Creation and Evaporation of Higher-Dimensional Black Holes

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Outline :

1. Introduction to Extra Dimensions
2. Creation of Black Holes
3. Evaporation of Black Holes
   - Hawking radiation on the Brane
   - Hawking radiation in the Bulk
4. Conclusions

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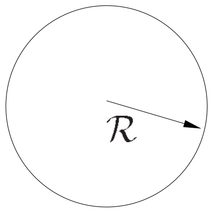
In collaboration with *John March-Russell* (*Oxford*) **PRD '02, '03**
and with *Chris Harris* (*Cambridge*) **JHEP '03**

Introduction

• Kaluza & Klein (1921/1926): Apart from the usual 3 spacelike dimensions, additional, compact space-like dimensions may exist in nature

Why we can not observe them? Because of their tiny size $\mathcal{R}$: for experiments at $r \gg \mathcal{R}$, the extra dimensions are 'invisible'



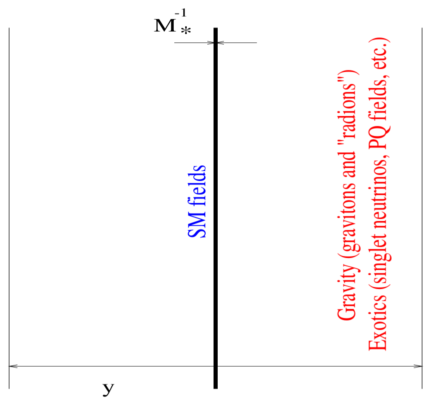
• Superstring Theory (1975-90) : Formulated in $D = 10$ dimensions with

$$\boxed{\mathcal{R} = l_P = 10^{-33}\ \text{cm}}$$

• Large Extra Dimensions (1998) : (Arkani-Hamed, Dimopoulos & Dvali)

- A 4D **Brane** with all the SM fields and scale for gravity $M_P = 10^{19}$ GeV
- A $(4+n)$D Extra Space (**Bulk**) with gravitons and scale for gravity M_*
- Then, we obtain:

$$\boxed{M_P^2 \simeq \mathcal{R}^n\, M_*^{2+n}}$$



If $\mathcal{R} \leq 1$ mm, then $M_* \geq 1$ TeV $\Rightarrow$ resolution of the hierarchy problem

(Antoniadis; Horava, Witten; Lykken; Dienes, Dudas, Gherghetta; Randall, Sundrum)

Introduction

Current limits on the fundamental energy scale

Type of Experiment/Analysis	$M_* \geq$	$M_* \geq$
Collider limits on the production of real or virtual KK gravitons	1.45 TeV $(n=2)$	0.6 TeV $(n=6)$
Torsion-balance Experiments	3.5 TeV $(n=2)$	
Overclosure of the Universe	8 TeV $(n=2)$	
Supernovae cooling rate	30 TeV $(n=2)$	2.5 TeV $(n=3)$
Non-thermal production of KK modes	35 TeV $(n=2)$	3 TeV $(n=6)$
Diffuse gamma-ray background	110 TeV $(n=2)$	5 TeV $(n=3)$
Thermal production of KK modes	167 TeV $(n=2)$	1.5 TeV $(n=5)$
Neutron star core halo	500 TeV $(n=2)$	30 TeV $(n=3)$
Neutron star surface temperature	1700 TeV $(n=2)$	60 TeV $(n=3)$
BH absence in neutrino cosmic rays		1-1.4 TeV $(n \geq 5)$

- For theories with a very low fundamental energy scale $M_* \sim 1$ TeV, the case with $n \leq 2$ (or, even with $n \leq 3$) has been excluded
- Nevertheless, models with $M_* \sim 1$ TeV and $n \geq 4$ are still viable

Creation of Black Holes

• Black Holes? During gravitational collapse on the brane, we expect the formation of a black hole, as in the 4D case, but now ...

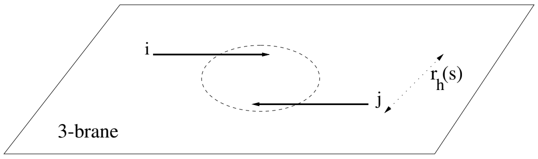
$\Downarrow$

• Large BH's with $r_H > \mathcal{R}$ are effectively 4D objects
• Small BH's with $r_H < \mathcal{R}$ are $(4+n)$-D objects

▷ Small $(4+n)$-D Black Holes can be created ...

- at the early universe: density perturbations, phase transitions
- nowadays: scattering of high energy particles at colliders and atmosphere

(Banks & Fischler; Giddings & Thomas; Dimopoulos & Landsberg)



For every center-of-mass-energy $\sqrt{s}$, there is a Schwarzschild radius r_H — if $b < r_H$, then a black hole will be created (Thorne)

For a black hole to be considered as a classical object, it must have:

• in 4 Dimensions: $M_{BH} > M_P \simeq 10^{19}$ GeV
• in $4+n$ Dimensions: $M_{BH} > M_* \geq 1$ TeV

Creation of Black Holes

There are two ways to study the creation of those microscopic black holes:

- the geometrical method: the creation of a closed-trapped surface around the colliding particles/shock-waves (Penrose; Eardley & Giddings; Yoshino & Nambu; Kohlprath and Veneziano)

$$b_{max} \sim 2^{-1/(D-3)}\, r_H \quad \Longrightarrow \quad \sigma \sim \pi b_{max}^2$$

Also:

$$M_{BH}(b, D) \geq (0.45 - 0.71)\,\sqrt{s}$$

- the perturbative method: calculation of the emitted gravitational radiation (D'Eath & Payne; Cardoso & Lemos; Berti, Cavaglia & Gualtieri)

$$E_{grav} \sim (16\% - 8\%)\sqrt{s} \qquad \text{for} \qquad (D = 4 - 10)$$

• Realistic Collision: The colliding particles may be composite particles $\Rightarrow$ we have to sum over all partons carrying enough energy to produce a microscopic, but classical, black hole (Giddings & Thomas; Dimopoulos & Landsberg)

$M_{BH} = 5$ TeV	$M_{BH} = 10$ TeV
$\sigma_{tot} \sim 10^5$ fb	$\sigma_{tot} \sim 10^1$ fb

• An over-estimate? It has been assumed that: $\sigma \sim \pi r_H^2$ and $M_{BH} \simeq \sqrt{s}$

Creation of Black Holes

▷ Stages of the life of the produced black hole: A highly asymmetric, rotating object that goes through the following: (Giddings & Thomas)

- Balding phase: shedding of all quantum numbers apart from (M, Q, A) – invisible energy (15% of the total energy?)
- Spin-down phase: Loss of angular momentum – Hawking radiation – visible energy (25% of the total energy?)
- Schwarzschild phase: Hawking radiation – visible energy (60% of the total energy?)
- Planck phase: when $M_{BH} \sim M_*$ – a few energetic quanta?

• The Spin-down phase: Very few results available

• The Schwarzschild phase: A spherically symmetric $(4+n)$-dimensional BH with line-element (Myers & Perry)

$$ds^2 = -\left[1 - \left(\frac{r_H}{r}\right)^{n+1}\right] dt^2 + \left[1 - \left(\frac{r_H}{r}\right)^{n+1}\right]^{-1} dr^2 + r^2 d\Omega^2_{2+n}$$

where

$$d\Omega^2_{2+n} = d\theta^2_{n+1} + \sin^2\theta_{n+1}\left(d\theta^2_n + \sin^2\theta_n\left(\,\ldots + \sin^2\theta_2\left(d\theta^2_1 + \sin^2\theta_1\, d\varphi^2\right)\ldots\right)\right)$$

Creation of Black Holes

What else a low M_* does for us? It modifies the Black-Hole Properties

(Argyres, Dimopoulos & March-Russell)

▷ Horizon Radius:

$$\boxed{r_H = \frac{1}{M_*}\left(\frac{M}{M_*}\right)^{\frac{1}{n+1}}\left(\frac{8\Gamma(\frac{n+3}{2})}{(n+2)\sqrt{\pi}^{(n+1)}}\right)^{1/(n+1)}}$$

Horizon radii for $M_* = 1\,$TeV and $M_{BH} = 5\,$TeV

n	1	2	3	4	5	6	7
r_H (10^{-4} fm)	4.06	2.63	2.22	2.07	2.00	1.99	1.99

For $M_{BH} = 5$ TeV and $n = 0$, $r_H = 10^{-35}$ fm !!

▷ Temperature:

$$\boxed{T_H = \frac{(1+n)}{4\pi\, r_H}}$$

Temperatures for $M_* = 1\,$TeV and $M_{BH} = 5\,$TeV

n	1	2	3	4	5	6	7
T_H (GeV)	77	179	282	379	470	553	629

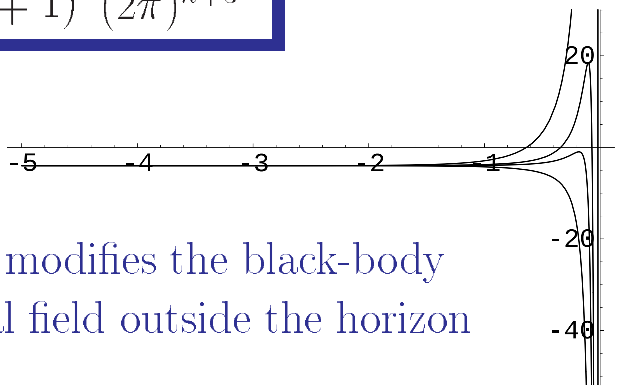
- Typical lifetime: $\tau = (1.7 - 0.5) \times 10^{-26}$ sec for $n = 1 - 7$

Evaporation of Black Holes

- Hawking Radiation: What is it? How is it produced?
  - creation of a virtual pair of particles just outside the horizon
  - the antiparticle falls into the BH whose mass decreases
  - the particle escapes to infinity where it gets observed

The Radiation Spectrum: A nearly black-body spectrum with emission rate

$$\boxed{\frac{dE(\omega)}{dt} = \frac{\sigma_n(\omega)\ \omega}{\left(\exp\left(\omega/T_{BH}\right) \mp 1\right)}\ \frac{d^{n+3}k}{(2\pi)^{n+3}}}$$



where $\sigma_n(\omega)$ is the greybody factor – it modifies the black-body spectrum due to the strong gravitational field outside the horizon

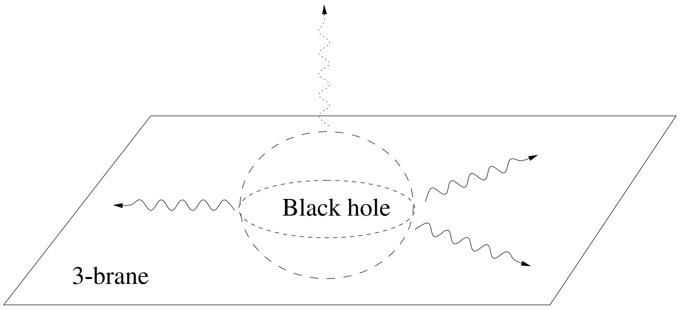
To find out how many particles reach infinity, we must solve the corresponding Equation of Motion and find the Absorption Coefficient $\mathcal{A}_\ell$

Then: (Gubser, Klebanov & Tseytlin)

$$\sigma_n(\omega) = \frac{2^n \pi^{(n+1)/2}\,\Gamma[(n+1)/2]}{n!\,\omega^{n+2}} \sum_\ell \frac{(2\ell+n+1)\,(\ell+n)!}{\ell!}\,|\mathcal{A}_\ell|^2$$

- 4D Case : Teukolsky & Press; Unruh; Sanchez; Page; McGibbon & Webber

Evaporation of Black Holes



▷ <u>Particles on the Brane :</u> (zero-mode) Scalars, Fermions, Gauge Bosons, (zero-mode) Gravitons ($\surd$)

▷ <u>Particles in the Bulk :</u> Gravitons, Scalars (?)

The brane particles live in a $4D$ background - a projection of the $(4+n)D$ one onto the brane : $\theta_n = \pi/2$ for $n \geq 2$

$$ds^2 = -\left[1 - \left(\frac{r_H}{r}\right)^{n+1}\right] dt^2 + \left[1 - \left(\frac{r_H}{r}\right)^{n+1}\right]^{-1} dr^2 + r^2\,(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Then, the corresponding $4D$ greybody factor takes the form

$$\sigma_n(\omega) = \frac{\pi}{\omega^2}\,(2\ell+1)\,|\mathcal{A}_\ell|^2$$

Nevertheless, $|\mathcal{A}_\ell|^2$ <u>does</u> depend on the number n of extra dimensions projected onto the brane

Hawking radiation on the Brane

- Visible Energy: $\left\{ \begin{array}{l} \text{Scalar fields } (s=0) \\ \text{Fermions } (s=1/2) \\ \text{Gauge Bosons } (s=1) \end{array} \right.$

- General (Master) Equation of Motion: Following the Newman-Penrose method, we assume:

$$\Psi_s = e^{-i\omega t}\, e^{im\varphi}\, \Delta^{-s}\, P_s(r)\, S^m_{s,j}(\theta)$$

where $Y^m_{s,j} = e^{im\varphi}\, S^m_{s,j}(\theta)$ are the spin-weighted spherical harmonics, and j the quantum number of the total angular momentum

The general equation of motion of a particle with arbitrary spin s in the curved $4D$ spacetime of the brane can be written as (Harris & P. K.)

$$\boxed{\Delta^s \frac{d}{dr}\left(\Delta^{1-s}\frac{dP_s}{dr}\right) + \left[\frac{\omega^2 r^2}{h} + 2i\omega s r - \frac{is\omega r^2 h'}{h} - \Lambda_{sj}\right] P_s(r) = 0}$$

where $\Delta = r^2\,[1-(\frac{r_H}{r})^{(n+1)}]$ and $\Lambda_{sj} = j(j+1) - s(s-1)$.

How can we solve this equation?

Hawking radiation on the Brane

- Analytically: An approximation method must be followed:

(P.K. & March-Russell)

  - find the solution in the The Near-Horizon regime $(r \simeq r_H)$
    – a solution of a hypergeometric equation
  - find the solution in the The Far-Field regime $(r \gg r_H)$
    – the solution of a confluent hypergeometric equation
  - match the two solutions in an intermediate zone

Once the complete solution $\Psi_s(r)$ is found, we may compute:

▷ the energy-momentum tensor: for $\Psi_{s=0} \equiv \phi\,,\;\; T_{\mu\nu} = \partial_\mu \phi \, \partial_\nu \phi$

▷ the energy flux:

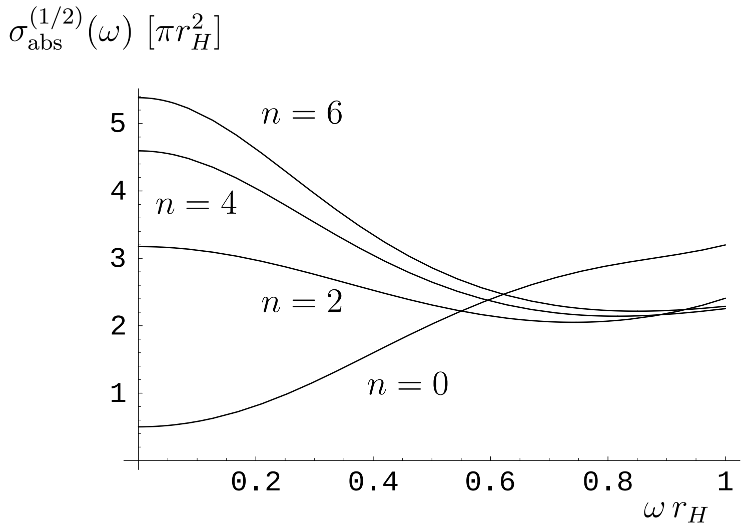
$$\mathcal{F} = \int d^2 s \, T^{0r}$$

▷ the Absorption Probability:

$$\boxed{|\mathcal{A}_\ell|^2 = \frac{\mathcal{F}_{\rm horizon}}{\mathcal{F}_{\rm infinity}}}$$

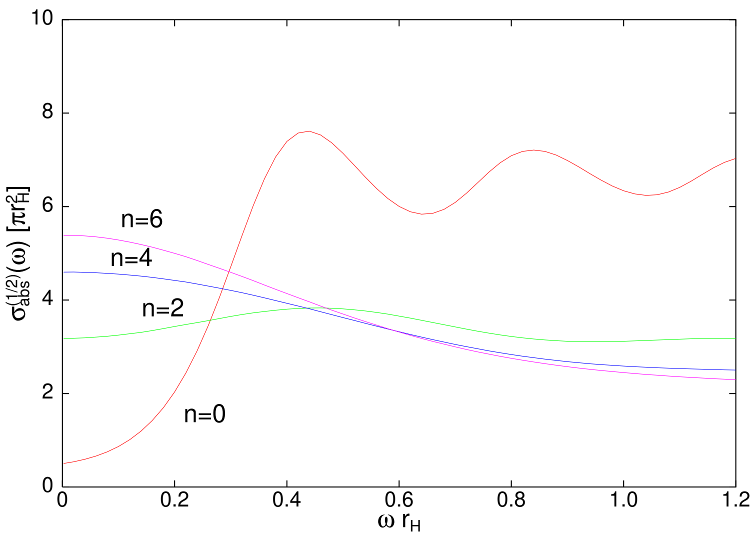
Hawking radiation on the brane

• E. g. for fermions, we find that $\sigma_\ell(\omega)$ has the behaviour



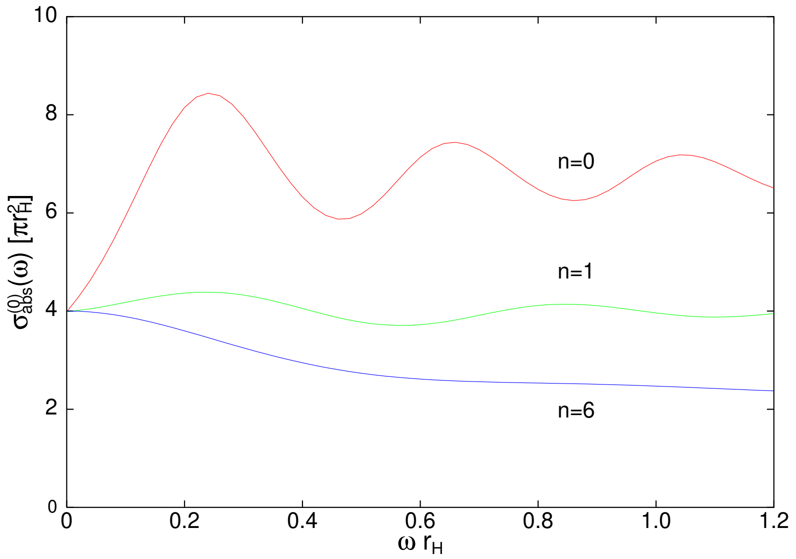
The analytic expressions are valid only at the low- and intermediate-energy regimes - during the matching we assumed that $\omega r_H \ll 1$

• Numerically: For results valid at all energy regimes, we have to use numerical analysis (Harris & P. K.)

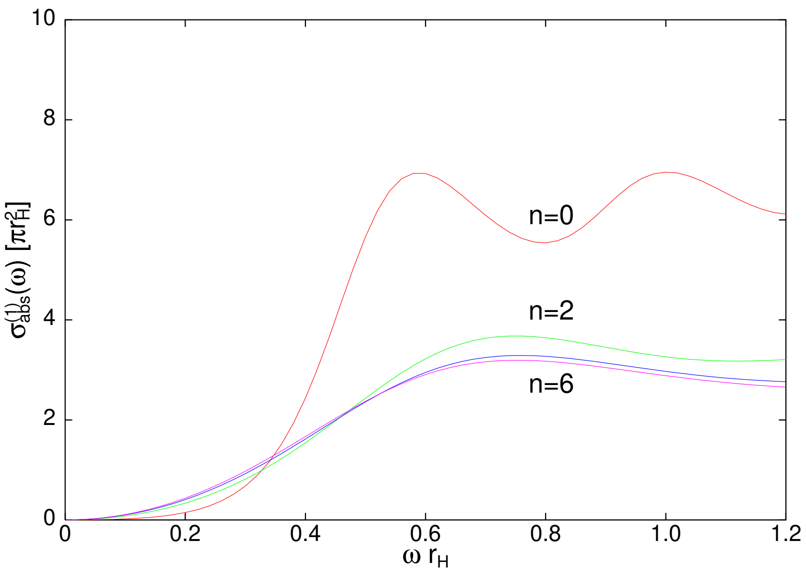


Hawking radiation on the brane

- In the same way, for scalar particles, we find:



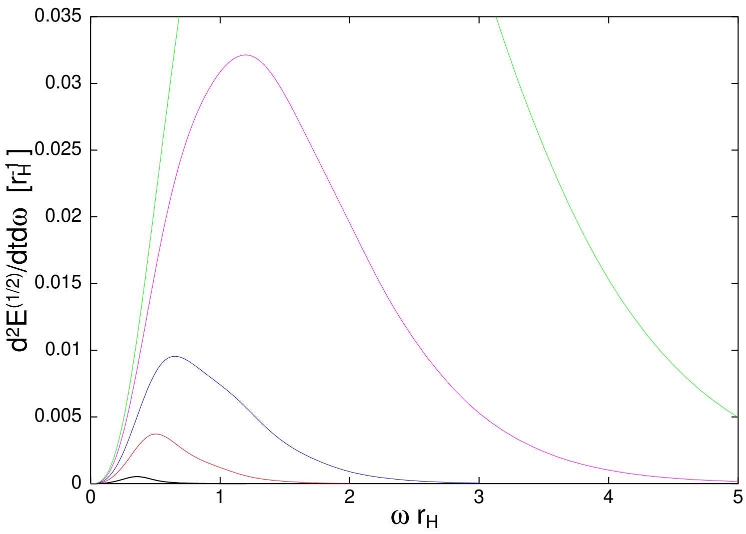
- ... and for gauge bosons:



- Different behaviour at low energies, same behaviour at high energies: $\hat{\sigma} \to \pi r_c^2(n)$ (Sanchez; Emparan, Horowitz & Myers)

Hawking radiation on the Brane

- Energy Emission Rate : Our numerical analysis gives for fermions:



- The amount of the emitted energy strongly depends on the number of spacelike dimensions that exist in nature – it increases as n increases

Total energy emissivities for different values of n

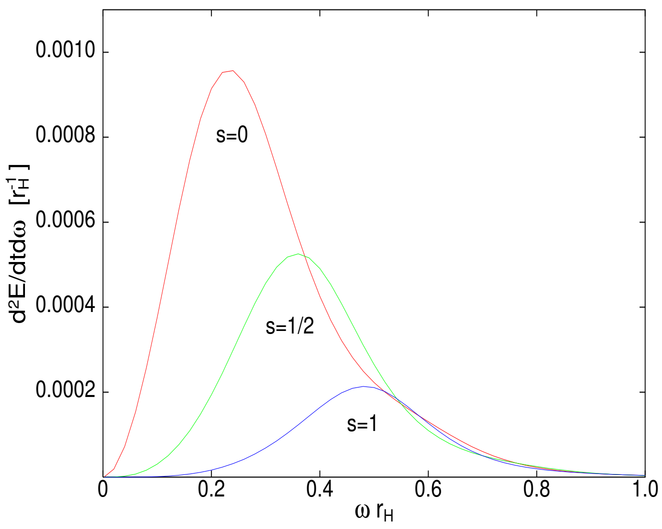
n	0	1	2	3	4	5	6	7
Scalars	1.0	8.94	36.0	99.8	222	429	749	1220
Fermions	1.0	14.2	59.5	162	352	664	1140	1830
Gauge Bosons	1.0	27.1	144	441	1020	2000	3530	5740

- Rotating Black Holes : Only an analytic, approximate study has been done for $n = 1$ (Ida, Oda & Park)

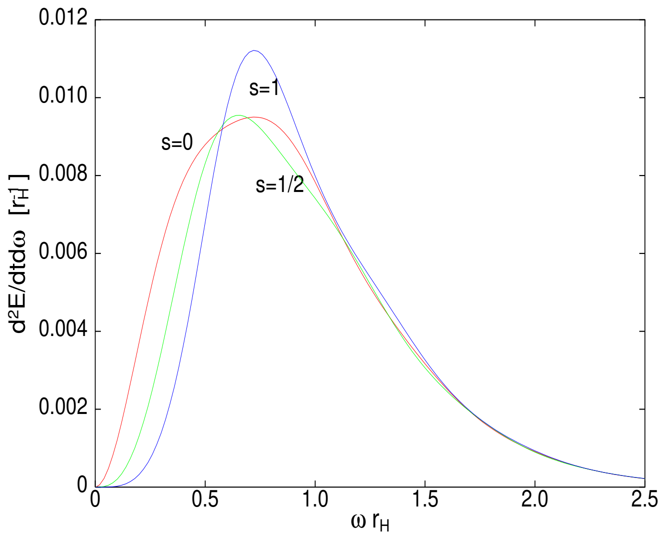
Hawking radiation on the Brane

- Relative Emission Rates: How do they change with n?

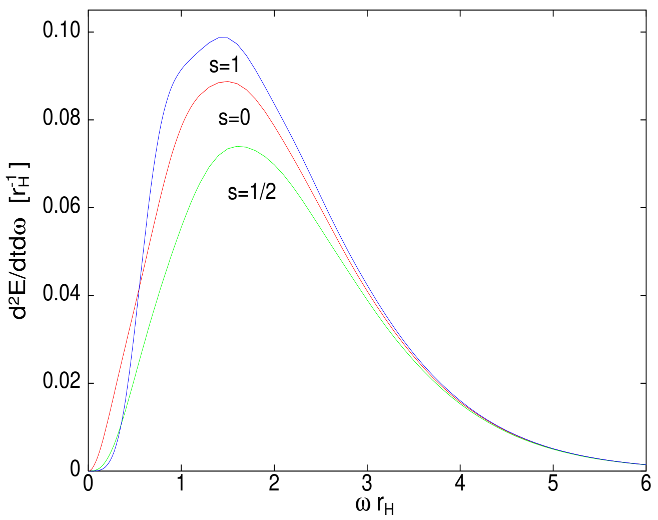
(Harris & P. K.)



$(n = 0)$ $\boxed{1 : 0.55 : 0.23}$



$(n = 2)$ $\boxed{1 : 0.91 : 0.91}$



$(n = 6)$ $\boxed{1 : 0.84 : 1.06}$

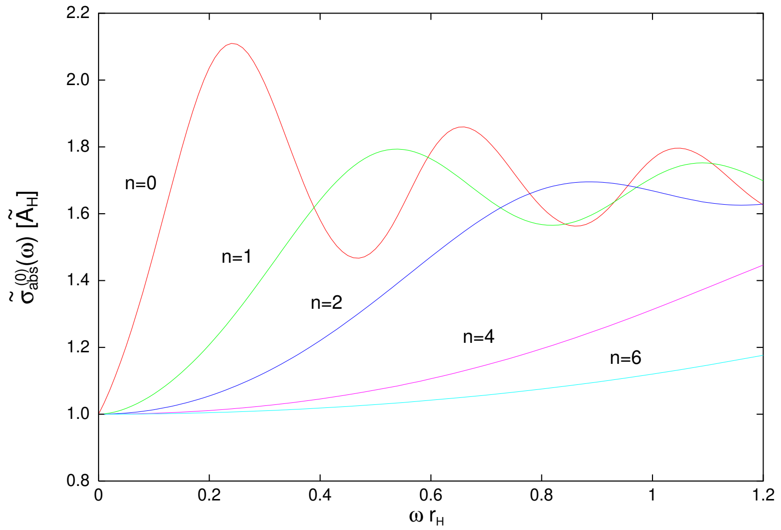
- The type of the emitted radiation also depends strongly on n

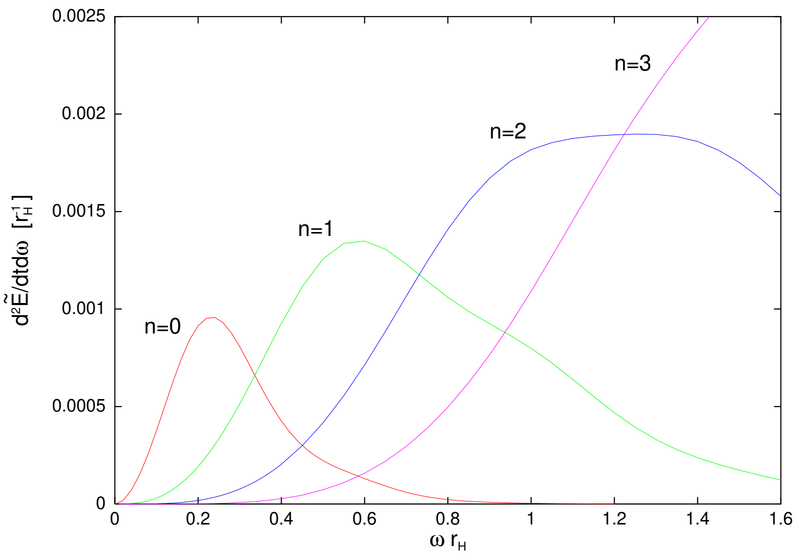
Hawking radiation in the Bulk

For the Schwarzschild phase of a $(4+n)$-dimensional Black Hole:

- Numerical Analysis: The exact results for scalar fields are:

(Harris & P. K.)





Hawking radiation in the Bulk

- Bulk or Brane? Where is most of the energy emitted?

(Emparan, Horowitz and Myers): Bulk-Brane $\sim$ 50%-50%

Bulk-to-Brane Relative Emissivity for Scalars

	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$
Bulk/Brane	1.0	0.40	0.24	0.22	0.24	0.33	0.52	0.93

No results for the emission spectra for scalars and gravitons have been derived for the spin-down phase

Conclusions

- The detection of Hawking radiation from a decaying black hole would be an extremely important result by itself, as the manifestation of a semi-classical phenonenon

- If Extra Dimensions exist, then
  - the production of extremely small black holes becomes possible
  - the detection of Hawking radiation becomes more likely
  - the emission spectra can help us determine the dimensionality of spacetime

- Possible signatures of the dimensionality of spacetime in the spectrum of Hawking radiation are:
  - the amount of the emitted radiation
  - the type of the emitted radiation

- For the exact number and types of particles emitted: Black Hole event generators

  (e.g. Dimopoulos & Landsberg; Harris, Richardson & Webber)

- One more reason for looking forward to the day when LHC is turned on....