An effective lagrangian approach to fermion-loop corrections

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The effort of including higher order corrections through the effective BBC approach, turned out to have serious problems, such as the **bad high energy behaviour** and a relatively **large discrepancy for small angle CC20 processes**.

We studied a **solution** to these problems that may provide a useful gauge invariant tool, leading to reliable results for four-fermion production processes.

Fermion-Loop Scheme

The Fermion-Loop Scheme includes all fermionic one loop corrections $O(\alpha N_c^f)$ of three point Standard model vertices and of the two point functions. After renormalization, all couplings are running and amplitudes can be written in a way similar to bare ones, but with bare couplings substituted by the running ones.

This subset of the higher order corrections is **gauge invariant**, so the scheme is able to give results for processes that include unstable gauge bosons.

BBC Scheme

(hep-ph/9909472)

The lagrangian :

$$\begin{aligned}
\boldsymbol{S_{NL}} = & -\tfrac{1}{4}\int d^4x d^4y \Sigma_1(x-y) B_{\mu\nu}(x) B^{\mu\nu}(y) \\
& -\tfrac{1}{4}\int d^4x d^4y \Sigma_2(x-y) Tr[U_2(y,x) F_{\mu\nu}(x) U_2(x,y) F^{\mu\nu}(y)] \\
& -\tfrac{2}{v^2}\tfrac{g_1}{g_2}\int d^4x d^4y \Sigma_3(x-y)[\Phi^\dagger F_{\mu\nu}\Phi](x) B^{\mu\nu}(y) \\
& -\tfrac{4}{v^4}\int d^4x d^4y \Sigma_4(x-y)[\Phi^\dagger F_{\mu\nu}\Phi](x)[\Phi^\dagger F_{\mu\nu}\Phi](y) \\
& +\int d^4x d^4y \Sigma_5(x-y)(D_\mu\Phi)^\dagger(x) U_2(x,y) U_1(x,y)(D_\mu\Phi)(y) \\
& +\tfrac{2}{v^2}\int d^4x d^4y \Sigma_6(x-y)(\Phi^\dagger D_\mu\Phi^\dagger)(x)(\Phi^\dagger D_\mu\Phi)^\dagger(y)
\end{aligned}$$

where :

$$U(x,y) = Pexp[-ig\int_x^y \boldsymbol{A_\mu}(\omega) d\omega^\mu]$$

the path-ordered exponetial.

The **BBC Scheme** introduces an effective non-local lagrangian, with gauge invariant terms, multiplied by **arbitrary two point functions $\boldsymbol{\Sigma_i(x-y)}$**, $i = 1, 6$. The new terms give contributions to the Standard Model propagators and vertices.

The values of the $\boldsymbol{\Sigma_i(x-y)}$ are estimated through comparison with **Fermion-Loop Scheme**, to give the same corrections to the gauge boson propagators.

The scheme provides a method to include in a gauge invariant way, the fermionic corrections to processes, **also in the higher point functions**, without doing the complete calculation, but only through the comparison of the two point functions .

Problems of BBC Scheme

1) **Bad high energy behaviour of the matrix element of the process $e^+e^- \to W^+W^-$.**

2)**For the process $e^+e^- \to e^-\bar{\nu}_e u\bar{d}$ the results for the total cross-section with the outgoing electron lost in the beam $|cos(\theta_e)| > 0.997$, differ than those of the Fermion-Loop more than 5% .**

First Problem : $s \to \infty$

- $\boldsymbol{W^+}$, $\boldsymbol{W^-}$ **longitudinal :** The validity of the Ward Identity guaranties the good high energy behaviour.

- $\boldsymbol{W^+}$, $\boldsymbol{W^-}$ **transverse :** A term proportional to

$$\left(\frac{\boldsymbol{\Sigma_2(p_+^2)-\Sigma_2(p_-^2)}}{\boldsymbol{p_+^2-p_-^2}}\right)\ \boldsymbol{s}$$

appears, where s the energy at the centre of mass. This term is responsible for the unphysical growth of the cross-section of the process.

Second Problem : $s \to 0$

In **Fermion-Loop scheme** all higher order corrections have been resummed, in the two-point functions and also in the vertices. In the **BBC scheme** contributions to the vertices come only from the two-point functions, since there are not three point non-local terms in the lagrangian. This does not seem to be enough for the $s \to 0$ limit.

The **Unitarity** is lost in the **BBC Scheme**, because of the truncation that is done. The Unitarity, in the sense of the **cut equations**, which is an intrinsic property of the feynman diagrams, does not seem to have a clear correspondance in a non-local theory. The cut equations connect a three point vertex with the tree level amplitudes. How this information is there in a non-local theory?

Suggestion

Two new vertices are implemented, one to restore the **bad high energy behaviour** of BBC and the other to restore the behaviour of the model at the area of $\boldsymbol{s \rightarrow 0}$.

The new vertices should be **zero mode**, since the two schemes obey the same Ward Identity.

We started investigating the zero mode solutions of a vertex that has the general form :

$$\begin{aligned}
V^{\mu\kappa\lambda}(q,p_+,p_-) =& x_1 p_+^\mu g^{\kappa\lambda} + x_2 p_-^\mu g^{\kappa\lambda} \\
&+ x_3 p_+^\kappa g^{\mu\lambda} + x_4 p_-^\kappa g^{\mu\lambda} \\
&+ x_5 p_+^\lambda g^{\mu\kappa} + x_6 p_-^\lambda g^{\mu\kappa} \\
&+ x_7 p_+^\mu p_-^\kappa p_-^\lambda + x_8 p_-^\mu p_+^\kappa p_+^\lambda \\
&+ x_9 p_+^\mu p_+^\kappa p_-^\lambda + x_{10} p_-^\mu p_+^\kappa p_-^\lambda \\
&+ x_{11} p_+^\mu p_-^\kappa p_+^\lambda + x_{12} p_-^\mu p_-^\kappa p_+^\lambda \\
&+ x_{13} p_+^\mu p_+^\kappa p_+^\lambda + x_{14} p_-^\mu p_-^\kappa p_-^\lambda
\end{aligned}$$

The $\boldsymbol{x_i}$ (i=1,..,14) are arbitrary coefficients, which will be specified from the zero mode identities :

$$\boldsymbol{q_\mu V^{\mu\kappa\lambda}(q, p_+, p_-) = 0}$$

$$\boldsymbol{p_{+k} V^{\mu\kappa\lambda}(q, p_+, p_-) = 0}$$

$$\boldsymbol{p_{-\lambda} V^{\mu\kappa\lambda}(q, p_+, p_-) = 0}$$

where $\boldsymbol{q^\mu = -p_+^\mu - p_-^\mu}$.

We end up with a system of 15 equations with 14 unknowns.

The rank of the 15×15 matrix (made up by adding a column of zeros) is 10, so we have **4 independent non-trivial zero-mode solutions**; that means that ten of them can be expressed in terms of the other four, which take arbitrary values.

We chose x_3, x_6, x_{12}, x_{13} as the **independent parameters**.

One solution of the system is :

$$
\begin{aligned}
x_1 &= \frac{c(b+c)}{a+c}x_{12} - a x_{13} - \frac{a}{a+c}x_3 + \frac{ab}{c(a+c)}x_6,\\
x_2 &= -c x_{12} + \frac{a(a+c)}{b+c}x_{13} + \frac{a}{b+c}x_3 - \frac{ab}{c(b+c)}x_6,\\
x_4 &= -\frac{a}{c}x_3,\\
x_5 &= -\frac{b}{c}x_6,\\
x_7 &= \frac{a}{b}x_{13} - \frac{1}{c}x_6,\\
x_8 &= -\frac{a+c}{b+c}x_{13} - \frac{1}{b+c}x_3 + \frac{b}{c(b+c)}x_6,\\
x_9 &= -\frac{c}{b}x_{13},\\
x_{10} &= \frac{c(a+c)}{b(b+c)}x_{13} - \frac{1}{b+c}x_3 - \frac{1}{b+c}x_6,\\
x_{11} &= -\frac{b+c}{a+c}x_{12} + \frac{a}{c(a+c)}x_3 + \frac{b}{c(a+c)}x_6,\\
x_{14} &= -\frac{a(a+c)}{b(b+c)}x_{13} + \frac{a}{c(b+c)}x_3 + \frac{a}{c(b+c)}x_6,
\end{aligned}
$$

where : $a = p_+^2$, $b = p_-^2$, $c = p_+ p_-$.

The equations have the **(anti-)symmetry** $p_+ \leftrightarrow -p_-$, as they should. Another way to parametrize the four linear independent solutions, is through the following **vertices** :

$$\begin{aligned} \boldsymbol{V_1^{\mu\kappa\lambda}(q, p_+, p_-)} &= \boldsymbol{-(p_+ p_-)[(qp_-)p_+^\mu - (qp_+)p_-^\mu]g^{\kappa\lambda}} \\ &\quad \boldsymbol{+[(qp_-)p_+^\mu - (qp_+)p_-^\mu]p_-^\kappa p_+^\lambda} \end{aligned}$$

$$\begin{aligned} \boldsymbol{V_2^{\mu\kappa\lambda}(q, p_+, p_-)} &= \boldsymbol{-[(qp_-)p_+^\mu - (qp_+)p_-^\mu]g^{\kappa\lambda}} \\ &\quad \boldsymbol{-[(p_+ p_-)p_+^\kappa - p_+^2 p_-^\kappa]g^{\mu\lambda}} \\ &\quad \boldsymbol{-[p_-^2 p_+^\lambda - (p_+ p_-)p_-^\lambda]g^{\mu\kappa}} \\ &\quad \boldsymbol{-p_+^\mu p_-^\kappa p_-^\lambda + p_-^\mu p_+^\kappa p_+^\lambda - [p_+^\mu - p_-^\mu]p_-^\kappa p_+^\lambda} \end{aligned}$$

$$
\begin{aligned}
V_3^{\mu\kappa\lambda}(q,p_+,p_-) = & -[(p_+p_-)^2 - p_+^2 p_-^2][(qp_-)p_+^\mu - (qp_+)p_-^\mu]g^{\kappa\lambda} \\
& -[(p_+p_-)^3 p_+^\kappa - p_+^2(p_+p_-)^2 p_-^\kappa]g^{\mu\lambda} \\
& -[p_-^2(p_+p_-)^2 p_+^\lambda - (p_+p_-)^3 p_-^\lambda]g^{\mu\kappa} \\
& -[p_+^2(qp_-) + (p_+p_-)^2]p_+^\mu p_-^\kappa p_-^\lambda \\
& +[p_-^2(qp_+) + (p_+p_-)^2]p_-^\mu p_+^\kappa p_+^\lambda \\
& +(p_+p_-)[(qp_-)p_+^\mu - (qp_+)p_-^\mu]p_+^\kappa p_-^\lambda \\
& -(p_+p_-)^2[p_+^\mu - p_-^\mu]p_-^\kappa p_+^\lambda \\
& -(qp_-)p_-^2 p_+^\mu p_+^\kappa p_+^\lambda + (qp_+)p_+^2 p_-^\mu p_-^\kappa p_-^\lambda \\
V_4^{\mu\kappa\lambda}(q,p_+,p_-) = & -(p_+p_-)[(qp_-)p_+^\mu - (qp_+)p_-^\mu]g^{\kappa\lambda} \\
& +p_-^2[(p_+p_-)p_+^\kappa - p_+^2 p_-^\kappa]g^{\mu\lambda} \\
& +p_+^2[p_-^2 p_+^\lambda - (p_+p_-)p_-^\lambda]g^{\mu\kappa} \\
& +(p_+p_-)[p_+^\mu - p_-^\mu]p_+^\kappa p_-^\lambda \\
& +[(qp_-)p_+^\mu - (qp_+)p_-^\mu]p_-^\kappa p_+^\lambda \\
& -p_-^2 p_+^\mu p_+^\kappa p_+^\lambda + p_+^2 p_-^\mu p_-^\kappa p_-^\lambda
\end{aligned}
$$

The first vertex can restore the bad high energy behaviour

- if multiplied by :

$$\frac{\boldsymbol{\Sigma_2(p_+^2)-\Sigma_2(p_-^2)}}{\boldsymbol{p_+^2-p_-^2}}$$

  has exactly the same high energy behaviour of the Σ_2 term, so **it can kill the divergence of that term**.

- if multiplied by :

$$\boldsymbol{f_\infty = \frac{q^4}{q^4+p_+^4+p_-^4-2q^2p_+^2-2q^2p_-^2-2p_+^2p_-^2}}$$

  its contribution is turned off everywhere else, exept the high energy limit, in which it is needed.($q^2 = s$)

For the other limit : $s \to 0$

we investigate the difference of the two $\gamma W^+ W^-$ vertices, the BBC and the Fermion-Loop. We construct the 14 differences $\boldsymbol{\Delta(i) = bbc(i) - fl(i)}$, i=1,14 of the corresponding coefficients.

- after reduction of the integrals, we ignore all terms proportional to C_0 that obviously are out of the spirit of the BBC 2-point function approach.
- we take the fermion masses equal to zero, in order to simplify a rather complicated result
- we take the limit $s \to 0$

We end up with an expression that has terms proportional to various B_0 and a term not multiplied by any integral.

We expressed the $\boldsymbol{B_0(p_i^2, 0, 0)}$ in terms of $\boldsymbol{\Sigma_2(p_i^2)}$. In a sense this is the simplest possible vertex, in terms of Σ_2, that carries a minimal (?) memory of the one-loop fermionic contribution. When implemented in the vertex the $\boldsymbol{\Sigma_2(p_i^2)}$ are computed, taking the fermion masses different from zero.

The Fermion-Loop vertex is taken from the Nuclear Physics B 500 paper.

If $s_1 = q^2$, $s_2 = p_+^2$, $s_3 = p_-^2$, in the limit $s_1 \to 0$, we end up with the following expressions:

$$\Delta(1) = 0$$

$$\begin{aligned}\Delta(3) = {} & \frac{8\,(-s_1+s_2+s_3)}{s_2-s_3} + \frac{-s_1+s_2+s_3}{s_3}\ \Sigma_2(s_1) \\ & + \frac{(-s_1+s_2+s_3)\,(-2\,s_2+3\,s_3)}{(s_2-s_3)^2}\ \Sigma_2(s_2) \\ & - \frac{(s_2-2\,s_3)^2\,(-s_1+s_2+s_3)}{(s_2-s_3)^2\,s_3}\ \Sigma_2(s_3)\end{aligned}$$

$$\begin{aligned}\Delta(4) = {} & \frac{16\,s_2}{s_2-s_3} + \frac{2\,s_2}{s_3}\ \Sigma_2(s_1) + \frac{2\,s_2\,(-2\,s_2+3\,s_3)}{(s_2-s_3)^2}\ \Sigma_2(s_2) \\ & - \frac{2\,s_2\,(s_2-2\,s_3)^2}{(s_2-s_3)^2\,s_3}\ \Sigma_2(s_3)\end{aligned}$$

$$
\begin{aligned}
\mathbf{\Delta}(7) = {} & \frac{-8\,(s_2+s_3)\,(-s_1+s_2+s_3)}{(s_2-s_3)^2\,s_3} - \frac{(s_2+s_3)\,(-s_1+s_2+s_3)}{s_2\,s_3{}^2}\;\Sigma_2(s_1) \\
& + \frac{(-s_1+s_2+s_3)\left(2\,s_2{}^3-7\,s_2{}^2\,s_3+4\,s_2\,s_3{}^2-s_3{}^3\right)}{s_2\,(s_2-s_3)^3\,s_3}\;\Sigma_2(s_2) \\
& + \frac{(-s_1+s_2+s_3)\left(s_2{}^3-4\,s_2{}^2\,s_3+7\,s_2\,s_3{}^2-2\,s_3{}^3\right)}{(s_2-s_3)^3\,s_3{}^2}\;\Sigma_2(s_3)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{\Delta}(9) = {} & \frac{-4\left(s_2{}^3+5\,s_2{}^2\,s_3+3\,s_2\,s_3{}^2-s_3{}^3+s_1{}^2\,(s_2+s_3)-2\,s_1\,s_2\,(s_2+3\,s_3)\right)}{s_2\,(s_2-s_3)^2\,s_3} \\
& -\left\{\frac{s_2{}^3+3\,s_2{}^2\,s_3+s_2\,s_3{}^2-s_3{}^3+s_1{}^2\,(s_2+s_3)-2\,s_1\,s_2\,(s_2+2\,s_3)}{2\,s_2{}^2\,s_3{}^2}\right\}\Sigma_2(s_1) \\
& +(-s_1+s_2+s_3)\left\{\frac{2\,s_2{}^4-13\,s_2{}^3\,s_3+13\,s_2{}^2\,s_3{}^2-7\,s_2\,s_3{}^3+s_3{}^4}{2\,s_2{}^2\,(s_2-s_3)^3\,s_3}\right. \\
& \qquad\qquad \left. + \frac{s_1\left(-2\,s_2{}^3+7\,s_2{}^2\,s_3-4\,s_2\,s_3{}^2+s_3{}^3\right)}{2\,s_2{}^2\,(s_2-s_3)^3\,s_3}\right\}\Sigma_2(s_2) \\
& +(-s_1+s_2+s_3)\left\{\frac{s_2{}^4-3\,s_2{}^3\,s_3+9\,s_2{}^2\,s_3{}^2-5\,s_2\,s_3{}^3+2\,s_3{}^4}{2\,s_2\,(s_2-s_3)^3\,s_3{}^2}\right. \\
& \qquad\qquad \left. - \frac{s_1\left(s_2{}^3-4\,s_2{}^2\,s_3+7\,s_2\,s_3{}^2-2\,s_3{}^3\right)}{2\,s_2\,(s_2-s_3)^3\,s_3{}^2}\right\}\Sigma_2(s_3)
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{\Delta}(11) = {} & \frac{-16\,(s_2+s_3)}{(s_2-s_3)^2} - \frac{2\,(s_2+s_3)}{s_2\,s_3}\;\Sigma_2(s_1) \\
& - \frac{2\left(-2\,s_2{}^3+7\,s_2{}^2\,s_3-4\,s_2\,s_3{}^2+s_3{}^3\right)}{s_2\,(s_2-s_3)^3}\;\Sigma_2(s_2) \\
& + \frac{2\left(s_2{}^3-4\,s_2{}^2\,s_3+7\,s_2\,s_3{}^2-2\,s_3{}^3\right)}{(s_2-s_3)^3\,s_3}\;\Sigma_2(s_3)
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{\Delta}(13) = {} & \frac{8\left(-s_2{}^2-4\,s_2\,s_3+s_3{}^2+s_1\,(s_2+s_3)\right)}{s_2\,(s_2-s_3)^2} \\
& + \left\{\frac{-s_2{}^2-2\,s_2\,s_3+s_3{}^2+s_1\,(s_2+s_3)}{s_2{}^2\,s_3}\right\}\Sigma_2(s_1) \\
& + \left\{\frac{2\,s_2{}^4-13\,s_2{}^3\,s_3+13\,s_2{}^2\,s_3{}^2-7\,s_2\,s_3{}^3+s_3{}^4 s_2{}^2\,(s_2-s_3)^3}{s_2{}^2\,(s_2-s_3)^3}\right. \\
& \qquad\qquad \left. + \frac{s_1\left(-2\,s_2{}^3+7\,s_2{}^2\,s_3-4\,s_2\,s_3{}^2+s_3{}^3\right)}{s_2{}^2\,(s_2-s_3)^3}\right\}\Sigma_2(s_2) \\
& + \left\{\frac{s_2{}^4-3\,s_2{}^3\,s_3+9\,s_2{}^2\,s_3{}^2-5\,s_2\,s_3{}^3+2\,s_3{}^4 s_2\,(s_2-s_3)^3\,s_3}{s_2\,(s_2-s_3)^3\,s_3}\right. \\
& \qquad\qquad \left. - \frac{s_1\left(s_2{}^3-4\,s_2{}^2\,s_3+7\,s_2\,s_3{}^2-2\,s_3{}^3\right)}{s_2\,(s_2-s_3)^3\,s_3}\right\}\Sigma_2(s_3)
\end{aligned}
$$

The differences that are not written, can be constructed from the above, with the change $s_2 \rightleftharpoons s_3$ The correspondence is:

$$\begin{aligned} &\boldsymbol{\Delta(2) \to -\Delta(1)} && \boldsymbol{\Delta(10) \to -\Delta(9)} \\ &\boldsymbol{\Delta(5) \to -\Delta(4)} && \boldsymbol{\Delta(12) \to -\Delta(11)} \\ &\boldsymbol{\Delta(6) \to -\Delta(3)} && \boldsymbol{\Delta(14) \to -\Delta(13)} \\ &\boldsymbol{\Delta(8) \to -\Delta(7)} && \end{aligned}$$

The vertex, that can restore the $s \to 0$ limit is :

$$\begin{aligned} \boldsymbol{V^{\mu\kappa\lambda}(q,p_+,p_-)} =& \boldsymbol{\Delta(1)p_+^{\mu}g^{\kappa\lambda} + \Delta(2)p_-^{\mu}g^{\kappa\lambda} + \Delta(3)p_+^{\kappa}g^{\mu\lambda}} \\ &\boldsymbol{+\Delta(4)p_-^{\kappa}g^{\mu\lambda} + \Delta(5)p_+^{\lambda}g^{\mu\kappa} + \Delta(6)p_-^{\lambda}g^{\mu\kappa}} \\ &\boldsymbol{+\Delta(7)p_+^{\mu}p_-^{\kappa}p_-^{\lambda} + \Delta(8)p_-^{\mu}p_+^{\kappa}p_+^{\lambda} + \Delta(9)p_+^{\mu}p_+^{\kappa}p_-^{\lambda}} \\ &\boldsymbol{+\Delta(10)p_-^{\mu}p_+^{\kappa}p_-^{\lambda} + \Delta(11)p_+^{\mu}p_-^{\kappa}p_+^{\lambda} + \Delta(12)p_-^{\mu}p_-^{\kappa}p_+^{\lambda}} \\ &\boldsymbol{+\Delta(13)p_+^{\mu}p_+^{\kappa}p_+^{\lambda} + \Delta(14)p_-^{\mu}p_-^{\kappa}p_-^{\lambda}} \end{aligned}$$

The above vertex gives the main contribution that comes from the Fermion-Loop at the $s \to 0$ limit, since if the fermion masses are not zero the extra contribution will be proportional to them, so it will be relatively small.

We multiply the vertex with the function:

$$\boldsymbol{f_0 = \frac{(s_2 - s_3)^2}{s_1^2 + s_2^2 + s_3^2 - 2s_1 s_2 - 2s_2 s_3 - 2s_1 s_3}}$$

in order to turn on the contribution of it, only in the area of $\boldsymbol{s_1 \to 0}$. The denominator of the function is nothing but the Gram determinant, that comes from the Fermion-Loop scheme.

For the Renormalization, we use the Complex Scheme.

Input Parameters	**solution, incl. bare quantities**
$M_W = 80.35 GeV$	$M_w = 81.945 GeV$
$M_Z = 91.1867 GeV$	$M_z = 93.735 GeV$
$Re\{a_l(m_z^2)\}^{-1} = 128.89$	$m_u = m_d = 0.2439\ GeV$
$a(0)^{-1} = 137.03599976$	$m_{top} = 145.9954\ GeV$
$G_F = 1.16639 \times 10^{-5} GeV^{-2}$	$\alpha = 1/135.55169$

Exept the Complex Scheme's conditions, we demand one more : the value of $\boldsymbol{\alpha(0)}$, that is the reason for having one more bare parameter to fix, **the quark masses**.

We collect a few results in the following tables.

$\sqrt{s}$	*fixed width*	*bbc old*	*bbc*	*fermionloop*
183*GeV*	89.17(26)	79.66(31)	85.22(37)	84.00(35)
				83.28(6)
189*GeV*	99.80(24)	89.17(34)	95.14(36)	94.69(36)
				93.79(7)
200*GeV*	120.98(31)	108.08(41)	115.46(44)	114.33(43)
				113.67(8)

Table 1 : Results for the process $e^+e^- \to e^-\bar{\nu}_e u\bar{d}$ (in fb) with $M(u\bar{d}) > 45GeV$ and $|cos(\theta_e)| > 0.997$. The second entry in ***fermionloop*** column represents the **hep-ph/0001212 results**.

$\boldsymbol{\theta_e^o}$	**183*GeV***	**189*GeV***	**200*GeV***
	0.49014	0.55253	0.67818
0.0^o - 0.1^o	0.45920	0.51798	0.63514
	0.45635	0.51429	0.63189
	0.07037	0.07878	0.09364
0.1^o - 0.2^o	0.06736	0.07523	0.08913
	0.06710	0.07448	0.08758
	0.04208	0.04553	0.05402
0.2^o - 0.3^o	0.03888	0.04303	0.05183
	0.03943	0.04347	0.05364
	0.02803	0.03237	0.03879
0.3^o - 0.4^o	0.02886	0.03196	0.03794
	0.02833	0.31119	0.03614

Table 2 : Results for the process $e^+e^- \to e^-\bar{\nu}_e u\bar{d}$ (in pb) with $M(u\bar{d}) > 45GeV$. First comes the ***fixed width*** result, then the ***bbc*** and finally the ***fermionloop***.

Conclusion :

With the implementation of these two new vertices, the new BBC $\gamma W^+ W^-$ vertex does not give terms with bad high energy behaviour and the whole scheme **can include the $\boldsymbol{O(\alpha)}$ fermionic corrections**, since for four fermion processes, the results that we get are pretty close to those of the Fermion-Loop scheme.

Maybe this is not a complete solution to the problem of the BBC scheme, but this new version of the scheme can still provide in a gauge invariant way, reliable results at least for four fermion processes.