SUSY contributions to $(g-2)_\mu$

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Muon $(g-2)$ workshop, Glasgow, 26/10/2007

Outline

1 Motivation

2 Overview

3 $\tan\beta$ enhancement and sign(μ)

4 SUSY one-loop and two-loop contributions

5 Conclusions

Observed deviation:

$$a_\mu^{\exp} - a_\mu^{\mathrm{SM,HMNT}} = (27.6 \pm 8.1) \times 10^{-10}$$

SUSY contributions:

$$a_\mu^{\mathrm{SUSY}} \approx 13 \times 10^{-10} \; \tan\beta \; \mathrm{sign}(\mu) \left(\frac{100\mathrm{GeV}}{M_{\mathrm{SUSY}}}\right)^2$$

e.g. $a_\mu^{\mathrm{SUSY}} = 26 \times 10^{-10}$ for

$$\left.\begin{array}{ll} \tan\beta = 2, & M_{\mathrm{SUSY}} = 100 \text{ GeV} \\ \tan\beta = 50, & M_{\mathrm{SUSY}} = 500 \text{ GeV} \end{array}\right. \quad (\mu > 0)$$

$\Rightarrow$ SUSY could easily be the origin of the observed deviation!

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SUSY vs generic BSM physics

Generic BSM physics with new, weakly interacting particles at M_{NP}:

- suppressed compared to SM weak contributions by $\left(\frac{M_W}{M_{\rm NP}}\right)^2$
- SM weak itself is only 15.4×10^{-10}

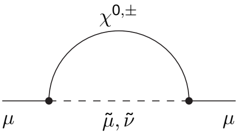
Two advantages of SUSY:

- $\tan\beta$-enhancement
- low SUSY masses possible

Status of SUSY prediction

1-Loop

$\propto \tan\beta$



complete

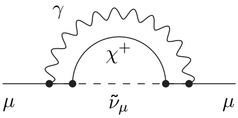
[Fayet '80],. . .

[Kosower et al '83],[Yuan et al '84],. . .

[Lopez et al '94],[Moroi '96]

2-Loop (SUSY 1L)

$\propto \tan\beta \, \log \frac{M_{\text{SUSY}}}{m_\mu}$

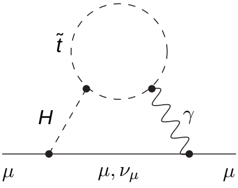


leading log

[Degrassi,Giudice '98]

2-Loop (SM 1L)

$\propto \tan\beta \, \mu \, m_t$



complete

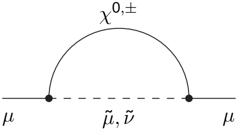
[Chen,Geng'01][Arhib,Baek '02]

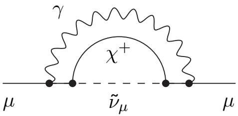
[Heinemeyer,DS,Weiglein '03]

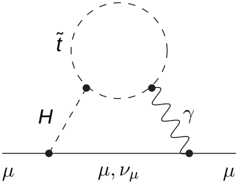
[Heinemeyer,DS,Weiglein '04]

Status of SUSY prediction

1-Loop	2-Loop (SUSY 1L)	2-Loop (SM 1L)
$\propto \tan\beta$	$\propto \tan\beta \; \log \frac{M_{\text{SUSY}}}{m_\mu}$	$\propto \tan\beta \, \mu \, m_t$







complete	leading log	complete
smuons/sneutrinos charginos/neutralinos	same plus SM-particles	stops/sbottoms charginos/neutralinos
Leading	$(-7 \ldots -9)\%$	qualitatively different

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$g-2$ in the MSSM

Key to understand $g-2$:

chiral symmetry

$g-2$ = chirality-flipping interaction

$$\bar{u}_R(p')\frac{\sigma_{\mu\nu}q^\nu}{2m_\mu}u_L(p) + (L \leftrightarrow R)$$

in each Feynman diagram we need to pick up one transition

$$\mu_L \to \mu_R \text{ or } \tilde{\mu}_L \to \tilde{\mu}_R$$

Chiral symmetry would forbid $g-2$

$g-2$ in the MSSM

Like in the SM, chiral symmetry broken by λ_μ, m_μ:

$$\begin{aligned}\mathcal{L}_{m,\ \text{Yukawa}} &= -m_\mu(\bar\mu_R\mu_L+\bar\mu_L\mu_R)\\ &- \lambda_\mu H_1^0(\bar\mu_R\mu_L+\bar\mu_L\mu_R)\end{aligned}$$

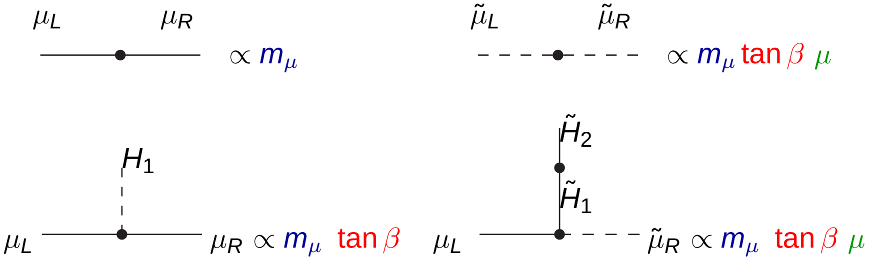
However, in the MSSM there is a second Higgs doublet H_2:

$$\tan\beta=\frac{\langle H_2\rangle}{\langle H_1\rangle},\qquad \mu = H_2 - H_1 \text{ transition}$$

If $\tan\beta$ is large: enhancement $\lambda_\mu\to\lambda_\mu^{\rm SM}\tan\beta \Rightarrow \lambda_\mu\propto m_\mu\tan\beta$
(and $\langle H_2\rangle\to\langle H^{\rm SM}\rangle$)

Chirality flips

Chirality-flipping interactions relevant for a_μ

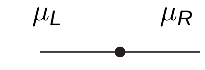


some terms

$\propto m_\mu$ $\quad \rightarrow a_\mu^{\text{SUSY}} \propto \frac{m_\mu^2}{M_{\text{SUSY}}^2}$

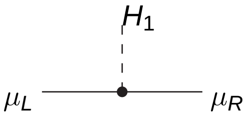
Chirality flips

Chirality-flipping interactions relevant for a_μ

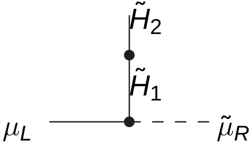


$\propto m_\mu$

$\propto m_\mu \tan\beta \; \mu$



$\propto m_\mu \; \tan\beta$



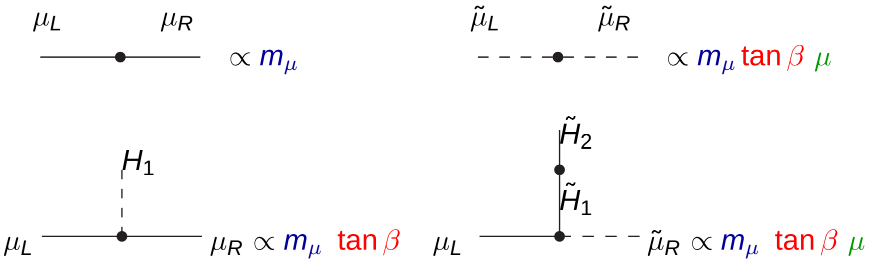
$\propto m_\mu \; \tan\beta \; \mu$

some terms

$$\propto m_\mu \; \tan\beta \; \mu \qquad \rightarrow a_\mu^{\mathrm{SUSY}} \propto \frac{m_\mu^2}{M_{\mathrm{SUSY}}^2} \tan\beta \, \mathrm{sign}(\mu)$$

Chirality flips

Chirality-flipping interactions relevant for a_μ

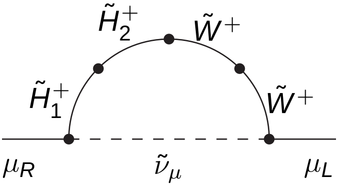


some terms

$$\propto m_\mu\ \tan\beta\ \mu \qquad \to a_\mu^{\rm SUSY} \propto \frac{m_\mu^2}{M_{\rm SUSY}^2}\ \tan\beta\, \mathrm{sign}(\mu)$$

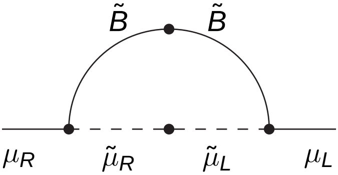
a_μ in the MSSM — leading contributions

(C)



$\propto m_\mu^2 \tan\beta \; \mu \, M_2 \, F(\mu, M_2, m_{\tilde{\mu}_L})$

(N1)



$\propto m_\mu^2 \tan\beta \; \mu \, M_1 \, F(M_1, m_{\tilde{\mu}_{L,R}})$

Leading contributions — alternative explanation

$\tan\beta\mu$ behaviour can be explained using effective field theory:

Two simplest gauge invariant operators contributing to a_μ in MSSM ($a = 1, 2, 3, '$)

$$\lambda_\mu \; \bar{\mu}_R \; \sigma^{\mu\nu}[LT^a H_1]F^a_{\mu\nu}$$

$$\mu\lambda_\mu \; \bar{\mu}_R \; \sigma^{\mu\nu}[LT^a H_2^C]F^a_{\mu\nu}$$

Factors λ_μ, μ necessary because of chiral/Peccei-Quinn symmetry.

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$$\lambda_\mu \, \bar{\mu}_R \, \sigma^{\mu\nu}[LT^a H_1]F^a_{\mu\nu} \longrightarrow \bar{\mu}_R \sigma^{\mu\nu} \mu_L F_{\mu\nu} \, (\lambda_\mu v_1) \qquad \longrightarrow m_\mu$$

$$\mu\lambda_\mu \, \bar{\mu}_R \, \sigma^{\mu\nu}[LT^a H_2^C]F^a_{\mu\nu} \longrightarrow \bar{\mu}_R \sigma^{\mu\nu} \mu_L F_{\mu\nu} \, (\lambda_\mu v_2 \mu) \qquad \longrightarrow m_\mu \tan\beta\mu$$

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Leading contributions — alternative explanation

tan $\beta\mu$ behaviour can be explained using effective field theory:

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$$\lambda_\mu \; \bar{\mu}_R \; \sigma^{\mu\nu} [L T^a H_1] F^a_{\mu\nu}$$

$$\mu \lambda_\mu \; \bar{\mu}_R \; \sigma^{\mu\nu} [L T^a H_2^C] F^a_{\mu\nu}$$

Factors λ_μ, μ necessary because of chiral/Peccei-Quinn symmetry.

Bottom line

$$a_\mu^{\rm SUSY} \approx \frac{\alpha}{\pi \; 8 s_W^2} \; \tan\beta \; {\rm sign}(\mu) \; \frac{m_\mu^2}{M_{\rm SUSY}^2}$$

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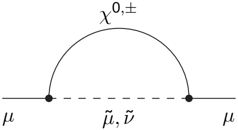


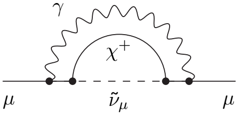
complete

smuons/sneutrinos
charginos/neutralinos

Leading

2-Loop (SUSY 1L)

$\propto \tan\beta \; \log \frac{M_{SUSY}}{m_\mu}$



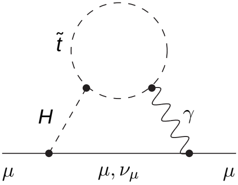
leading log

same plus
SM-particles

$(-7\ldots-9)\%$

2-Loop (SM 1L)

$\propto \tan\beta \, \mu \, m_t$



complete

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qualitatively different

Discussion of SUSY 1-Loop contributions

One-loop plus two-loop leading QED-logs

$$13 \times 10^{-10} \left(\frac{100\,\mathrm{GeV}}{M_{\mathrm{SUSY}}} \right)^2 \tan\beta \; \mathrm{sign}(\mu M_2) \; \left(1 - \frac{4\alpha}{\pi} \log \frac{M_{\mathrm{SUSY}}}{m_\mu} \right)$$

- $M_{\mathrm{SUSY}} =$ combination of smuon masses, μ, gaugino masses $M_{1,2}$.
- Generally, increasing a mass parameter leads to a suppression
- Exception: $\mu \to \infty$ with fixed smuon, gaugino masses: increases a_μ^{SUSY} due to diagram with bino exchange (N1)

- The observed deviation of 27×10^{-10} can be easily explained, large $\tan\beta$, rather small M_{SUSY} preferred, but no particular mass pattern required.

Discussion of SUSY 2-Loop contributions

Largest non-logarithmic two-loop contributions: from closed chargino loop or closed stop loop:

$$a_\mu^{(\chi V H)} \approx \; 11 \times 10^{-10} \left(\frac{\tan\beta}{50}\right) \left(\frac{100 \text{ GeV}}{\{\mu, M_2, M_H\}}\right)^2 \text{sign}(\mu M_2),$$

$$a_\mu^{(\tilde{t}\gamma H)} \approx -13 \times 10^{-10} \left(\frac{\tan\beta}{50}\right) \left(\frac{m_t}{m_{\tilde{t}}}\right) \left(\frac{\mu}{20 M_H}\right) \text{sign}(X_t)$$

- for degenerate masses: only 2% of the one-loop contributions
- large contributions of up to 10×10^{-10} possible for:
  - very light M_H, M_2, μ, large $\tan\beta$
  - very light M_H, $m_{\tilde{t}}$, large $\tan\beta$ and μ

Summary: SUSY prediction

- 1-loop and most 2-loop contributions known
- remaining theory uncertainty of SUSY prediction: [DS '06]

$$\delta a_\mu^{\rm SUSY} \approx 3 \times 10^{-10}$$

based on:

  - unknown two-loop contributions with $f\tilde{f}$ loops (can be compared to SM t/b loop contributions): 0.5×10^{-10}
  - unknown further two-loop contributions (would go beyond the QED-logarithms; partial evaluation in [Feng, Li, Lin, Maalampi, Song '06]): 2.5×10^{-10}

- Currently under investigation: $(\tan\beta)^2$-enhanced two-loop contribution from renormalization constant $\delta m_\mu \propto \tan\beta$

[Marquetti, Mertens, Nierste, DS]

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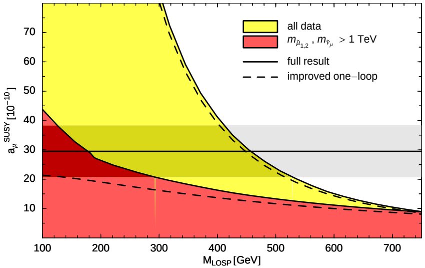
Conclusions

- Case for new physics gets stronger!
- SUSY with low mass scale $\sim 200 \ldots 600$ GeV fits very well and large parameter regions already excluded



- SUSY contributions enhanced by tan βsign(μ)
- SUSY theory prediction reliable, $\delta a_\mu^{\rm SUSY} \approx 3 \times 10^{-10}$
- so far, all one-loop and most two-loop contributions known
- further improvements possible