MULTI-JET CALCULATIONS AT NLO LEVEL

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Contents:

INTRODUCTION, MOTIVATION

In the point of the QCD the multi-jet cross section can be used to

- test the prediction of perturbative QCD
- determine the $\alpha_s(Q^2)$ strong coupling and the parton distrubution function of proton ($f_a(x,Q^2)$)

In the point of the new physics search when QCD particles are presented the QCD contributions could be very important both to the signal and background process (e.g.: $pp \to H + 0, 1\mathrm{jet} \to \gamma\gamma + 0, 1\mathrm{jet}$).

Because of the large strong coupling the LO order predictions strongly depend on the unphysical renormalization and factorization scales thus the higher order corrections are important.

NLO Cross Section

$$\sigma(p_A, p_B) = \sum_{a,b} \int_0^1 d\eta_a d\eta_b f_a(\eta_b, \mu_F^2) f_b(\eta_b, \mu_F^2)$$
$$\times \left[\hat{\sigma}_{a,b}^{LO}(\eta_a p_A, \eta_b p_B) + \hat{\sigma}_{a,b}^{NLO}(\eta_a p_A, \eta_b p_B)\right]$$

where

$$\hat{\sigma}_{a,b}^{LO}(p_a, p_b) = \int_m d\hat{\sigma}_a^B(p_a, p_b) = \int d\Gamma^{(m)} |M_{a,b}|^2 F_J^{(m)}(p_a, p_b, p_1, .., p_m)$$

and the NLO correction

$$\hat{\sigma}_a^{NLO}(p_a, p_b) = \int_{m+1} \hat{\sigma}_{a,b}^R(p_a, p_b) + \int_m \hat{\sigma}_{a,b}^V(p_a, p_b) + \int_m \hat{\sigma}_{a,b}^C(p_a, p_b)$$

This integrals (R,V,C) are separately divergent but their sum is finite in $d = 4$ dimension.

NLO CROSS SECTION: SUBTRACTION TERM

We used the dipole method to regularize the divergent integrals [Catani- Seymour]

$$d\sigma_{a,b}^{NLO} = [d\sigma_{a,b}^{R} - d\sigma_{a,b}^{A}] + d\sigma_{a,b}^{A} + d\sigma_{a,b}^{V} + d\sigma_{a,b}^{C} \quad ,$$

where $d\sigma_{a,b}^{A}$ is a proper approximation of $d\sigma_{a,b}^{R}$ and

- it has to exactly match the singular behaviour of $d\sigma_{a,b}^{R}$
- it has to be exactly integrable in $d = 4 - 2\epsilon$ dimension over the single parton subspaces leading to soft and collinear divergences

$$\sigma_{a,b}^{NLO} = \int_{m+1} [d\sigma_{a,b}^{R}|_{\epsilon=0} - d\sigma_{a,b}^{A}|_{\epsilon=0}] + \int_{m} [d\sigma_{a,b}^{V} + d\sigma_{a,b}^{C} + \int_{1} d\sigma_{a,b}^{A}]|_{\epsilon=0}$$

The cancelation of the divergences is guaranteed only for that jet observables which fulfill the condition

$$F_J^{(m+1)} \longrightarrow F_J^{(m)}$$

in the kinematically denegerate phase space regions (soft and collinear).

NLO CROSS SECTION: DIPOLE TERMS

$$d\sigma^{A}_{a,b} = \sum_{\{m+1\}} d\Gamma^{(m+1)}(p_a, p_b, p_1, ..., p_{m+1}) \frac{1}{S_{\{m+1\}}}$$

$$\cdot \Bigg\{ \sum_{\substack{\text{pairs} \\ i,j}} \sum_{k \neq i,j} \mathcal{D}_{ij,k}(p_a, p_b, p_1, ..., p_{m+1}) F_J^{(m)}(p_a, p_b, p_1, .., \tilde{p}_{ij}, \tilde{p}_k, .., p_{m+1})$$

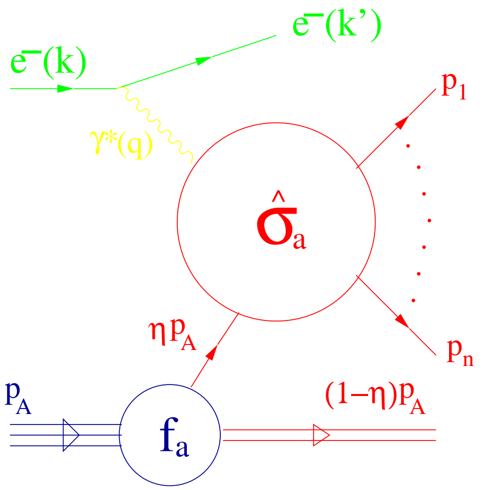
$$+ \sum_{\substack{\text{pairs} \\ i,j}} \Big[\mathcal{D}^{a}_{ij}(p_a, p_b, p_1, ..., p_{m+1}) F_J^{(m)}(\tilde{p}_a, p_b, p_1, .., \tilde{p}_{ij}, .., p_{m+1}) + (a \leftrightarrow b) \Big]$$

$$+ \sum_{i \neq k} \Big[\mathcal{D}^{ai}_{k}(p_a, p_b, p_1, ..., p_{m+1}) F_J^{(m)}(\tilde{p}_a, p_b, p_1, .., \tilde{p}_k, .., p_{m+1}) + (a \leftrightarrow b) \Big]$$

$$+ \sum_{i} \Big[\mathcal{D}^{ai,b}(p_a, p_b, p_1, ..., p_{m+1}) F_J^{(m)}(\tilde{p}_a, p_b, \tilde{p}_1, ..., \tilde{p}_{m+1}) + (a \leftrightarrow b) \Big] \Bigg\}$$

In $h\bar{h} \to 3\text{jet}$ process case there are 26 subprocesses and the number of the corresponding dipole terms is $26 \times (6 + 4 + 4) = 364$.

THREE-JET LEPTOPRODUCTION AT NLO



$$q = k - k'$$
$$Q^2 = -q^2 = s x_B y$$
$$x_B = \frac{Q^2}{2 p_A q}$$
$$y = \frac{p_A q}{p_A k}$$

Kinematic region of H1:

$5\,\mathrm{GeV}^2 < Q^2 < 5000\,\mathrm{GeV}^2$

$0 < x_{\mathrm{Bj}} < 1$

$0.2 < y < 0.6$

Hard scale:

$$Q_{\mathrm{H.S.}} = \frac{1}{3} \sum_j E_{T,\mathrm{jet}}^{\mathrm{Breit}}(j)$$

PDF and α_s:

LO: CTEQ5L and one-loop α_s with $\alpha_s(M_{Z^0}) = 0.127$

NLO: CTEQ5M1 and two-loop α_s with $\alpha_s(M_{Z^0}) = 0.118$

THREE-JET LEPTOPRODUCTION AT NLO: MATRIX ELEMENTS

Five parton tree level matrix elements: $0 \to q\bar{q}ggg$ and $0 \to q\bar{q}Q\bar{Q}g$

Berends, Giele & Kuijf

Hagiwara, Zeppenfeld

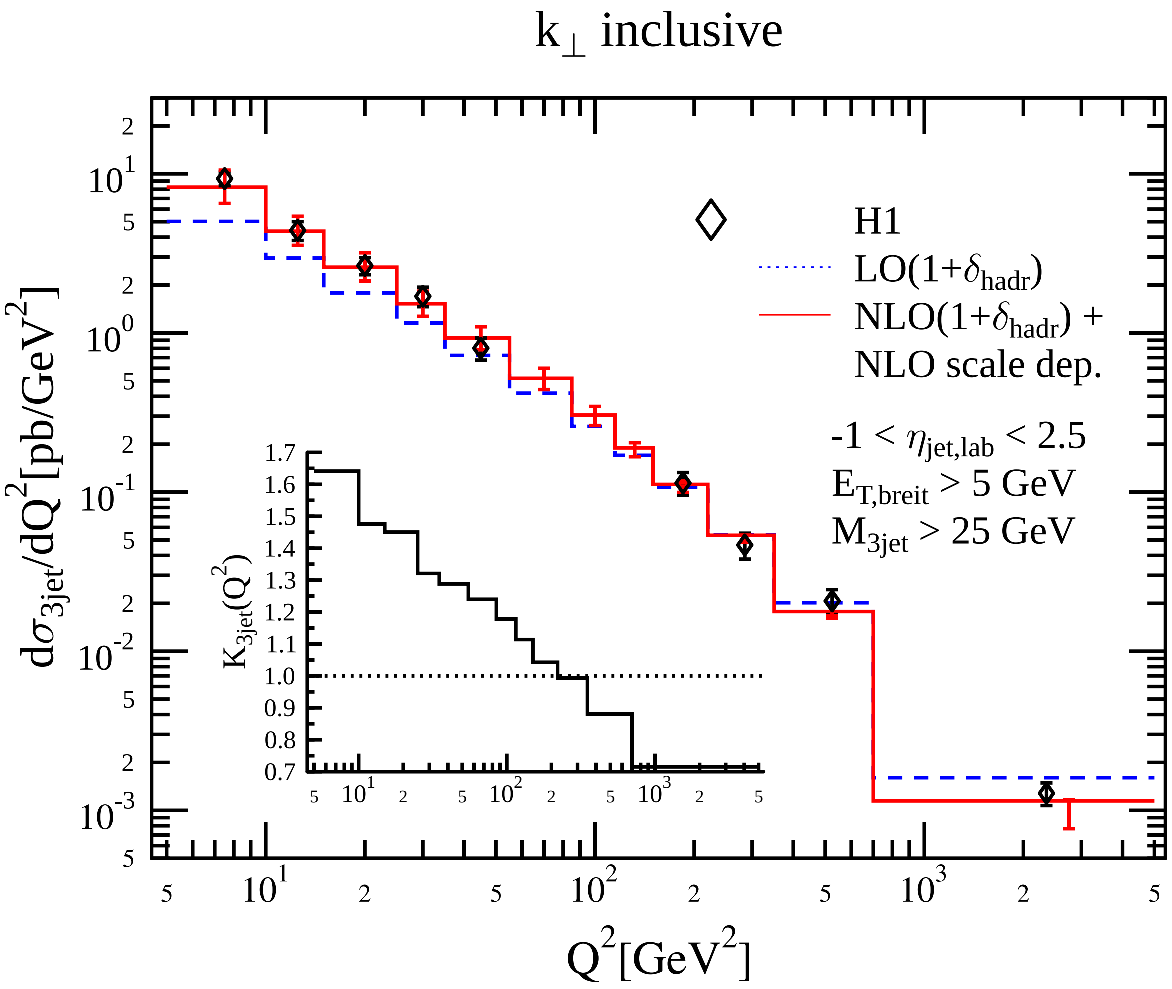
Z. Nagy

Four parton tree and 1-loop level matrix elements: $0 \to q\bar{q}gg$ and $0 \to q\bar{q}Q\bar{Q}$

Glover, Campbell, Miller

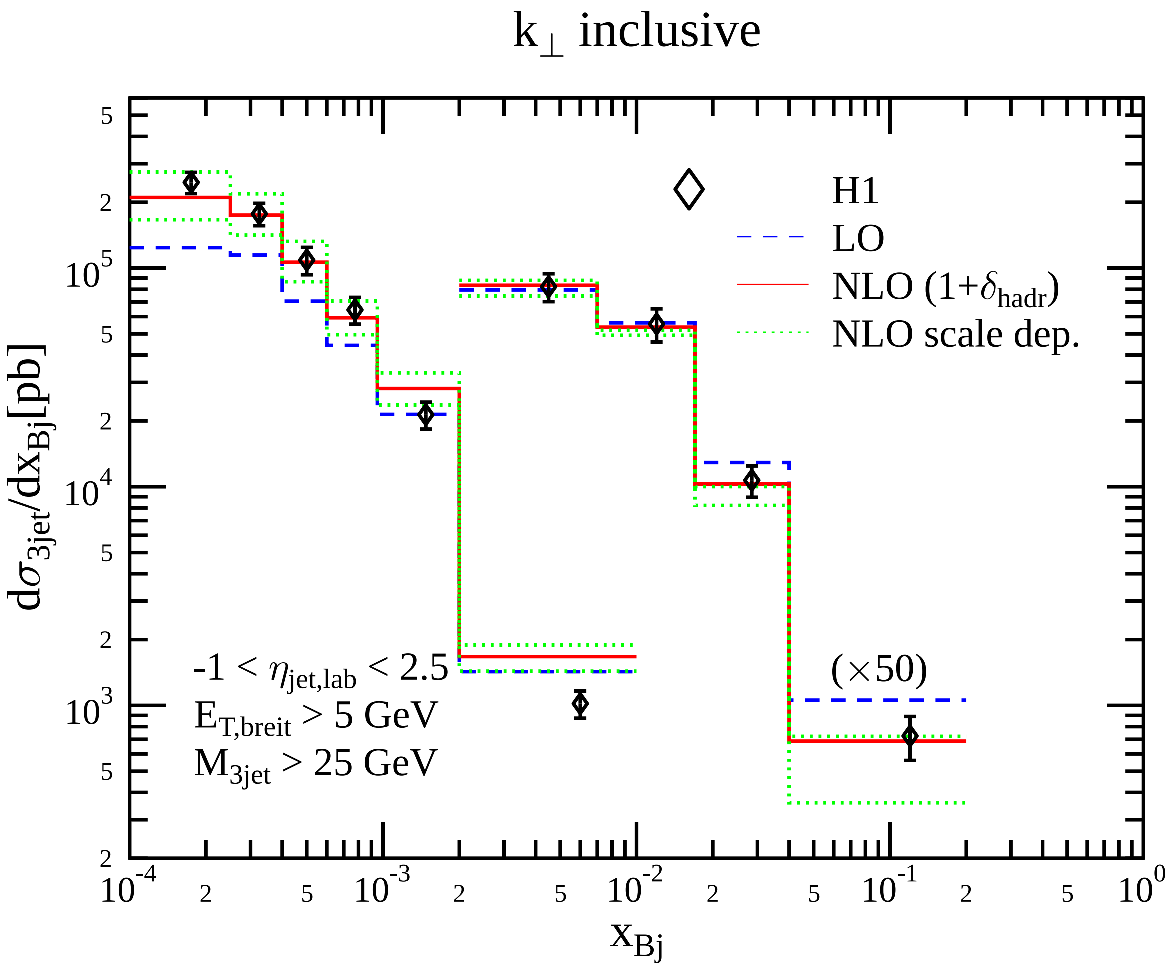
Bern, Dixon, Kosower

THREE-JET LEPTOPRODUCTION AT NLO



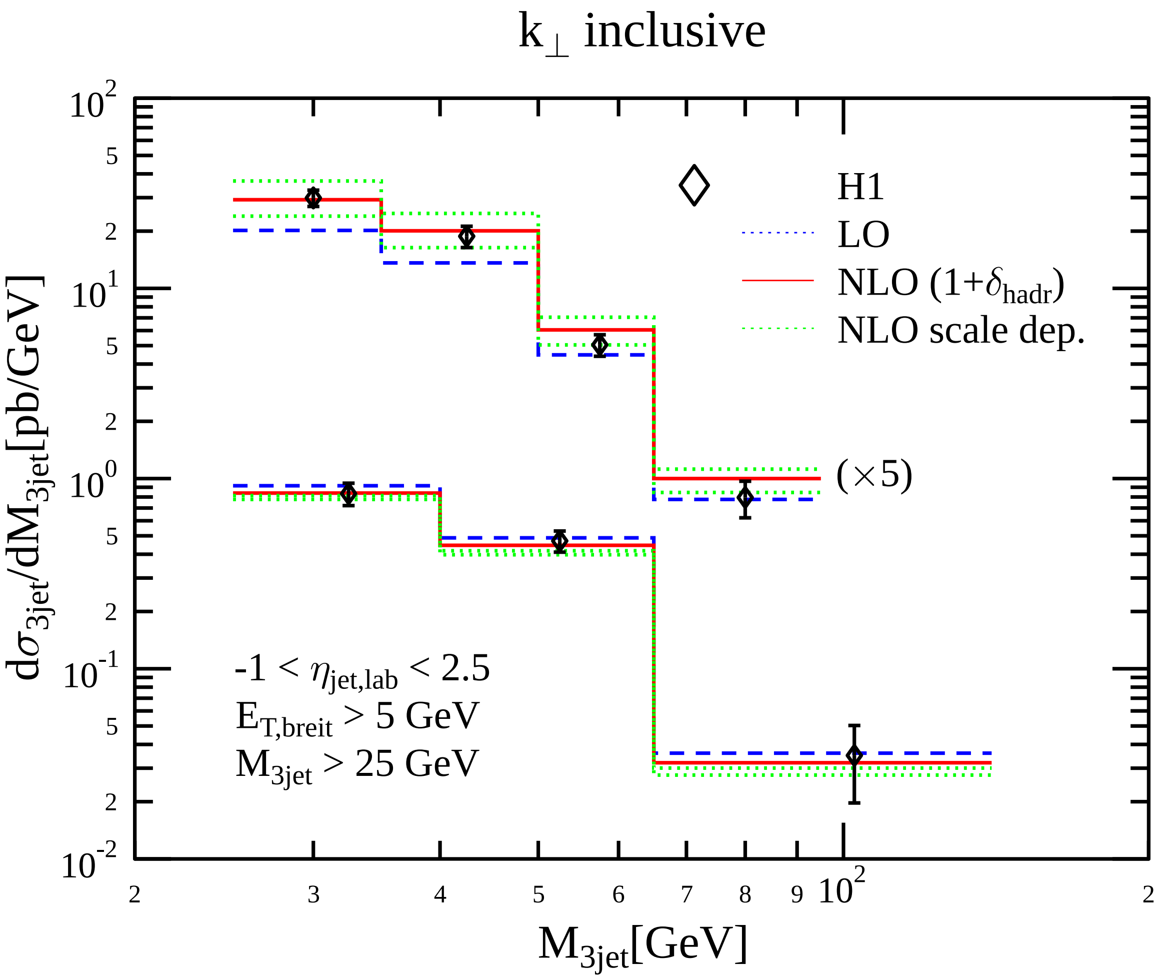
Differential (3+1)-jet cross section as a function of the momentum transfer

THREE-JET LEPTOPRODUCTION AT NLO



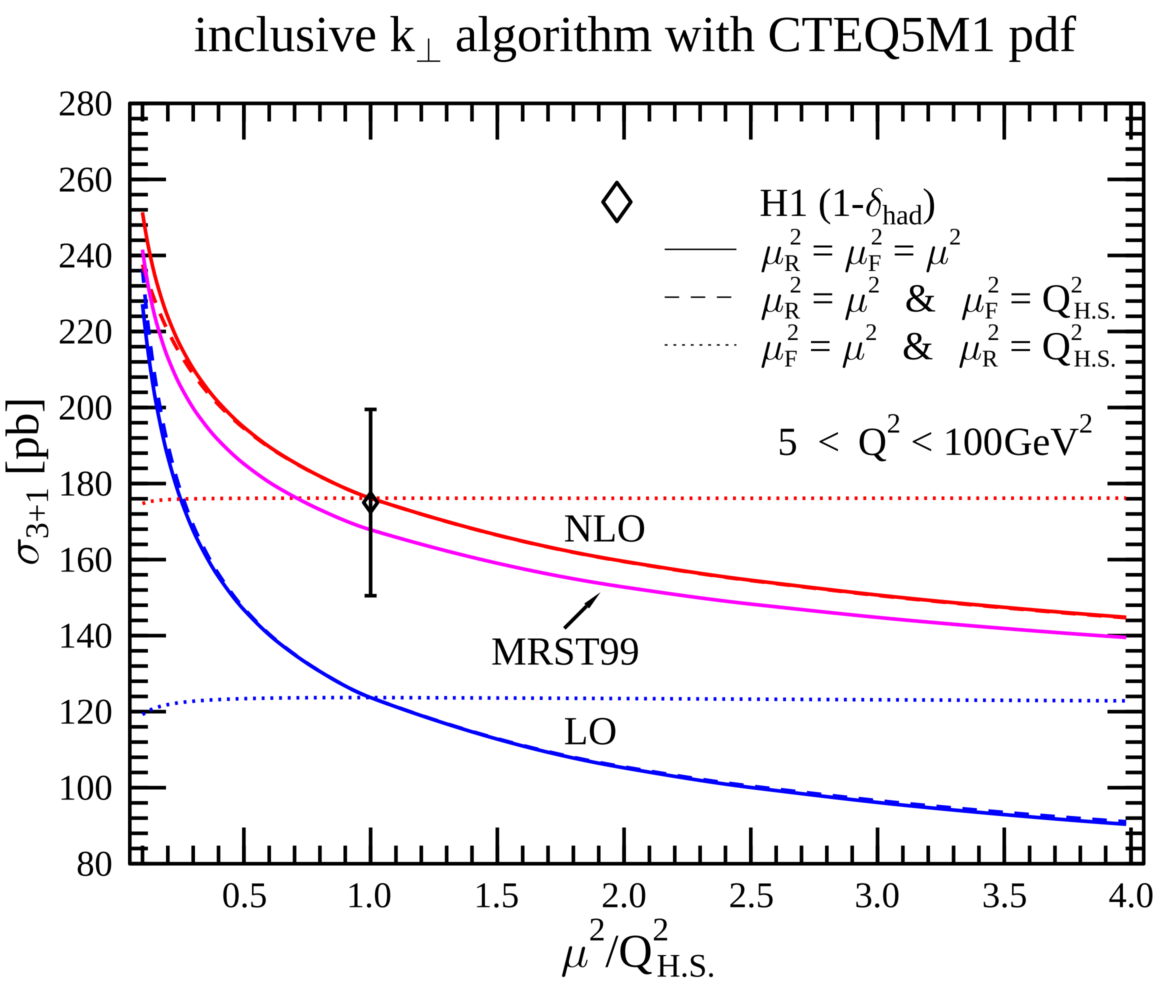
Differential (3+1)-jet cross section as a function of the Bjorken variable

THREE-JET LEPTOPRODUCTION AT NLO



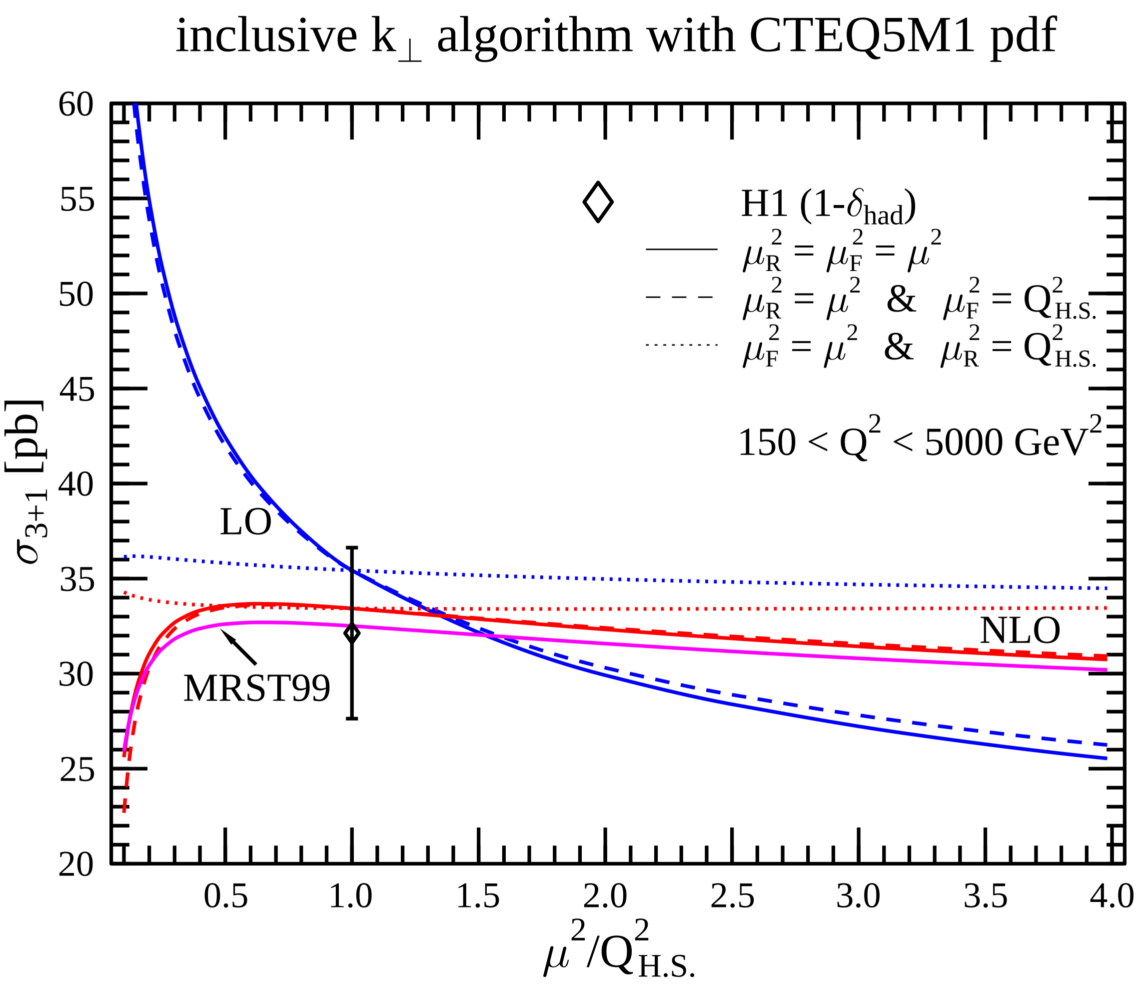
Differential (3+1)-jet cross section as a function of the 3-jet invariant mass

THREE-JET LEPTOPRODUCTION AT NLO



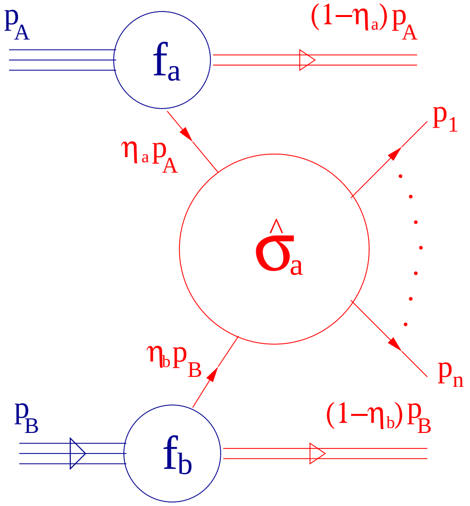
Scale dependence of the inclusive (3+1)-jet cross section at low Q^2

THREE-JET LEPTOPRODUCTION AT NLO



Scale dependence of the inclusive (3+1)-jet cross section at high Q^2

THREE-JET IN HADRON-HADRON COLLISIONS



Kinematic region:

$$|\eta_{\rm jet}| < 4.0 \qquad E_{T,\rm jet} > 50\,{\rm GeV}$$

Hard scale:

$$Q_{\rm H.S.} = E_{T,\rm jet}^{(1)}$$

PDF and α_s:

LO: CTEQ5L and one-loop α_s with $\alpha_s(M_{Z^0}) = 0.127$

NLO: CTEQ5M1 and two-loop α_s with $\alpha_s(M_{Z^0}) = 0.118$

THREE-JET IN HADRON-HADRON COLLISIONS: MATRIX ELEMENTS

Six parton tree level matrix elements: $0 \to gggggg$, $0 \to q\bar{q}gggg$, $0 \to q\bar{q}Q\bar{Q}gg$ and $0 \to q\bar{q}Q\bar{Q}r\bar{r}$

Gunion, Kunszt

Berends, Giele & Kuijf

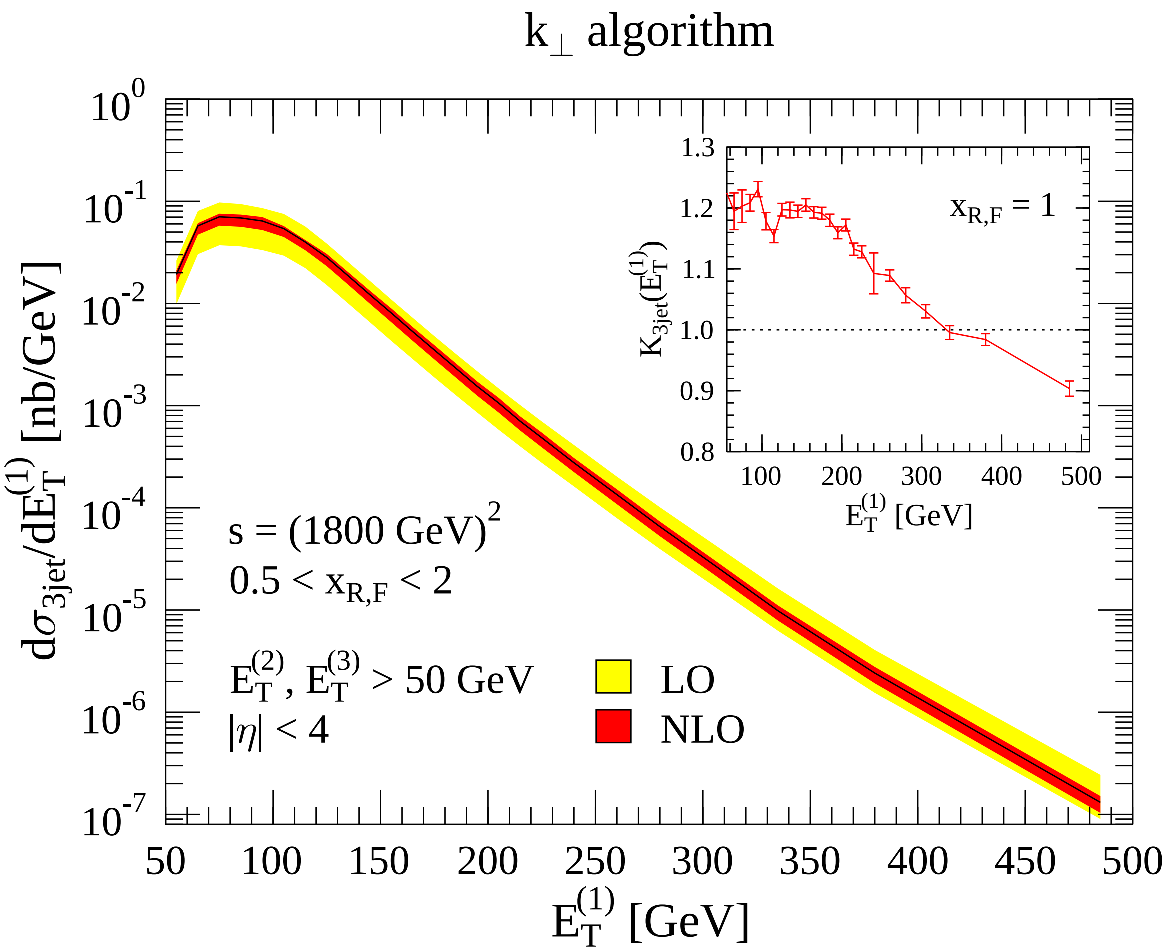
Five parton tree and 1-loop level matrix elements: $0 \to ggggg$, $0 \to q\bar{q}ggg$

Bern, Dixon, Kosower

and $0 \to q\bar{q}Q\bar{Q}g$

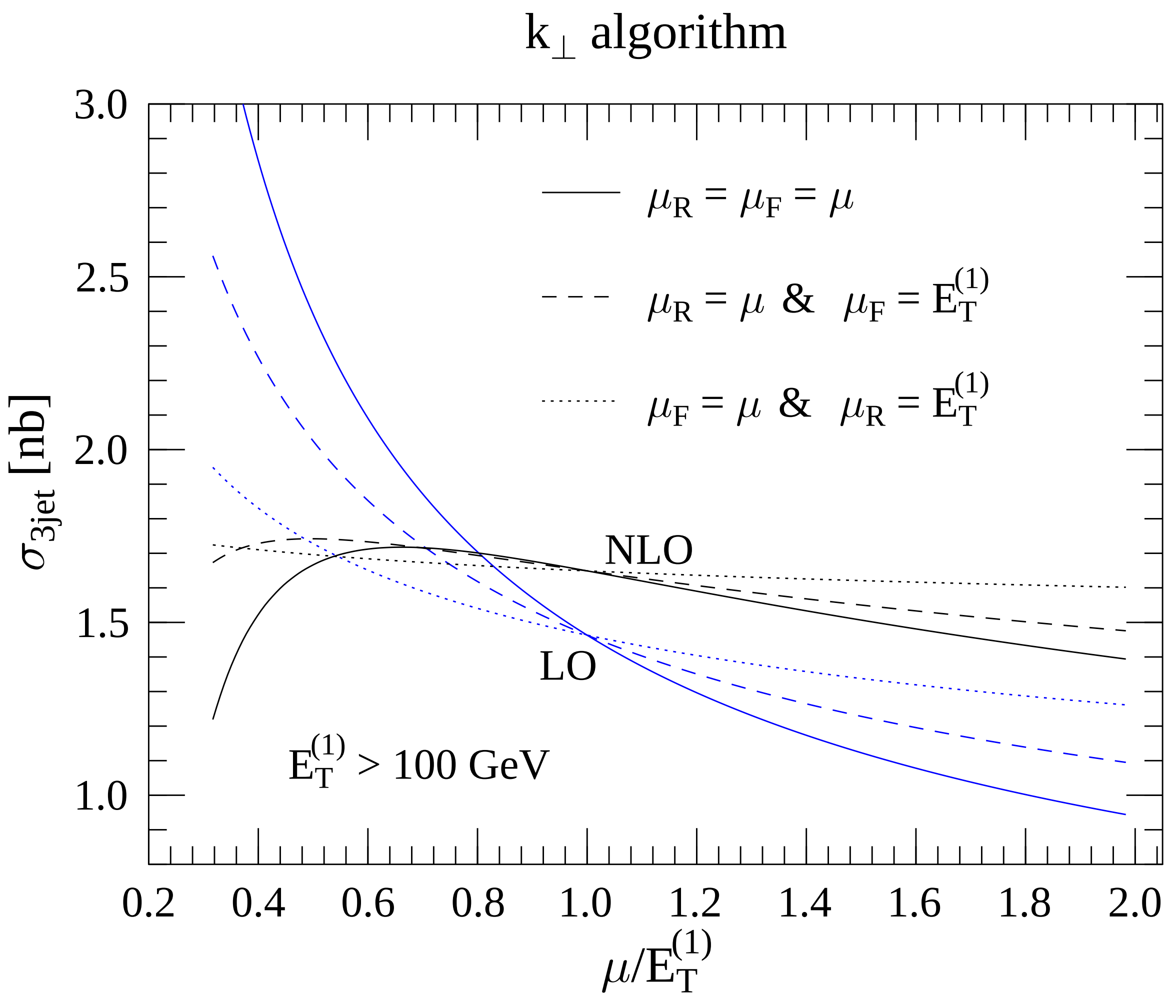
Kunszt, Signer, Trócsányi

THREE-JET IN HADRON HADRON COLLISIONS AT NLO



Differential 3-jet cross section as a function of tranverse energy of the leading jet

THREE-JET IN HADRON HADRON COLLISIONS AT NLO



Scale dependence of the 3-jet cross section

NLOJET++

A program for calculating next-to-leading order jet cross sections.

1. It knows several process:
   (a) 3,4-jet in e^+e^- at NLO and 5-jet at LO
   (b) 2,3jet production in DIS at NLO and 4-jet at LO
   (c) 1,2,3-jet in hadron-hadron collision at NLO and 4-jet at LO
   (d) 1,2,3-jet in photoproduction at NLO and 4-jet at LO
   (e) 2γ + 1-jet in hadron-hadron collision at NLO
2. It implements the dipole method generally process independet way.
   It's *easy* to apply for another process when the QCD corrections are important.
   (not only for pure QCD process).
3. It's written in C++ but there are Fortran interfaces to user defined parts.
4. http://www.cpt.dur.ac.uk/˜nagyz/nlo++/