EW Constraints on Minimal 5-Dimensional Extensions of the Standard Model: from LEP to Next Linear Colliders

Apostolos Pilaftsis

Department of Physics and Astronomy, University of Manchester, Manchester M13 9PL, United Kingdom

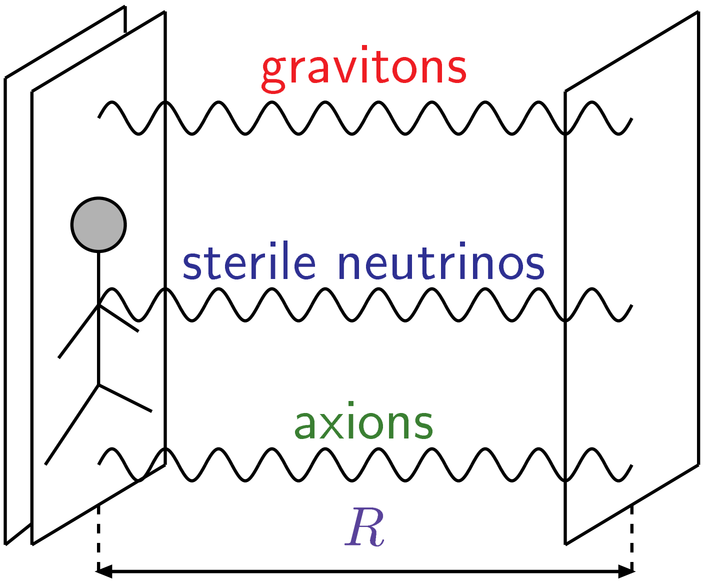
- **Introduction – Motivation**
- **Minimal 5-dimensional models and quantization of the theory in the generalized R_ξ gauge.**
- **KK effects on electroweak observables**
- **LEP2, LEP1, and lower-energy constraints**
- **Potential constraints from Next Linear Colliders**
- **Conclusions**

*Talk based on

Alexander Mück, A.P. and Reinhold Rückl, Phys. Rev. **D65** (2002) 085037; hep-ph/0312186 (to appear in NPB)

• Introduction – Motivation

- **Low-scale quantum-gravity model of Arkani-Hamed, Dimopoulos and Dvali** [Phys. Lett. **B429** (1998) 263; Phys. Rev. **D59** (1999) 086004]:



The Planck mass $M_{\rm P} \approx 1.9 \times 10^{16}$ TeV is related to the quantum gravity scale M_F and the compactification radius R through

$$M_{\rm P} \;\approx\; M_F\,(R\,M_F)^{n/2}\,.$$

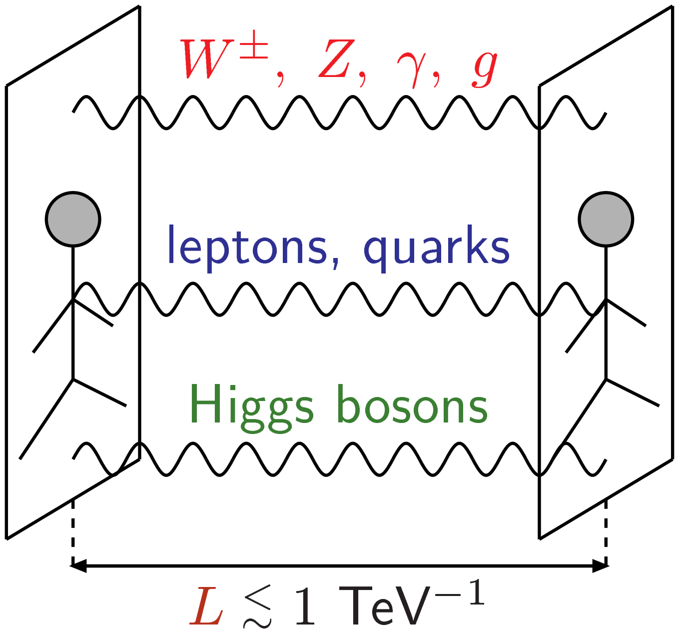
For $n = 1$ and $M_F \sim 1$ TeV, one finds $R \sim L_\odot \sim 10^{13}$ cm; this scenario is ruled out. For $2 \leq n \leq 6$ and $M_F \sim 1$ TeV, we have

$$R \;\sim\; 10^{-3}\,{\rm mm}\,,\; 1\,\mathring{A} \;\; \ldots$$

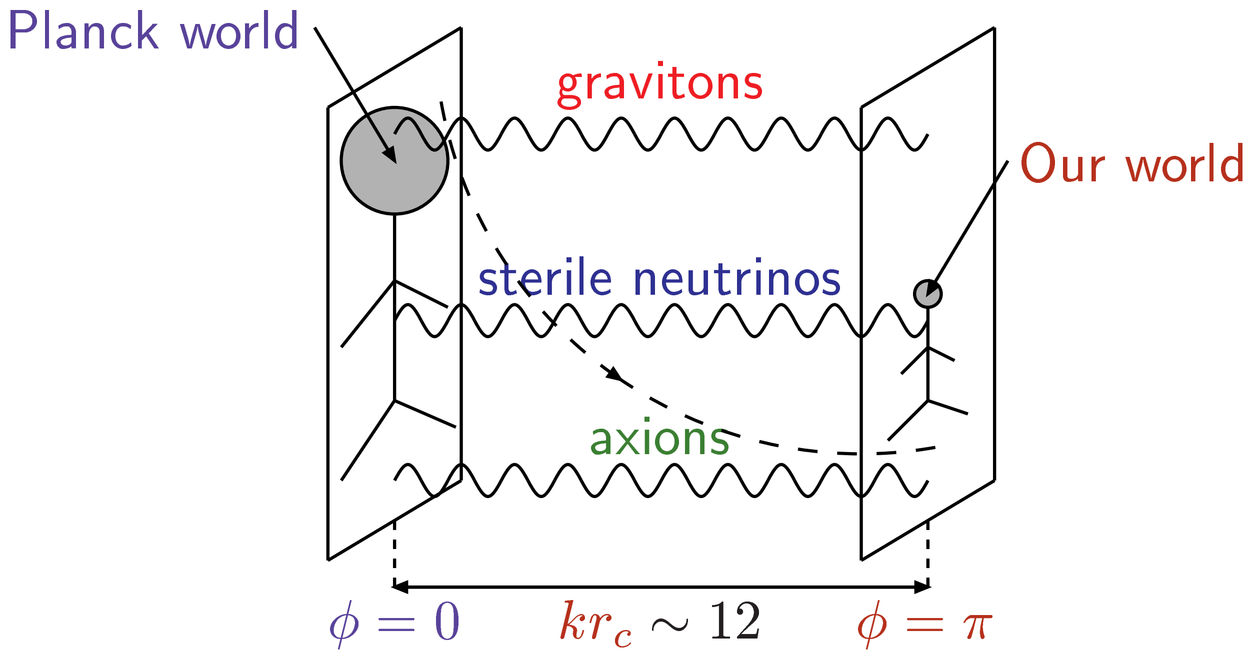
Astrophysical limits require that $M_F \gtrsim 100$ TeV for $n = 2$, whilst $M_F \gtrsim 1$ TeV, for $n > 2$.

[e.g. L.J. Hall and D. Smith, Phys. Rev. **D60** (1999) 085008]

- **Low compactification scale of gauge interactions** [I. Antoniadis, Phys. Lett. **B246** (1990) 377; J.D. Lykken, Phys. Rev. **D54** (1996) 3693; K.R. Dienes, E. Dudas and T. Gherghetta, Phys. Lett. **B436** (1998) 55; Nucl. Phys. **B537** (1999) 47]:



- **Randall-Sundrum higher-dimensional model** [Phys. Rev. Lett. **83** (1999) 3370, also motivated by P. Hořava and E. Witten, Nucl. Phys. **B460** (1996) 506; Nucl. Phys. **B475** (1996) 94]:



$$ds^2 = e^{-2kr_c|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 d\phi^2$$

Literature on phenomenological studies:

- P. Nath and M. Yamaguchi, Phys. Rev. **D60** (1999) 116006; Phys. Lett. **B466** (1999) 100.
- W.J. Marciano, Phys. Rev. **D60** (1999) 093006;
  M. Masip and A. Pomarol, Phys. Rev. **D60** (1999) 096005.
- R. Casalbuoni, S. De Curtis, D. Dominici and R. Gatto, Phys. Lett. **B462** (1999) 48;
  C. Carone, Phys. Rev. **D61** (2000) 015008.
- T. Rizzo and J. Wells, Phys. Rev. **D61** (2000) 016007;
  A. Strumia, Phys. Lett. **B466** (1999) 107.
- A. Delgado, A. Pomarol and M. Quiros, Phys. Rev. **D60** (1999) 095008; JHEP **0001** (2000) 030.
- I. Antoniadis and K. Benakli, Int. J. Mod. Phys. **A15** (2000) 4237;
  I. Antoniadis, K. Benakli and A. Laugier, JHEP **0105** (2001) 044.
- K. Cheung and G. Landsberg, Phys. Rev. D (to be published) hep-ph/0110346.
- A. Mück, A.P., R. Rückl, Phys. Rev. **D65** (2002) 085037; and hep-ph/0312186 (to appear in NPB).

⋮

• Minimal 5-dimensional models and quantization of the theory in the generalized R_ξ gauge

5D-QED on S^1/Z_2:

$$\mathcal{L}(x,y) \;=\; \mathcal{L}_{\rm inv}(x,y) \;+\; \mathcal{L}_{\rm GF}(x,y)\,,$$

where

$$\begin{aligned}
\mathcal{L}_{\rm inv}(x,y) &= -\frac{1}{4}F_{MN}(x,y)F^{MN}(x,y) \\
F_{MN}(x,y) &= \partial_M A_N(x,y) - \partial_N A_M(x,y)
\end{aligned}$$

Requiring that $\mathcal{L}_{\rm inv}$ remains invariant under

$$A_M(x,y) \;\to\; A_M(x,y) + \partial_M \Theta(x,y)$$

and under S^1/Z_2 implies:

$$\begin{aligned}
A_M(x,y) &= A_M(x,y+2\pi R)\,, \\
A_\mu(x,y) &= \pm A_\mu(x,-y)\,, \\
A_5(x,y) &= \mp A_5(x,-y)\,, \\
\Theta(x,y) &= \Theta(x,y+2\pi R)\,, \\
\Theta(x,y) &= \pm\Theta(x,-y)\,.
\end{aligned}$$

For conventional QED with a massless mode, A_μ should be even under Z_2, i.e. $A_\mu(x,y) = A_\mu(x,-y)$.

Gauge-fixing Lagrangian for the R_ξ gauge:

$$\mathcal{L}_{\rm GF}(x,y) \; = \; -\frac{1}{2\xi}(\partial^\mu A_\mu \; - \; \xi\, \partial_5 A_5)^2 \, .$$

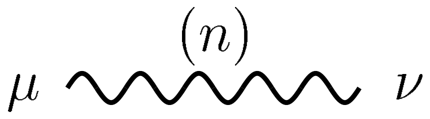
Remarks:

– $\mathcal{L}_{\rm GF}(x,y)$ is S^1/Z_2 and 4D-Lorentz invariant, but explicitly breaks the 5D-Lorentz and gauge symmetries.

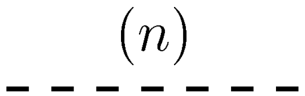
– 5D-Lorentz invariance will be broken by bulk radiative effects.

see, e.g., H. Georgi, A.K. Grant and G. Hailu, Phys. Lett. **B506** (2001) 207, . . .

Propagators of KK photons and would-be Goldstone bosons:



$$= \frac{i}{k^2 - \left(\frac{n}{R}\right)^2} \left[-g^{\mu\nu} \; + \; \frac{(1-\xi) k^\mu\, k^\nu}{k^2 - \xi \left(\frac{n}{R}\right)^2} \right]$$



$$= \frac{i}{k^2 - \xi \left(\frac{n}{R}\right)^2}$$

Note that $\mathcal{L}_{\rm GF} = -\frac{1}{2\xi}(\partial^M A_M)^2$ does not admit the unitary gauge where all unphysical modes decouple when $\xi \to \infty$.

Abelian 2-Higgs Model:

$$\begin{aligned}\mathcal{L}(x,y) &= -\frac{1}{4}F^{MN}F_{MN} + (D_M\Phi_1)^*(D^M\Phi_1)\\ &\quad + \delta(y)(D_\mu\Phi_2)^*(D^\mu\Phi_2)\\ &\quad - V(\Phi_1,\Phi_2) + \mathcal{L}_{\rm GF}(x,y)\end{aligned}$$

$$\begin{aligned}\Phi_1(x,y) &= \frac{1}{\sqrt{2}}\left(\frac{v_1}{\sqrt{2\pi R}} + h_1(x,y) + i\,\chi_1(x,y)\right),\\ \Phi_2(x) &= \frac{1}{\sqrt{2}}\left(v_2 + h_2(x) + i\,\chi_2(x)\right),\end{aligned}$$

where $v_1 = v\cos\beta$ and $v_2 = v\sin\beta$.

KK gauge-boson masss matrices:

$$M_A^2 = M_{\rm brane}^2 + M_{\rm bulk}^2\,.$$

$\sin\beta = 1 \;\Rightarrow\; M_A^2 = M_{\rm brane}^2$:Brane Higgs

$\sin\beta = 0 \;\Rightarrow\; M_A^2 = M_{\rm bulk}^2$:Bulk Higgs

Appropriate gauge-fixing Lagrangian for the R_ξ gauge:

$$\begin{aligned}\mathcal{L}_{\rm GF}(x,y) \;&=\; -\frac{1}{2\xi}\,\Big[\,\partial_\mu A^\mu \;-\; \xi\,\Big(\,\partial_5\,A_5 \;+\; e_5\frac{v}{\sqrt{2\pi R}}\cos\beta\,\chi_1 \\ &\qquad +\, e_5 v\sin\beta\,\chi_2\,\delta(y)\Big)\Big]^2\,,\end{aligned}$$

where the δ-function on S^1/Z_2 may be represented as

$$\delta(y) \;=\; \frac{1}{2\pi R} \;+\; \sum_{n=1}^{\infty}\frac{1}{\pi R}\;\cos\left(\frac{ny}{R}\right)\;,$$

with the properties $\delta(y) = \delta(y + 2\pi R)$ and $\delta(y) = \delta(-y)$.

The same gauge-fixing procedure can be generalized to a $\mathrm{SU}(2)_L$-bulk, $\mathrm{U}(1)_Y$-bulk model with brane and bulk Higgs doublets.

Localization of fermions on the brane:

We assume that all fermion fields that carry SM charges are localized at the $y = 0$ fixed point of the S^1/Z_2 orbifold:

$$\mathcal{L}_{\rm int}(x) = \bar{\Psi}\,\gamma^\mu\,(g_V + g_A\gamma^5)\,\Psi\;\Big(A_{(0)\mu} \;+\; \sqrt{2}\sum_{n=1}^{\infty}A_{(n)\mu}\Big)\,,$$

where g_V and g_A are given by the SM quantum numbers of the fermion Ψ.

SU(2)$_L$-bulk, U(1)$_Y$-brane model:

$$\begin{aligned}
\mathcal{L}(x,y) &= -\frac{1}{4}F^a_{MN}F^{aMN} + \delta(y)\,[\,-\frac{1}{4}B_{\mu\nu}\,B^{\mu\nu} \\
&\quad + (D_\mu\,\Phi)^\dagger\,(D^\mu\,\Phi) - V(\Phi) + \mathcal{L}_{\mathrm{fermions}}(x)\,] \\
&\quad + \mathcal{L}_{\mathrm{GF}}(x,y) + \mathcal{L}_{\mathrm{FP}}(x,y)
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{L}_{\mathrm{GF}}(x,y) &= -\frac{1}{2\xi}\,(F^a(A^a))^2 - \frac{1}{2\xi}\,(F(B))^2\,\delta(y)\,, \\
\mathcal{L}_{\mathrm{FP}}(x,y) &= \bar{c}^a(x,y)\,\frac{\delta F^a(A^a(x,y))}{\delta\Theta^b(x,y)}\,c^b(x,y) \\
&\quad + \delta(y)\,\bar{c}(x)\,\frac{\delta F(B(x))}{\delta\Theta(x)}\,c(x)\,,
\end{aligned}$$

with

$$\begin{aligned}
F^a(A^a) &= \partial_\mu A^{a\mu} - \xi\,[\partial_5 A^a_5 \\
&\quad - \delta(y)\; ig_5\,(\Phi^\dagger\tau^a\langle\Phi\rangle - \langle\Phi\rangle^\dagger\tau^a\Phi)]\,, \\
F(B) &= \partial_\mu B^\mu + \xi\,i\,\frac{g'}{2}(\Phi^\dagger\langle\Phi\rangle - \langle\Phi\rangle^\dagger)\,.
\end{aligned}$$

• KK effects on electroweak observables

$$\mathcal{O}^{\mathrm{HDSM}} \;=\; \mathcal{O}^{\mathrm{SM}}\,(1\;+\;\Delta_{\mathcal{O}}^{\mathrm{HDSM}}\,X\,)\,,$$

with $X \;=\; \frac{1}{3}\pi^2 m_Z^2 R^2$.

Input parameters:

$$\alpha_{\mathrm{em}},\qquad M_Z = M_Z^{\mathrm{SM}}(1+\Delta_Z X),\qquad G_F = G_F^{\mathrm{SM}}(1+\Delta_G X)$$

Introduce effective weak-mixing angle $\hat{\theta}_w$ by enforcing the tree-level relation

$$G_F \;=\; \frac{\pi\alpha_{\mathrm{em}}}{\sqrt{2}\sin^2\hat{\theta}_w\,\cos^2\hat{\theta}_w\,M_Z^2}\,,$$

i.e. $\sin^2\hat{\theta}_w \;=\sin^2\theta_w\,(1\;+\;\Delta_\theta\,X)$.

Example:

$$\Delta^{\mathrm{HDSM}}_{\Gamma_Z(\nu\overline{\nu})} = \begin{cases} \sin^2\hat{\theta}_w\,(\sin^2\beta-1)^2-1 & \text{:SU(2)}_L\text{-bulk, U(1)}_Y\text{-bulk} \\ -\sin^2\hat{\theta}_w & \text{:SU(2)}_L\text{-brane, U(1)}_Y\text{-bulk} \\ -\cos^2\hat{\theta}_w & \text{:SU(2)}_L\text{-bulk, U(1)}_Y\text{-brane} \end{cases}$$

Observable	Exp. Value ($\mathcal{O}^{\rm EXP}$)	SM Prediction ($\mathcal{O}^{\rm SM}$)
M_W	80.448(62) GeV	80.378(20) GeV
$\Gamma_Z(\rm had)$	1.7439(20) GeV	1.7422(15) GeV
$\Gamma_Z(l^+l^-)$	83.96(9) MeV	84.00(3) MeV
$\Gamma_Z(\nu\bar{\nu})$	498.8(15) MeV	501.65(15) MeV
$Q_W(\rm Cs)$	- 72.06(44)	-73.09(03)
R_e	20.803(49)	20.740(18)
R_μ	20.786(33)	20.741(18)
R_τ	20.764(45)	20.786(18)
R_b	0.21642(73)	0.2158(2)
R_c	0.1674(38)	0.1723(1)
A_e	0.15108(218)	0.1475(13)
A_μ	0.137(16)	0.1475(13)
A_τ	0.1425(44)	0.1475(13)
A_b	0.911(25)	0.9348(1)
A_c	0.630(26)	0.6679(6)
A_s	0.85(9)	0.9357(1)
$A_{\rm FB}^{(0,e)}$	0.0145(24)	0.0163(3)
$A_{\rm FB}^{(0,\mu)}$	0.0167(13)	0.0163(3)
$A_{\rm FB}^{(0,\tau)}$	0.0188(17)	0.0163(3)
$A_{\rm FB}^{(0,b)}$	0.0988(20)	0.1034(9)
$A_{\rm FB}^{(0,c)}$	0.0692(37)	0.0739(7)
$A_{\rm FB}^{(0,s)}$	0.0976(114)	0.1035(9)

• **Methods of our statistical analysis**

– Simple χ^2-test:

$$\chi^2(R) \;=\; \sum_{i,j} (\mathcal{O}_i^{\rm exp} \;-\; \mathcal{O}_i^{\rm HDSM}) V_{ij}^{-1} (\mathcal{O}_j^{\rm exp} \;-\; \mathcal{O}_j^{\rm HDSM})$$

where $V_{ij} = \Delta\mathcal{O}_i \Delta\mathcal{O}_j \rho_{ij}$ is the so-called covariance matrix, $\Delta\mathcal{O}_i$ is the uncertainty of a given observable $\mathcal{O}_i$, and ρ_{ij} is the so-called correlation matrix.

– $\Delta\chi^2$ at the $n\sigma$ CL:

$$\chi^2(R) \;-\; \chi^2_{\rm min} \;<\; n^2\,,$$

where $\chi^2_{\rm min}$ is the minimum of χ^2 for $R > 0$.

– Bayesian with a flat prior in the physical region:

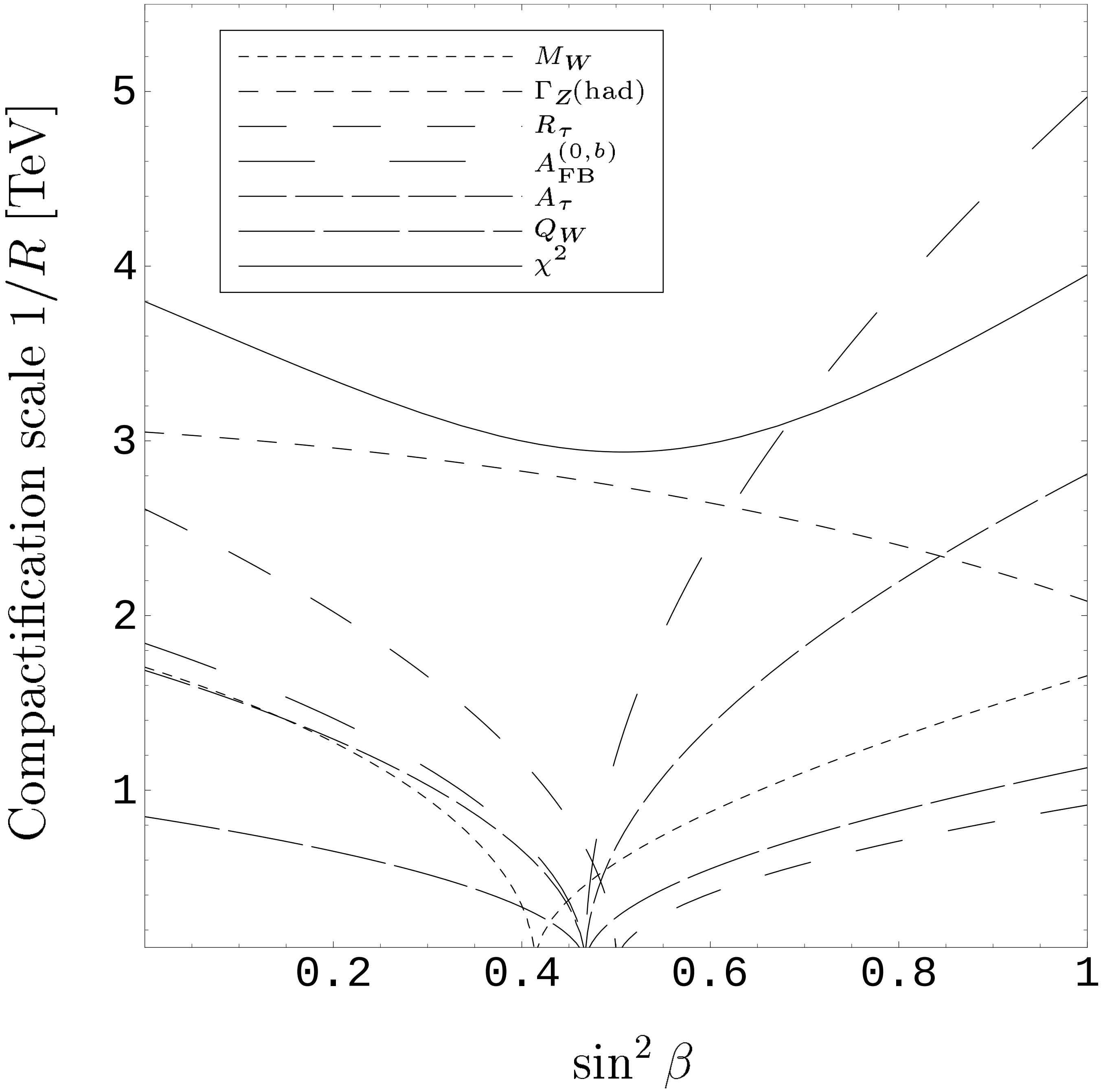
For instance, at the 95% CL, the bound X_{95} is obtained by means of

$$0.95 \;=\; \frac{\int_0^{X_{95}} dX \;\, P(X)}{\int_0^{\infty} dX \;\, P(X)}\;,$$

where $P(X) = \exp[-(\chi^2(X) - \chi^2_{\rm min})/2]$.

• LEP1 and lower-energy constraints

Lower bounds on $1/R$ at the 3σ CL for the SU(2)$_L$-bulk, U(1)$_Y$-bulk model.



3σ limits on $1/R$ [TeV] in brane-bulk and bulk-brane models:

Observable	$U(1)_Y$ in bulk	$SU(2)_L$ in bulk
M_W	1.2	1.2
$\Gamma_Z(\text{had})$	0.8	2.3
$Q_W(\text{Cs})$	0.4	0.8
$A_{\text{FB}}^{(0,b)}$	4.4	2.4
A_τ	2.5	1.4
R_τ	1.0	0.5
global analysis	3.5	2.6

	$SU(2)_L$-brane, $U(1)_Y$-bulk	$SU(2)_L$-bulk, $U(1)_Y$-brane	$SU(2)_L$-bulk, $U(1)_Y$-bulk (brane Higgs)	$SU(2)_L$-bulk, $U(1)_Y$-bulk (bulk Higgs)
2σ	4.3	3.0	4.7	4.6
3σ	3.5	2.6	4.0	3.8
5σ	2.7	2.1	3.1	3.0

Lower bounds on $1/R$ from LEP1 precision observables including (not including) correlations at the 2σ CL

Model	best X	$\Delta\chi^2$	χ^2 test	Bayesian
SU(2)$_L$-brane, U(1)$_Y$-bulk	-0.000052 (-0.00013)	4.2 (4.3)	4.3 (5.7)	4.2 (4.3)
SU(2)$_L$-bulk, U(1)$_Y$-brane	0.00089 (0.00083)	2.9 (3.0)	3.8 (3.4)	3.0 (3.2)
SU(2)$_L$-bulk, U(1)$_Y$-bulk (brane Higgs)	0.00016 (0.00015)	4.6 (4.7)	4.7 (5.8)	4.7 (4.8)
SU(2)$_L$-bulk, U(1)$_Y$-bulk (bulk Higgs)	0.000023 (0.00015)	4.6 (4.6)	4.7 (5.6)	4.6 (4.6)

• LEP2 constraints combined

• Hadronic cross-sections only:

2σ lower limits on $1/R$ [TeV], including (not including) correlations

Model	best X	$\Delta\chi^2$	χ^2 test	Bayesian
$SU(2)_L$-brane, $U(1)_Y$-bulk	-0.0075 (-0.0089)	2.6 (4.0)	1.8 (3.9)	2.3 (3.4)
$SU(2)_L$-bulk, $U(1)_Y$-brane	-0.0026 (-0.0029)	4.7 (7.3)	3.3 (7.0)	4.1 (6.1)
$SU(2)_L$-bulk, $U(1)_Y$-bulk (brane Higgs)	-0.0020 (-0.0022)	5.4 (8.3)	3.8 (8.0)	4.7 (7.0)*
$SU(2)_L$-bulk, $U(1)_Y$-bulk (bulk Higgs)	-0.0017 (-0.0019)	5.8 (9.0)	4.1 (8.6)	5.1 (7.6)*

*K. Cheung and G. Landsberg, hep-ph/0110346.

- **Bhabha scattering and (muon) ((tau)) production only:**

2σ lower limits on $1/R$ [TeV]

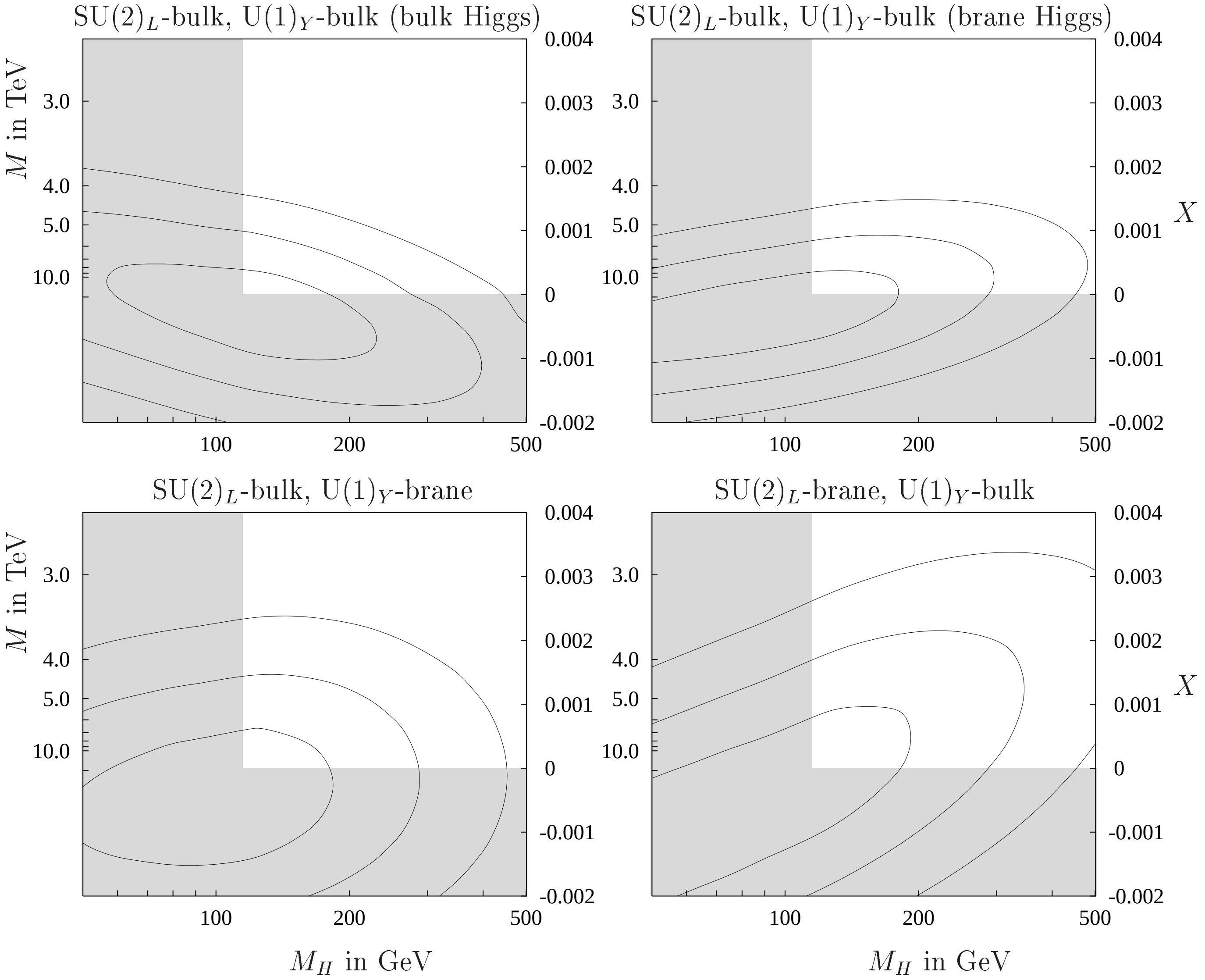
Model	best X	$\Delta\chi^2$	χ^2 test	Bayesian
SU(2)$_L$-brane, U(1)$_Y$-bulk	-0.0024 (0.0026) $((0.0013))$	3.0 (2.0) ((2.0))	2.3 (1.8) ((1.5))	2.8 (2.0) ((2.0))
SU(2)$_L$-bulk, U(1)$_Y$-brane	-0.0063 (0.0042) $((0.0021))$	2.0 (1.5) ((1.5))	1.5 (1.4) ((1.2))	1.8 (1.6) ((1.6))
SU(2)$_L$-bulk, U(1)$_Y$-bulk (brane Higgs)	-0.0017 (0.0016) $((0.0008))$	3.6 (2.5) ((2.5))	2.7 (2.3) ((1.9))	3.3 (2.6) ((2.6))
SU(2)$_L$-bulk, U(1)$_Y$-bulk (bulk Higgs)	-0.0018 (0.0015) $((0.0008))$	3.5 (2.5) ((2.5))	2.7 (2.3) ((2.0))	3.2 (2.6) ((2.6))

• **Combined LEP1 and LEP2 constraints on $1/R$ [TeV], including correlations at the 2σ CL**

Model	best X	$\Delta\chi^2$	χ^2 test	Bayesian
SU(2)$_L$-brane, U(1)$_Y$-bulk	-0.00022	4.7	2.7	4.6
SU(2)$_L$-bulk, U(1)$_Y$-brane	-0.00032	4.3	2.4	4.2
SU(2)$_L$-bulk, U(1)$_Y$-bulk (brane Higgs)	-0.00017	6.1	3.4	5.9
SU(2)$_L$-bulk, U(1)$_Y$-bulk (bulk Higgs)	-0.00032	6.4	3.4	6.1

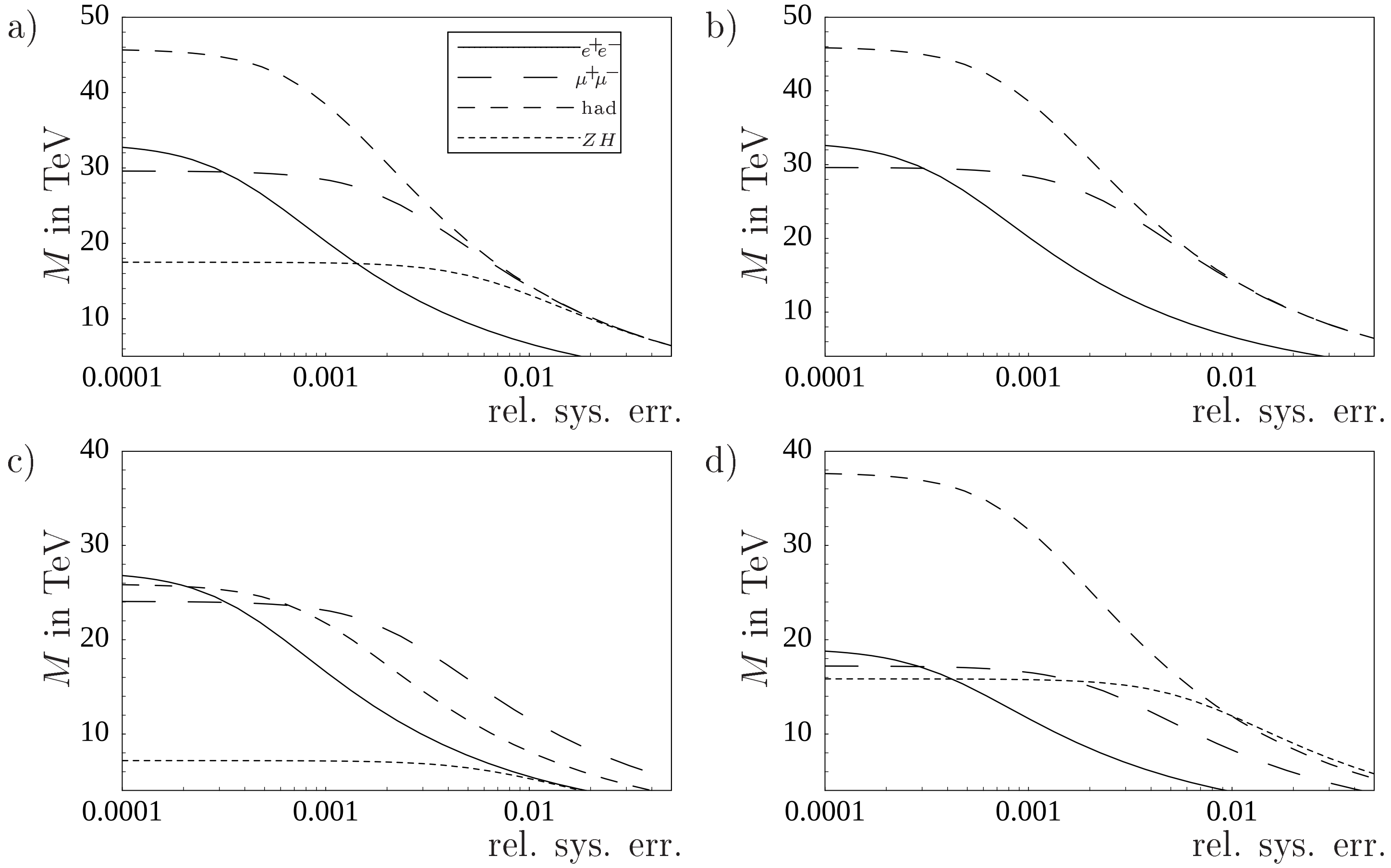
- **Combined LEP1 and LEP2 constraints on $1/R$ [TeV]**

Model	2σ	3σ	5σ
$SU(2)_L$-brane, $U(1)_Y$-bulk	4.6	3.7	2.9
$SU(2)_L$-bulk, $U(1)_Y$-brane	4.2	3.4	2.6
$SU(2)_L$-bulk, $U(1)_Y$-bulk (brane Higgs)	5.9	4.7	3.6
$SU(2)_L$-bulk, $U(1)_Y$-bulk (bulk Higgs)	6.1	4.8	3.6

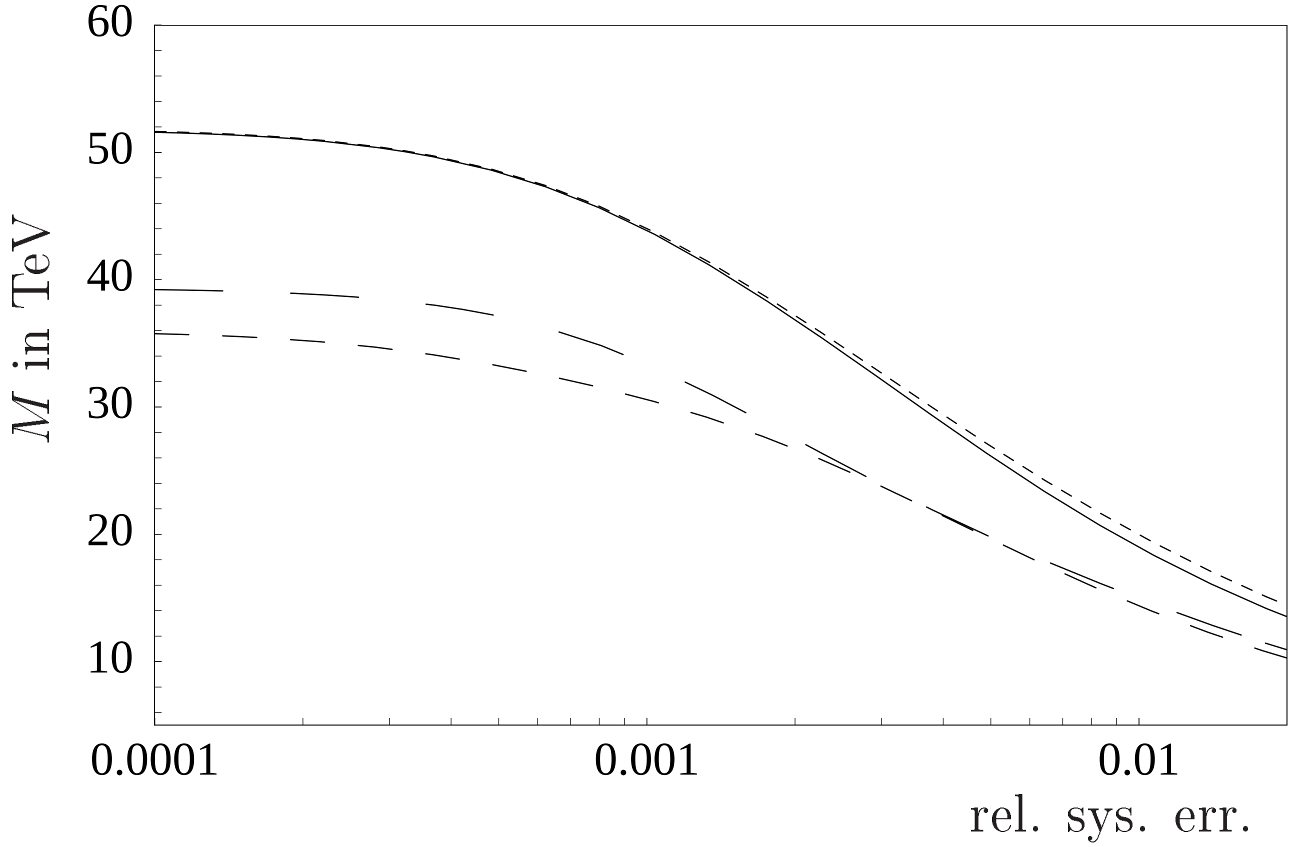


Contours of $\Delta\chi^2 = 1, 4, 9$ derived from the combined analysis of LEP2 data and electroweak precision measurements. The shaded regions of the parameter space correspond to $m_H < 114$ GeV and/or $M^2 < 0$.

• Linear Colliders



Expected sensitivity to M as a function of relative systematic error at $\sqrt{s} = 800$ GeV and for an integrated luminosity of 1000 fb^{-1}: (a) bulk-bulk model with brane Higgs, (b) bulk-bulk model with bulk Higgs, (c) brane-bulk model, and (d) bulk-brane model.



Combined sensitivity to the compactification scale M as a function of relative systematic error at $\sqrt{s} = 800$ GeV and for an integrated luminosity of 1000 fb^{-1} in the models bulk-bulk with brane Higgs (short dashed), bulk-bulk with bulk Higgs (solid), brane-bulk (dashed), and bulk-brane (long dashed).

The GigaZ option:

	brane-bulk	bulk-brane	bulk-bulk (brane Higgs)	bulk-bulk (bulk Higgs)
A_{LR}	12.4	6.8	14.2	12.4

2σ bound on the compactification scale M in TeV which can be obtained with the GigaZ option at TESLA if the measured value of A_{LR} coincides with the SM expectation.

• Conclusions

- Minimal 5-dimensional models with localized gauge and Higgs interactions can be consistently quantized in the generalized R_ξ gauge, before compactification.

- In 5-dimensional models with localized fermions, combined LEP1 (and low-energy observables) and LEP2 electroweak data place stringent lower limits on $1/R$. At the 2σ CL, including correlations, we obtain:

  (i) ≈ 6 TeV, for the SU(2)$_Y$-bulk, U(1)$_Y$-bulk model, with a brane or a bulk Higgs boson.

  (ii) ≈ 4.6 TeV, for SU(2)$_Y$-brane, U(1)$_Y$-bulk model.

  (iii) ≈ 4.2 TeV, for SU(2)$_Y$-bulk, U(1)$_Y$-brane model.

- TEVATRON data appear unlikely to significantly influence our derived bounds.

- Uncertainties: (i) Higgs-boson mass; (ii) Loop effects; (iii) Model-dependent winding number contributions (relevant when M_s and $1/R$ are comparable)