Durham, April 2002

Six-fermion production in e^+e^- annihilation

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1 Motivation

General consideration:

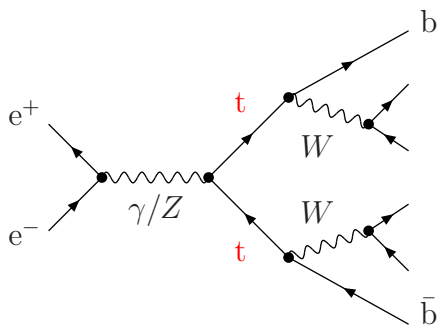
Many interesting resonance processes lead to $6f$ final states

$\longrightarrow$ full $\mathrm{ee} \longrightarrow 6f$ matrix-element calculation required

- for a consistent inclusion of off-shell effects of resonances
- to check equivalent vector-boson approximation in $\mathrm{VV} \longrightarrow \mathrm{VV}$

Interesting subprocesses:

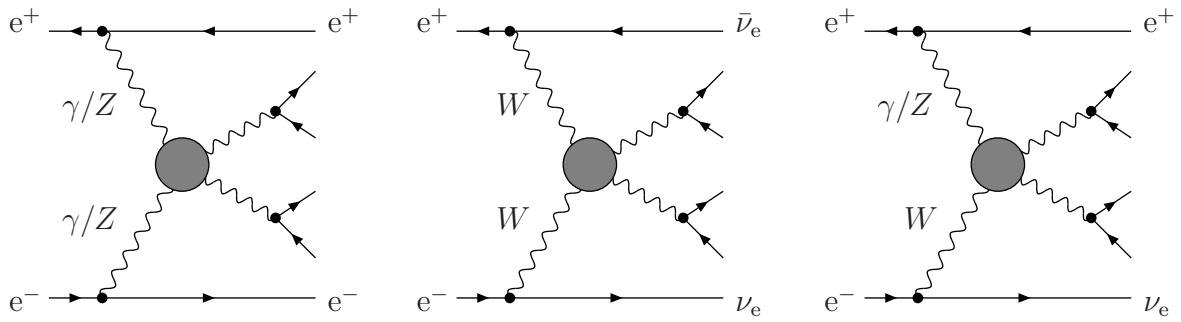
$\mathrm{t\bar{t}}$ production:



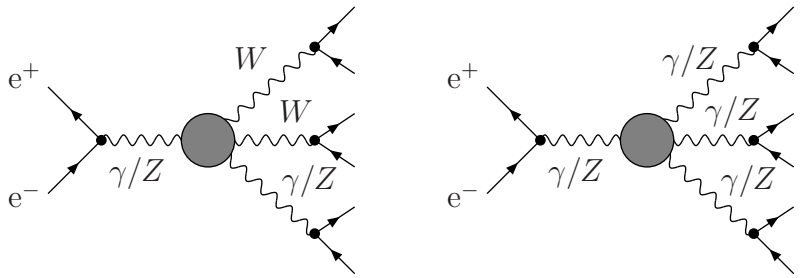
- mass and widths of the top quark
- top-quark couplings to gauge bosons
- top spin physics
- QCD effects in the $\mathrm{t\bar{t}}$ system (mainly near $\mathrm{t\bar{t}}$ threshold)

Interesting subprocesses: (continued)

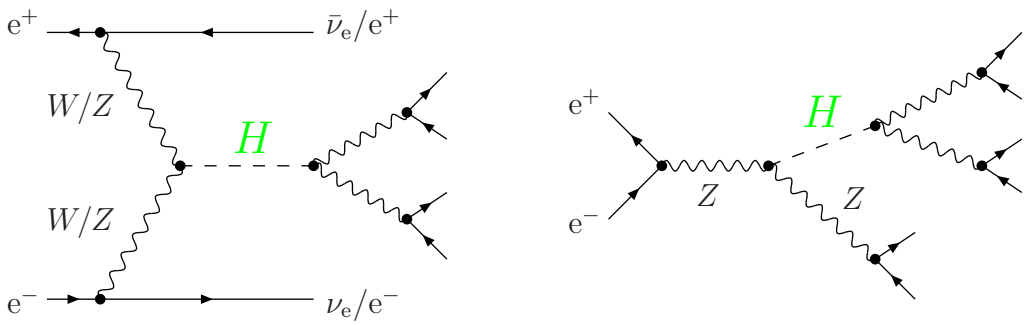
Vector-boson scattering:



Triple gauge-boson production:



- quartic gauge-boson couplings WWWW, ZZWW, etc.
- longitudinal W's and Z's at high energies
  → window to mechanism of spontaneous symmetry breaking (equivalence theorem!)
- Higgs production channels for intermediate Higgs masses:



Theoretical status in $e^+e^- \to 6f$

Numerical case studies presented by

- Accomando et al. '97,'98
  $e^+e^- \to t\bar{t}, ZH, WWZ \to 6f$
- Yuasa et al. '97, Kolodziej '01
  $e^+e^- \to t\bar{t} \to 6f$
- Montagna et al. '97–'99
  $e^+e^- \to t\bar{t}, ZH, \nu_e\bar{\nu}_e H \to 6f$
- Gangemi '00, Chierici et al. '01
  $e^+e^- \to \nu_e\bar{\nu}_e(WW \to WW) \to 6f$

Generic multi-particle event generators:

- GRACE (Ishikawa et al. '92)
- PHEGAS & HELAC (Papadopoulos, Kanaki '00)
- WHIZARD & MADGRAPH/O'MEGA
  (Kilian '01; Stelzer, Long '94 / Moretti et al. '01)

Why more work is needed

- Case studies not exhaustive, since
  - many final states not considered yet
  - no detailed *tuned comparison* available
- Generic multi-purpose generators not sufficient, since
  - good performance not guaranteed without detailed tests
    $\to$ convergence, efficiency, speed, limitations ?!
  - often not suited for refinements
    $\to$ radiative corrections to subprocesses, etc.

2 Calculational framework — the new Monte Carlo generator LUSIFER

What is LUSIFER ? – a LUcid approach to SIx-FERmion production

S.D., Roth '02

Features similar to (tree-level part of) RACOONWW:

- Monte Carlo event generator for $e^+e^- \to 6f$ (also $4f, 4f+\gamma$)
- ALL $6f$ final states supported, full treel-level amplitudes
- massless fermions, except for (internal) top quarks
- multi-channel integration technique
- initial-state radiation (ISR) via leading-log structure functions

Emphasis put on stability, speed, and flexibility

Main complications in $e^+e^- \to 6f$

- fast evaluation of diagrams

  # diagrams (U-gauge):

264	$\mu^-\bar{\nu}_\mu u\bar{d}b\bar{b}$
1016	$\nu_e\bar{\nu}_e u\bar{u}u\bar{u}$
⋮	
13896	$e^-e^+e^-e^+e^-e^+$

- extremely rich peaking structure in integrand

  → different channels (=14-dim. phase-space parametrizations) for each diagram

- gauge invariance in presence of unstable particles

$e^+e^- \to 6f$ with LUSIFER

"Generic" calculation of amplitudes

- Analytic part: helicity amplitudes for "generic diagrams"

  Evaluation with Weyl–van der Waerden spinors
  and use of discrete symmetries

  $\to$ one/two compact generic functions per generic diagram
  for all final states and polarizations !

- Combinatorics: field insertions for generic diagrams

  $\to$ permutations of external fermions and
  insertions of internal fermion and boson fields

- Square amplitude: squared diagrams and interferences

  $\to$ Fermi signs and colour algebra
  (gluon diagrams implemented for up to four quarks)

Phase-space integration

- "multi-channel" Monte Carlo integration Berends,Daverveldt,Kleiss '85 Hilgart,Kleiss,Le Diberder '93

  one channel per diagram, mappings for all propagators
  $\to$ $\mathcal{O}(10^2$–$10^4)$ different channels

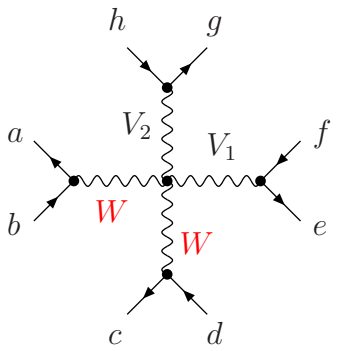
- "adaptive weight optimization" Kleiss,Pittau '94

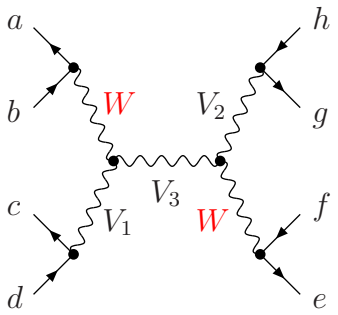
Introduction of decay widths

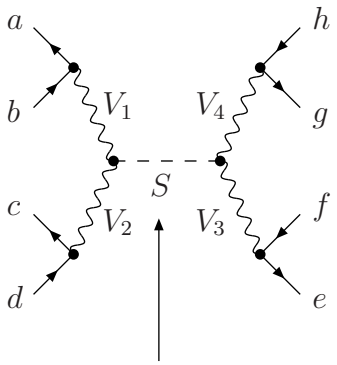
optional: constant/running width, complex-mass scheme

$\to$ important for controlling gauge-invariance violations

Generic diagrams with 4 fermion currents:



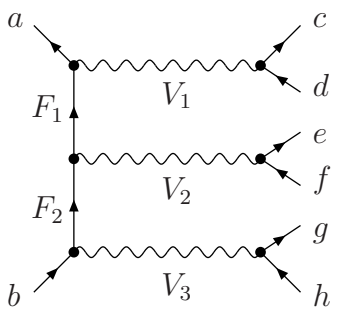


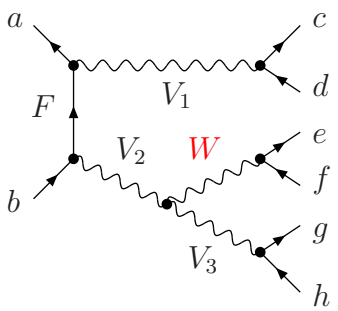


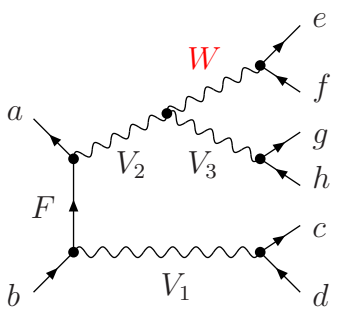
$V_{...} = \gamma, Z, W$

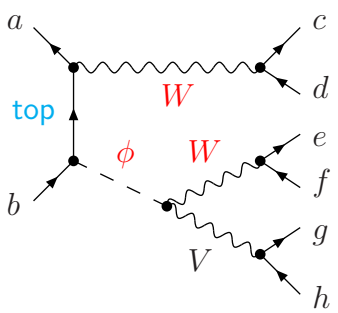
S = Higgs, Goldstone ϕ

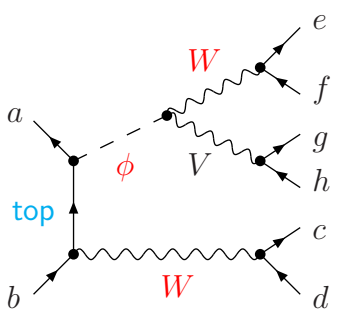
Generic diagrams with 3 fermion currents:





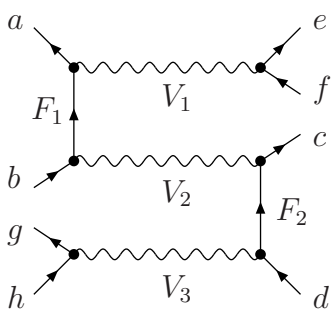


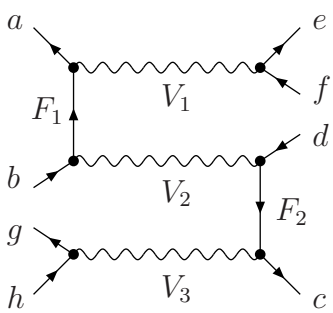


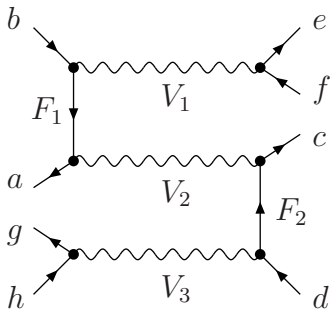


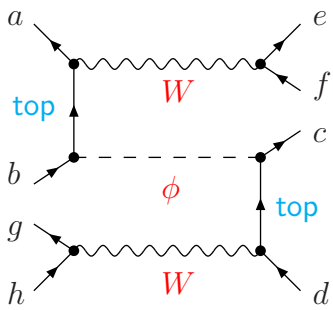
$F_{...}$ = massless fermion, top

Generic diagrams with 2 fermion currents:









3 Numerical results (Still preliminary !)

3.1 Results for $\mathrm{t\bar{t}}$ production

Integrated cross sections:

$\sqrt{s} = 500\,\mathrm{GeV}$, no gluon diagrams, phase-space cuts applied

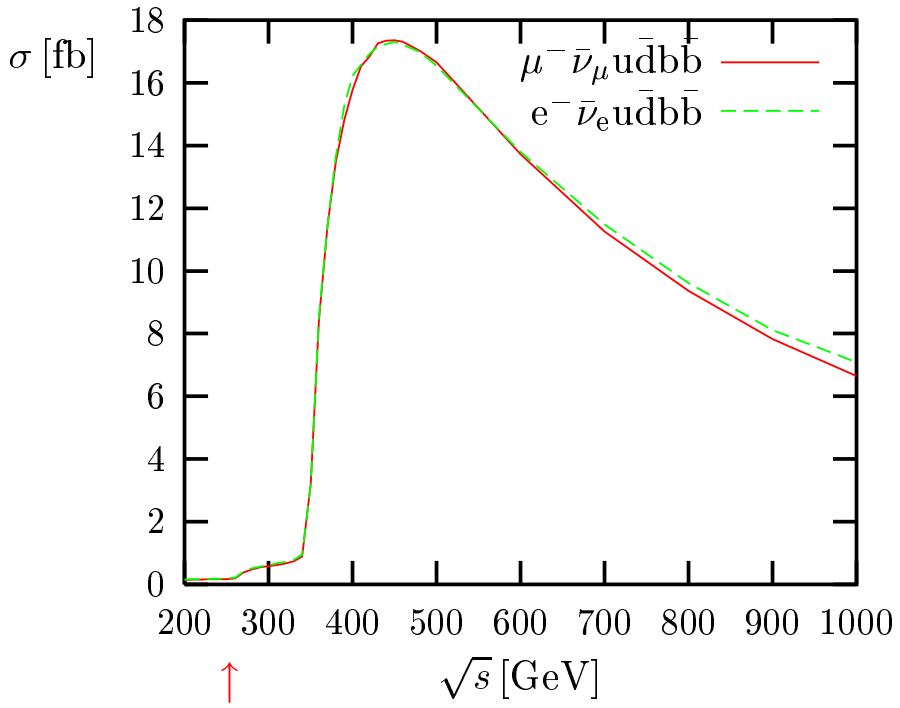
$\sigma[\mathrm{fb}]$	Lusifer		Whizard & Madg.	
$\mathrm{e^+e^-} \to$	Born	Born + ISR	Born	Born + ISR
$\mu^-\bar{\nu}_\mu\nu_\tau\tau^+\mathrm{b\bar{b}}$	5.800(4)	5.584(4)	5.796(3)	5.589(3)
$\mathrm{e^-}\bar{\nu}_\mathrm{e}\nu_\mu\mu^+\mathrm{b\bar{b}}$	5.819(5)	5.604(4)	5.827(3)	5.607(3)
$\mathrm{e^-e^+}\nu_\mathrm{e}\bar{\nu}_\mathrm{e}\mathrm{b\bar{b}}$	5.853(7)	5.647(7)	5.875(3)	5.651(4)
$\mu^-\bar{\nu}_\mu\mathrm{u\bar{d}b\bar{b}}$	17.095(11)	16.454(10)	17.103(8)	16.463(9)
$\mathrm{e^-}\bar{\nu}_\mathrm{e}\mathrm{u\bar{d}b\bar{b}}$	17.187(21)	16.511(12)	17.148(8)	16.529(9)

Good agreement !

- ISR reduces cross section by $\sim 4\%$
- Electron background diagrams $\lesssim 1\%$ for $\theta(\mathrm{e}^\pm, \mathrm{beam}) > 5^\circ$

BUT: Effects can be more pronounced in distributions or affected by varying cuts

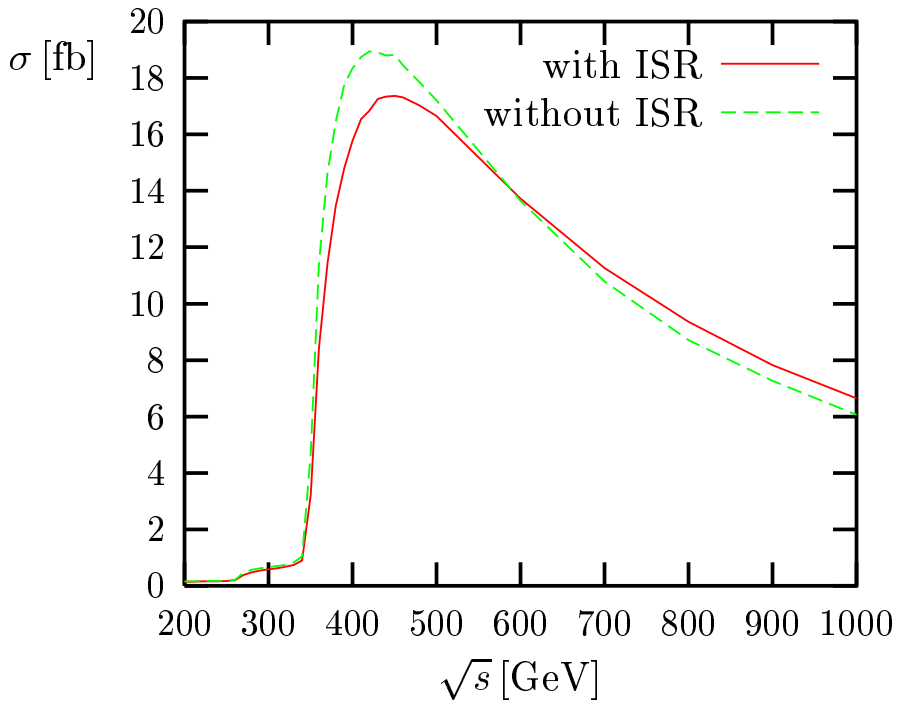
Integrated cross sections:



phase-space cuts
no gluon diagrams
with ISR

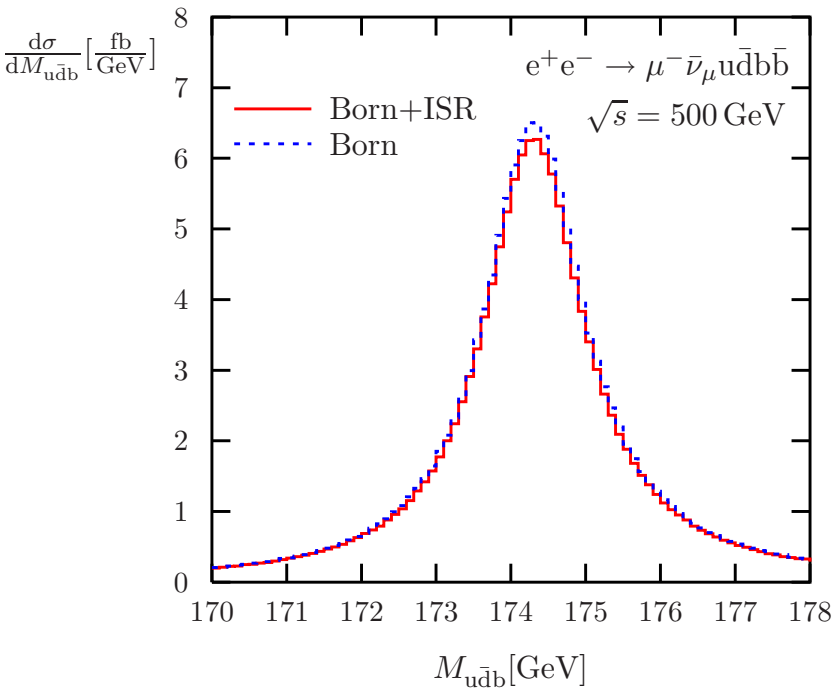
↑ WWZ threshold

Impact of ISR for $e^+e^- \to \mu^-\bar{\nu}_\mu u\bar{d}b\bar{b}$:



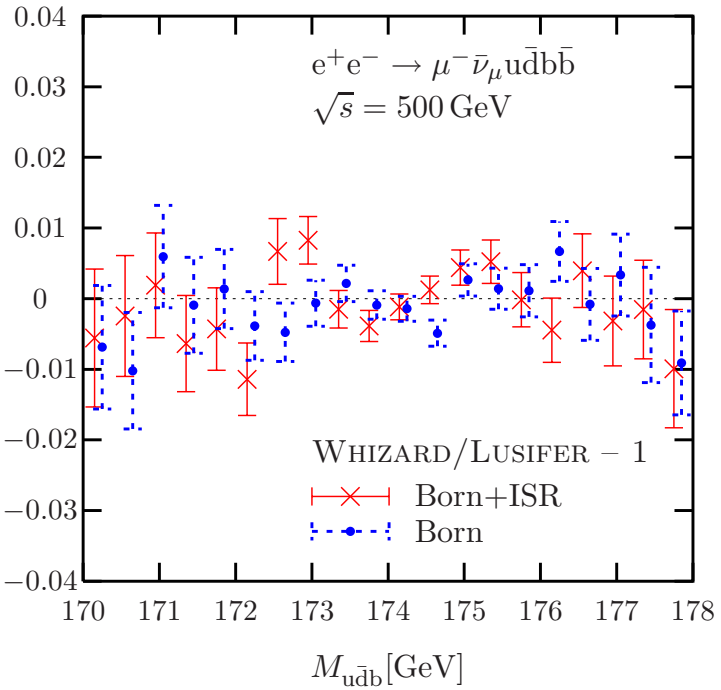
phase-space cuts
no gluon diagrams

Invariant-mass distribution of the top resonance:



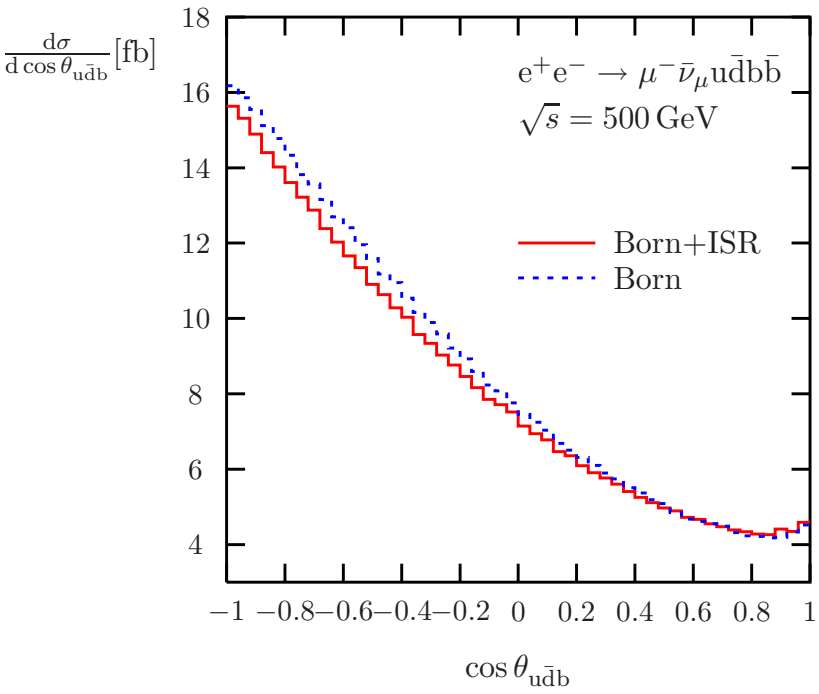
Rescaling of spectrum by ISR

Comparison with package WHIZARD & MADGRAPH:



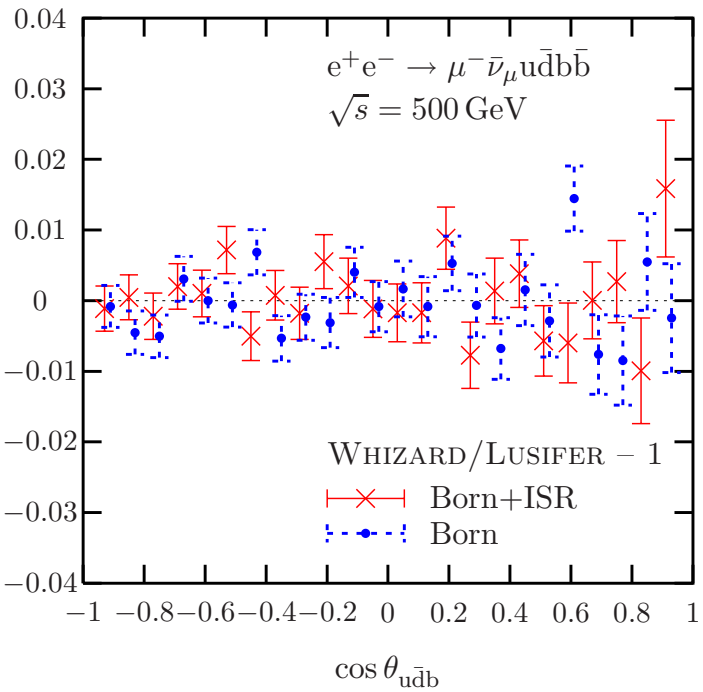
Good agreement !

Production-angle distribution of the top resonance:



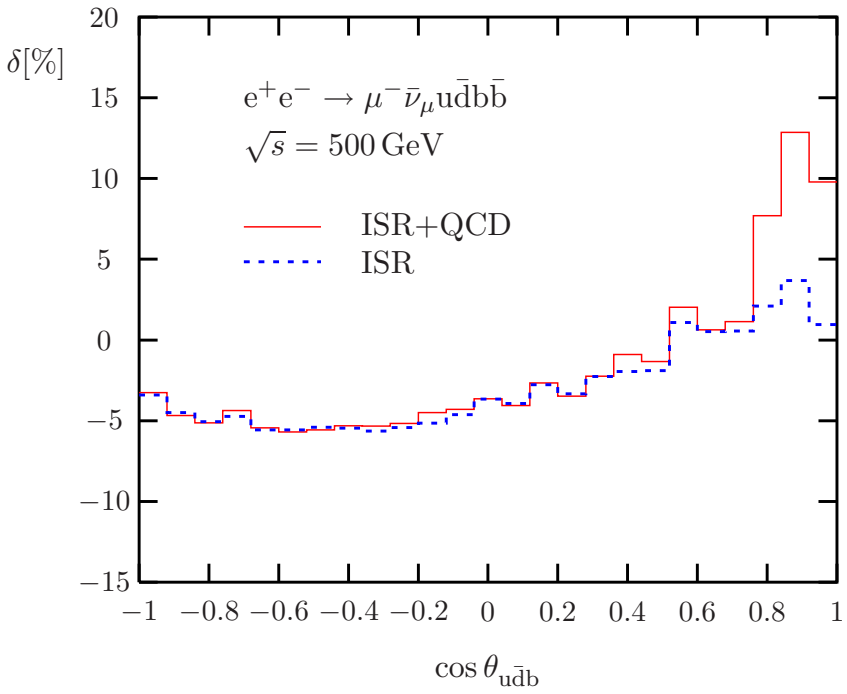
Distortion of spectrum by ISR

Comparison with package WHIZARD & MADGRAPH:

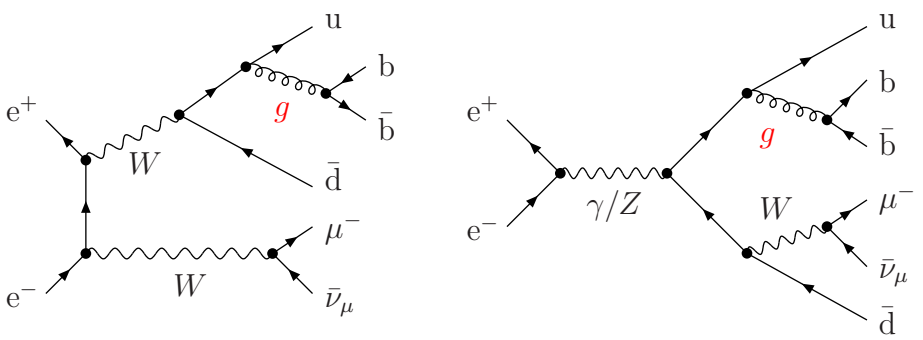


Good agreement !

ISR and QCD background to production-angle distribution:



Typical QCD background diagrams:

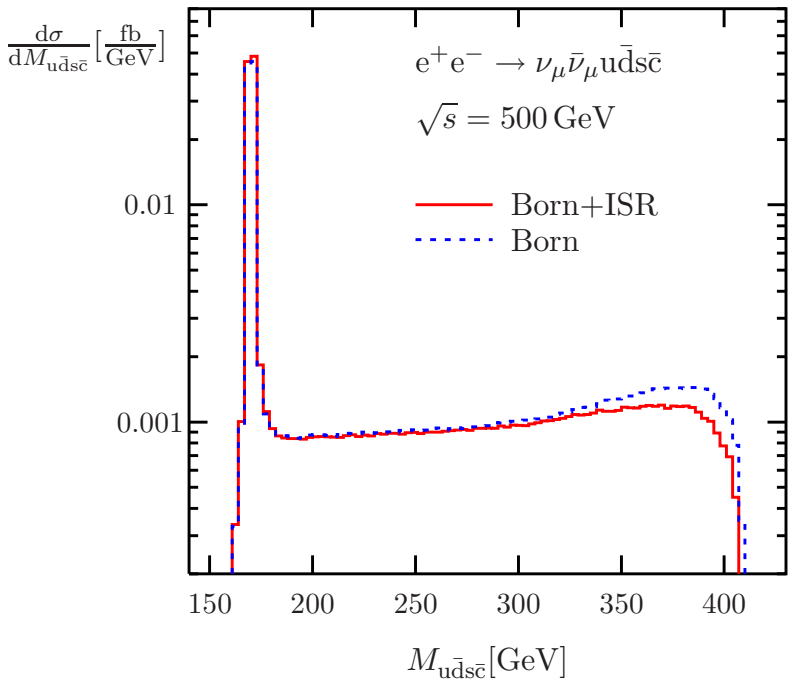


Invariant-mass cut: $M_{2q} > 10\,\mathrm{GeV}$ for all quark pairs

3.2 Results for Higgs production

Results for a $e^+e^- \to ZH \to ZWW$ channel:

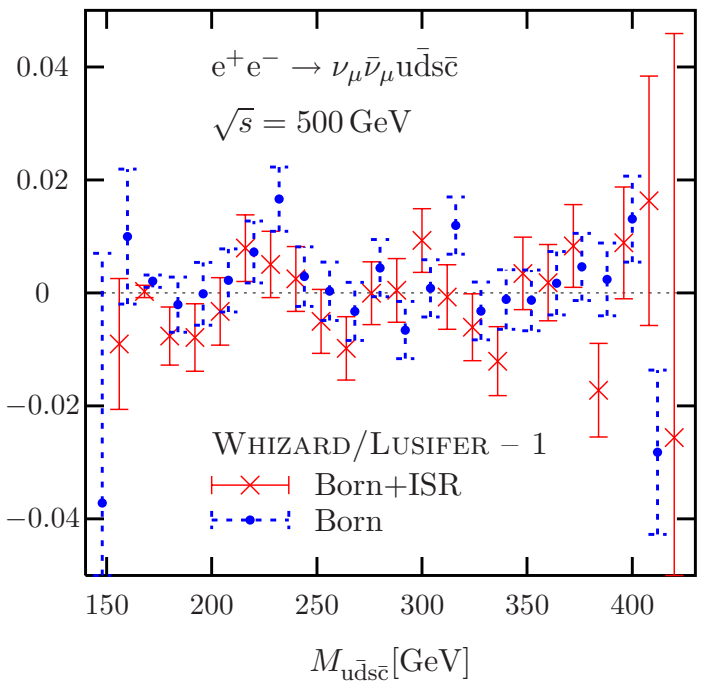
Invariant-mass distribution of the Higgs resonance:



$M_H = 170\,GeV$

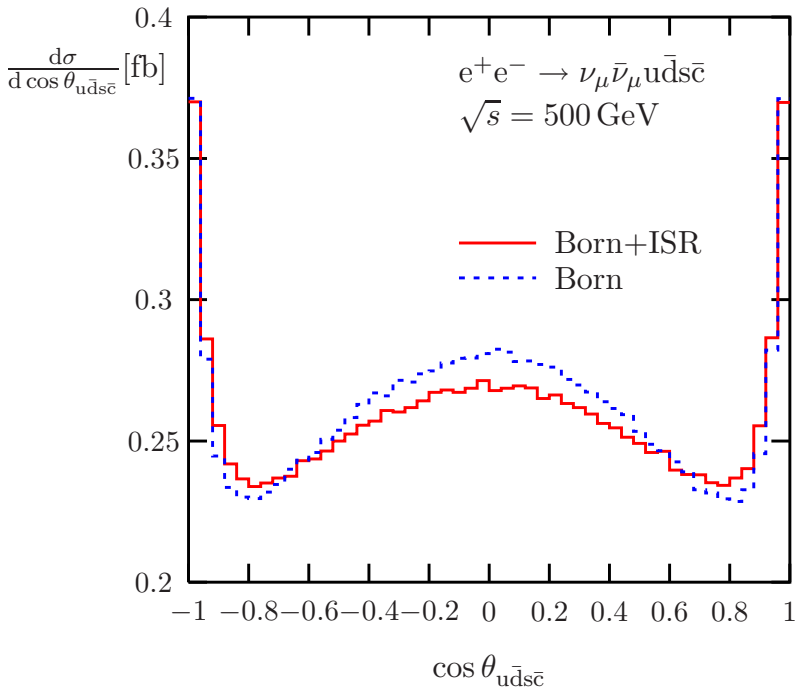
phase-space cuts
no gluon diagrams

Comparison with package WHIZARD & MADGRAPH:



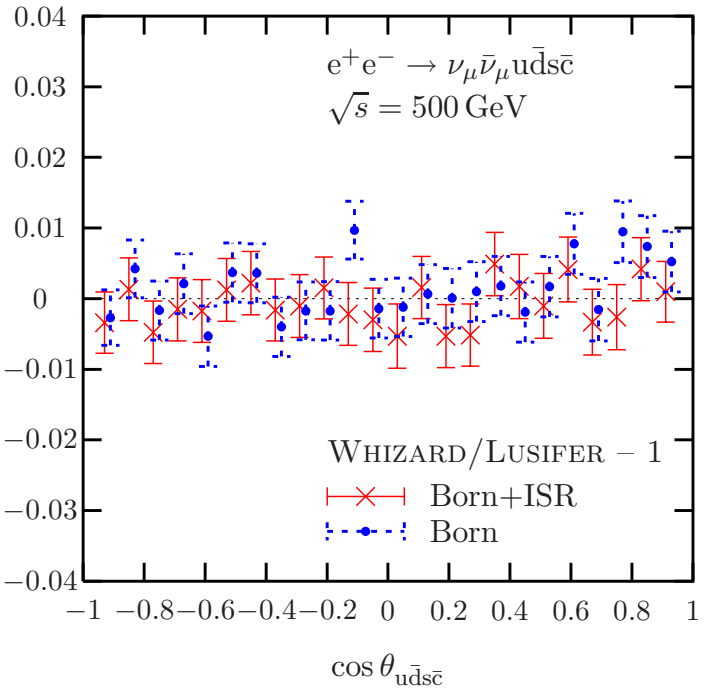
Good agreement !

Production-angle distribution of the Higgs resonance:



Distortion of spectrum by ISR

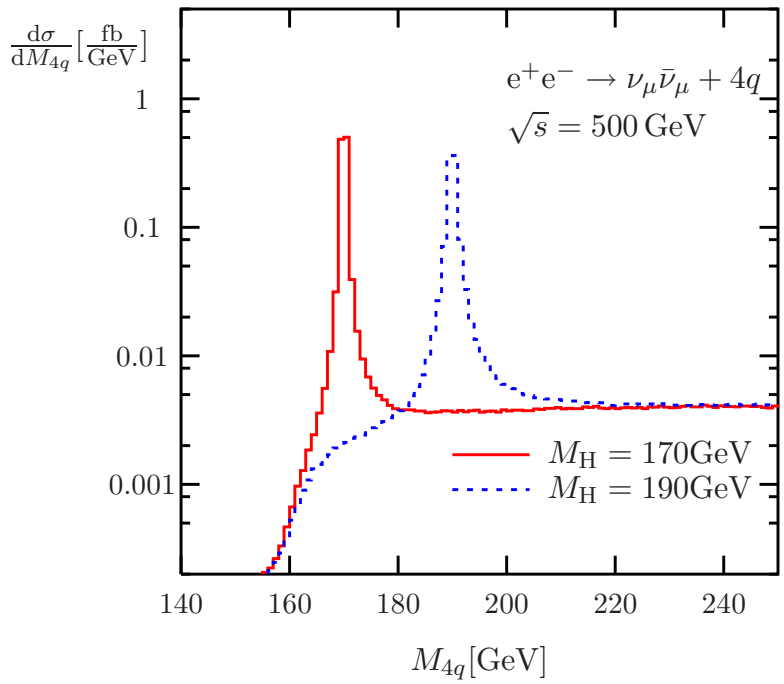
Comparison with package WHIZARD & MADGRAPH:



Good agreement !

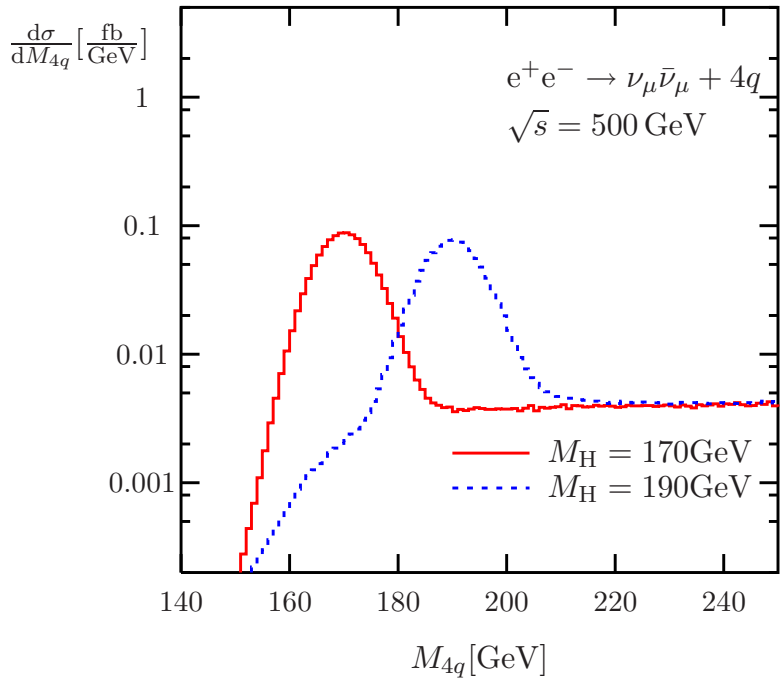
Results for $e^+e^- \to \nu_\mu \bar{\nu}_\mu + 4q$:

Invariant-mass distribution for various Higgs masses:



phase-space cuts
no gluon diagrams
no ISR

Gaussian smearing of $4q$ invariant mass ($\delta M_{4q} = 5\,GeV$):



phase-space cuts
no gluon diagrams
no ISR

3.3 Finite decay widths and gauge-invariance violation

Example of a $\mathrm{WW} \to \mathrm{WW}$ channel:

Cross section for $\mathrm{e}^+\mathrm{e}^- \to \nu_\mathrm{e}\bar{\nu}_\mathrm{e}\mu^-\bar{\nu}_\mu\mathrm{u}\bar{\mathrm{d}}$ with increasing CM energy:

$\sqrt{s}/\,\mathrm{GeV}$	500	800	2000	10000
LUSIFER ('t Hooft–Feynman gauge)				
fixed width	1.633(1)	4.105(4)	6.96(2)	1.73(2)
step width	1.633(1)	4.104(4)	6.96(2)	1.75(2)
running width	1.640(1)	4.132(4)	7.79(1)	1667(4)
complex mass	1.633(1)	4.104(3)	6.94(1)	1.74(2)
WHIZARD&MADGRAPH (unitary gauge)				
step width	1.631(1)	4.105(4)	11.74(1)	26.57(8)

- Crude violations of gauge cancellations for
  - running width $\Gamma(k^2) = \Gamma \times k^2/M^2 \times \theta(k^2)$
  - step width $\Gamma(k^2) = \Gamma \times \theta(k^2)$ in unitary gauge

  $\to$ totally wrong high-energy behaviour
- Fixed width and step width in 'tHF gauge in agreement with gauge-invariant result of complex-mass scheme

Note: Similar results for other channels !
(top-quark pairs, WWZ production, etc.)

3.4 More tuned comparisons of various cross sections

LUSIFER compared with WHIZARD & MADGRAPH
for many final states:

- in general statistical errors comparable
- LUSIFER faster than WHIZARD
- WHIZARD has convergence problems for some $6f$ final states
  $\rightarrow$ error sometimes underestimated (typically $f_1\bar{f}_1 f_2\bar{f}_2 f_3\bar{f}_3$)
- no MADGRAPH amplitude in WHIZARD for some $6f$ final states (e.g. $\mathrm{e^-e^+e^-e^+}f\bar{f}$, $\mu^-\mu^+\mu^-\mu^+\mu^-\mu^+$)

In summary: Good agreement between the programs !

Some examples: ($\sqrt{s} = 500\,\mathrm{GeV}$, phase-space cuts applied)

σ[fb]	LUSIFER		WHIZARD & MADG.	
$\mathrm{e^+e^-} \rightarrow$	Born	Born + ISR	Born	Born + ISR
$\mu^-\mu^+\nu_\mu\bar{\nu}_\mu\tau^-\tau^+$	0.04230(8)	0.04197(9)	0.04212(12)	0.04165(33)
$\mathrm{e^-e^+e^-}\bar{\nu}_\mathrm{e}\nu_\mu\mu^+$	0.1860(3)	0.1709(4)	0.1862(4)	0.1712(5)
$\nu_\mathrm{e}\bar{\nu}_\mathrm{e}\mu^-\mu^+\tau^-\tau^+$	0.00737(2)	0.00679(2)	0.00730(3)	0.00650(7)
$\mu^-\bar{\nu}_\mu\nu_\tau\bar{\nu}_\tau\mathrm{u\bar{d}}$	0.1841(1)	0.1820(1)	0.1843(1)	0.1820(1)
$\mathrm{e^-e^+e^-}\bar{\nu}_\mathrm{e}\mathrm{u\bar{d}}$	0.5509(7)	0.5046(7)	0.5514(13)	0.5061(10)
$\mathrm{e^-e^+}\nu_\mathrm{e}\bar{\nu}_\mathrm{e}(\mathrm{u\bar{u}}+\mathrm{d\bar{d}})$	0.3579(6)	0.3436(7)	0.3592(6)*	0.3437(4)*
$\mathrm{e^-e^+}\mu^-\mu^+(\mathrm{u\bar{u}}+\mathrm{d\bar{d}})$	0.01547(6)	0.01489(5)	0.01523(10)*	0.01396(27)*

* factor 2 more WHIZARD events

4 Summary and outlook

$e^+e^- \to 6f$ interesting at future colliders:

- top-quark pair production $e^+e^- \to t\bar{t}$
- vector-boson scattering $WW \to WW$, etc.
- triple-gauge-boson production $e^+e^- \to WWZ$
- Higgs-boson production with intermediate M_H, $e^+e^- \to ZH/\nu_e\bar{\nu}_e H$

Status of predictions based on full $e^+e^- \to 6f$ amplitudes:

Case studies and generic generators available.

NEW: Monte Carlo event generator LUSIFER

- all final states (massless external fermions)
- emphasis put on stability, speed, and flexibility
- ISR via structure functions
  (inclusion of beamstrahlung straighforward)
- tuned comparison against WHIZARD&MADGRAPH successful
  (whenever possible)

Prospects for the future:

- public version of LUSIFER
  $\to$ opportunity for phenomenological studies
- inclusion of non-universal radiative corrections to subprocesses
  (for example as done in RACOONWW)