Theory of the muon g2: QED and beyond

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The present experimental values:

$$a_e = 1159652180.85\ (76) \times 10^{-12}$$

Odom et al., PRL97 (2006) 030801

$$a_\mu = 116592080\ (63) \times 10^{-11}$$

E821 – Final Report: PRD73 (2006) 072003

0.7 parts per billion for a_e

0.5 parts per million for a_μ

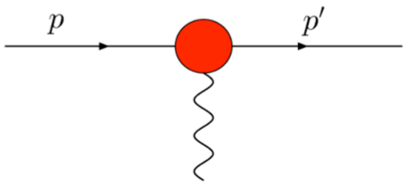
- The Dirac theory predicts for a lepton l=e,μ,τ:

$$\vec{\mu}_l = g_l \left(\frac{e}{2m_l c}\right) \vec{s} \qquad g_l = 2$$

- QFT predicts deviations from the Dirac value:

$$g_l = 2\,(1 + a_l)$$

- Study the photon-lepton vertex:



$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m} F_2(q^2) + \ldots\right] u(p)$$

$$F_1(0) = 1 \qquad F_2(0) = a_l$$

The QED contribution to a_μ

a_μ^{QED} = $(1/2)(\alpha/\pi)$ Schwinger 1948

+ $0.765857410\ (27)\ (\alpha/\pi)^2$

Sommerfield; Petermann; Suura & Wichmann '57; Elend '66; MP '04

+ $24.05050964\ (43)\ (\alpha/\pi)^3$

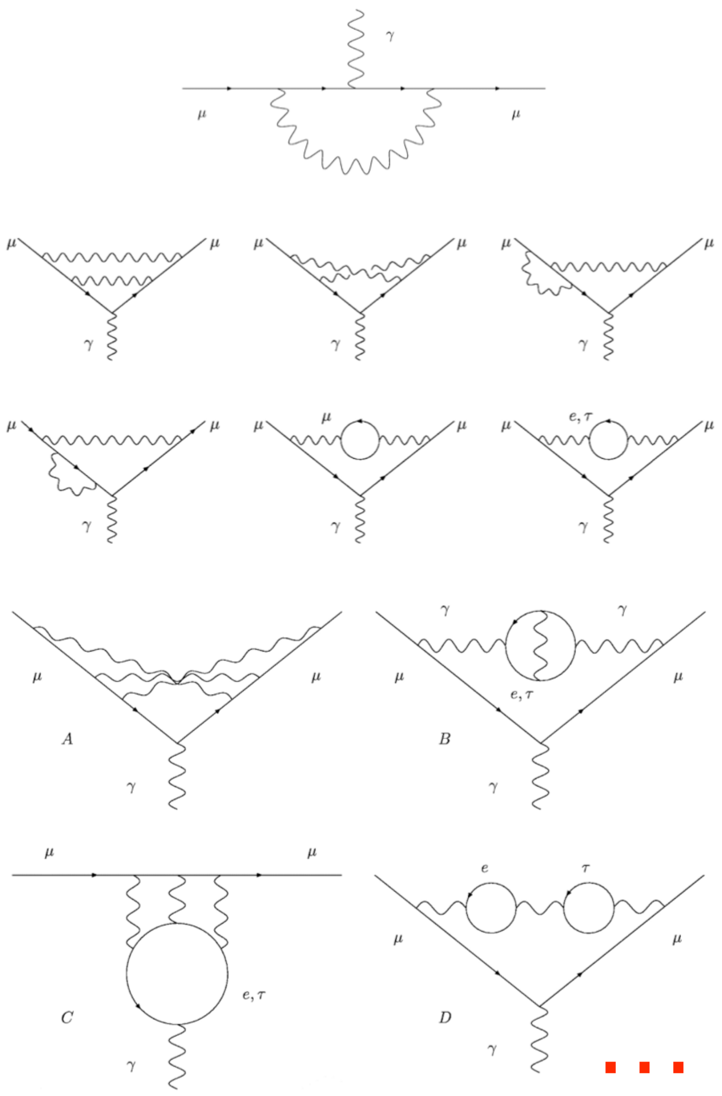
Barbieri, Lautrup, de Rafael, Laporta, Remiddi; … Czarnecki, Skrzypek; MP '04; Friot, Greynat, de Rafael '05

+ $130.810\ (8)\ (\alpha/\pi)^4$ Revised!

Kinoshita & Lindquist '81, … , Kinoshita & Nio '04, '05; Aoyama, Hayakawa, Kinoshita & Nio, June 2007

+ $663\ (20)\ (\alpha/\pi)^5$ In progress: see Hayakawa's talk

Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta, Karshenboim,…, Kataev, Kinoshita & Nio March '06.



Adding up, we get:

a_μ^{QED} = 116584718.10 (14) (08) x 10^{-11}

mainly from 5-loop unc ↵ ↳ from new $\delta\alpha$

with α = 1/137.035999068 (96) [0.7 ppb]

[The electron g-2…

a_e^{SM} = $(1/2)(\alpha/\pi)$ - $0.328\ 478\ 444\ 002\ 90(60)\ (\alpha/\pi)^2$

Schwinger 1948 Sommerfield; Petermann; Suura & Wichmann '57; Elend '66; MP '06

$A_2^{(4)}\ (m_e/m_\mu) = 5.197\ 386\ 70\ (28) \times 10^{-7}$

$A_2^{(4)}\ (m_e/m_\tau) = 1.837\ 62\ (60) \times 10^{-9}$

+ $1.181\ 234\ 016\ 827\ (19)\ (\alpha/\pi)^3$

Kinoshita, Barbieri, Laporta, Remiddi, … , Li, Samuel; Mohr & Taylor '05; MP '06

$A_2^{(6)}\ (m_e/m_\mu) = -7.373\ 941\ 64\ (29) \times 10^{-6}$

$A_2^{(6)}\ (m_e/m_\tau) = -6.5819\ (19) \times 10^{-8}$

$A_3^{(6)}\ (m_e/m_\mu,\ m_e/m_\tau) = 1.909\ 45\ (62) \times 10^{-13}$

- ~~1.7283~~ $(35)\ (\alpha/\pi)^4$ New revised value: -1.9144 (35)

Kinoshita & Lindquist '81, … , Kinoshita & Nio '05; Aoyama, Hayakawa, Kinoshita & Nio, June '07

+ $0.0\ (3.8)\ (\alpha/\pi)^5$ In progress (12672 diagrams!)

Mohr & Taylor '05; Aoyama, Hayakawa, Kinoshita & Nio, in progress.

+ $1.671\ (19) \times 10^{-12}$ Hadronic

Mohr & Taylor '05; Davier & Hoecker '98, Krause '97, Knecht '03

+ $0.0297\ (5) \times 10^{-12}$ Electroweak

Mohr & Taylor '05; Czarnecki, Krause, Marciano '96

... and the best determination of alpha]

- The new measurement of the electron g-2 is:

  a_e^{exp} = 1159652180.85 (76) x 10^{-12} Odom et al, PRL97 (2006) 030801

  vs. old (factor of 6 improvement, 1.7σ difference):

  a_e^{exp} = 1159652188.3 (4.2) x 10^{-12} Van Dyck et al, PRL59 (1987) 26

  Equating $a_e^{SM}(\alpha) = a_e^{exp}$ → best determination of alpha to date:

  α^{-1} = 137.035 999 ~~709~~ (12)(30)(2)(90) [0.7ppb]   Gabrielse et al, '06; MP '06

  ~~709~~ → 068; (12) → δC_4^{qed}; (30) → δC_5^{qed}; (2) → δa_e^{had}; (90) → δa_e^{exp}

- Compare it with other determinations (independent of a_e):

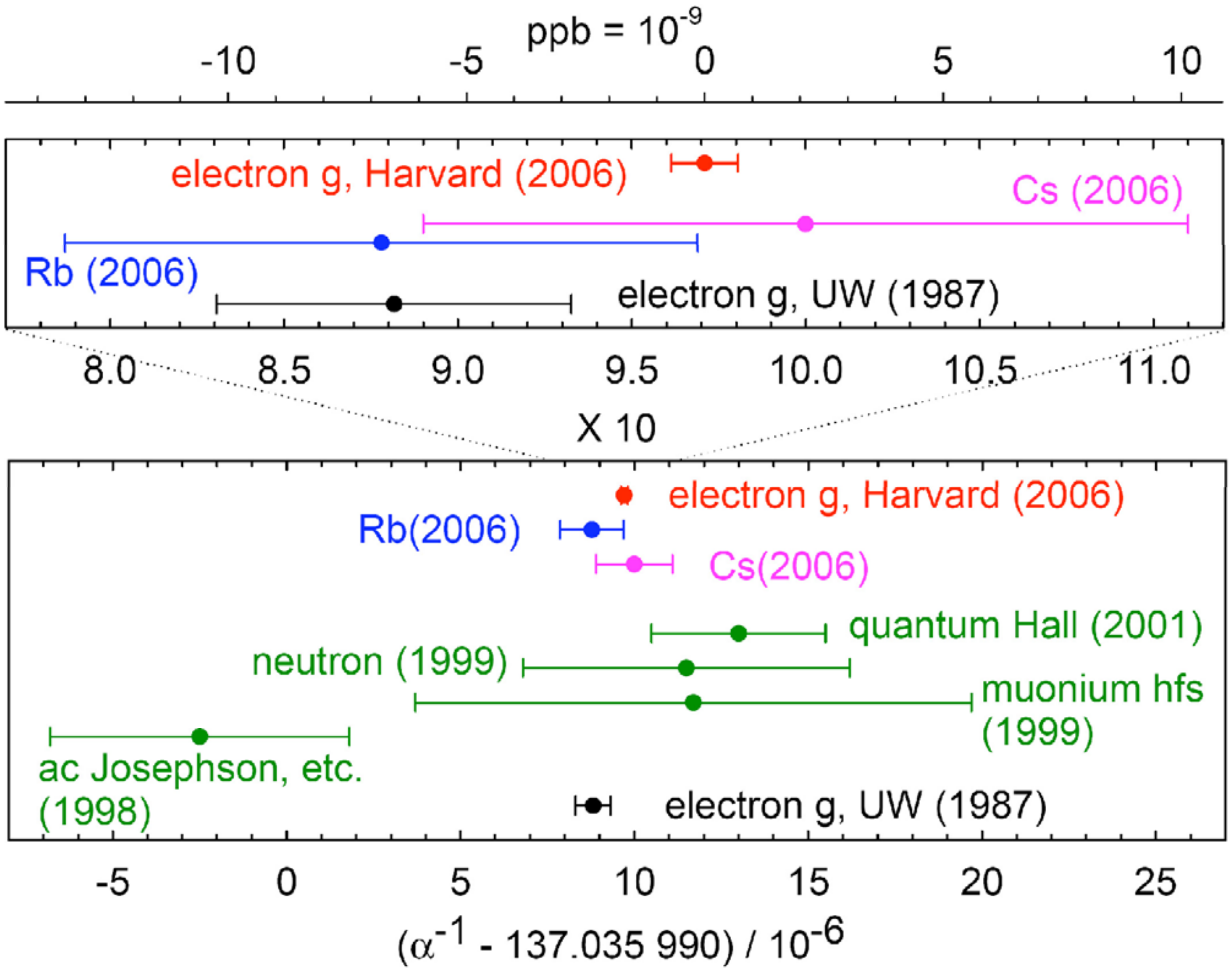
α^{-1} = 137.036 000 00	(110)	[8.0 ppb]	PRA73 (2006) 032504 (Cs)
α^{-1} = 137.035 998 78	(91)	[6.7 ppb]	PRL96 (2006) 033001 (Rb)

  Δ = +0.8 and -0.3 σ → beautiful test of QED at 4-loop level!

  Old values were:

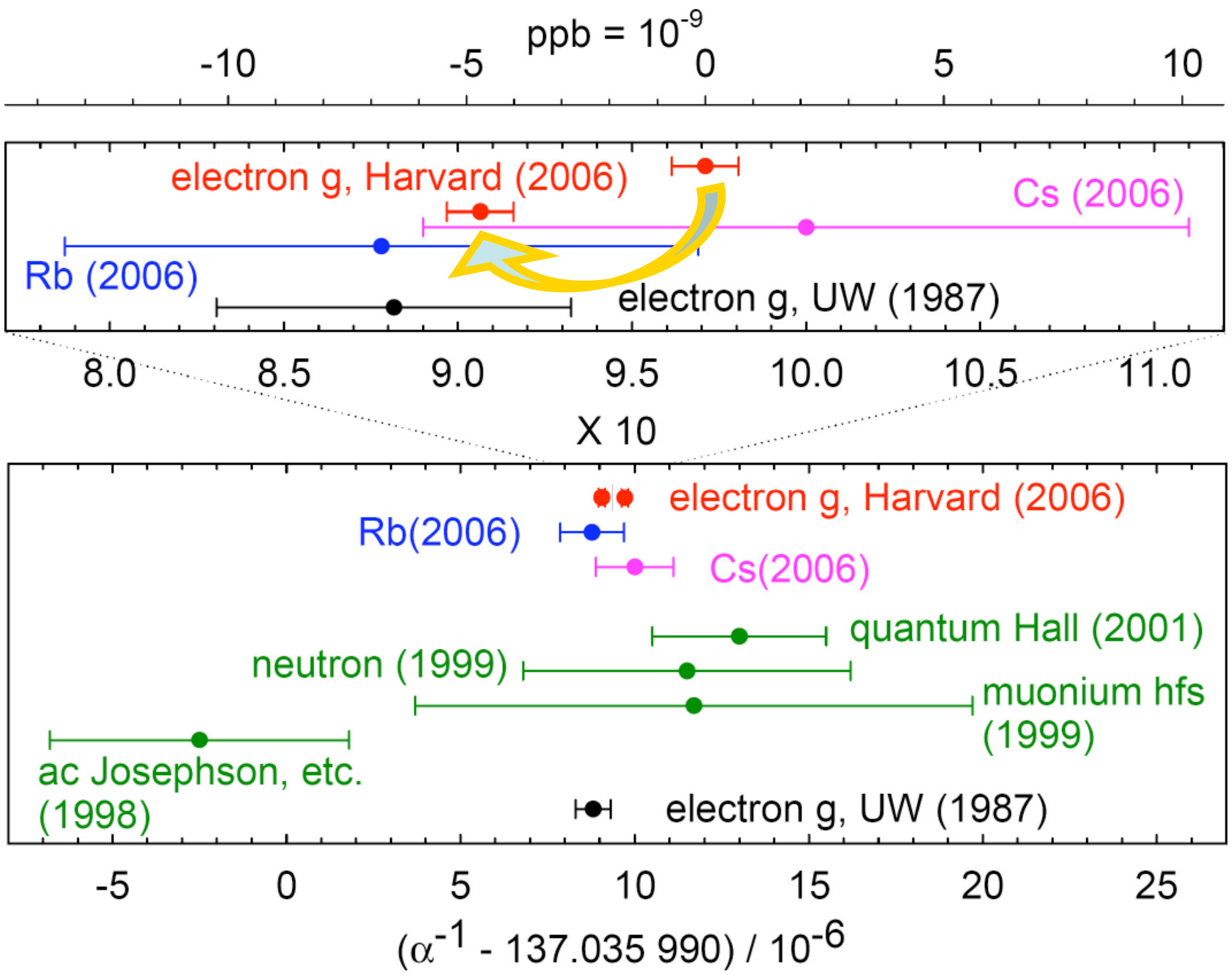
α^{-1} = 137.035 998 83	(50)	[3.6 ppb]	CODATA '98 based on UW '87
α^{-1} = 137.035 999 11	(46)	[3.3 ppb]	CODATA '02 = PDG'04 = PDG '06

Old and new determinations of alpha



Gabrielse, Hanneke, Kinoshita, Nio & Odom, PRL97 (2006) 030802

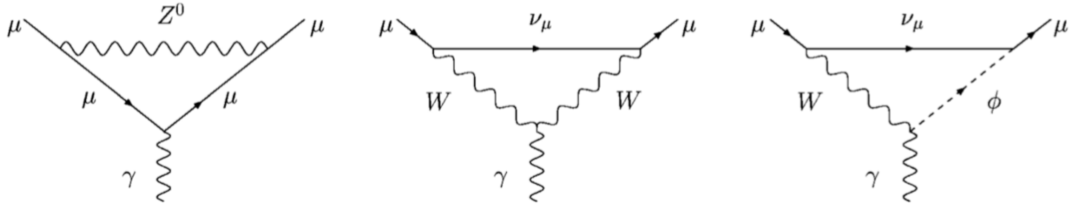
Old and new determinations of alpha



Gabrielse, Hanneke, Kinoshita, Nio & Odom, PRL97 (2006) 030802

The Electroweak contribution a_μ (see de Rafael's talk)

- **One-Loop Term:**



$$a_\mu^{\rm EW}(\text{1-loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1+\frac{1}{5}\left(1-4\sin^2\theta_W\right)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195\times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda.

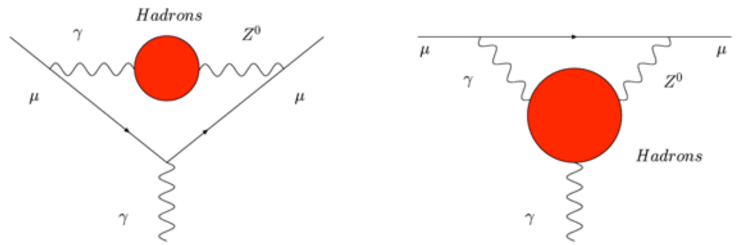
- **One-Loop plus Higher-OrderTerms:**

$a_\mu^{\rm EW} = 154\ (2)\ (1) \times 10^{-11}$

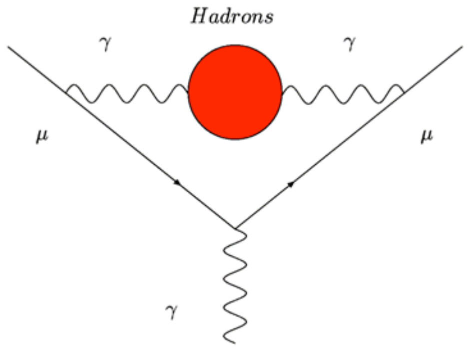
Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano, Vainshtein '02; Degrassi, Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk, Czarnecki '05; Vainshtein '03.

Higgs mass, M_top error, three-loop nonleading logs

Hadronic loop uncertainties:



The leading hadronic contribution



Bouchiat & Michel '61; Gourdin & de Rafael '69

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)s/m_\mu^2}$$

$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^\infty ds\, K(s)\, \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^\infty \frac{ds}{s} K(s)\, R(s)$$

$a_\mu^{\rm HLO}$ = 6909 $(39)_{\rm exp}$ $(19)_{\rm rad}$ $(7)_{\rm qcd}$ × 10^{-11}	S. Eidelman, ICHEP06; M. Davier, TAU06
= 6921 (56) × 10^{-11}	F. Jegerlehner, hep-ph/0608329
= 6944 $(48)_{\rm exp}$ $(10)_{\rm rad}$ × 10^{-11}	de Troconiz, Yndurain, PRD71 (2005) 73008
= 6894 $(42)_{\rm exp}$ $(18)_{\rm rad}$ × 10^{-11}	Hagiwara,Martin,Nomura,Teubner, PLB649(2007)173
= 7110 (58) × 10^{-11} (τ data)	Davier, Eidelman, Hoecker, Zhang, EPJC31 (2003) 503

Will be discussed in the talks of Czyz, Davier, Eidelman, Fedotovich & Venanzoni
Long-lasting discrepancy between e+e- and τ data. Unaccounted isospin-breaking corrections? See Davier's talk.

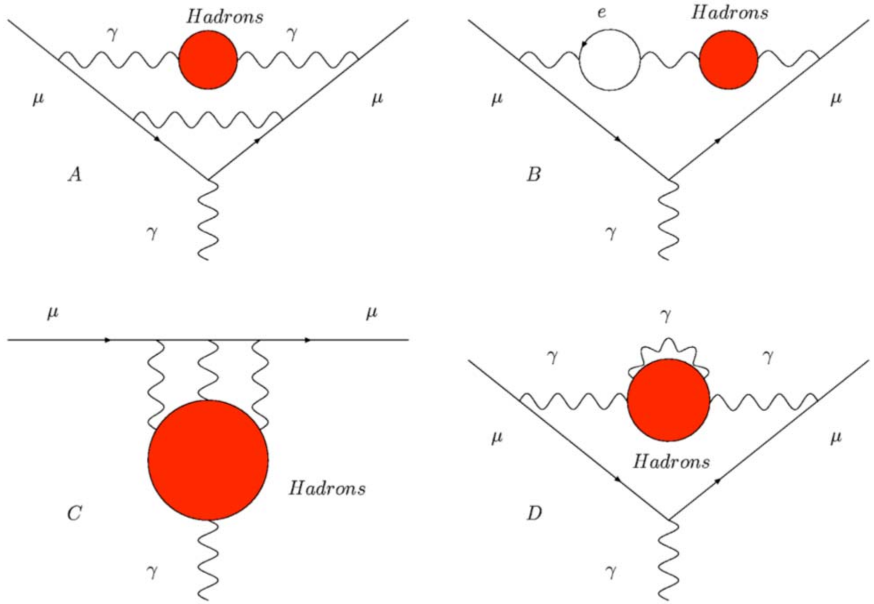
• Vacuum Polarization

$O(\alpha^3)$ contribution of diagrams containing hadronic vacuum polarization insertions:

$a_\mu^{HHO}(vp) = -98\ (1) \times 10^{-11}$

Krause '96, Alemany et al. '98, Hagiwara et al. '03,'06

Shifts by ~ -3×10^{-11} if tau data are used instead of the e^+e^- ones Davier & Marciano '04



• Light-by-Light

The contribution of the hadronic l-b-l diagrams had a troubled life. The latest values vary between:

$a_\mu^{HHO}(lbl) = +\ 80\ (40) \times 10^{-11}$ Knecht et al. 2002

$a_\mu^{HHO}(lbl) = +136\ (25) \times 10^{-11}$ Melnikov & Vainshtein '03

$a_\mu^{HHO}(lbl) = +110\ (40) \times 10^{-11}$ Bijnens & Prades '07

based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02;

The ultimate limitation? See talks of Fischer, Hayakawa, Prades, Rakow and Vainshtein

The muon g-2: Standard Model vs. Experiment

Adding up all the above contribution we get the following SM predictions for a_μ and comparisons with the measured value:

	$a_\mu^{\mathrm{SM}} \times 10^{11}$	$\Delta \times 10^{11}$	σ
[1]	116 591 793 (60)	287 (87)	3.3
[2]	116 591 805 (69)	275 (93)	2.9
[3]	116 591 828 (63)	252 (89)	2.8
[4]	116 591 778 (61)	302 (88)	3.4
[5]	116 591 991 (70)	89 (95)	0.9

with $a_\mu^{\mathrm{HHO}}(\mathrm{lbl}) = 110\ (40) \times 10^{-11}$.

[1] S. Eidelman at ICHEP06 & Davier at TAU06 (DEHZ06, update of ref. [5]).
[2] F. Jegerlehner, hep-ph/0608329, August 2006.
[3] J.F. de Troconiz and F.J. Yndurain, PRD71 (2005) 073008.
[4] Hagiwara, Martin, Nomura, Teubner, PLB649 (2007) 173.
[5] Davier, Eidelman, Hoecker and Zhang, EPJC31 (2003) 503 (τ data).

The th. error is now the same (or even smaller) as the exp. one!

The muon g-2 and the mass of the Higgs:
An exercise

How do we explain Δa_μ?

- The g-2 discrepancy could be explained in many ways: LBL, QED, EW, HHO-VP, g-2 EXP, HLO and Physics beyond the SM.

- Can Δa_μ be really due to inconsistencies in the hadronic $\sigma(s)$?

- An upward shift of $\sigma(s)$ also induces an increase in $\Delta\alpha_{had}^{(5)}(M_Z)$.

- Consider

$$a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\, \sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3},$$

$$b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\, \sigma(s), \qquad g(s) = \frac{M_Z^2}{M_Z^2 - s^2},$$

and the increase

$$\sigma(s) \to \sigma(s)\,(1+\epsilon) = \sigma(s) + \Delta\sigma(s),$$

in the range:

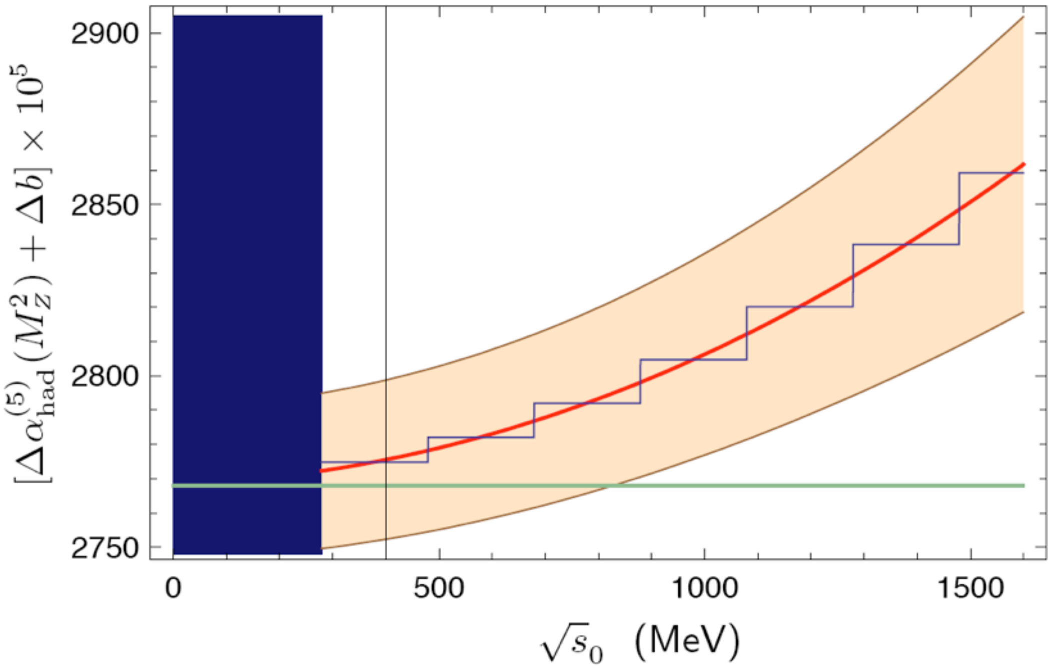
$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2].$$

Shift induced in $\Delta\alpha_{had}^{(5)}(M_Z)$

- If this shift $\Delta\sigma(s)$ in $[\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$ is fixed to solve the g-2 discrepancy, then the value of $\Delta\alpha_{had}^{(5)}(M_Z)$ increases by:

$$\Delta b(\sqrt{s_0}, \delta) = \Delta a_\mu \frac{\int_{\sqrt{s_0}-\delta/2}^{\sqrt{s_0}+\delta/2} 2t\, g(t^2)\, \sigma(t^2)\, dt}{\int_{\sqrt{s_0}-\delta/2}^{\sqrt{s_0}+\delta/2} 2t\, f(t^2)\, \sigma(t^2)\, dt}.$$

- Adding this shift to $\Delta\alpha_{had}^{(5)}(M_Z)$=0.02768(22) [HMNT06] , with Δa_μ= 302(88) x 10^{-11} [HMNT06], we have:



Bounds on the Higgs mass - 1

- The dependence of SM predictions on the Higgs mass, via loops, provides a tool to set bounds on its value.

- Comparing the theoretical predictions of M_W and $\sin^2\theta_{\rm eff}^{\rm lept}$

  [convenient formulae of Degrassi, Gambino, MP, Sirlin '98; Degrassi, Gambino '00; Ferroglia, Ossola, MP, Sirlin '02; Awramik, Czakon, Freitas, Weiglein '04 & '06, in terms of M_H, M_{top}, $\Delta\alpha_{had}^{(5)}(M_Z)$ and $\alpha_s(M_Z)$]

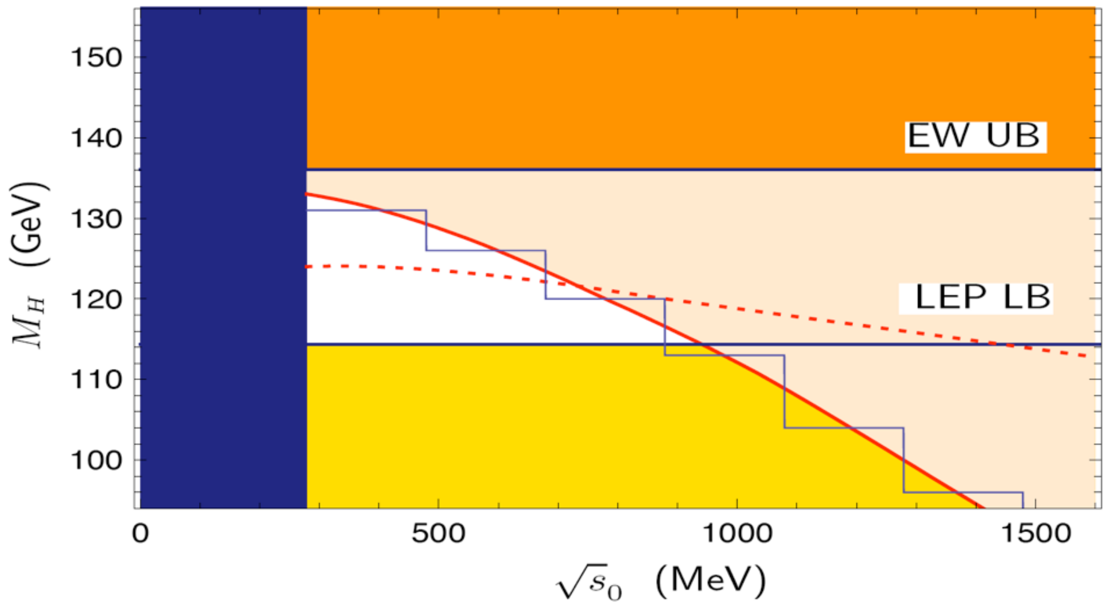
with	M_W = 80.398 (25) GeV	[LEP+Tevatron]
	$\sin^2\theta_{\rm eff}^{\rm lept}$ = 0.23153 (16)	[LEP+SLC]
and	$\Delta\alpha_{had}^{(5)}(M_Z)$ = 0.02768 (22)	[HMNT '06]
	M_{top} = 170.9 (1.8) GeV	[Tevatron '07]
	$\alpha_s(M_Z)$ = 0.118 (2)	[PDG '06]

we get: $M_H = 79^{+32}_{-23}$ GeV & $M_H < 136$ GeV 95%CL

- The value of $\Delta\alpha_{had}^{(5)}(M_Z)$ is a key ingredient of these EW fits…

Bounds on the Higgs mass - 2

- How much does the M_H upper bound change when we shift $\sigma(s)$ (and thus $\Delta\alpha_{had}^{(5)}(M_Z)$) to accomodate the g-2 discrepancy?



- Conflict with the LEP bound above ~1 GeV. If an increase $\Delta\sigma(s)$ should occur, it should be below ~1 GeV.

- Focus on region below ~1 GeV: are such shifts realistic?? The required $\varepsilon=\Delta\sigma/\sigma$ depends on the choice of energy range, but it's $\varepsilon > 50\%$ for any range < ~500 MeV --> option ruled out! If $\Delta\sigma$ occurs in 0.5-1 GeV, then M_H upper bound is less than ~130GeV

Conclusions

- Beautiful examples of interplay between theory and experiment: g_e probed at <ppt! $\rightarrow$ α and extraordinary test of QED's validity; g_μ probed at <ppb $\rightarrow$ test of the full SM and great opportunity to unveil (or just constrain) "New Physics" effects!
- The discrepancy Δa_μ is more than 3 σ if e^+e^- data are used. With tau data the deviation is ~ 1 σ only! The e^+e^- vs tau puzzle is still unsolved. Unaccounted isospin breaking corrections?
- The QED sector appears to be solid, with very small uncertainty and ready for the E969 challenge! The hadronic sector needs more work…
- It is hard to explain Δa_μ modifying the hadronic cross section: larger than 50% increase required for a shift below ~500 MeV, conflict with LEP direct M_H bound for a shift above ~1 GeV. If the shift occurs in the 0.5-1 GeV region, then the M_H upper bound is less than ~130 GeV.

The End

The effective fine-structure constant at the scale s is given by:

$$\alpha(s) = \frac{\alpha}{1 - \Delta\alpha} \qquad \text{with} \qquad \Delta\alpha = \Delta\alpha_{lep} + \Delta\alpha^{(5)}_{had} + \Delta\alpha_{top}$$

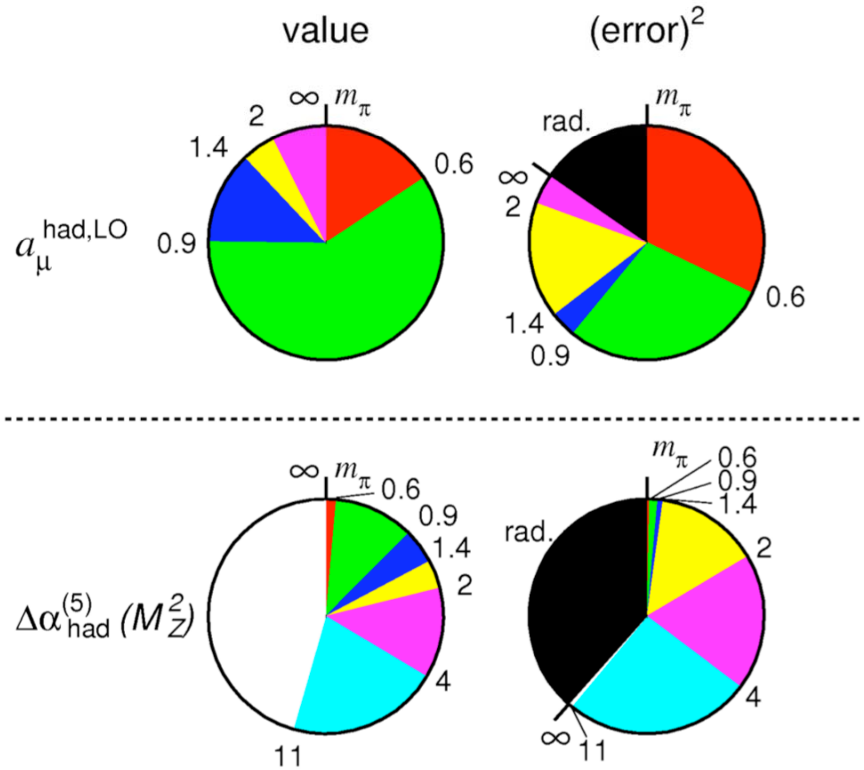
The light quarks part is determined by:

$$\Delta\alpha^{(5)}_{had}(M_z^2) = -\frac{\alpha M_z^2}{3\pi}\int_{s_{thr}}^{\infty} ds \frac{R(s)}{s(s - M_z^2 - i\epsilon)}.$$

Progress due to significant improvement of the data (mostly CMD-2 and BES):

$\Delta\alpha_{had}^{(5)}(M_z^2)$ =

0.02800 (70)	Eidelman, Jegerlehner'95
0.02775 (17)	Kuhn, Steinhauser 1998
0.02761 (36)	Burkhardt, Pietrzyk 2001
0.02755 (23)	Hagivara et al., 2004
0.02758 (35)	Burkhardt, Pietrzyk 6-05
0.02768 (22)	Hagiwara et al. 11-2006



Hagivara et al., PRD69 (2004) 093003