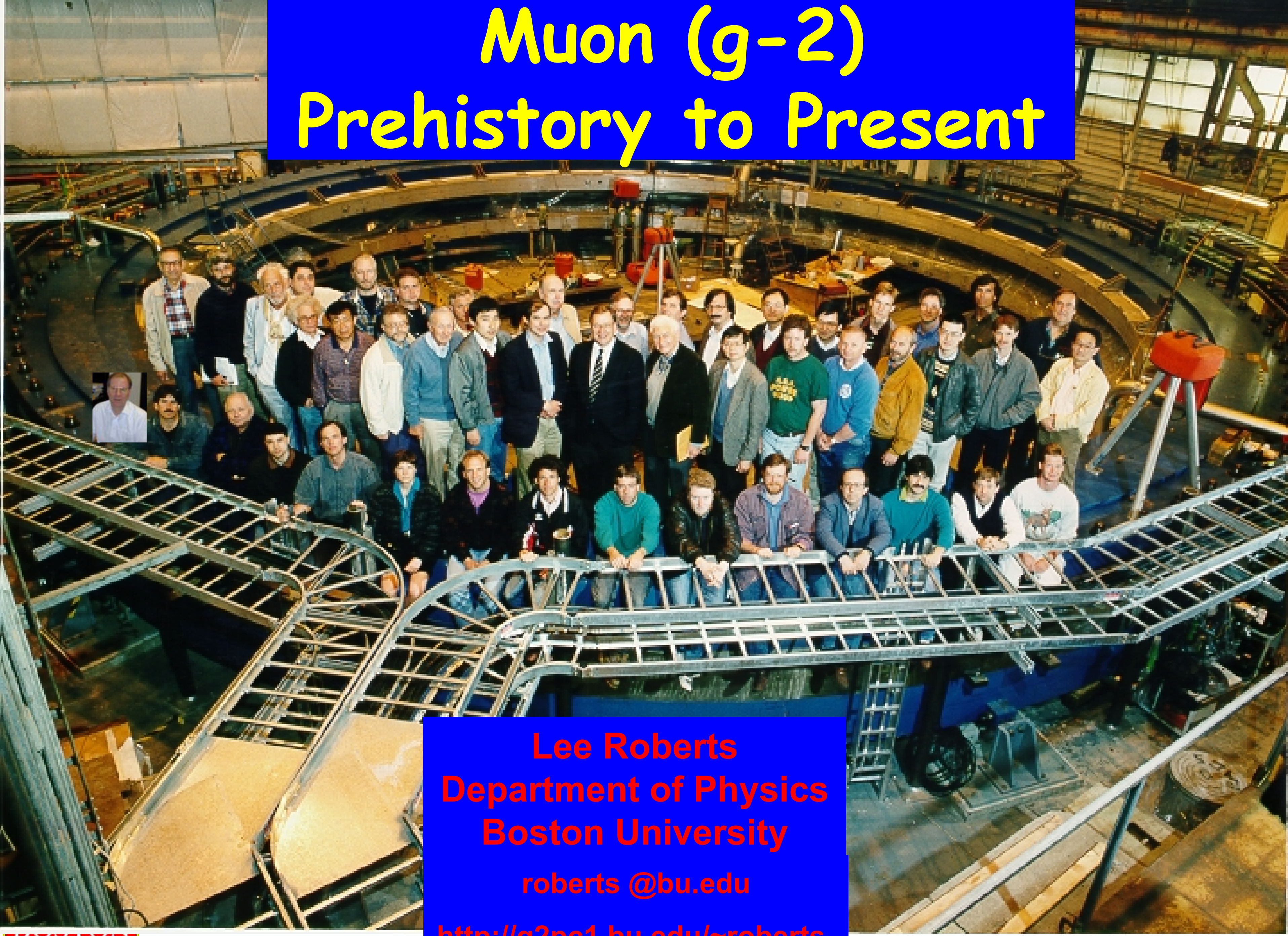


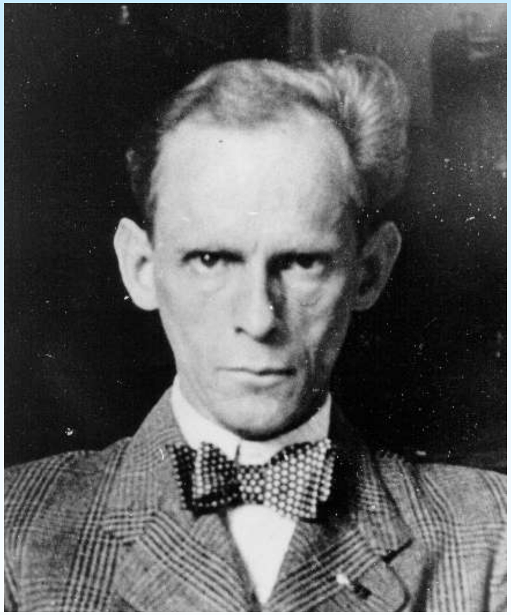
Muon (g-2) Prehistory to Present

Lee Roberts
Department of Physics
Boston University
roberts @bu.edu
http://g2pc1.bu.edu/~roberts

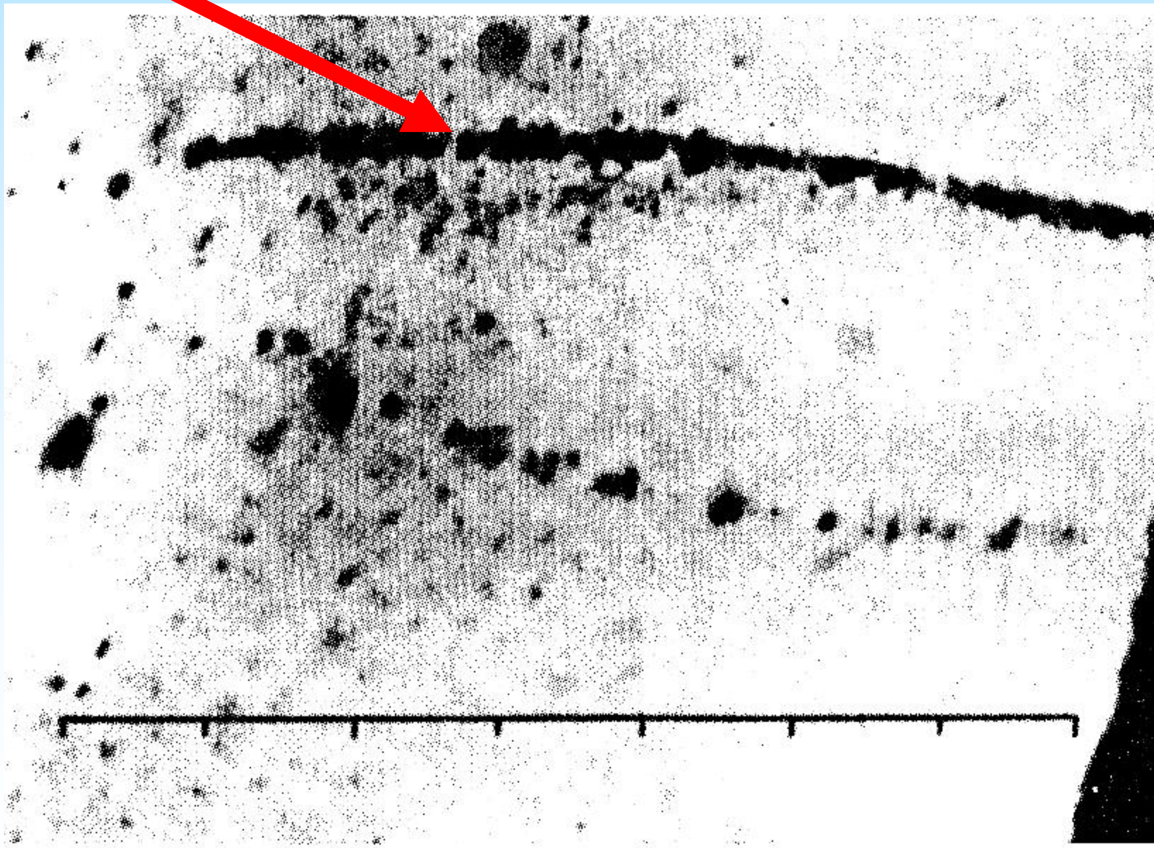
First published observation of the muon came from cosmic rays:

Paul Kunze,

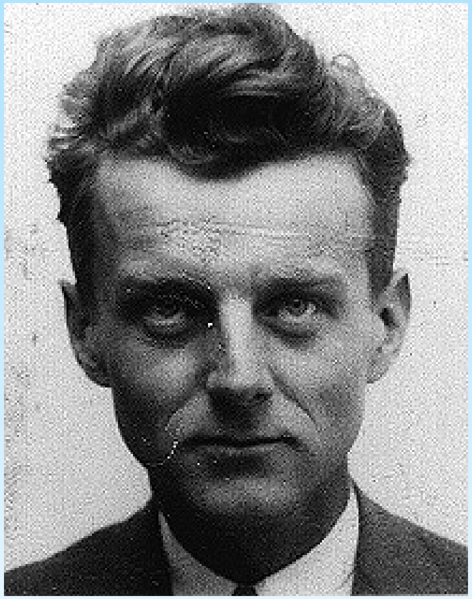
Z. Phys. 83, 1 (1933)



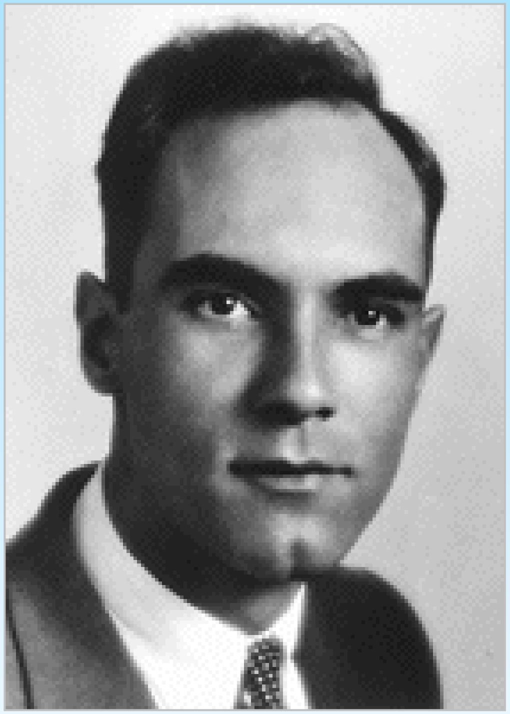
“a particle of uncertain nature”



Identified in 1936



Study of cosmic rays by Seth Neddermeyer and Carl Anderson



MAY 15, 1937 PHYSICAL REVIEW VOLUME 51

Note on the Nature of Cosmic-Ray Particles

SETH H. NEDDERMEYER AND CARL D. ANDERSON
California Institute of Technology, Pasadena, California

(Received March 30, 1937)

MEASUREMENTS[1] of the energy loss of particles occurring in the cosmic-ray showers have shown that this loss is proportional massive than protons but more penetrating than electrons obeying the Bethe-Heitler theory, we have taken about 6000 counter-tripped photo-

Confirmed by Street and Stevenson

NOVEMBER 1, 1937 PHYSICAL REVIEW VOLUME 52

LETTERS TO THE EDITOR

Prompt publication of brief reports of important discoveries in physics may be secured by addressing them to this department. Closing dates for this department are, for the first issue of the month, the eighteenth of the preceding month, for the second issue, the third of the month. Because of the late closing dates for the section no proof can be shown to authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents.

Communications should not in general exceed 600 words in length.

New Evidence for the Existence of a Particle of Mass Intermediate Between the Proton and Electron

Anderson and Neddermyer[1] have shown that for energies

between those of the proton and electron. If this is true, it should be possible to distinguish clearly such a particle from an electron or proton by observing its track density

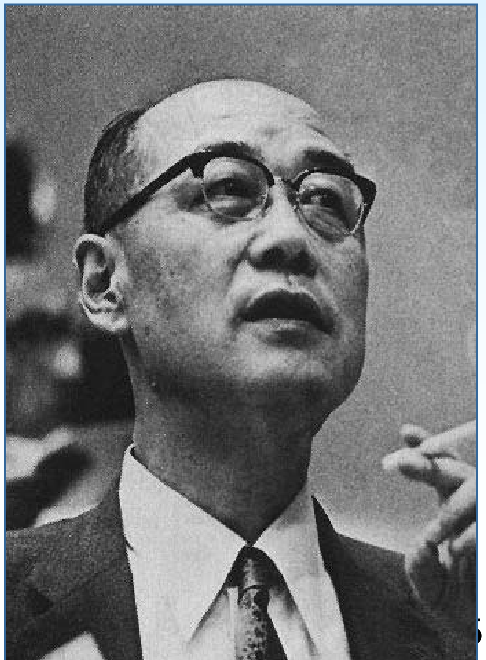
tracks of high energy particles.

J. C. Street
E. C. Stevenson

Research Laboratory of Physics,
Harvard University,
Cambridge, Massachusetts,
October 6, 1937.

[1] Anderson and Neddermeyer, Phys. Rev. **50**, 263 (1936).
[2] Street and Stevenson, Phys. Rev. **51**, 1005 (1937).
[3] Neddermeyer and Anderson, Phys. Rev. **51**, 885 (1937).

It took 10 years to conclude that the muon interacted too weakly with matter to be the "Yukawa" particle which was postulated to carry the nuclear force



Measurement of Magnetic Dipole Moments

ANNALEN DER PHYSIK.

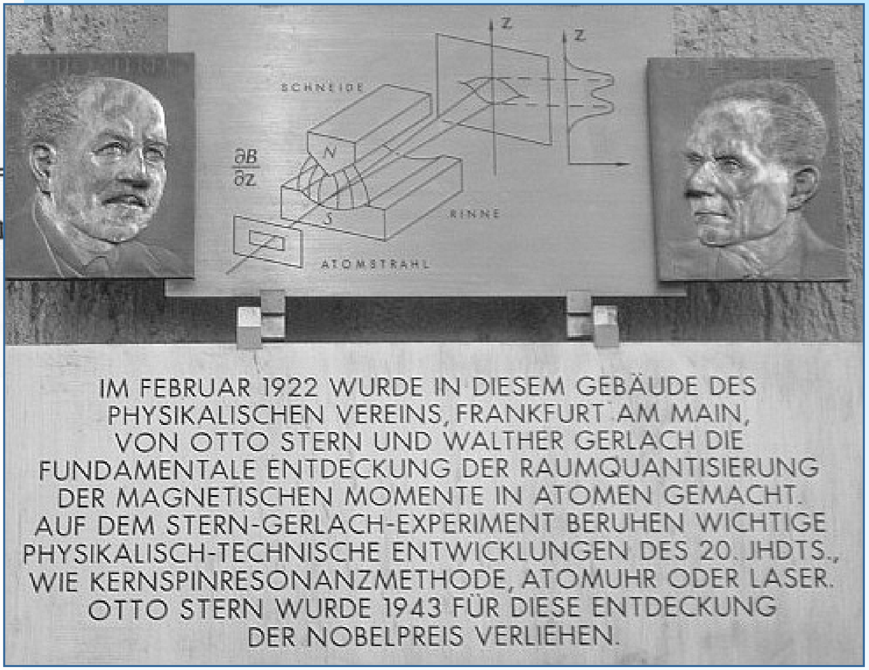
VIERTE FOLGE. BAND 74.

1. *Über die Richtungsquantelung im Magnetfeld*[1]
von Walther Gerlach und Otto Stern.

(Hierzu Tafel III.)

Nr. der Aufnahme	Entfernung des unabgelenkten Strahles von der Schneide	Mittlere Ablenkung des abgestoßenen Strahles	
		berechnet	beobachtet
15	0,32 mm	$0{,}10_1$ mm	$0{,}10_3$ mm
14	0,21 mm	$0{,}14_8$ mm	0,15 mm

Die Genauigkeit der Messungen schätzen wir auf 10 Proz. Innerhalb dieser Fehlergrenzen zeigen also die Versuche, daß das Silberatom im Normalzustand ein Bohrsches Magneton hat.



(in modern language)

(and in English)

$$\vec{\mu}_s = g_s \left(\frac{e\hbar}{2m} \right) \vec{s}$$

$$\Rightarrow g = 2$$

PHYSICAL REVIEW VOLUME 74, NUMBER 3 AUGUST 1, 1948

The Magnetic Moment of the Electron†

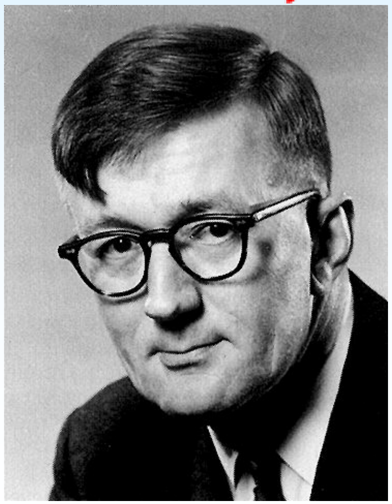
P. KUSCH AND H. M. FOLEY
Department of Physics, Columbia University, New York, New York

(Received April 19, 1948)

A comparison of the g_J values of Ga in the $^2P_{3/2}$ and $^2P_{\frac{1}{2}}$ states, In in the $^2P_{\frac{1}{2}}$ state, and Na in the $^2S_{\frac{1}{2}}$ state has been made by a measurement of the frequencies of lines in the *hfs* spectra in a constant magnetic field. The ratios of the g_J values depart from the values obtained on the basis of the assumption that the electron spin gyromagnetic ratio is 2 and that the orbital electron gyromagnetic ratio is 1. Except for small residual effects, the results can be described by the statement that $g_L=1$ and $g_S=2(1.00119\pm0.00005)$. The possibility that the observed effects may be explained by perturbations is precluded by the consistency of the result as obtained by various comparisons and also on the basis of theoretical considerations.

$$g_L = 1 \text{ and } g_S = 2(1.00119 \pm 0.00005)$$

$$a = \frac{\alpha}{2\pi} = 0.001161$$



First muon spin rotation experiment

Observations of the Failure of Conservation of Parity and Charge Conjugation in Meson Decays: the Magnetic Moment of the Free Muon*

RICHARD L. GARWIN,† LEON M. LEDERMAN, AND MARCEL WEINRICH

Physics Department, Nevis Cyclotron Laboratories, Columbia University, Irvington-on-Hudson, New York, New York

(Received January 15, 1957)

LEE and Yang[1–3] have proposed that the long held space-time principles of invariance under charge conjugation, time reversal, and space reflection (parity) are violated by the "weak" interactions responsible for decay of nuclei, mesons, and strange particles. Their hypothesis, born out of the $\tau-\theta$ puzzle,[4] was accompanied by the suggestion that confirmation should be sought (among other places) in the study of the successive reactions

$$\pi^+ \rightarrow \mu^+ + \nu, \quad (1)$$

$$\mu^+ \rightarrow e^+ + 2\nu. \quad (2)$$

They have pointed out that parity nonconservation implies a polarization of the spin of the muon emitted from stopped pions in (1) along the direction of motion and that furthermore, the angular distribution of electrons in (2) should serve as an analyzer for the muon polarization. They also point out that the longitudinal

VIII. Negative muons stopped in carbon show an asymmetry (also leaked backwards) of $a \sim -1/20$, i.e., about 15% of that for μ^+.

IX. The magnetic moment of the μ^-, bound in carbon, is found to be negative and agrees within limited accuracy with that of the μ^+.[8]

X. Large asymmetries are found for the e^+ from polarized μ^+ beams stopped in polyethylene and calcium. Nuclear emulsion (as a target in Fig. 1) yields an asymmetry of about half that observed in carbon.

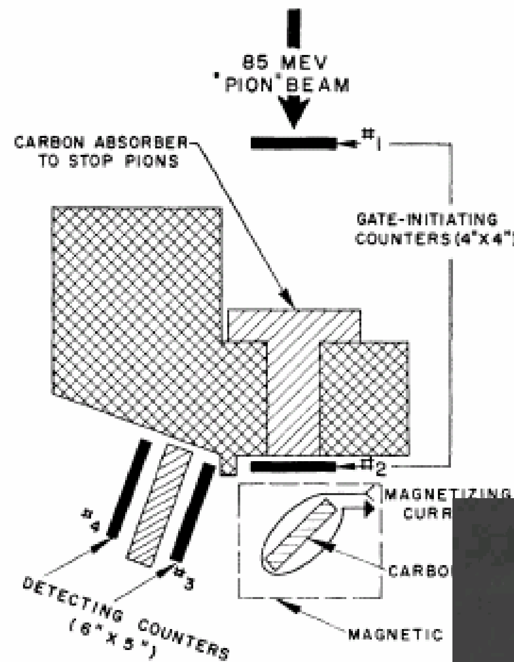


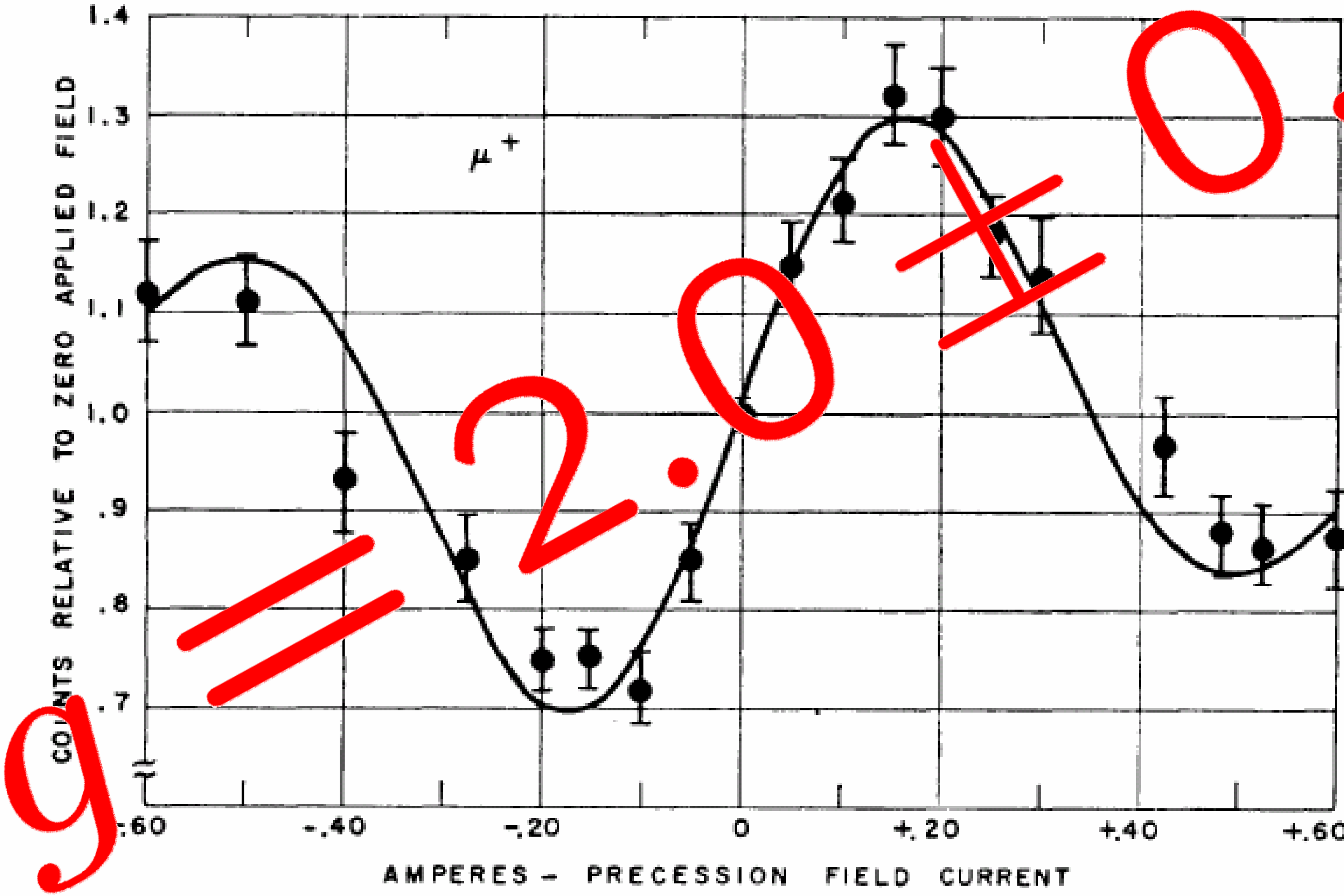
FIG. 1. Experimental arrangement. The ma close wound directly on the carbon to provide field of 79 gauss per ampere.

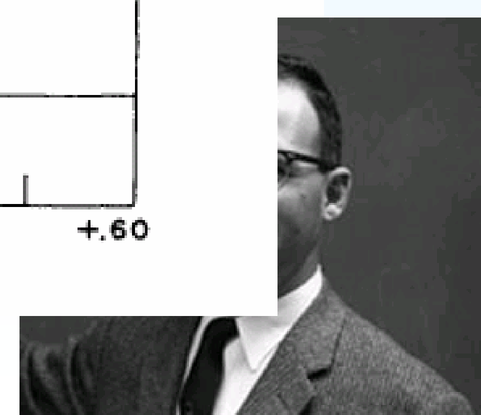
First muon spin rotation experiment

Observations of the Failure of Conservation of Parity and Charge Conjugation in

VIII. Negative muons stopped in carbon show an asymmetry (also leaked backwards) of $a \sim -1/20$, i.e., about 15% of that for μ^+.

IX. The magnetic moment of the μ^-, bound in carbon, is found to be negative and agrees with





PHYSICAL REVIEW VOLUME 118, NUMBER 1 APRIL 1, 1960

Accurate Determination of the μ^+ Magnetic Moment*

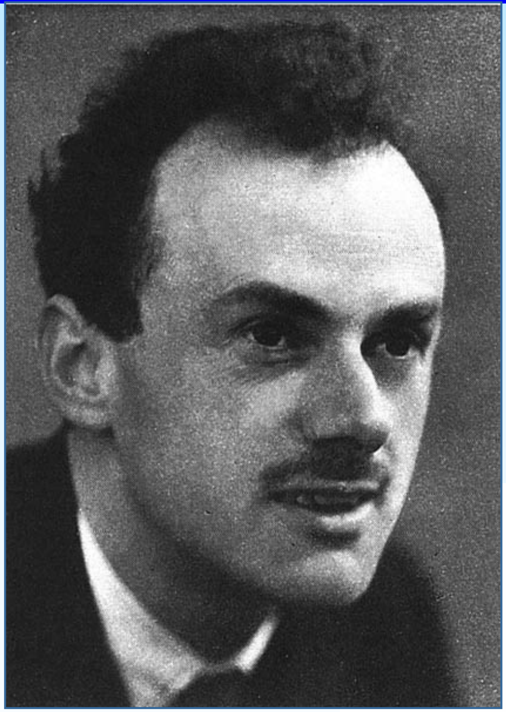
R. L. GARWIN,† D. P. HUTCHINSON, S. PENMAN,‡ AND G. SHAPIRO§
Columbia University, New York, New York

(Received August 4, 1959)

Note added in proof.—Experiments which have recently been reported to us [J. Lathrop, et al. and A. Bearden et al., Phys. Rev. Letters (to be published)] indicate a mass value of $M_\mu = 206.76_{-0.02}^{+0.03} M_e$. This yields a value of $g_\mu = 2(1.00113_{-0.00012}^{+0.00016})$. Although the assigned errors are now slightly greater than above, it is to be noted that the new result represents a direct measurement, rather than a lower limit. The agreement

$$a = \frac{\alpha}{2\pi} = 0.001161$$

Theory of Magnetic and Electric Dipole Moments



The Quantum Theory of the Electron.

By P. A. M. Dirac, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received January 2, 1928.)

§ 4. *The Hamiltonian for an Arbitrary Field.*

To obtain the Hamiltonian for an electron in an electromagnetic field with scalar potential A_0 and vector potential $\mathbf{A}$, we adopt the usual procedure of substituting $p_0 + e/c \,.\, A_0$ for p_0 and $\mathbf{p} + e/c \,.\, \mathbf{A}$ for $\mathbf{p}$ in the Hamiltonian for no field. From equation (9) we thus obtain

$$\left[p_0 + \frac{e}{c}A_0 + \rho_1\left(\boldsymbol{\sigma}, \mathbf{p} + \frac{e}{c}\mathbf{A}\right) + \rho_3 mc\right]\psi = 0. \tag{14}$$

This differs from (1) by the two extra terms

$$\frac{eh}{c}(\boldsymbol{\sigma}, \mathbf{H}) + \frac{ieh}{c}\rho_1(\boldsymbol{\sigma}, \mathbf{E})$$

in F. These two terms, when divided by the factor $2m$, can be regarded as the additional potential energy of the electron due to its new degree of freedom. The electron will therefore behave as though it has a magnetic moment $eh/2mc \,.\, \boldsymbol{\sigma}$ and an electric moment $ieh/2mc \,.\, \rho_1\boldsymbol{\sigma}$. This magnetic moment is just that assumed in the spinning electron model. The electric moment, being a pure imaginary, we should not expect to appear in the model. It is doubtful whether

Schwinger

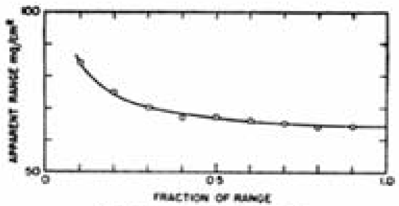


FIG. 2. Feather plot for Ca^{45}.

12,000 counts per minute, and the contribution due to gamma-rays and other unabsorbed contaminants was less than one part in 3000 with the strongest source, thus indicating the absence of any appreciable amount of gamma-radiation. The absorption curve obtained with the strongest source is shown in Fig. 1. The Feather plot, shown in Fig. 2, gives a range of 64 ± 1 mg/cm².

Glendenin[4] has shown that a reliable range-energy curve for the low energy region can be derived from the data of Marshall and Ward[5] for monoenergetic electrons and beta-ray spectrograph data on low energy beta-emitters. Glendenin's curve is identical with that of Marshall and Ward below 0.5 Mev. Using this range-energy curve, we have found that the Ca^{45} beta-radiation has a maximum energy of 260 ± 5 kev. We have found no evidence of any harder beta-radiation, or of any gamma-radiation at all in the course of this investigation.[6]

Acknowledgments.—This work has been supported with funds from the Office of Naval Research. The authors wish to express their appreciation to Miss Jacqueline Becker for her assistance in making the counts.

1 Walke, Thompson, and Holt, Phys. Rev. 57, 171 (1940).
2 Solomon, Gould, and Anfinsen, Phys. Rev. 72, 1097 (1947).
3 Feather, Proc. Camb. Phil. Soc. 35, 599 (1938).
4 Glendenin, Nucleonics, in press for January, 1948.
5 Marshall and Ward, Can. J. Research 15, 39 (1939).
6 This result is in good agreement with a value of 250 kev, given in *Radioisotopes, Catalog and Price List No. 2*, revised September, 1947, distributed by Isotopes Branch, United States Atomic Energy Commission. Unfortunately, the Atomic Energy Commission's result is not supported by any published experimental evidence.

On Quantum-Electrodynamics and the Magnetic Moment of the Electron

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts
December 30, 1947

ATTEMPTS to evaluate radiative corrections to electron phenomena have heretofore been beset by divergence difficulties, attributable to self-energy and vacuum polarization effects. Electrodynamics unquestionably requires revision at ultra-relativistic energies, but is presumably accurate at moderate relativistic energies. It would be desirable, therefore, to isolate those aspects of the current theory that essentially involve high energies, and are subject to modification by a more satisfactory theory, from aspects that involve only moderate energies and are thus relatively trustworthy. This goal has been achieved by transforming the Hamiltonian of current hole theory electrodynamics to exhibit explicitly the logarithmically divergent self-energy of a free electron, which arises from the virtual emission and absorption of light quanta. The electromagnetic self-energy of a free electron can be ascribed to an electromagnetic mass, which must be added to the mechanical mass of the electron. Indeed, the only meaningful statements of the theory involve this combination of masses, which is the experimental mass of a free electron. It might appear, from this point of view, that the divergence of the electromagnetic mass is unobjectionable, since the individual contributions to the experimental mass are unobservable. However, the transformation of the Hamiltonian is based on the assumption of a weak interaction between matter and radiation, which requires that the electromagnetic mass be a small correction ($\sim(e^2/\hbar c)m_0$) to the mechanical mass m_0.

The new Hamiltonian is superior to the original one in essentially three ways: it involves the experimental electron mass, rather than the unobservable mechanical mass; an electron now interacts with the radiation field only in the presence of an external field, that is, only an accelerated electron can emit or absorb a light quantum;* the interaction energy of an electron with an external field is now subject to a *finite* radiative correction. In connection with the last point, it is important to note that the inclusion of the electromagnetic mass with the mechanical mass does not avoid all divergences; the polarization of the vacuum produces a logarithmically divergent term proportional to the interaction energy of the electron in an external field. However, it has long been recognized that such a term is equivalent to altering the value of the electron charge by a constant factor, only the final value being properly identified with the experimental charge. Thus the interaction between matter and radiation produces a renormalization of the electron charge and mass, all divergences being contained in the renormalization factors.

The simplest example of a radiative correction is that for the energy of an electron in an external magnetic field. The detailed application of the theory shows that the radiative correction to the magnetic interaction energy corresponds to an additional magnetic moment associated with the electron spin, of magnitude $\delta\mu/\mu=(\frac{1}{2}\pi)e^2/\hbar c=0.001162$. It is indeed gratifying that recently acquired experimental data confirm this prediction. Measurements on the hyperfine splitting of the ground states of atomic hydrogen and deuterium[1] have yielded values that are definitely larger than those to be expected from the directly measured nuclear moments and an electron moment of one Bohr magneton. These discrepancies can be accounted for by a small additional electron spin magnetic moment.[2] Recalling that the nuclear moments have been calibrated in terms of the electron moment, we find the additional moment necessary to account for the measured hydrogen and deuterium hyperfine structures to be $\delta\mu/\mu=0.00126\pm0.00019$ and $\delta\mu/\mu=0.00131\pm0.00025$, respectively. These values are not in disagreement with the theoretical prediction. More precise conformation is provided by measurement of the g values for the $^2S_{\frac{1}{2}}$, $^2P_{\frac{1}{2}}$, and $^2P_{3/2}$ states of sodium and gallium.[3] To account for these results, it is necessary to ascribe the following additional spin magnetic moment to the electron, $\delta\mu/\mu=0.00118\pm0.00003$.

$$\delta\mu/\mu = \frac{\alpha}{2\pi} = 0.001161$$

The new Hamiltonian is superior to the original one in essentially three ways:

- it involves the experimental electron mass, rather than the unobservable mechanical mass;
- an electron now interacts with the radiation field only in the presence of an external field…
- the interaction of an electron with an external field is now subject to a *finite* radiative correction.

Schwinger

$$\delta\mu/\mu = \frac{\alpha}{2\pi} = 0.001161$$

It is indeed gratifying that recently acquired experimental data confirm this prediction.

These discrepancies can be accounted for by a small additional electron spin magnetic moment.

On Quantum-Electrodynamics and the Magnetic Moment of the Electron

Julian Schwinger
Harvard University, Cambridge, Massachusetts
December 30, 1947

ATTEMPTS to evaluate radiative corrections to electron phenomena have heretofore been beset by divergence difficulties, attributable to self-energy and vacuum polarization effects. Electrodynamics unquestionably requires revision at ultra-relativistic energies, but is presumably accurate at moderate relativistic energies. It would be desirable, therefore, to isolate those aspects of the current theory that essentially involve high energies, and are subject to modification by a more satisfactory theory, from aspects that involve only moderate energies and are thus relatively trustworthy. This goal has been achieved by transforming the Hamiltonian of current hole theory electrodynamics to exhibit explicitly the logarithmically divergent self-energy of a free electron, which arises from

…corresponds to an additional magnetic moment associated with the electron spin, of magnitude $\delta\mu/\mu=(\frac{1}{2\pi})e^2/\hbar c = 0.001162$. It is indeed gratifying that recently acquired experimental data confirm this prediction. Measurements on the hyperfine splitting of the ground states of atomic hydrogen and deuterium[1] have yielded values that are definitely larger than those to be expected from the directly measured nuclear moments and an electron moment of one Bohr magneton. These discrepancies can be accounted for by a small additional electron spin magnetic moment.[2] Recalling that the nuclear moments have been calibrated in terms of the electron moment, we find the additional moment necessary to account for the measured hydrogen and deuterium hyperfine structures to be $\delta\mu/\mu=0.00126\pm0.00019$ and $\delta\mu/\mu=0.00131\pm0.00025$, respectively. These values are not in disagreement with the theoretical prediction. More precise conformation is provided by measurement of the g values for the $^2S_{\frac{1}{2}}$, $^2P_{\frac{1}{2}}$, and $^2P_{3/2}$ states of sodium and gallium.[3] To account for these results, it is necessary to ascribe the following additional spin magnetic moment to the electron, $\delta\mu/\mu=0.00118\pm0.00003$.

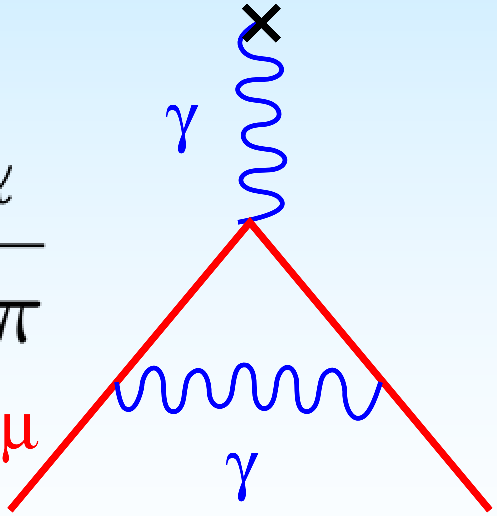
The magnetic dipole moment directed along spin.

$$\vec{\mu}_s = g_s \left(\frac{e\hbar}{2m}\right) \vec{s}$$ **Dirac Theory: g_s = 2**

$$\mu = (1+a)\frac{e\hbar}{2m}$$ Dirac + Pauli moment $$a = \frac{g-2}{2}$$

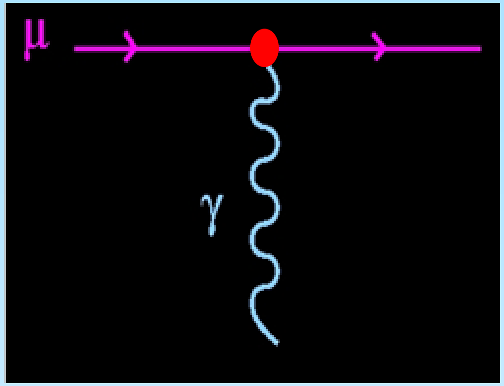
The Schwinger term dominates the value of a

$$a = \frac{\alpha}{2\pi}$$



e vrs. μ : relative contribution of heavier things $$\left(\frac{m_\mu}{m_e}\right)^2 \simeq 42,000$$

Modern notation: Magnetic Dipole Moment:



chiral changing

$$\Gamma_\mu = eF_1\ \bar{\psi}_R \gamma_\mu \psi_R + \frac{ie}{2m} F_2\ \bar{\psi}_R \sigma_{\mu\nu} q^\nu \psi_L$$

$$F_1(0) = 1 \quad F_2(0) = a_\mu$$

Spin Motion in a Magnetic Field

Momentum turns with ω_C, cyclotron frequency

Spin turns with ω_S

$$\omega_C = \frac{eB}{mc\gamma} \qquad \omega_S = \frac{geB}{2mc} + (1-\gamma)\frac{eB}{\gamma mc}$$

Spin turns relative to the momentum with ω_a

$$\omega_a = \omega_S - \omega_C = \left(\frac{g-2}{2}\right)\frac{eB}{mc} = a\frac{eB}{mc}$$

We measure the difference frequency, ω_a, between the spin and momentum precession

With an electric quadrupole field for vertical focusing

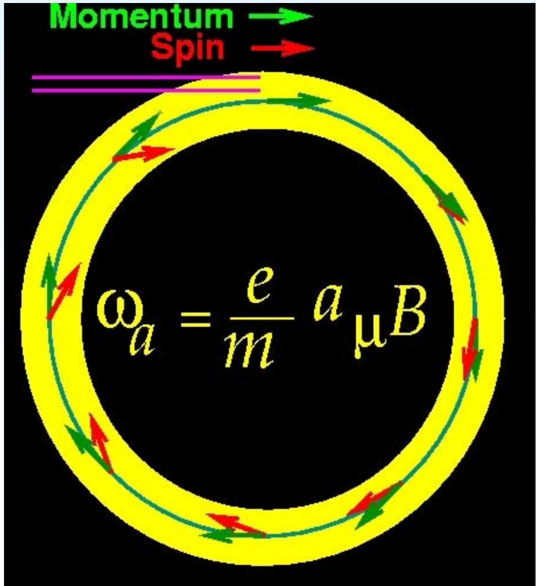
$$\vec{\omega}_a = -\frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta} \times \vec{E}}{c}\right]$$

0

$$B \Rightarrow \langle B \rangle_{\mu-\text{dist}}$$

$$\gamma_{\text{magic}} = 29.3$$

$$p_{\text{magic}} = 3.09 \text{ GeV/c}$$



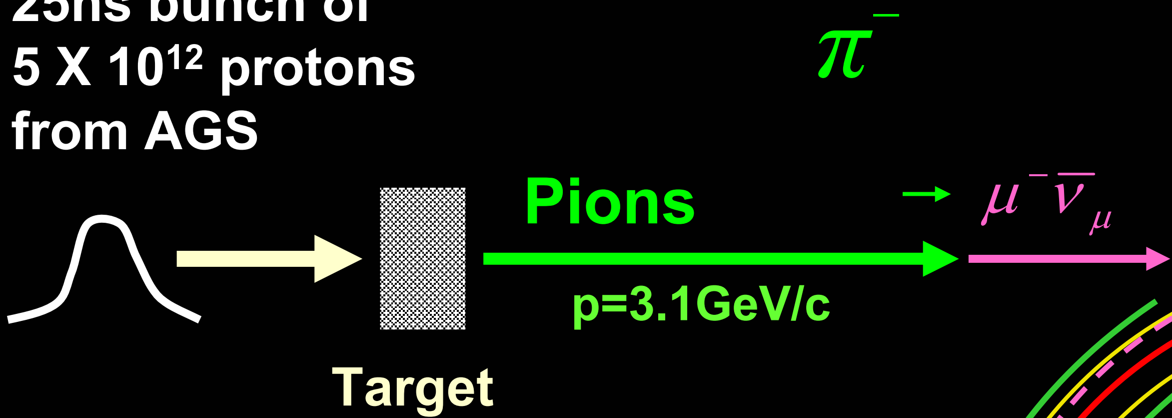
Experimental Technique

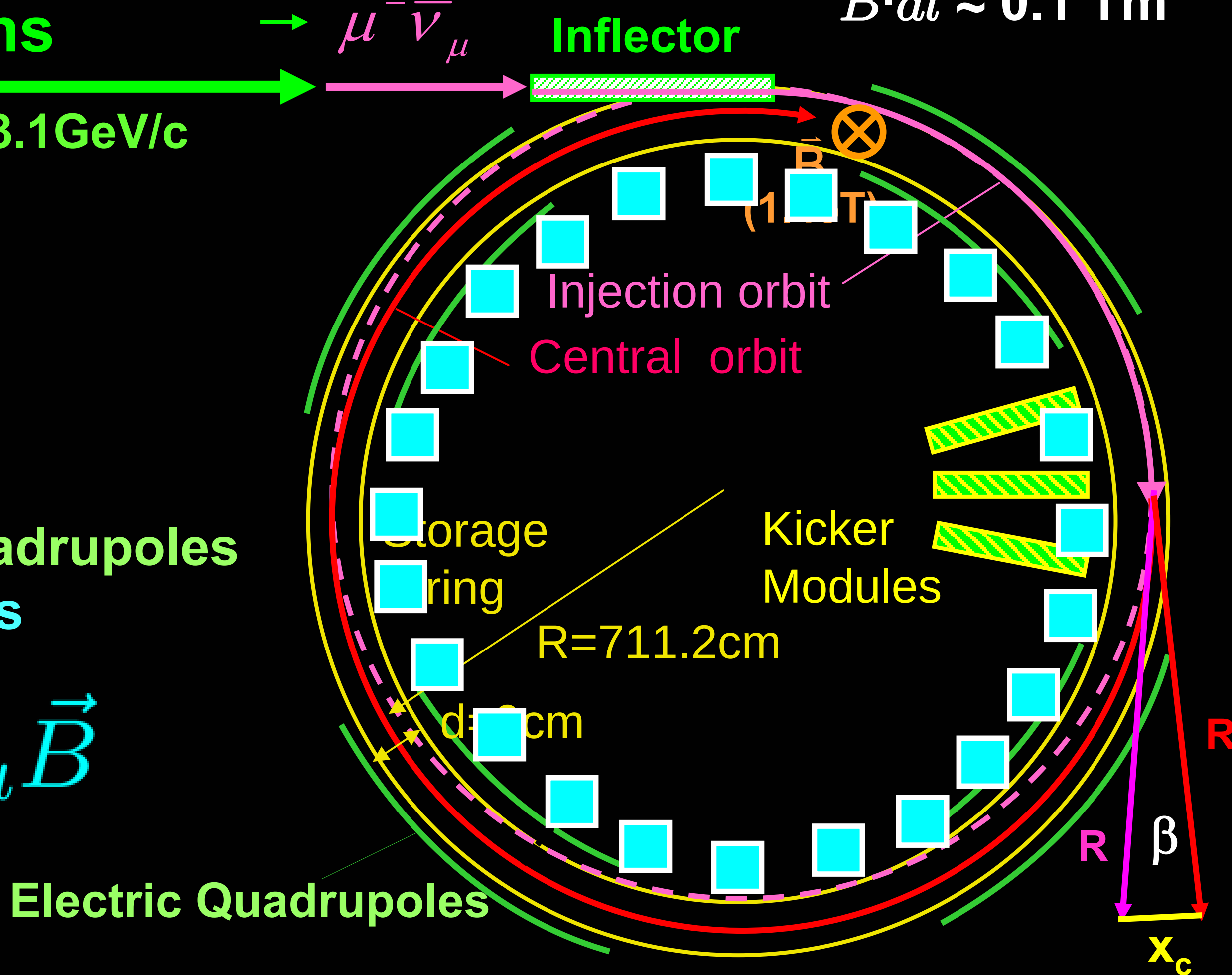
25ns bunch of
5 X 10^{12} protons
from AGS

$x_c \approx 77$ mm

$\beta \approx 10$ mrad

$B \cdot dl \approx 0.1$ Tm



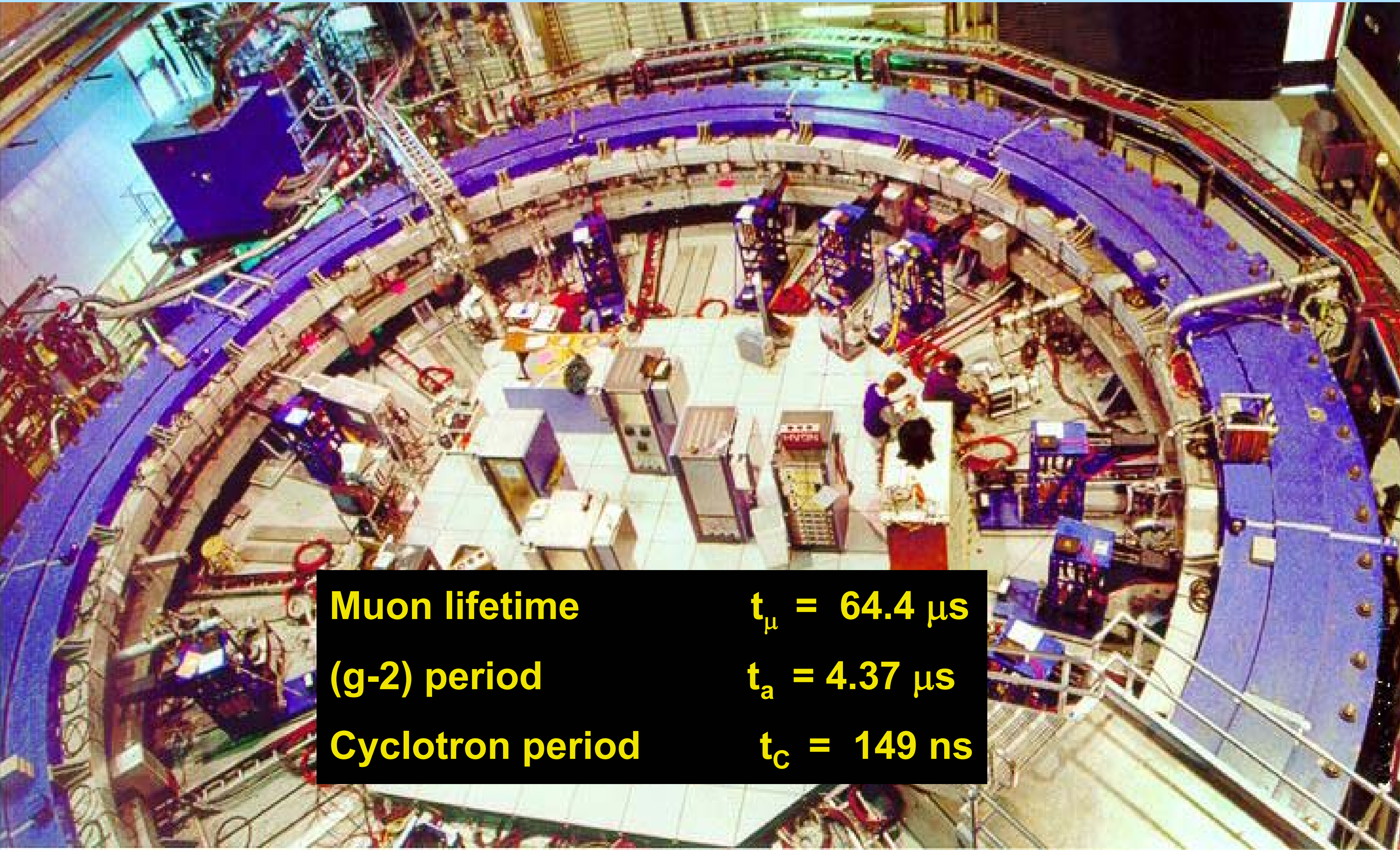


- Muon polarization
- Muon storage ring
- injection & kicking
- focus with Electric Quadrupoles
- 24 electron calorimeters

$$\vec{\omega}_a = -\frac{e}{m} a_\mu \vec{B}$$

(thanks to Q. Peng)

muon (g-2) storage ring

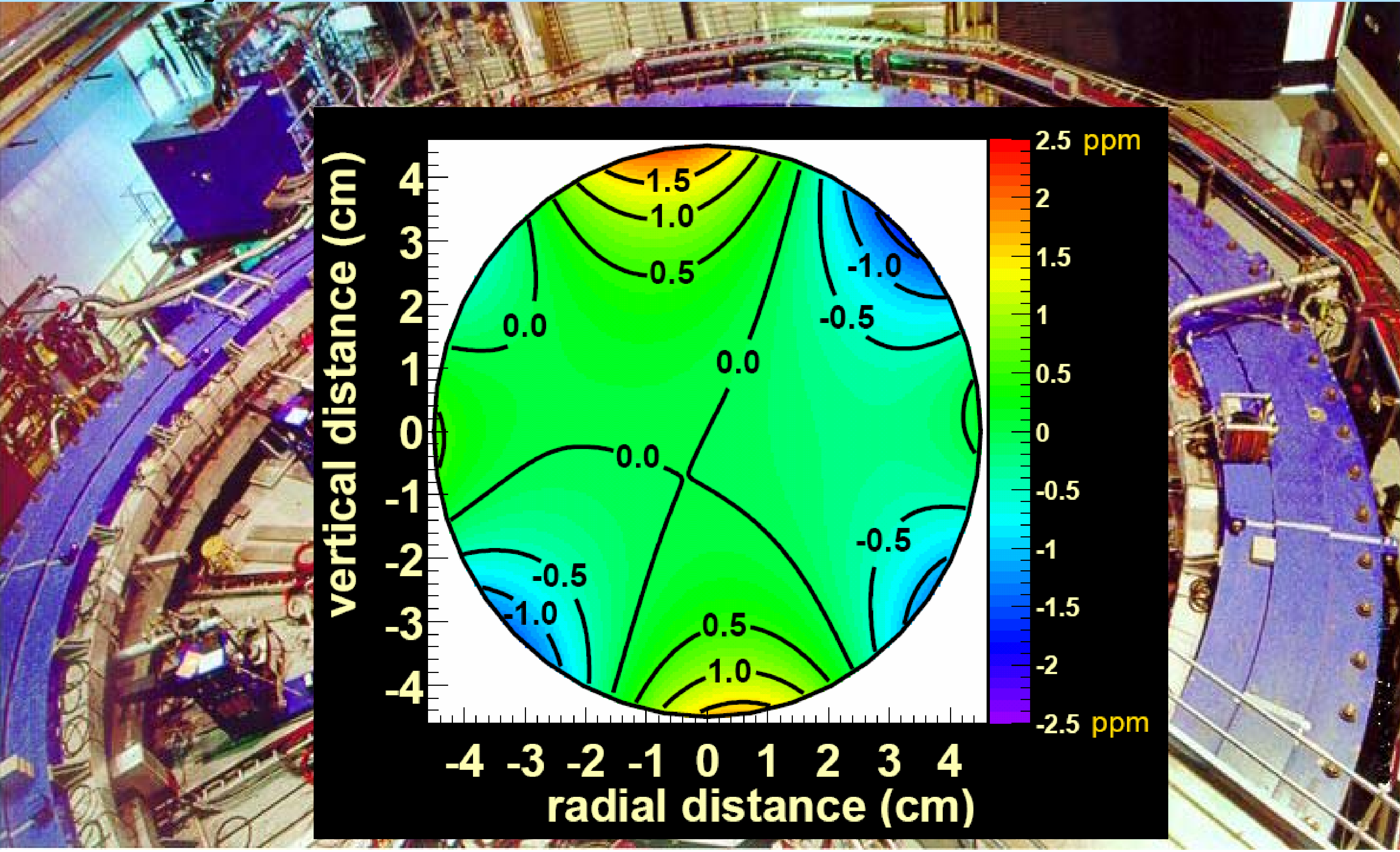


Muon lifetime $t_{\mu} = 64.4\ \mu s$

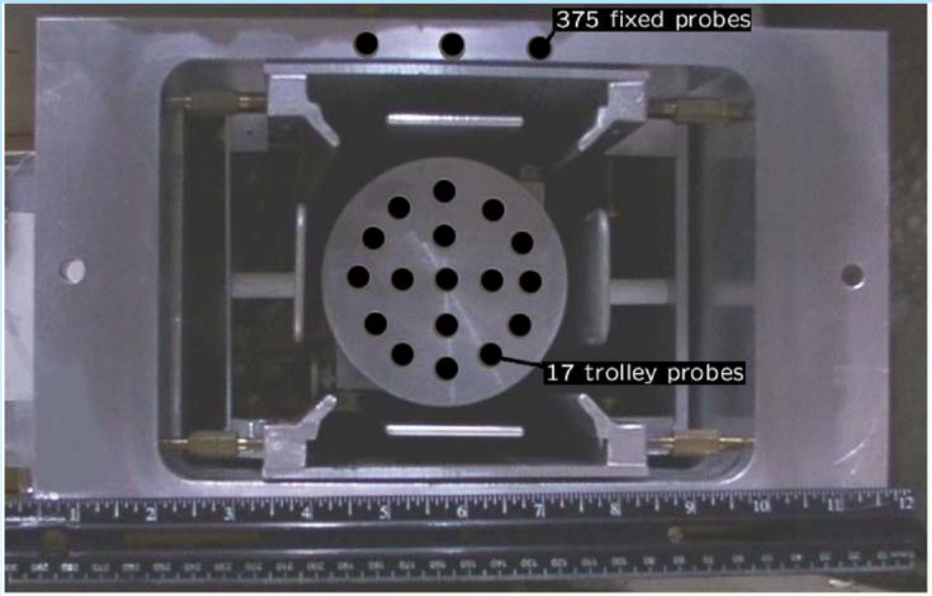
(g-2) period $t_{a} = 4.37\ \mu s$

Cyclotron period $t_{C} = 149$ ns

Field averaged over azimuth in the storage ring (0.5ppm contours)



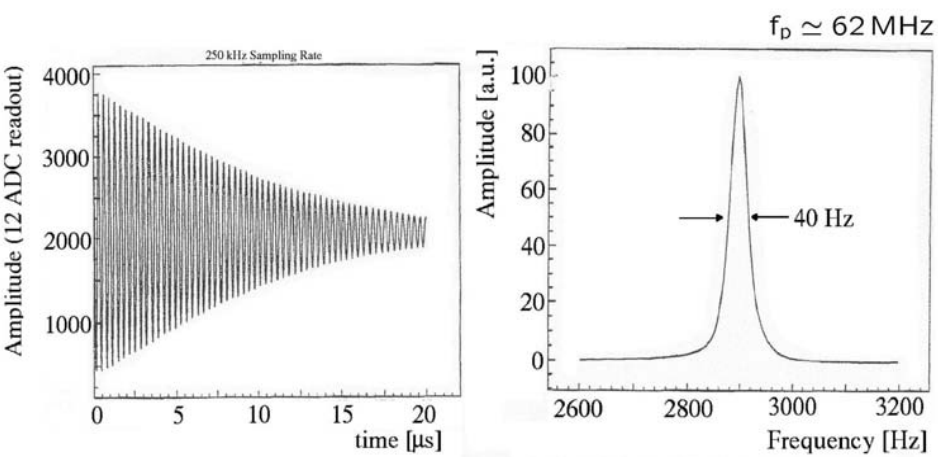
The magnetic field is measured and controlled using pulsed NMR and the free-induction decay.



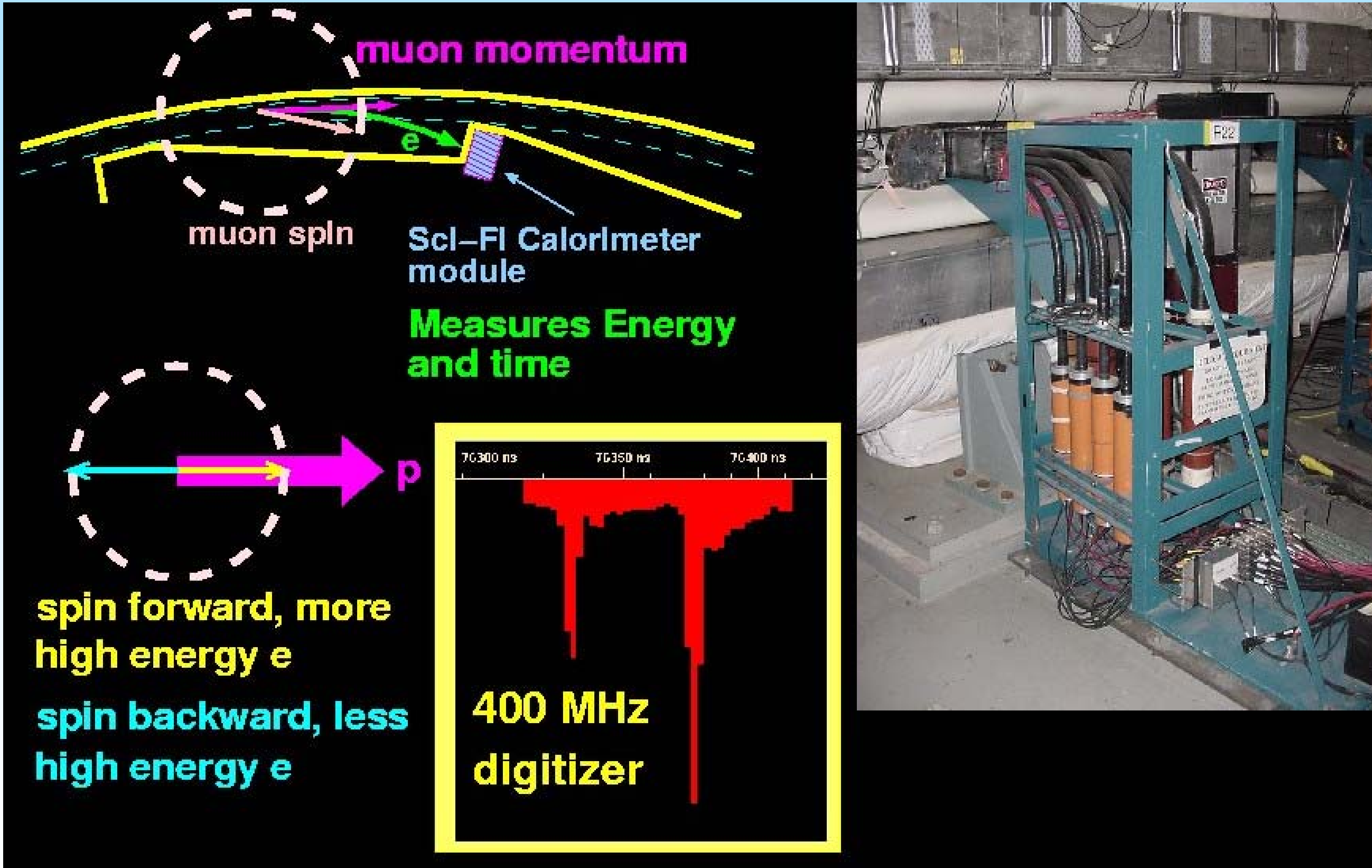
- Calibration to a spherical water sample that ties the field to the Larmor frequency of the free proton ω_p.
- So we measure ω_a and ω_p

$$a_\mu = \frac{\frac{\omega_a}{\omega_p}}{\frac{\mu_\mu}{\mu_p} - \frac{\omega_a}{\omega_p}}$$

Free induction decay signals:



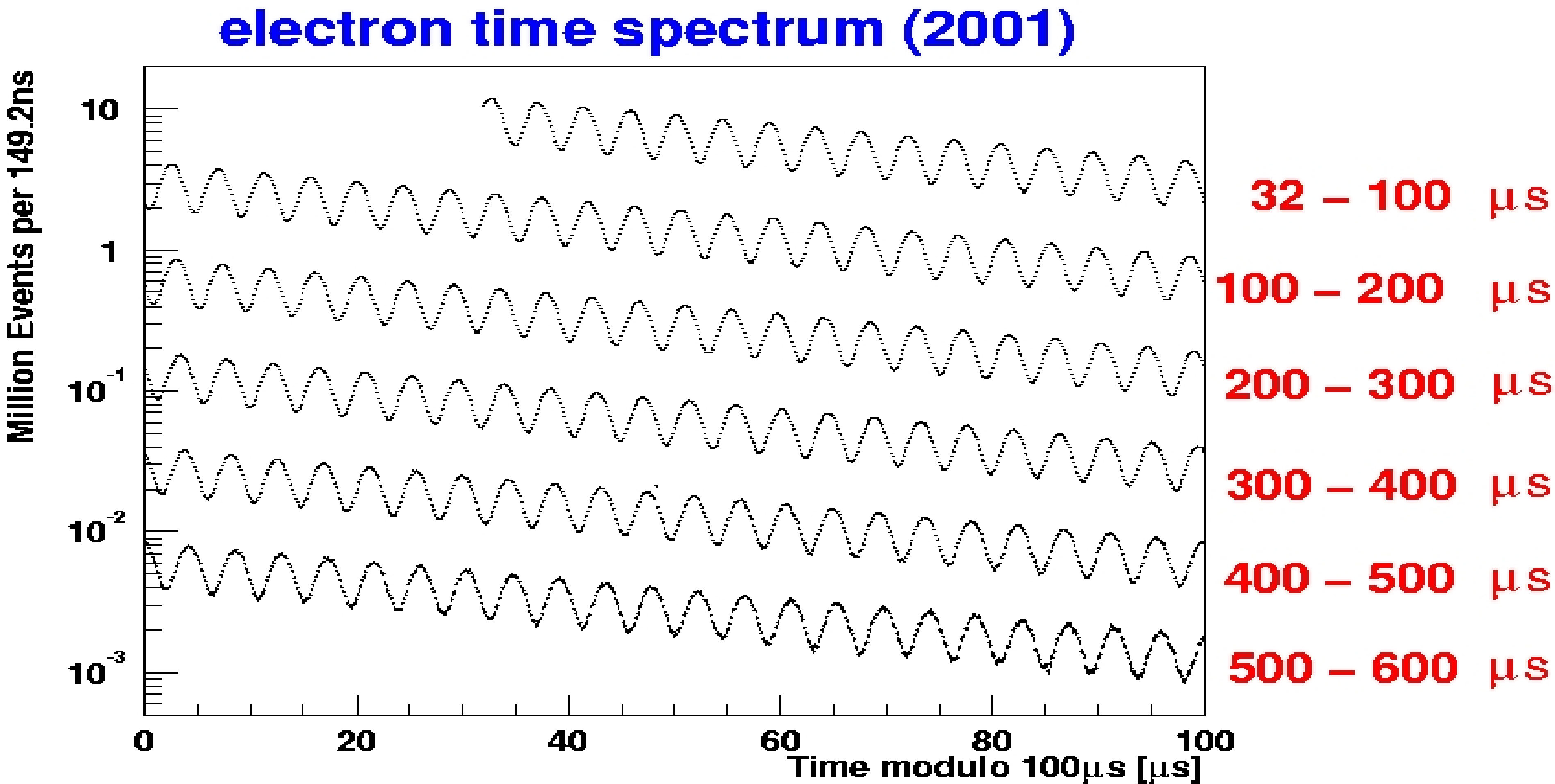
Detectors and vacuum chamber



We count high-energy electrons as a function of time.

$$4 \times 10^9 \quad e,\ E_{e^-} \geq 1.8\ \text{GeV}$$

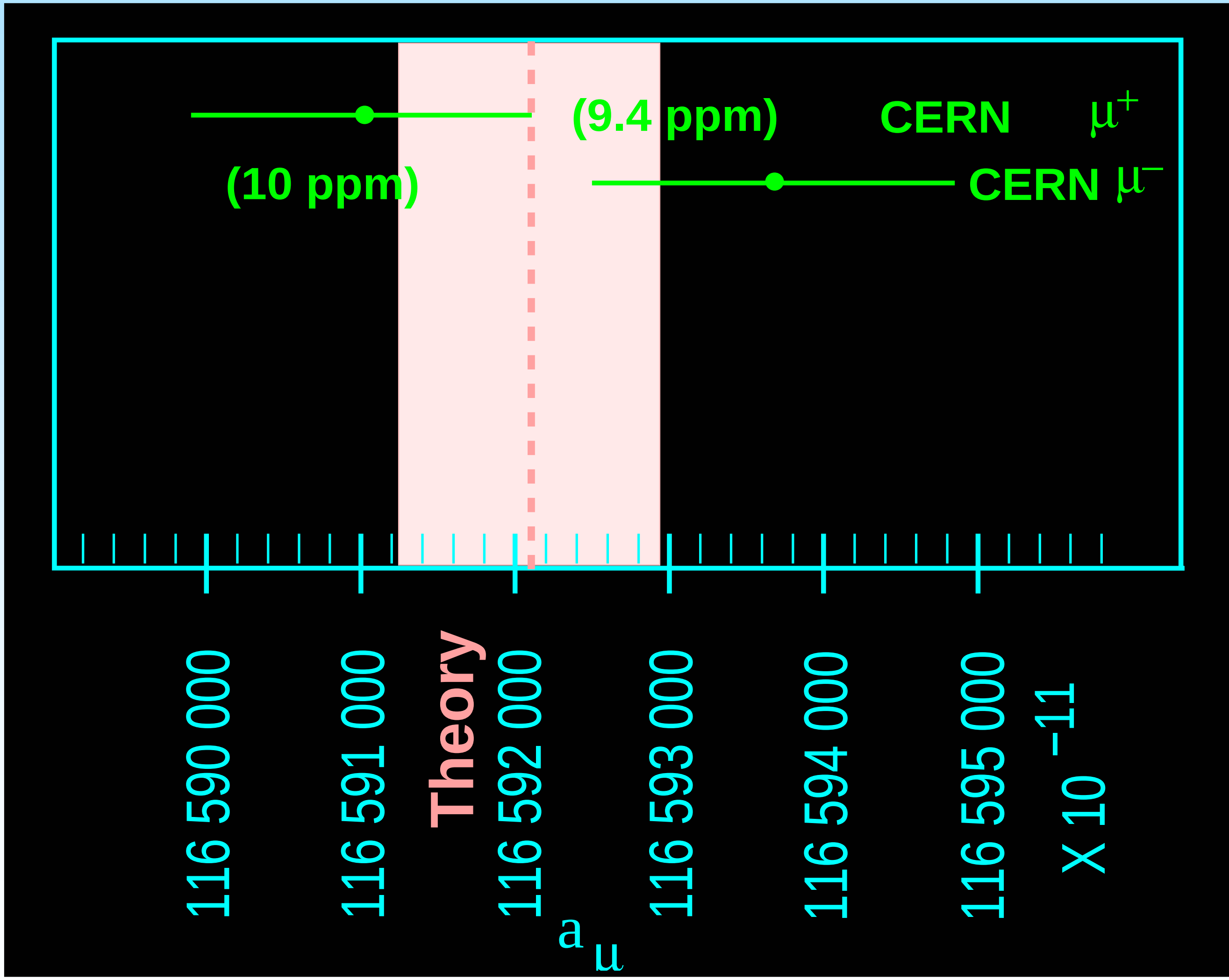
$$f(t) \simeq N_0 e^{-\lambda t}[1 + A\cos\omega_a t + \phi)]$$



When we started in 1983, theory and experiment were known to about 10 ppm.

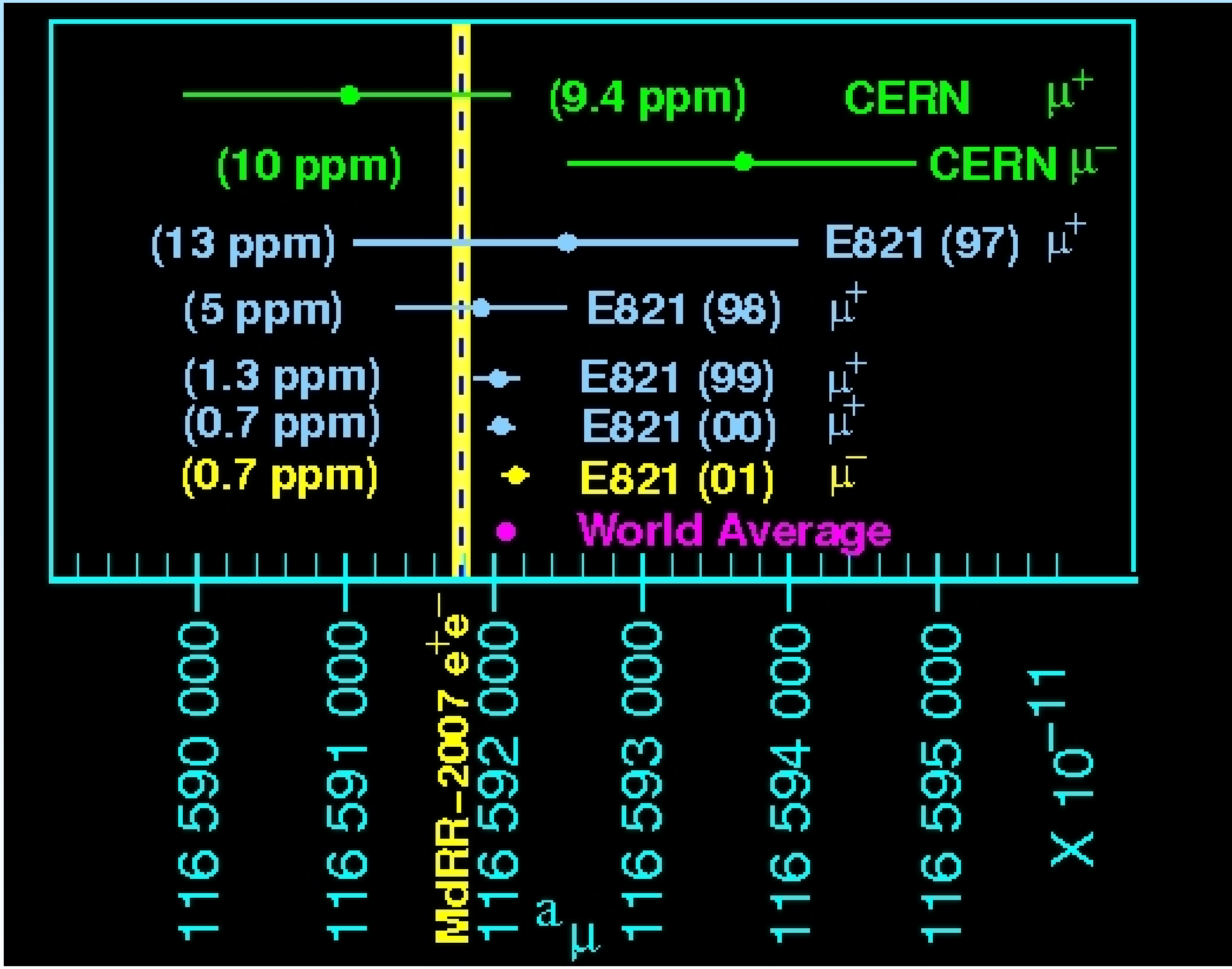
Theory uncertainty was ~ 9 ppm

Experimental uncertainty was 7.3 ppm



E821 achieved 0.5 ppm and the e^+e^- based theory is also at the 0.6 ppm level. Difference is 3.4σ

MdRR=Miller, de Rafael, Roberts, Rep. Prog. Phys. **70** (2007) 795



$$\Delta a_\mu^{(\text{today})} = (29.5 \pm 8.8) \times 10^{-10}$$

Today:

- Muon (g-2), with a precision of 0.5 ppm, has a ~3.4 σ discrepancy with the standard model, using the e^+e^- data.
  - Upgrade, E969 or otherwise, waits for funding

To be continued in the next talk by Dave Hertzog