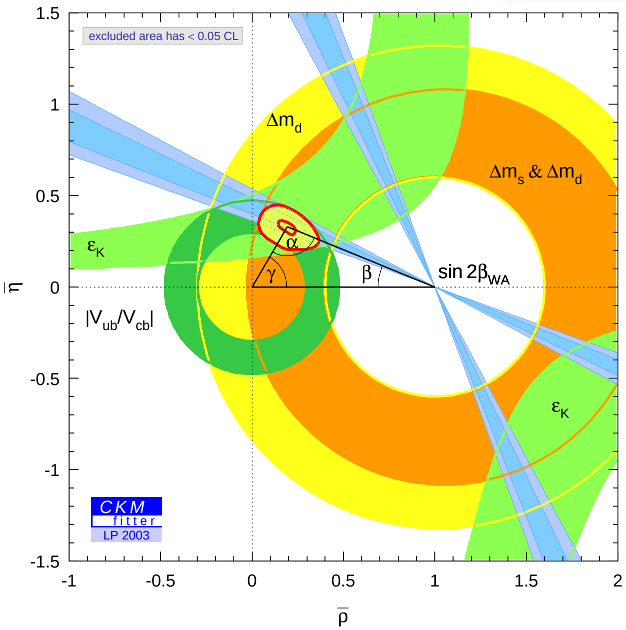
$\bar{B} \to X_s\gamma$ and $\bar{B} \to X_s l^+ l^-$

- Tests of the Flavour Sector
- Inclusive Decays
- Resum the Logs
- $\bar{B} \to X_s\gamma$ and m_c
- Towards NNLO
- NNLO Analysis of $\bar{B} \to X_s l^+ l^-$
- Electroweak corrections
- Conclusions

Advances in Heavy Quark Physics

Tests of the Flavour Sector



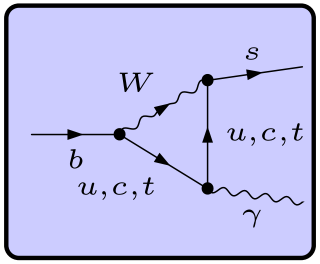
- Unitarity triangle fit [Höcker et al. '03] already constrains new sources of flavour and CP violation
- Only the $\Delta F = 2$ constraints test the quantum level, but they suffer from large hadronic uncertainties

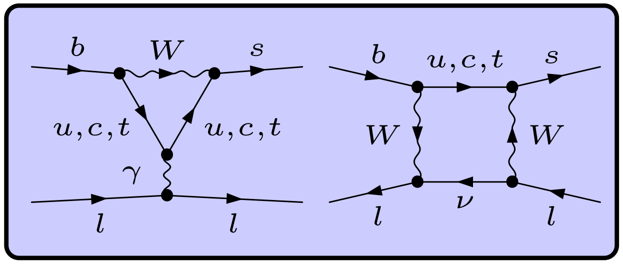
Find theoretically clean decays

Test $\Delta F = 1$ decays up to quantum level

Inclusive $\bar{B} \to X_s \gamma$ and $\bar{B} \to X_s l^+ l^-$ decays

FCNC Decays $b \to s\gamma$ and $b \to sl^+l^-$



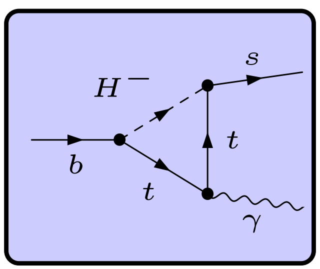


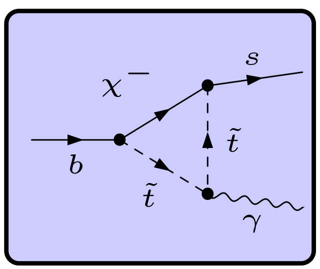
$\propto V_{tb}V_{ts}^* = \mathcal{O}(\lambda^2)$

In the SM forbidden at tree level & CKM suppressed

Precision test of the flavour sector

Enhanced sensitivity to new physics

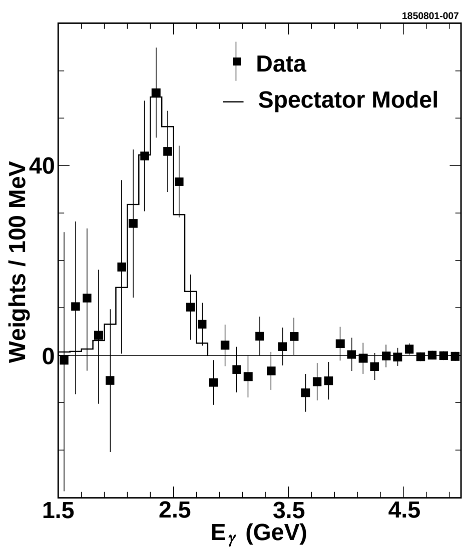


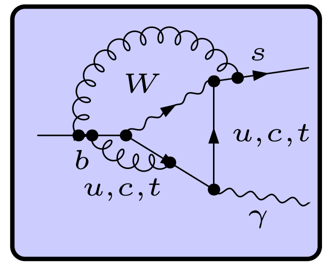


- Charged Higgs contribution enhance $b \to s\gamma$
- Different new physics contributions have to cancel

Two Problems: Bound States and Large Logs

- We can only observe decays of bound states ⇒ decay at parton level may not approximate the hadronic decay
- Study inclusive $\bar{B} \to X_s \gamma$ and $\bar{B} \to X_s l^+ l^-$ decays
- For $\bar{B} \to X_s \gamma$ we know only the integral over the spectrum





For n gluons we have

$$\left(\frac{\alpha_s}{4\pi}\right)^n \log^n \frac{m_b^2}{M_W^2} (LL)$$

$$\left(\frac{\alpha_s}{4\pi}\right)^n \log^{n-1} \frac{m_b^2}{M_W^2} (NLL)$$

- Large logs ⇒ straightforward perturbation theory unreliable
- Use renormalization group to resum leading and next-to-leading logs

Inclusive $\bar{B} \to X_s\gamma$ and $\bar{B} \to X_s l^+ l^-$ Decays

Sum over all X_s final states

$m_b \gg \Lambda_{QCD}$ hadron binding energy

Contribution of external states drops out

- For $m_b \to \infty$ is $\Gamma[\bar{B} \to X_s\gamma] \approx \Gamma[b \to s\gamma] + \Gamma[b \to s\gamma g]^\delta + \ldots$ [Chay et al. '90, Manohar et al. '93]
- $1/m_b^2$ and $1/m_c^2$ corrections can be added systematically [Falk et al. '93, Bigi '92, Voloshin '97, Khodjamirian et al. '00]
- Treatment of $\bar{B} \to X_s l^+ l^-$ is similar to $\bar{B} \to X_s\gamma$ [Ali et al. '96, Bauer et al. '99, Chen et al. '97, Buchalla et al. '97]

$\Rightarrow$ High precision is possible!

Effective Field Theories

At high scales $\mu_0 \sim M_W$ the full theory contains heavy $W, t, \ldots$ and light $g, b, \ldots$ fields:

$$\mathcal{L}_{\rm full} = \mathcal{L}_H(h, l) + \mathcal{L}(l).$$

At a low scale $\mu < \mu_0$ we obtain an effective Lagrangian:

$$\mathcal{L}_{\rm eff} = \mathcal{L}(l) + \delta\mathcal{L}(l)$$

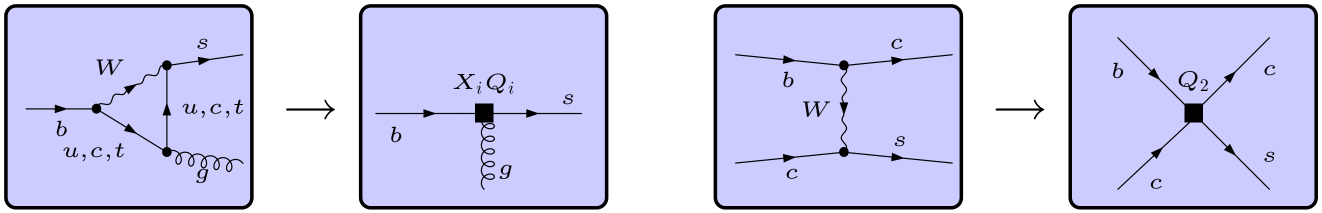
The calculation takes three steps

- Matching of $\mathcal{L}_{\rm full}$ and $\mathcal{L}_{\rm eff}$ at μ_0 gives $\delta\mathcal{L}(L)$
- With the help of the Renormalization Group Equation (RGE) we can relate the effective Lagrangian at the high scale to the low scale one

$$\mathcal{L}_{\rm eff} \quad \text{at} \quad \mu_0 \to \mathcal{L}_{\rm eff} \quad \text{at} \quad \mu$$

- Calculation of the matrix elements

QCD Matching



$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}\times\text{QED}} + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum C_i(\mu)\, Q_i$$

- Current-current

$$Q_1 = (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L)\,,$$
$$Q_2 = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)\,,$$

- QCD Penguin

$$Q_3 = (\bar{s}_L \gamma_\mu b_L) \sum\nolimits_q (\bar{q}\gamma^\mu q)\,,$$
$$Q_4 = (\bar{s}_L \gamma_\mu T^a b_L) \sum\nolimits_q (\bar{q}\gamma^\mu T^a q)\,, \ldots$$

- Magnetic

$$Q_7 = e/g^2\; m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}\,,$$
$$Q_8 = 1/g\; m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}\,,$$

- Semileptonic

$$Q_9 = e^2/g^2\; (\bar{s}_L \gamma_\mu b_L) \sum\nolimits_\ell (\bar{\ell}\gamma^\mu \ell)\,,$$
$$Q_{10} = e^2/g^2\; (\bar{s}_L \gamma_\mu b_L) \sum\nolimits_\ell (\bar{\ell}\gamma^\mu \gamma_5 \ell)\,.$$

Scale Dependence of the Wilson Coefficients

- Wilson coefficients are renormalized

$$C_{i,B} = Z_{ji} C_j$$

and the renormalization constants are expanded

$$Z_{ij} = \delta_{ij} + \sum_{k=1}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^k Z_{ij}^{(k)} \qquad Z_{ij}^{(k)} = \sum_{l=0}^{k} \frac{1}{\epsilon^l} Z_{ij}^{(k,l)}$$

- The scale dependence of the Wilson coefficients

$$\mu \frac{d}{d\mu} C_i(\mu) = \gamma_{ji} C_j(\mu)$$

is given by the anomalous dimension matrix

$$\gamma_{ij} = Z_{ik}\, \mu \frac{d}{d\mu} Z_{kj}^{-1} \qquad = (-\epsilon + \beta(\alpha_s)) Z_{ik} \frac{d}{d\alpha_s} Z_{kj}^{-1}$$

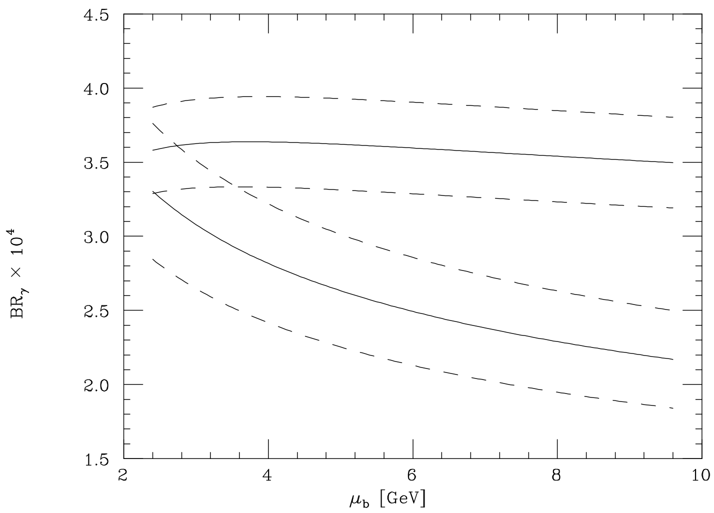
The scale dependence is governed by $Z_{ij}^{(k,l)}$

Scale Dependence of $\bar{B} \to X_s\gamma$

- At LO the branching ratio can be written

$$\mathrm{BR}[\bar{B} \to X_s\gamma]_{E_\gamma > E_0} =$$
$$\mathrm{BR}[\bar{B} \to X_c e\bar{\nu}]_{\mathrm{exp}} \times$$
$$\frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha_{QED}}{\pi g(z)} |C_7^{(0)eff}(\mu)|^2$$

- At NLO we get a $\alpha_s \log(\mu_b/m_b)$ term form the matrix elements
- This reduces the scale uncertainty drastically



In $\bar{B} \to X_s\gamma$ a pecularity arisis

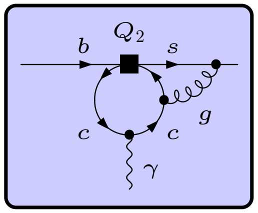
The one loop mixing into the magnetic Operators vanishes

Leading Order needs two loop calculation

NLO needs

- 2-loop matching [Adel, Yao '93; Greub, Hurth '97; Buras et al. '98]
- 3-loop running [Chetyrkin, Misiak, Münz '98; Gambino, Gorbahn, Haisch '03]
- 2-loop matrix elements [Greub, Hurth, Wyler '96, Buras et al. '01, Asatrian et al. '04]
- Bresmstrahlung [Ali, Greub '93; Pott '96]

charm mass dependence starts at NLO



First charm depenendent Matrix Element is 2 loop:

- Formally, any definition for m_c can be used
- Gambino Misiak pointed out to use m_c in $\overline{\mathrm{MS}}$ at $\mu \sim m_b/2$

Beyond NLO QCD

Electroweak corrections

- No $\ln(m_b^2/m_e^2)$ if one uses $\alpha_{\rm em}^{\rm onshell}$ as overall normalisation [Czarnecki, Marciano '98]
- $\alpha_e/\alpha_s \ln(m_W^2/m_b^2)$ negligible [Kagan, Neubert '99; Baranowski Misiak '00]
- Matching reduces $\Gamma[b \to s\gamma]$ by -1.5% for $M_{\rm Higgs} = 115 GeV$ [Gambino, Haisch '00 '01]

Nonperturbative corrections

- $1/m_b^2$ amounts to -3%, $1/m_c^2$ to $+2.5\%$

The dependence on the defintion of m_c (formally NNLO)

- if we use m_c in $\overline{\rm MS}$ at $\mu \sim m_b/2$ we get $+10\%$:

$$\mathrm{BR}(\bar{B} \to X_s\gamma)_{\rm th} = (3.70 \pm 0.30) \times 10^{-4}$$

Bounds on the Charged Higgs Mass

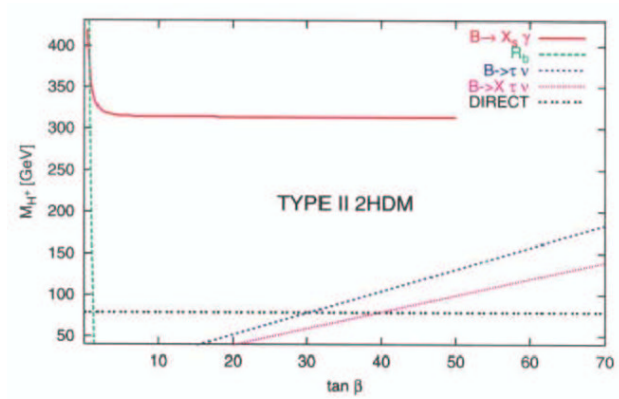
Type II 2HDM

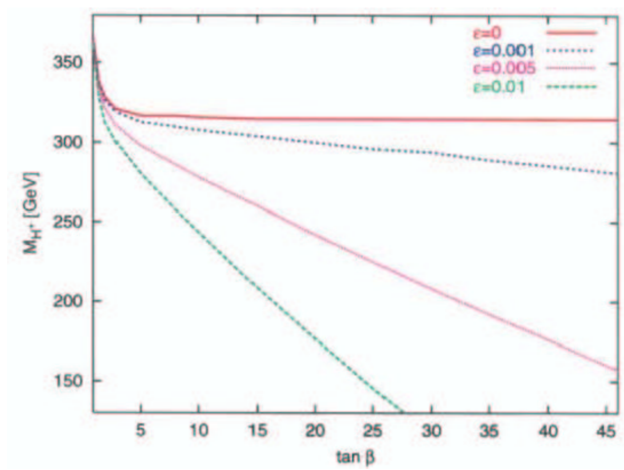
- Always positive contribution to the branching ratio
- Lower bound on m_H saturates for $\tan_\beta \sim 5$
- If one takes pole mass interpretation bound gets weaker $m_H > 280 GeV$

2HDM II $\neq$ MSSM

- non decoupling effects are parametrized by

$$\epsilon = \frac{\alpha_s}{6\pi}\frac{\mu}{m_{\tilde{g}}} f(m_{\tilde{b}_i}, m_{\tilde{g}})$$





Towards a NNLO prediction of $b \to s\gamma$

To settle the m_c dependence we have to go to NNLO, which requires the follwing calculations:

- 2-loop matching of the 4-quark operators [Bobeth, Misiak, Urban '00]
- 3-loop matching of the magnetic operators [Misiak, Steinhauser '04]
- 3-loop mixing of the 4-quark operators [Gorbahn, Haisch in preparation]
- 4-loop mixing into the magnetic operators and 3-loop selfmixing [Gorbahn, Haisch, . . .]
- 3-loop matrix elements of the 4-quark operators [Bieri, Greub, Steinhauser '03; Misiak, Steinhauser]
- 2-loop matrix elements of the magnetic moment operators [Greub, Hurth, Asatrian]

Implications for $\bar{B} \to X_s \gamma$

- The complete NLO prediction of $\bar{B} \to X_s \gamma$ has been done independently by at least two groups
- This is in particular important since the LO analysis suffers from 25% scale uncertainties [Buras '93]
- The NLO SM prediction of $\bar{B} \to X_s \gamma$ is in good agreement with experiment

$$\mathrm{BR}_{\mathrm{th}} = (3.70 \pm 0.30) \times 10^{-4} \sim (3.34 \pm 0.38) = \mathrm{BR}_{\mathrm{exp}}$$

- With improving experimental results the definition of the charm quark mass must be solved
- This means a NNLO calculation is becoming necessary and has been started recently
- This is also important to stringently constraint new Physics

The $\bar{B} \to X_s l^+ l^-$ decay

- Belle and BaBar have recently aounced a clear evidence of $\bar{B} \to X_s l^+ l^-$

$$\mathrm{BR}_{\mathrm{exp}}(\bar{B} \to X_s l^+ l^-) = 6.2 \pm 1.1^{+1.6}_{-1.3} \times 10^{-6}$$

- Non-perturbative corrections can be controlled by
  - the heavy quark expansion for $\Lambda_{\mathrm{QCD}}/m_b$
  - kinematical cuts to avoid $c\bar{c}$ intermidate states $(B \to X_s \bar{c}c \to X_s l^+ l^-)$:

$$\text{low}: q^2 \equiv m^2_{l^+l^-} \in [1\mathrm{GeV}^2, 6\mathrm{GeV}^2]; \qquad \text{high}: q^2 > 14.4\mathrm{GeV}^2; \qquad \text{use}: \hat{s} = q^2/m_b^2$$

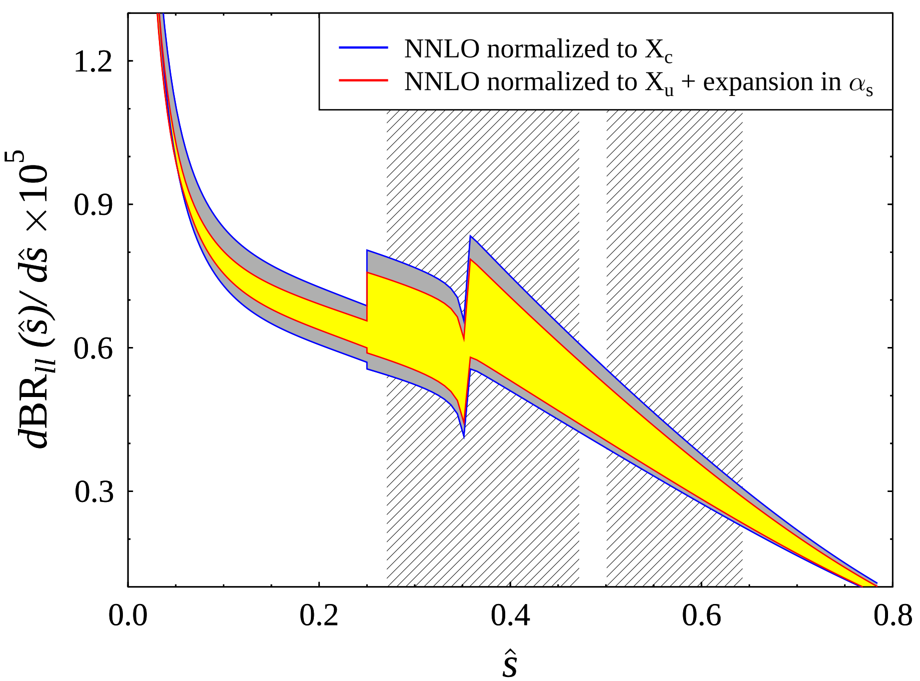
- To cancel m_b^5 dependence and avoid charm mass dependence normalise

$$\mathrm{BR}_{ll} = \frac{\mathrm{BR}[\bar{B} \to X_u l \bar{\nu}]}{C} \left| \frac{V_{ub}}{V_{cb}} \right|^2 \int_{0.05}^{0.25} d\hat{s} \frac{d\Gamma[\bar{B} \to X_s l^+ l^-]/d\hat{s}}{\Gamma[\bar{B} \to X_u l \bar{\nu}]}$$

Completing the NNLO Analysis of $\bar{B} \to X_s l^+ l^-$

Recently the NNLO Calculation has been (nearly) completed

- 2-loop matching conditions [Bobeth, Misiak, Urban '00]
- 2-loop matrix elements of Q_1,Q_2 and bremsstrahlung [Asatrian et al. '02 '03; Ghinculov et al. '03]
- 2-loop matrix element of Q_9 [Bobeth, Gambino, Gorbahn, Haisch '03]
- 3-loop evolution [Gambino, Gorbahn, Haisch '03]
- 2-loop matrix elements of Q_1 and Q_2 for the high q^2 region [Ghinculov, Hurth, Isidori, Yao '03]



Electroweak corrections $\bar{B} \to X_s l^+ l^-$

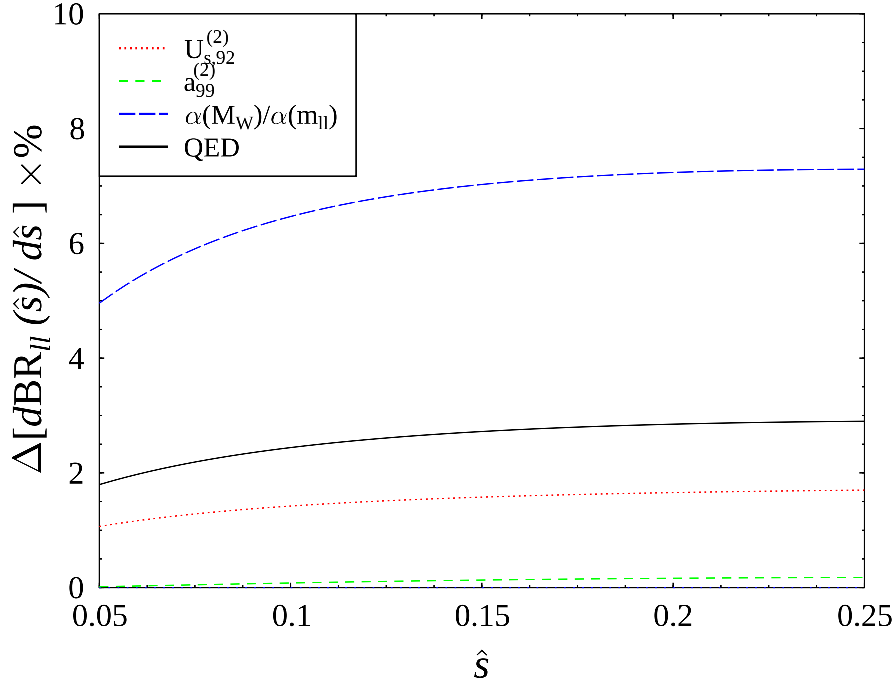
- Matching corrections known [Gambino, Haisch '00 '01]
- 2-loop QED QCD evolutiuon [Bobeth, Gambino, Gorbahn, Haisch '03]

Contributions	BR_{ll}
NLO	$(1.53 \pm 0.27)10^{-6}$
Low q^2	$(1.53 \pm 0.20)10^{-6}$

For the high q^2 region [Isidori '04]

Contributions	$\mathrm{BR}_{ll} \quad (q^2 > 14.4\mathrm{GeV}^2)$
Without QED	$(4.04 \pm 0.78)10^{-7}$

Errors come mainly of parametric nature



Conclusions

$\bar{B} \to X_s l^+ l^-$

- The extrapolated $\mathrm{BR} = 4.2 \pm 0.7 \times 10^{-6}$ agrees with $\mathrm{BR}_{\mathrm{exp}} = 6.2 \pm 1.1^{+1.6}_{-1.3} \times 10^{-6}$
- The NNLO calculation of $\bar{B} \to X_s l^+ l^-$ is completed
- The theory predicion for the clean windows can not be directly confronted with the experimental result
- Future experiments should measure in both regions seperately

$\bar{B} \to X_s \gamma$

- The Standard Model is consistent with the current experimental data
- The main uncertainty of the theory resides in the perturbative side (m_c)
- NNLO calculation will solve this