GOLEM: Automation in Loop Calculations

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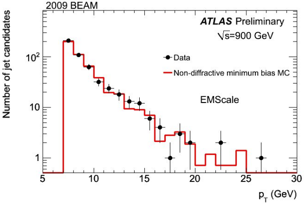
Thursday 8th April, 2010

1. LHC, NLO
2. GOLEM
3. ZZ+jet
4. Neutralino pairs
5. Summary and Outlook

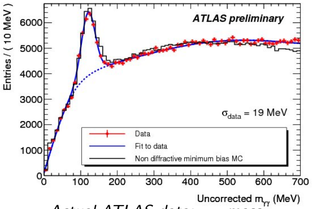
LHC data are on their way

- The LHC is running – a new era in particle physics is dawning.
- Backgrounds huge problem – new physics needs accurate calculation (NLO) of signal-background ratio.
- Experimentalists need, e.g. $pp \longrightarrow t\bar{t} + 2jets$; $pp \longrightarrow VVb\bar{b}$. Preferably public code, numerically robust and fast.
- First task, "rediscovery" of Standard Model, has begun!

Background: a genuine event from CMS



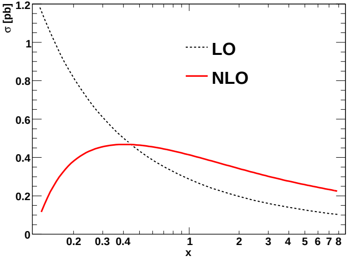
Actual ATLAS data: Jet p_T

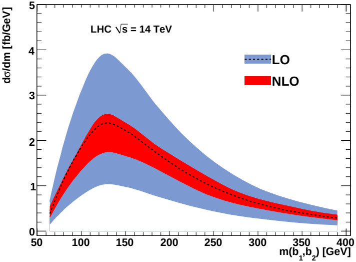


Actual ATLAS data: π_0 mass

Need for NLO: example

- Example here is $q\bar{q} \to b\bar{b}b\bar{b}$, for $m_b = 0$
- Important background to many Higgs and BSM channels – precision required
- LO gives little more than order of magnitude. Reduction in scale variation uncertainty:

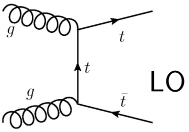


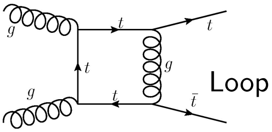


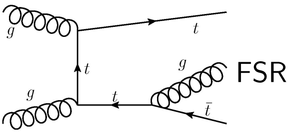
Factorisation scale $\mu_F = 100 GeV$, renormalisation scale μ_R variable: $\mu_R = x\mu_0$ where $\mu_0 = \sqrt{\sum_j p_T^2(b_j)}$ bands from $\mu_0/4$ to $2\mu_0$

NLO calculations

$$\mathcal{A}_{total} = \mathcal{A}_{LO} + (\underbrace{\mathcal{A}^{virtual}}_{Loop\ \ part} + \underbrace{\mathcal{A}^{real}}_{ISR\ \ and\ \ FSR})_{NLO} + \dots$$







- Real corrections and dipole subtraction performed by other tools
- Task of GOLEM: calculate $\mathcal{A}^{virt}$
- ($\mathcal{A}^{LO}$ of hard process several programmes, including GOLEM)

GOLEM

GOLEM (General One Loop Evaluator of Matrix elements)

T. Binoth, G. Cullen, N. Greiner, A. Guffanti, J.-P. Guillet, G. Heinrich, S. Karg, N. Kauer, T. Kleinschmidt, T. Reiter, MR, I. Wigmore

Aim: Automated tool which can easily be interfaced with leading order matrix element generators

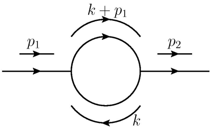
GOLEM consists of:

- Method – calculating using Feynman diagrams.
- Library for integrals
- Interface and integrated approach – Golem 2.0

GOLEM Method

- Expressions from Feynman diagrams
- Helicity projection
  - Subexpressions are smaller (more manageable)
  - Scalar: no non-commuting behaviour
  - Have spin information
  - Gauge invariant in massless case
  - By symmetry only a subset needs calculating, except as a check.
- Tensor reduction: e.g. rank $r = 1$, legs $N = 2$

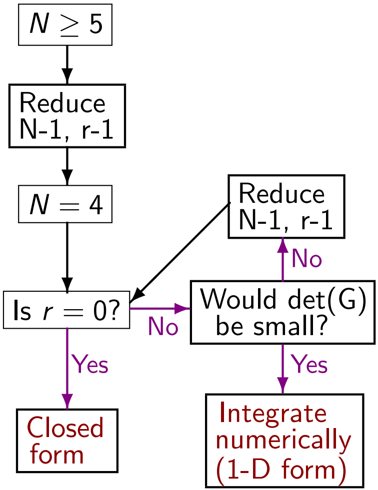
$$\mathcal{I} = \int \frac{d^d k}{i\pi^{\frac{d}{2}}} \frac{k \cdot p_1}{k^2 (k + p_1)^2}$$



Can write top $2k \cdot p_1 = (k + p_1)^2 - k^2 - p_1^2$
More terms, but get cancellations:
$\longrightarrow$ simpler integrals

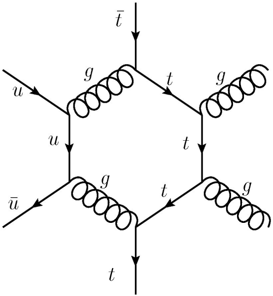
GOLEM Method: Tensor Reduction

- Tensor reduction: algebraic process, rank reduced at cost of more terms.
- If dangerous terms (*small Gram determinants*) produced, instead integrate numerically: *seminumerical* process.
  - These points slower, but give greater stability.
  - Numerical evaluation of 1-D parameter integrals in massless case
- For full phase space integration, seminumerical much faster than fully numerical.



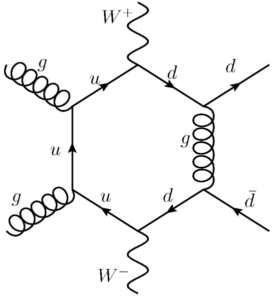
GOLEM Library: Current Capability

- Fortran 95 library.
- Up to six external legs, massive or massless
  - Links LoopTools for finite cases
- Recently expanded to include internal masses
  - Example shown $u\bar{u} \rightarrow t\bar{t}jj$
- Still in the testing phase
  - Plan to make public in May 2010
- Option of one-dimensional formulation of integrals (Slide 8) not yet available in all new cases
- Extension to complex masses under construction



GOLEM Library: Public Version

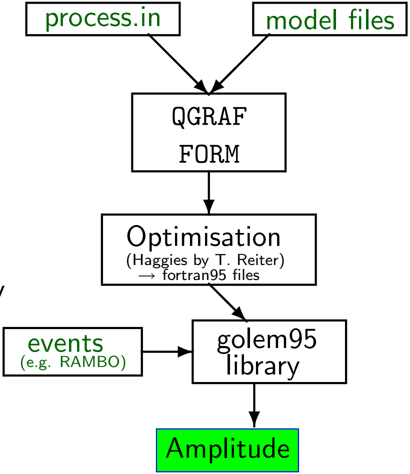
- Public version (*Oct 2008*)
  - Six external legs, massive or massless.
  - Internal legs must be massless
  - Example shown $gg \to W^+W^-jj$ (massless light quarks)



Available at *http://lappweb.in2p3.fr/lapth/Golem/golem95.html*

Golem 2.0

- Feed in:
  - process – standard input file demonstrating all options and exclusions
  - model file (SM already present)
  - kinematic point (e.g. from RAMBO or Monte Carlo generator)
- Expressions QGRAF, calculated by GOLEM (black box!)
- Model files: CalcHEP/LanHEP ready, FeynRules ongoing
  - MSSM: see later
- Can also draw diagrams



Binoth Les Houches Accord

Standardisation regime agreed at Les Houches 2009

- improve ease-of-use, portability and comparability
- modular structure
- early days!

Monte Carlo tool (MC) | One-Loop Provider (OLP)

initialisation:
process info
colour, helicity sum
model parameters
fix scheme . . .

order →

copy/confirm

← contract

runtime:
events ⇄ $A_2, A_1, A_0, |Born|^2$

Standard Interface

ZZ + jet production

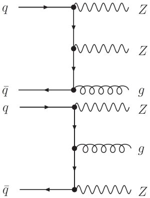
arXiv: 0911.3181

T. Binoth, T. Gleisberg, S. Karg, N. Kauer, G. Sanguinetti

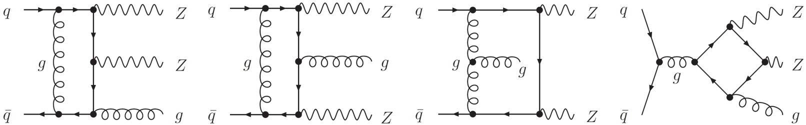
Investigation of ZZj production at LHC and Tevatron, to NLO in QCD

- GOLEM for virtual part
- SHERPA, MadGraph/MadDipole, HELAC for dipole subtraction and real part

Cut on hardest jet: $p_T > 50\text{GeV}$



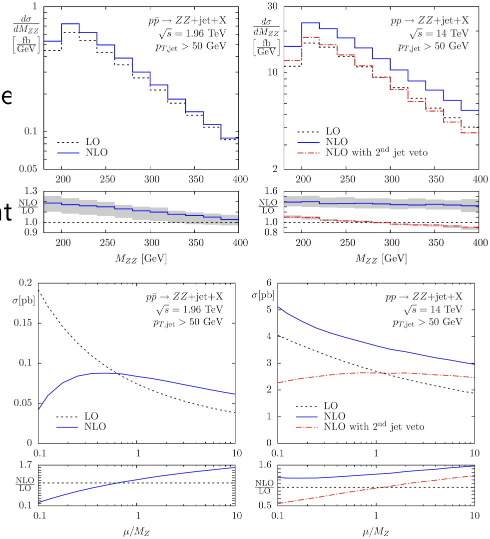
LO



Virtual

ZZ + jet production (cross sections)

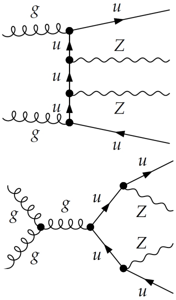
- $\frac{d\sigma}{dM_{ZZ}}$ against M_{ZZ} and scale variation of σ shown
- Tevatron $\sqrt{s} = 1.96$GeV left, LHC $\sqrt{s} = 14$GeV right
- Tevatron NLO scale variation reduced
- LHC NLO qualitatively unchanged (blue)
- With 2nd jet veto (red) good reduction



ZZ + jet production (2nd jet)

For improvement, LHC needs 2nd jet veto [1]

- Real corrections have 2 jets
- New channels open involving e.g. initial state gluons
- Tree graphs: LO-like large influence
  - uncompensated logs of μ_F
- Significant at $\sqrt{s} = 14$GeV, (not at $\sqrt{s} = 1.96$GeV)
- LHC needs veto on 2nd jet ($p_T > 50$GeV) to have NLO-like improvement
- Demonstrates caution required moving from Tevatron to LHC



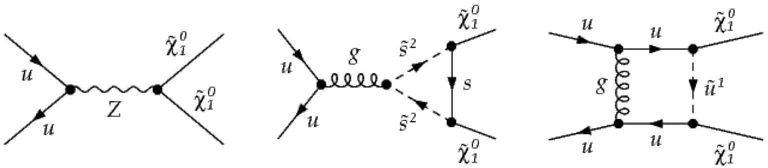
[1] Observed previously for WWj (Dittmaier et al; Campbell, Ellis, Zanderighi) and $W\gamma j$ (Zeppenfeld et al)

Neutralino pairs

Under construction: QCD corrections to SUSY processes, such as neutralino pair production (with jets)

1. With Golem 2.0
2. With alternative method using FeynArts' expression generation

Testing ground for extensions of Golem 2.0



Summary and Outlook

- In principle applicable to all processes
  - IR and UV divergences regulated by dimensional regularisation
- Golem 2.0: new version, greater automation and flexibility
  - Public release this year
- Move towards six-leg amplitudes with several mass scales
  - Public release May 2010
- Can be expanded to NLO outside SM, ongoing

Backup Slides

References

Slide 5: examples of dipole-subtraction programmes are MadDipole, HELAC dipole, Sherpa, TevJet and AutoDipole.
Slide 4's material is from Binoth et al. arXiv: 0910.4379
Slide 12: for details see arXiv: 1001.1307
Slide 20 (Majorana and treating as Dirac) – Feynman rules for fermion-number-violating interactions Denner, Eck, Hahn, Küblbeck
Slide 16 – good explanation of regularisation and reduction schemes: Signer and Stöckinger arXiv: 0807.4424

For the most recent update on GOLEM, see Les Houches Proceedings arXiv: 1003.1241

Authors

FeynArts/FormCalc: T. Hahn
FORM: J. Vermaseren
Golem: T. Binoth, J.-Ph. Guillllet, G. Heinrich, E. Pilon, T. Reiter
QGRAF: P. Nogueira

Neutralino pairs (2)

Regularisation and reduction

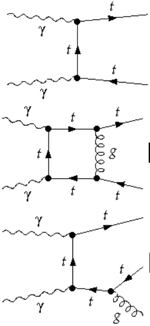
- 't Hooft-Veltman regularisation used so far breaks SUSY
- Either restoration terms or use different scheme

Any purists look away now

Majorana spinors:

- Neutralinos are Majorana particles – QGRAF/Golem 2.0 as it stands cannot deal with them directly
- Can be treated as Dirac if we choose a direction and are careful about a relative sign
- Implemented but in testing phase

Feyman Diagrams to Integrals



- Well-known pictorial representations of amplitudes.
- Diagrams correspond to terms in perturbative expansion: calculate directly.
- Expressions: *FeynArts*[1]; *QGRAF*; also by hand (check).
- Amplitude:

$$\mathcal{A}_{total} = \mathcal{A}_{LO} + (\underbrace{\mathcal{A}^{virtual}}_{Loop\ \ part} + \underbrace{\mathcal{A}^{real}}_{ISR\ \ and\ \ FSR})_{NLO} + \ldots$$

- Object: calculate $\mathcal{A}^{virt}_{NLO}$
- Linear combination of tensor integrals of different ranks

$$\mathcal{A}^{virt}_{NLO} = \sum_n \sum_i c^i_{\mu_1,\mu_2\ldots,\mu_n} \mathcal{I}_i^{\mu_1,\mu_2\ldots,\mu_n}$$

as amplitude must be scalar: all free indices of different γ^μ must be contracted.

[1] Also able to do calculation, but we only use its expression generation

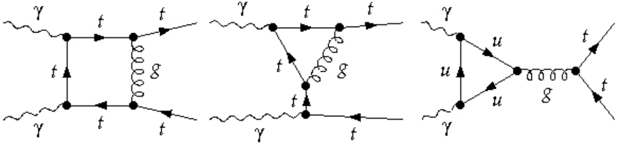
- e.g. from a one-loop diagram might get:

$$\mathcal{A}^{virt}_{NLO} = \underbrace{-e^4 \overline{v}_{p_4} \gamma_{\nu_1} \gamma_{\nu_2} \gamma_{\nu_3} u_{p_3}}_{c_{\nu_1,\nu_2,\nu_3}} \underbrace{\int \frac{d^d k}{i\pi^{\frac{d}{2}}} \frac{k_1^{\nu_1} k_2^{\nu_2} k_3^{\nu_3}}{(k_1^2 - m_1^2)...(k_4^2 - m_4^2)}}_{\mathcal{I}^{\nu_1,\nu_2,\nu_3}}$$

  (details of momenta glossed over: ks are combinations of external momenta, ps)
- *Golem* performs tensor reduction on $\mathcal{I}$s: implementation of algebraic process, rank reduced at cost of more terms.
- If dangerous terms (*small Gram determinants*) produced, integrals instead done numerically: *seminumerical* process.
- Seminumerical much faster than fully numerical.

Interfacing

- Expressions produced by *FeynArts*/*FormCalc*.
- Process using FORM: spinor algebra on the coefficients, analytical steps, cancellations.
- Pass to *Golem* (FORTRAN)
- Nearly complete: leading order with one spinor line
  Analytical control for check where expression by hand
  e.g. leading order $\gamma\gamma \to t\bar{t}$
- Started: one one-loop case



Form Factors

Strip off Lorentz structure, deal with integrals which are Lorentz scalars with coefficients of higher rank. Find *form factors* A, B, C (no higher)

$$
\begin{aligned}
\mathcal{I}_N^{n,\mu_1\ldots\mu_r}(\mathcal{S}) = & \sum_{l_1\ldots,l_r\in\mathcal{S}} p_{l_1}^{\mu_1}\ldots p_{l_r}^{\mu_r} A^{N,r}_{l_1,\ldots,l_r}(\mathcal{S}) \\
& + \sum_{l_1\ldots,l_{r-2}\in\mathcal{S}} [g^{\cdot\cdot} p^{\cdot}_{l_1}\ldots p^{\cdot}_{l_{r-2}}]^{\{\mu_1\ldots\mu_r\}} B^{N,r}_{l_1\ldots,l_{r-2}}(\mathcal{S}) \\
& + \sum_{l_1\ldots,l_{r-4}\in\mathcal{S}} [g^{\cdot\cdot} g^{\cdot\cdot} p^{\cdot}_{l_1}\ldots p^{\cdot}_{l_{r-4}}]^{\{\mu_1\ldots\mu_r\}} C^{N,r}_{l_1\ldots,l_{r-4}}(\mathcal{S})
\end{aligned}
$$

Where $\mathcal{S}_{ij} = (r_i - r_j)^2 - m_i^2 - m_j^2$, N is the number of legs, n is the number of dimensions (e.g. $4 - 2\epsilon$), and the notation represents distributing the r μs between the ps and gs.

Feynman parametrisation

- To combine multiple denominators D into a single one, use *Feynman parametrisation*.
  Example for 2 Ds, each to a power ν_i:

$$\frac{1}{D_1^{\nu_1} D_2^{\nu_2}} = \frac{\Gamma(\nu_1+\nu_2)}{\Gamma(\nu_1)\cdot\Gamma(\nu_2)} \int\limits_0^\infty dz_1 dz_2 z_1^{\nu_1-1} z_2^{\nu_2-1} \frac{\delta(1-z_1-z_2)}{[z_1 D_1 + z_2 D_2]^{(\nu_1+\nu_2)}}$$

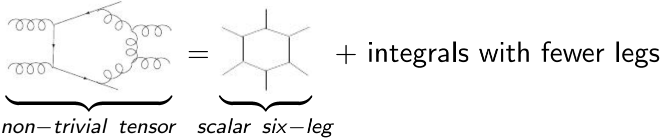
- Rank r tensor integral in n dimensions transforms into sets of $(n+2m)$ dimensional integrals with $r-2m$ Feynman parameters (m integer). Lorentz indices separated off.

  $$\mathcal{I}_N^{n,\mu_1\ldots\mu_r}(\mathcal{S}) \to \sum_m X_m^{\mu_1\ldots\mu_r} \mathcal{I}_N^{n+2m}(j_1,\ldots,j_{r-2m};\mathcal{S})$$

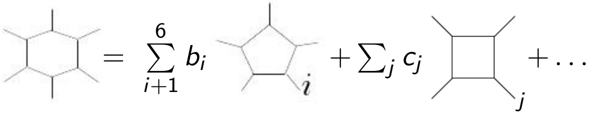
  where the js label Feynman parameters, and X is an object composed of the momenta and the metric tensor.

Tensor Reduction

- First isolate Lorentz structure:



- Reduction: some steps (e.g. from $N = 6$) are not dangerous:



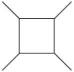
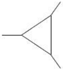
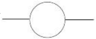
- Tensor integrals (with feynman parameters) with $N \leq 4$ introduce $\frac{1}{\mathcal{B}}$:

$$\mathcal{I}_4^{n+2}(j_1) = \frac{1}{\mathcal{B}}\{b_{j_1}\mathcal{I}_4^n + \sum_i c_i \mathcal{I}_3^n\}$$

$$\mathcal{B} = (-1)^{N+1} det(\mathcal{G})/det(\mathcal{S}); \quad \mathcal{G}_{ij} = 2r_i \cdot r_j;$$

$$\mathcal{S}_{ij} = (r_i - r_j)^2 - m_i^2 - m_j^2$$

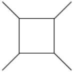
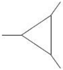
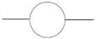
- Higher powers of $\mathcal{B}$ with more Feynman parameters.
- Small $\mathcal{B}$ gives instability.
  If $\mathcal{B} \leq 0.005$, don't reduce and work numerically.
  Otherwise, reduce to a well-known set of basis integrals:

$\mathcal{A}^{virt}_{NLO} = a$  $+b$  $+c$  $+\mathcal{R}$

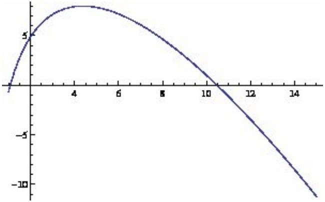
$\mathcal{R}$ is called the rational part.

Unitarity Cuts

- Alternative method: use analytic structure of different types of basis integral to read off coefficients for amplitude.

$\mathcal{A}^{virt}_{NLO} = a$  $+b$  $+c$  $+\mathcal{R}$

- Good if few mass scales, high symmetry.
  - Compuation grows $\mathcal{O}(N^9)$ with N #(external legs).
- Seminumerical Feynman: Compuation grows $\mathcal{O}(2^N\Gamma(N))$,
  - But multiple mass scales and low symmetry (e.g. broken SUSY) easier, also colour part "for free".
  - Faster for low N.



$log(N^9/2^N\Gamma(N))$

ZZ + jet production

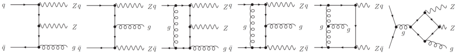
arXiv: 0911.3181

T. Binoth, T. Gleisberg, S. Karg, N. Kauer, G. Sanguinetti

Investigation of ZZj production at LHC and Tevatron, to NLO in QCD

- GOLEM for virtual part
- SHERPA, MadGraph/MadDipole, HELAC for dipole subtraction and real part

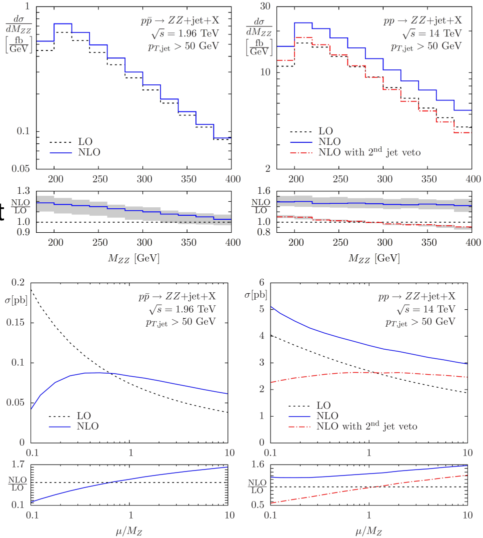
Cut on hardest jet: $p_T > 50$GeV



The point about the jet veto is that at the LHC, the contribution from ZZ+2 jets (which is part of the real corrections) is large (new partonic channels opening up, which contain gluons in the initial state and therefore play a minor role at the Tevatron), and contains uncompensated logs of the factorisation scale as it is "tree level". The jet veto suppresses these contributions.

ZZ + jet production (cross sections)

- $\frac{d\sigma}{M_{Z}Z}$ against M_{ZZ} and scale variation of σ shown
- Tevatron $\sqrt{s} = 1.96$GeV left, LHC $\sqrt{s} = 14$GeV right
- Tevatron NLO scale variation reduced
- LHC NLO qualitatively unchanged (blue)
- With 2nd jet veto (red) good reduction



ZZ + jet production (2nd jet)

For improvement, LHC needs 2nd jet veto

- Real corrections have 2 jets
- New channels open involving e.g. initial state gluons
- Tree graphs: LO-like large influence
  - uncompensated logs of μ_F
- Significant at $\sqrt{s} = 14\text{GeV}$, (not at $\sqrt{s} = 2\text{GeV}$)
- LHC needs veto on 2nd jet ($p_T > 50\text{GeV}$) to have NLO-like improvement
- Demonstrates improvements with judicious cuts!