M_{\mathrm{W}}^2} + \delta_{VV'}Q_V^2\log\frac{M_{\mathrm{W}}^2}{\lambda^2} + T_{VV'}\log\frac{m_{\mathrm{t}}^2}{M_{\mathrm{W}}^2}\right.$$
$$\left. + \delta_{VV'}\frac{M_V^2}{24 s_{\mathrm{w}}^2 M_{\mathrm{W}}^2}\log\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}^2}\right\} - \frac{1}{2}\delta_{VA}\delta_{V'A}\Delta\alpha(M_{\mathrm{W}}^2)$$

leading part **non-diagonal and non-symmetric owing to mixing effects**

$$\frac{\alpha}{8\pi}K\log\frac{\mu^2}{M_{\mathrm{W}}^2} = \frac{\alpha}{8\pi}\log\frac{\mu^2}{M_{\mathrm{W}}^2}U\, b^{\mathrm{ew}}\, U^{-1} + \delta U\, U^{-1}$$

related to

$$b^{\mathrm{ew}} = \begin{pmatrix} b_1 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 \\ 0 & 0 & b_2 & 0 \\ 0 & 0 & 0 & b_2 \end{pmatrix}, \qquad U = \begin{pmatrix} c_{\mathrm{w}} & -s_{\mathrm{w}} & 0 & 0 \\ s_{\mathrm{w}} & c_{\mathrm{w}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{-\mathrm{i}}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{\mathrm{i}}{\sqrt{2}} \end{pmatrix}$$

$$\boxed{\delta^{\mathrm{C}} = \tfrac{1}{2}\delta Z + \delta^{\mathrm{coll}}}$$

Higgs bosons: H

$$\delta^{\mathrm{C}}_{H} = \frac{\alpha}{4\pi}\left\{2C^{\mathrm{ew}}_{\Phi}\log\frac{s}{M_{\mathrm{H}}^2} - \frac{3}{4s_{\mathrm{w}}^2}\frac{m_{\mathrm{t}}^2}{M_{\mathrm{W}}^2}\log\frac{s}{M_{\mathrm{H,t}}^2}\right\}$$

Longitudinal gauge bosons: $V_{\mathrm{L}} = \mathrm{Z_L}, \mathrm{W^\pm_L}$
GBET $\Rightarrow$

$$\begin{aligned}\delta^{\mathrm{C}}_{V_{\mathrm{L}}} &= \frac{1}{2}\delta Z_{VV} + \delta^{\mathrm{coll}}_{\Phi\Phi} + \frac{\delta M_V}{M_V} + \mathrm{i}^{1+Q_V}\frac{\Sigma^{V\Phi}_{\mathrm{L}}(M_V^2)}{M_V} - \frac{\Sigma^{VV}_{\mathrm{L}}(M_V^2)}{M_V^2} \\ &= \frac{\alpha}{4\pi}\left\{2C^{\mathrm{ew}}_{\Phi}\log\frac{s}{M_{\mathrm{W}}^2} - \frac{3}{4s_{\mathrm{w}}^2}\frac{m_{\mathrm{t}}^2}{M_{\mathrm{W}}^2}\log\frac{s}{m_{\mathrm{t}}^2} + Q_V^2\log\frac{M_{\mathrm{W}}^2}{\lambda^2}\right. \\ &\qquad\left. + \frac{M_V^2}{8s_{\mathrm{w}}^2 M_{\mathrm{W}}^2}\log\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}^2}\right\}\end{aligned}$$

In **1-loop logarithmic approximation**

$$\mathrm{Z_L}, \mathrm{W^\pm_L} \simeq \chi, \phi^\pm$$

as naively expected from the LO GBET

Conclusions

- Universality of EW leading logs proved
  - for non-suppressed matrix elements
  - at one loop
- Factorization of collinear singularities proved
  - within a generic **SB** gauge theory
  - with collinear WI's
- General results presented include all large logs
  - **Running couplings**: single logs
  - **Mass singularities**: double, single and angular-dependent logs

EWLC to WZ and Wγ production at the LHC

- Implement EWLC into Monte Carlo for

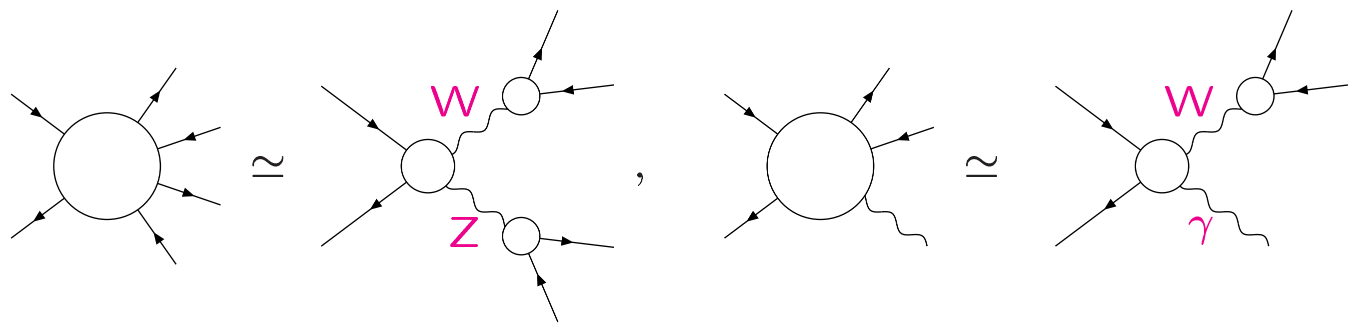
$$\mathrm{pp} \to \mathrm{W}^{\pm}\mathrm{Z} \to l\nu_l l'\bar{l'}, \quad \mathrm{pp} \to \mathrm{W}^{\pm}\gamma \to l\nu_l\gamma, \quad l,l' = \mathrm{e},\mu$$

- Investigate region of high P_{T} and large invariant mass $\sqrt{s} = M_{VV'}$ $\Rightarrow$ sensitive to anomalous couplings

Hadronic-partonic cross secctions

$$\mathrm{d}\sigma(P_1,P_2) = \sum_{i,j=\mathrm{u,d,c,d}} \int \mathrm{d}x_1\mathrm{d}x_2\ f_i(x_1,Q^2)f_j(x_2,Q^2)\,\mathrm{d}\hat{\sigma}^{ij}(x_1P_1,x_2P_2)$$

Leading-pole approximation



- Check reliability at Born level

- Check that $s,t,u \gg M_{\mathrm{W}}^2$

$\Rightarrow$ implement corrections in LPA

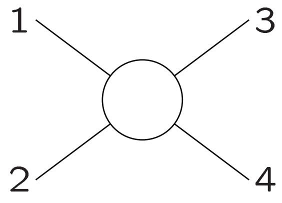
Corrections in leading-pole approximation

$$\delta\mathcal{M}^{qq'\to \mathrm{W}\gamma_{\lambda'}\to l\nu_l\gamma_{\lambda'}} = \mathrm{i}\frac{\sum_\lambda \delta\mathcal{M}^{qq'\to \mathrm{W}_\lambda\gamma_{\lambda'}}\mathcal{M}_0^{\mathrm{W}_\lambda\to l\nu_l}}{p_\mathrm{W}^2 - M_\mathrm{W}^2 + \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W}}$$

$$\delta\mathcal{M}^{qq'\to \mathrm{WZ}\to l\nu_l l'\bar{l}'} = -\frac{\sum_{\lambda,\lambda'} \delta\mathcal{M}^{qq'\to \mathrm{W}_\lambda \mathrm{Z}_{\lambda'}}\mathcal{M}_0^{\mathrm{W}_\lambda\to l\nu_l}\mathcal{M}_0^{\mathrm{Z}_{\lambda'}\to l'\bar{l}'}}{(p_\mathrm{W}^2 - M_\mathrm{W}^2 + \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W})\,(p_\mathrm{Z}^2 - M_\mathrm{Z}^2 + \mathrm{i}M_\mathrm{Z}\Gamma_\mathrm{Z})}$$

Theoretical error:

- non-factorizable corrections $\sim \mathcal{O}(1\%)$
- non-logarithmic $m_\mathrm{t}, M_\mathrm{H}$ corrections $\sim \mathcal{O}(1\%)$

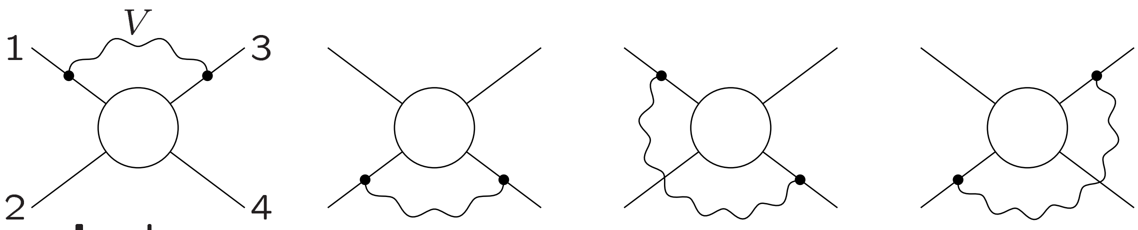


One-loop corrections

(omitting purely EM corrections)

Mass singularities: **double** logs

$$\delta\mathcal{M} = -\frac{\alpha}{8\pi}\log^2\frac{s}{M_\mathrm{W}^2}\sum_{k=1}^{4} C^\mathrm{ew}(k)\mathcal{M}_0$$
$$+\frac{\alpha}{2\pi}\log\frac{s}{M_\mathrm{W}^2}\sum_{V=\gamma,Z,W^\pm}\left\{\log\frac{|t|}{s}\left[I^{\bar{V}}(1)I^{V}(3)+I^{\bar{V}}(2)I^{V}(4)\right]\right.$$
$$\left.+\log\frac{|u|}{s}\left[I^{\bar{V}}(1)I^{V}(4)+I^{\bar{V}}(2)I^{V}(3)\right]\right\}\mathcal{M}_0$$



single logs

$$\delta\mathcal{M} = \sum_{k=1}^{4}\delta^\mathrm{C}(k)\mathcal{M}_0$$

Running couplings with $\alpha = \alpha(M_\mathrm{Z})$

$$\delta\mathcal{M} = -\frac{\alpha}{4\pi}\log\frac{s}{M_\mathrm{W}^2}\sum_{i=1}^{2} b_i\frac{\partial\mathcal{M}_0}{\partial\ln g_i^2}$$

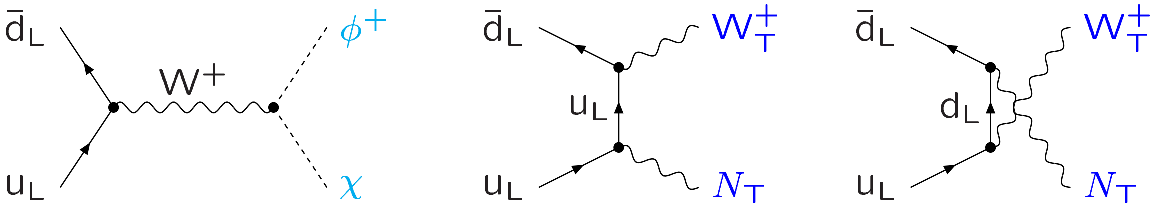
Needed ingredients:

- in general $I^V, C^\mathrm{ew}, \delta^\mathrm{coll}$ non diagonal
  $\Rightarrow$ SU(2)-transformed $\mathcal{M}_0$'s

- analytic form for $\mathcal{M}_0$'s in $M^2/s \to 0$ limit
  $\Rightarrow$ very compact expressions for $\delta\mathcal{M}/\mathcal{M}_0$!

Polarized born amplitudes for $\bar{\mathrm{d}}_\mathrm{L}\mathrm{u}_\mathrm{L} \to \mathrm{W}^+_\lambda N_{\lambda'}$

with $\mathrm{W}^+_\lambda = \mathrm{W}^+_\mathrm{L}, \mathrm{W}^+_\mathrm{T}$, $\quad N_{\lambda'} = \mathrm{Z}_\mathrm{L}, \mathrm{Z}_\mathrm{T}, \gamma_\mathrm{T}$



A) purely longitudinal polarizations $(\lambda = \lambda' = \mathrm{L})$

$$\mathcal{M}_0^{\bar{\mathrm{d}}_\mathrm{L}\mathrm{u}_\mathrm{L}\phi^-\chi} = -\frac{\mathrm{i}e^2}{2\sqrt{2}s_\mathrm{w}^2}\frac{A_s}{s}, \qquad \mathcal{M}_0^{\bar{q}_\mathrm{L}q_\mathrm{L}H\chi} = e^2 I^Z_{q_\mathrm{L}} I^Z_{\chi H}\frac{A_s}{s}$$

$$\mathcal{M}_0^{\bar{q}_\mathrm{L}q_\mathrm{L}\phi^-\phi^+} = -e^2\left(\frac{T^3_{q_\mathrm{L}}}{2s_\mathrm{w}^2} + \frac{Y_{q_\mathrm{L}}}{4c_\mathrm{w}^2}\right)\frac{A_s}{s}$$

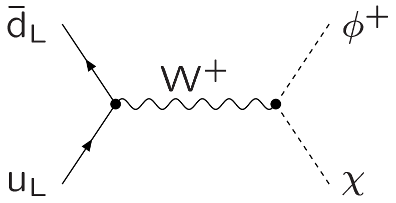
B) purely transverse polarizations $(\lambda = \lambda' = \mathrm{T})$

$$\mathcal{M}_0^{\bar{\mathrm{d}}_\mathrm{L}\mathrm{u}_\mathrm{L}\mathrm{W}^-_\mathrm{T}N_\mathrm{T}} = \frac{e^2}{\sqrt{2}s_\mathrm{w}}\left(I^N_{\mathrm{u}_\mathrm{L}}\frac{A_t}{t} + I^N_{\mathrm{d}_\mathrm{L}}\frac{A_t}{u}\right)$$

$$\mathcal{M}_0^{\bar{q}_\mathrm{L}q_\mathrm{L}N'_\mathrm{T}N_\mathrm{T}} = e^2 I^{N'}_{q_\mathrm{L}} I^N_{q_\mathrm{L}}\left(\frac{A_t}{t} + \frac{A_t}{u}\right)$$

$$\mathcal{M}_0^{\bar{q}_\mathrm{L}q_\mathrm{L}\mathrm{W}^-_\mathrm{T}\mathrm{W}^+_\mathrm{T}} = \frac{e^2}{2s_\mathrm{w}^2}\frac{A_t}{r}, \qquad q = \mathrm{d}\,(\mathrm{u}) \qquad r = t\,(u)$$

C) mixed polarizations $(\lambda \neq \lambda')$: suppressed



A) Corrections for **longitudinal** polarizations

Mass singularities: **double** logs

$$\frac{\delta\mathcal{M}}{\mathcal{M}_0} = -\frac{\alpha}{4\pi}\log^2\frac{s}{M_W^2}\left[C^{ew}_{q_L} + C^{ew}_{\Phi}\right] - \frac{\alpha}{2\pi s_w^2}\log\frac{s}{M_W^2}\left\{\left(\log\frac{|u|}{s} + \log\frac{|t|}{s}\right) - \frac{s_w^2}{c_w^2}Y_{q_L}\log\frac{t}{u}\right\}$$

1 channel $\Rightarrow$ simple angular dependence

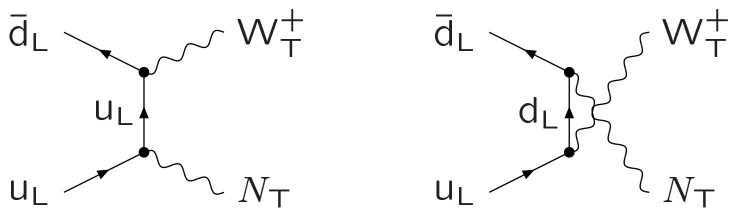
single logs

$$\frac{\delta\mathcal{M}}{\mathcal{M}_0} = \frac{\alpha}{4\pi}\log\frac{s}{M_W^2}\left[3C^{ew}_{q_L} + 4C^{ew}_{\Phi} - \frac{3}{2s_w^2}\frac{m_t^2}{M_W^2}\right]$$

Running couplings

$$\frac{\delta\mathcal{M}}{\mathcal{M}_0} = -\frac{\alpha}{4\pi}\log\frac{s}{M_W^2}\, b_2$$

1 coupling $\Rightarrow$ simple form



B) Corrections for **transverse** polarizations

Mass singularities: **double** logs

$$\frac{\delta\mathcal{M}}{\mathcal{M}_0} = -\frac{\alpha}{8\pi}\log^2\frac{s}{M_\mathrm{W}^2}\left\{2C^\mathrm{ew}_{q_\mathrm{L}} + C^\mathrm{ew}_W\left[1 + F^N\cos\theta\right]\right\}$$
$$-\frac{\alpha}{2\pi s_\mathrm{w}^2}\log\frac{s}{M_\mathrm{W}^2}\left\{\left(\log\frac{|t|}{s} + \log\frac{|u|}{s}\right) + F^N\log\frac{t}{u}\right\}$$

2 channels $\Rightarrow$ non-trivial angular dependence!

$$\cos\theta = \frac{t-u}{s},\qquad F^A = \frac{1}{\cos\theta + Y_{q_\mathrm{L}}},\qquad F^Z = \frac{c_\mathrm{w}^2}{c_\mathrm{w}^2\cos\theta - s_\mathrm{w}^2 Y_{q_\mathrm{L}}}$$

single logs

$$\frac{\delta\mathcal{M}}{\mathcal{M}_0} = \frac{\alpha}{4\pi}\log\frac{s}{M_\mathrm{W}^2}\left\{3C^\mathrm{ew}_{q_\mathrm{L}} + \frac{1}{2}\left[b_2 + \sum_{N'=A,Z} K_{N'N}\frac{\mathcal{M}_0^{\bar{\mathrm{d}}_\mathrm{L}\mathrm{u}_\mathrm{L}\to\mathrm{W}^\mp_\mathrm{T}N'_\mathrm{T}}}{\mathcal{M}_0}\right]\right\}$$

$\gamma_\mathrm{T}, \mathrm{Z}_\mathrm{T} \Rightarrow$ mixing effects!

Running couplings

$$\frac{\delta\mathcal{M}}{\mathcal{M}_0} = -\frac{\alpha}{8\pi}\log\frac{s}{M_\mathrm{W}^2}\left[b_2 + \sum_{N'=A,Z} K_{N'N}\frac{\mathcal{M}_0^{\bar{\mathrm{d}}_\mathrm{L}\mathrm{u}_\mathrm{L}\to\mathrm{W}^\mp_\mathrm{T}N'_\mathrm{T}}}{\mathcal{M}_0}\right]$$

Cancellation $\Rightarrow$ scale for real-V_T coupling is M_V !