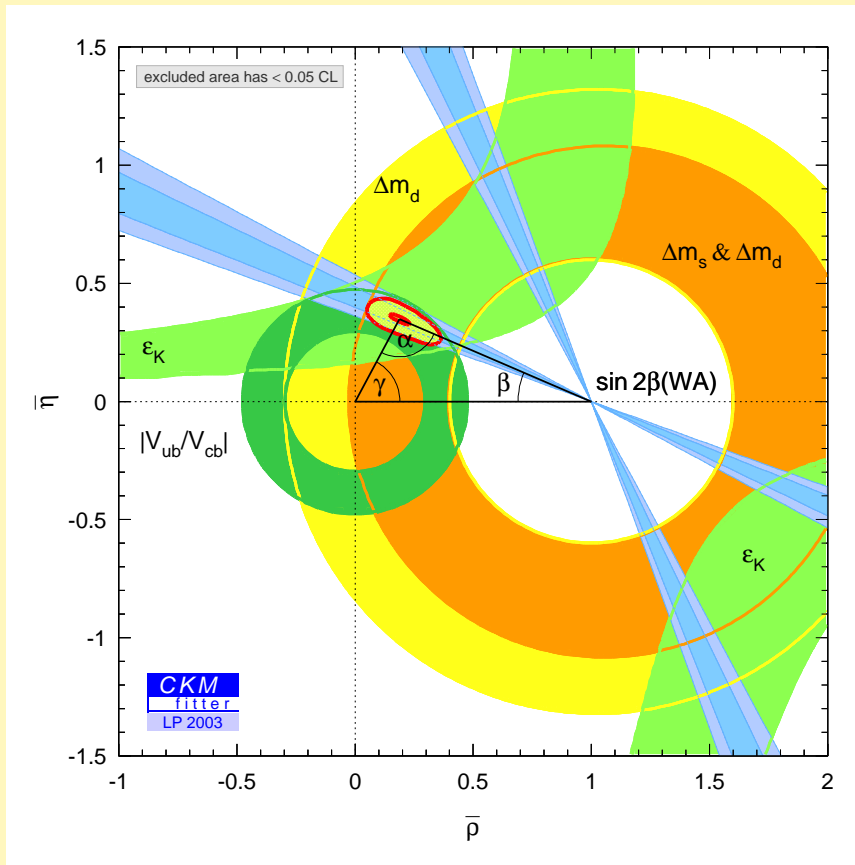


Theoretical Overview of B Physics

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(Some) Goals of B Physics

- precision determination of CKM matrix elements
 - ▶ $|V_{cb}|$ from $B \rightarrow D(*)e\nu$ and $B \rightarrow X_c e\nu$
 - ▶ $|V_{ub}|$ from $B \rightarrow \pi e\nu$ and $B \rightarrow X_u e\nu$
- new sources of CP violation?
 \rightsquigarrow info about scalar sector of SM and BSM
- new physics in flavour changing neutral currents (FCNC)?
- B physics at B factories complementary to LHC:
 once new physics (e.g. SUSY) is established, measure couplings
 (to which LHC is not that sensitive)

Deserves more study!

Theoretical Tools

- treatment of large scales (m_W , m_t , m_{SUSY} , even m_b) by effective field theory methods
- resummation of large (perturbative) QCD logarithms by renormalization group methods
 \rightsquigarrow talk by M. Gorbahn
- treatment of low-energy nonperturbative QCD effects
 - ▶ on the lattice (*calculation from first principles*)
 \rightsquigarrow talk by K. Foley
 - ▶ by QCD sum rules
 (*QCD-based calculation with a certain degree of model dependence*)
 - ▶ within heavy quark expansion
 (*parton model for inclusive decays, QCD factorisation for exclusive decays*)

What We **CANNOT** Expect from B Physics

- **discovery of SUSY!**

(FCNC) B decays sensitive to new physics, but interpretation model-dependent.

Instead, once SUSY with sparticle masses < 1 TeV found at LHC, use B decays to **constrain couplings**.

- **new physics from small effects:**

QCD effects (perturbative and nonperturbative) relevant in most cases, but in general not known to accuracy better than $\sim 10\%$

→ new-physics **unambiguously detectable** in B decays
only if effects larger $\sim 10\%$

Vita brevis, Ars longa. . .

Time limited → selection of topics:

- Effective Field Theory Description
- $B \rightarrow \pi, \rho, \eta, K, K^*$ Form Factors
- CP Violation
- QCD Factorisation

E(ffective) F(ield) T(heory)

- hierarchy of scales $\mu \sim m_b \ll m_W, m_t \ll m_{\text{new physics}}$
- separate scales:
$$\mathcal{H}_{\text{eff}} = \sum_i C_i(\text{large scale}/\mu, \alpha_s) O_i(\mu) + O(1/\text{large scale})$$
- Wilson coefficients C_i to encode short-distance (QCD + EW + NP) effects
- operators O_i to encode long-distance nonperturbative QCD effects
- complications: (cf. talk by M. Gorbahn)
 - ✂ generation of new operators by radiative corrections (penguin etc. operators)
 - ✂ large corrections $\sim \alpha_s \ln(m_W^2/\mu^2)$ etc. to C_i :
need for resummation (renormalisation group improvement)
 - ✂ operator mixing: large anomalous dimension matrices, complicated μ -dependence

Outline

- Effective Field Theory Description
- $B \rightarrow \pi, \eta, K$ Form Factors
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Setting the Stage for $B \rightarrow \pi$

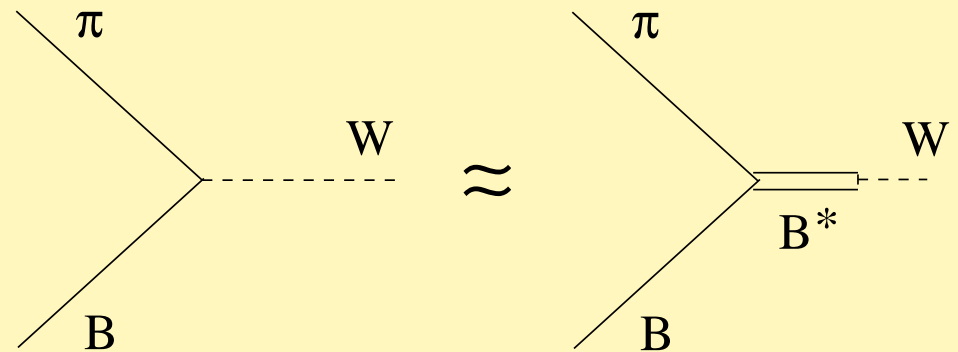
Definition of **form factors** ($0 \leq q^2 \leq (m_B - m_\pi)^2$):

$$\langle \pi | \bar{u} \gamma_\mu b | B \rangle = f_+(q^2)(p_{B\mu} + p_{\pi\mu}) + f_-(q^2)q_\mu \quad [q_\mu = p_{B\mu} - p_{\pi\mu}]$$

$B \rightarrow \pi e \nu$: f_- suppressed by m_e^2/m_B^2 , i.e. get $|V_{ub}|$ from experiment once f_+ is known

Naïve expectation: f_+ dominated by B^* -pole ($m_{B^*} = 5.32$ GeV):

$$f_+(q^2) \propto \frac{1}{m_{B^*}^2 - q^2}$$



Correct expression:
$$f_+(q^2) = \frac{c}{m_{B^*}^2 - q^2} + \int_{(m_B + m_\pi)^2}^{\infty} dt \frac{\rho(t)}{t - q^2}$$

Why $f_+^{B \rightarrow \pi}$?

- physical observable:
 - ◆ no dependence on renormalisation scale μ
 - ◆ direct comparison between theory and experiment
- q^2 dependence accessible experimentally, need only normalisation
 \rightsquigarrow additional check of theory
- clean experimental signature
(as compared to inclusive decays with huge $b \rightarrow c$ background)

But:

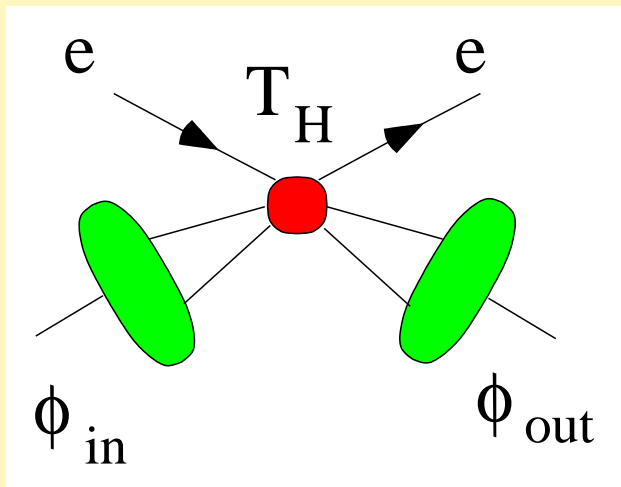
- challenging, as large range of momentum transfer: $0 \leq q^2 \leq 25 \text{ GeV}^2$
- accessible in different theoretical limits:
 - large momentum transfer $m_b \rightarrow \infty$ (\rightarrow QCD sum rules, SCET)
 - small energy (\rightarrow lattice, talk by K. Foley)

Factorisation à la Brodsky/Lepage

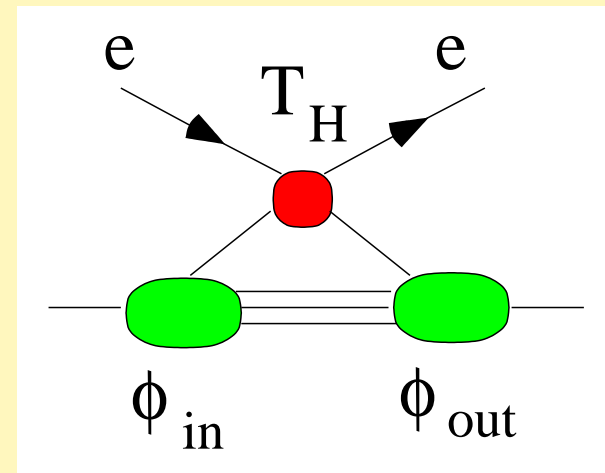
At **large momentum transfer** Q^2 , exclusive QCD processes dominated by states with “valence” quark content; process amplitude **factorises**:

$$M = \prod_j \phi_{\text{out},j}(n_j) \otimes T_H(n_j, n_i) \otimes \prod_i \phi_{\text{in},i}(n_i)$$

$\phi(u)$, $0 \leq u \leq 1$: probability amplitude for collinear quarks with momentum up and $(1-u)p$, resp., to form hadron with momentum p ($p^2 \ll Q^2$)



Purely **hard** process: dominant in “classical” applications of pQCD, e.g. EM π FF; explicitly $O(\alpha_s)$



Soft (Feynman) mechanism: strongly asymmetric kinematical configuration of partons

Apply to B Physics: $Q^2 \sim m_b^2 \rightarrow \infty$

Both hard and soft mechanisms contribute!

↪ **SCET** (soft collinear effective theory) (Bauer/Fleming/Pirjol/Stewart)

- identify (uncalculable) soft/nonperturbative terms order by order in $1/m_b$ expansion
- construct perturbatively calculable (hard) relations between form factors (to given accuracy in $1/m_b$) with uncalculable soft terms *dropping out*

↪ **QCD sum rules on the light-cone** (Ball/Braun)

- calculate both soft and hard terms within the same method, using the techniques of QCD sum rules
- obtain **numerical predictions** (and estimates of theoretical systematic (model-dependent) uncertainty)

QCD Sum Rules on the Light-Cone

$$i \int d^4y e^{iqy} \langle \pi(p) | T[\bar{u}\gamma_\mu b](y) [m_b \bar{b} i \gamma_5 d](0) | 0 \rangle \stackrel{\text{LCE}}{=} \sum_n T_H^{(n)} \otimes \phi_\pi^{(n)}$$

LCE = light-cone expansion

- $\phi_\pi^{(n)}$: π distribution amplitudes (DAs)
- n : twist
- $T_H^{(n)}$: perturbative amplitudes

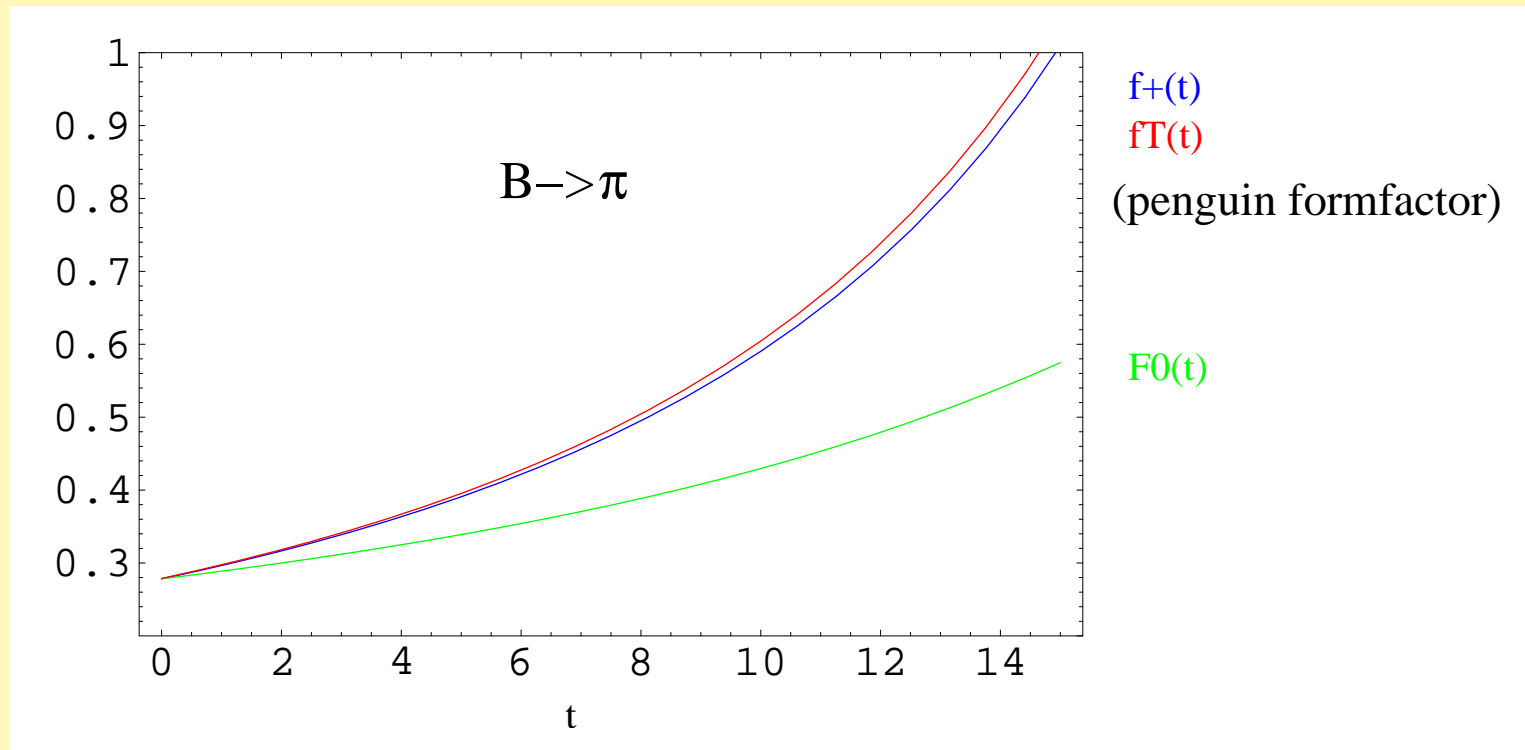
$$= 2p_\mu \left(f_+(q^2) \frac{m_B^2 f_B}{m_B^2 - p_B^2} + \text{higher poles and cuts} \right) + \text{terms contrib. to other FF}$$

- ↪ avoid B-meson DA as B described not as real particle, but via analytic continuation
- ↪ LC-expansion starts at $O(1)$, not $O(\alpha_s)$ → **soft terms included**

And now, Ladies and Gentlemen:

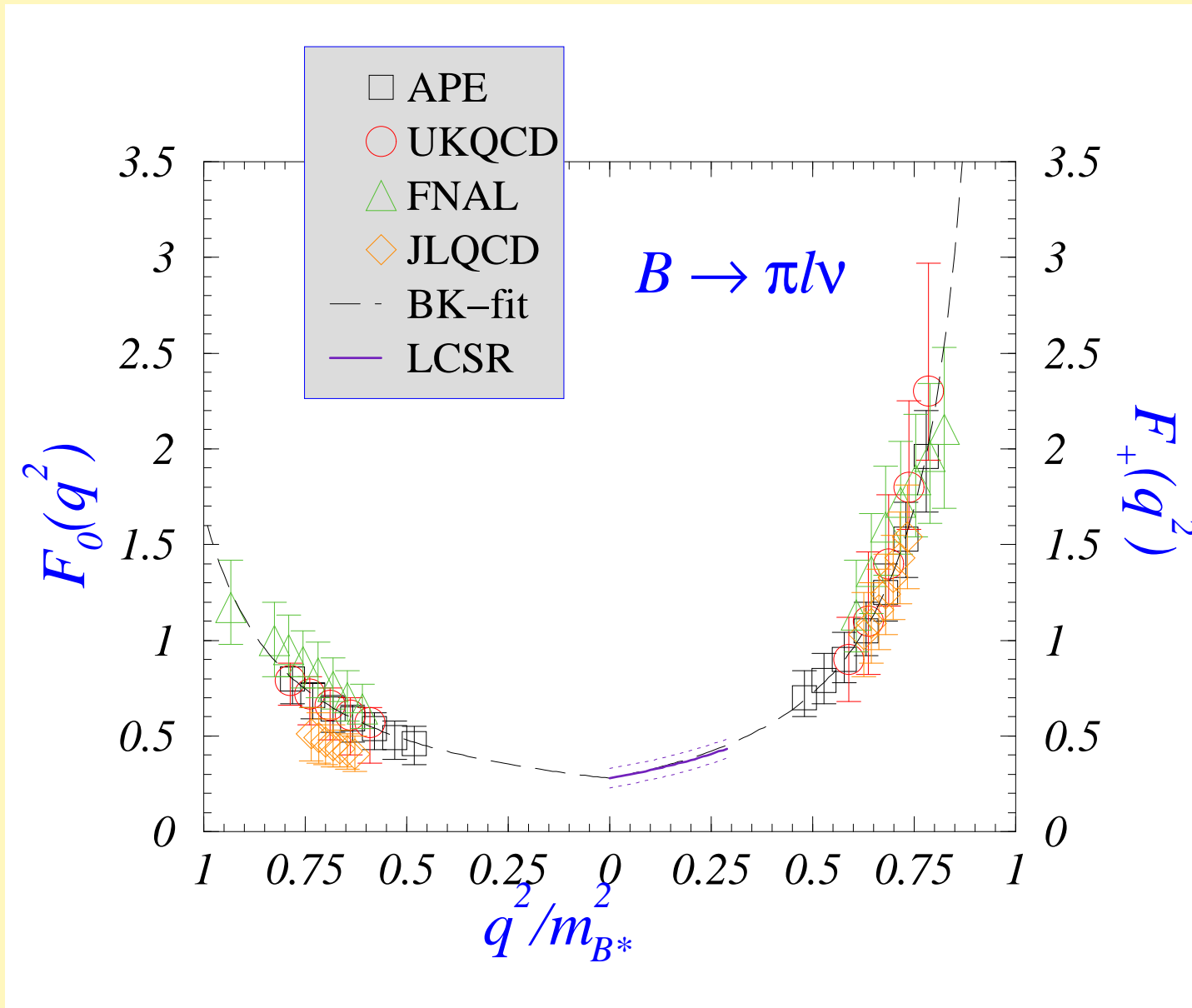
New (& Preliminary) Results!

Ball/Zwicky 2004



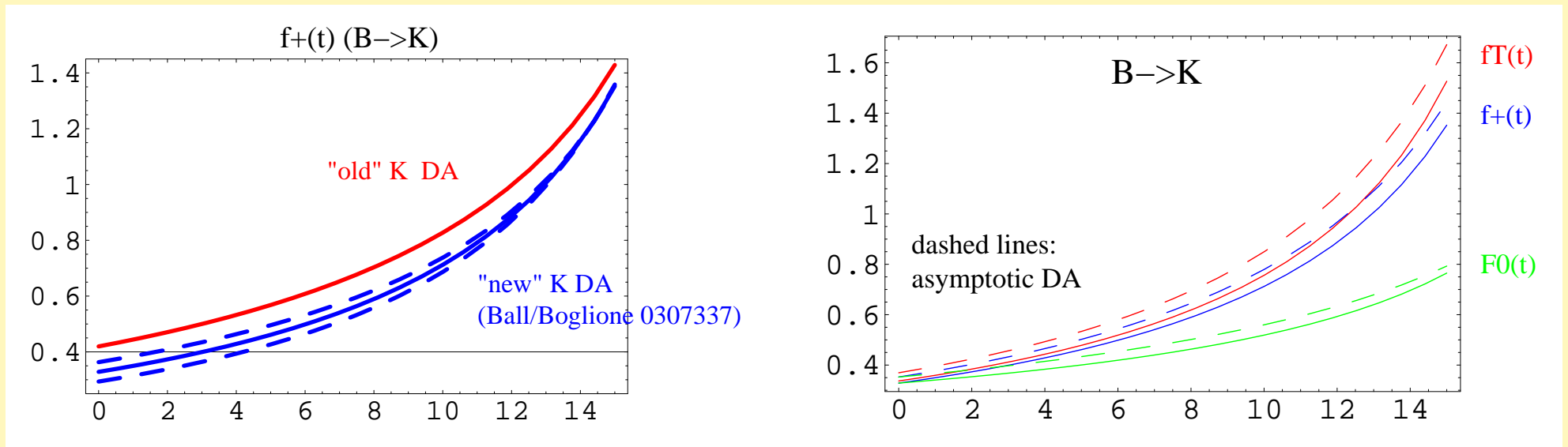
- LCSRs only valid for $E_\pi \gg \Lambda_{QCD}$, i.e. $t = m_B^2 - 2m_B E_\pi < m_B^2$.
Choose $E_\pi^{min} \approx 1.2$ GeV.

Compare to lattice: (Becirevic 2002)



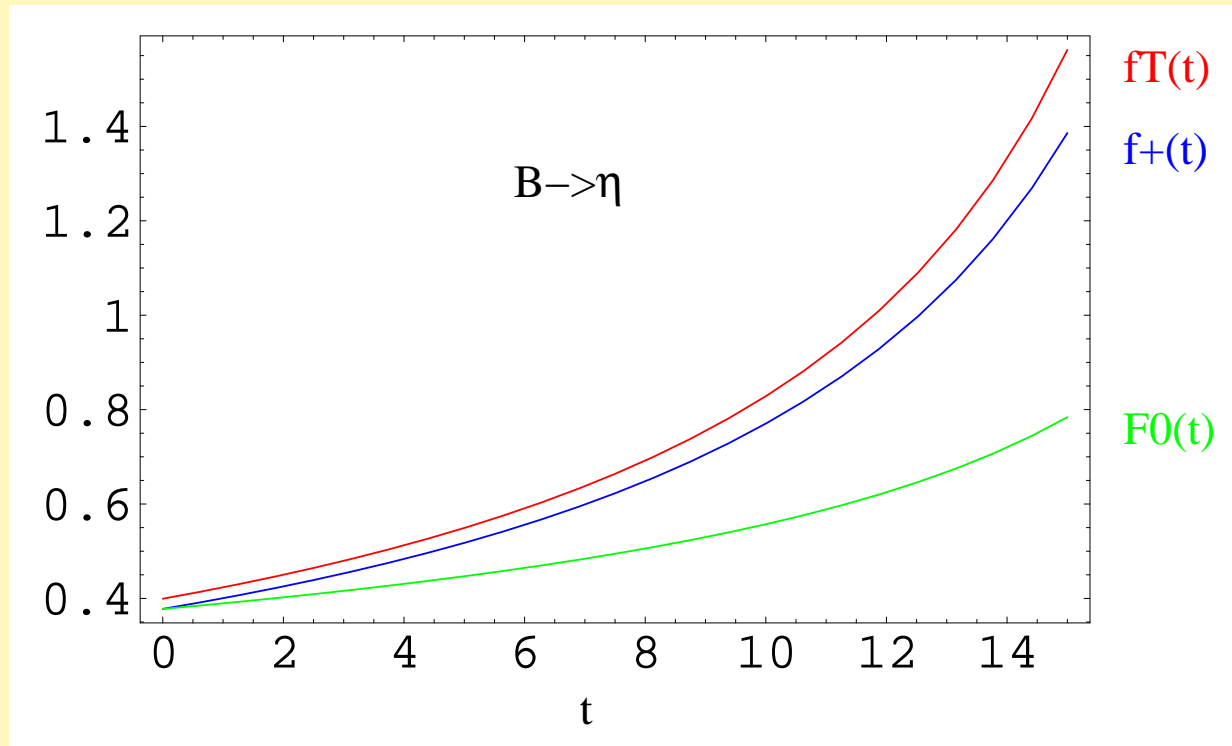
$B \rightarrow K$

Recent redetermination of distribution amplitude ϕ for K: $f_+^{B \rightarrow K}$ becomes smaller!



Ball/Zwicky 2004

$$B \rightarrow \eta$$



Ball/Zwicky 2004

Only flavour octet included.

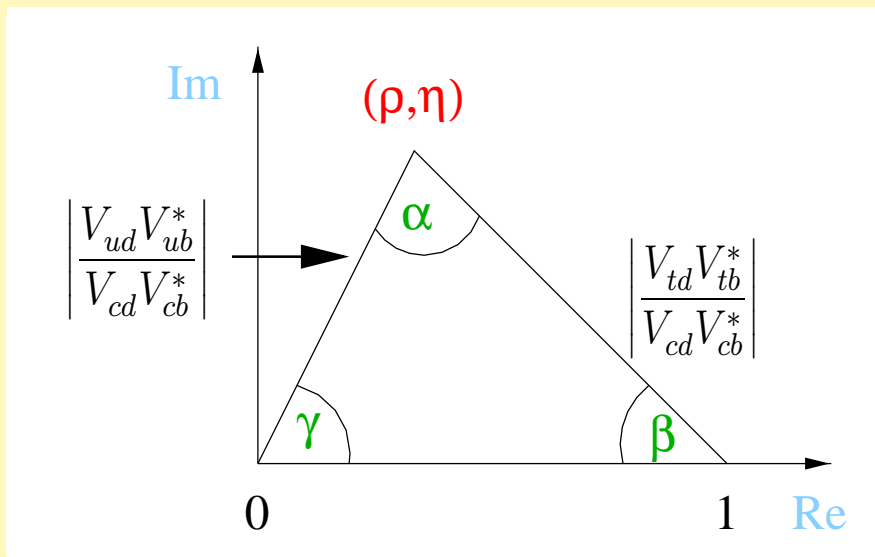
Outline

- Effective Field Theory Description
- $B \rightarrow \pi, \eta, K$ Form Factors
- **CP Violation**
- QCD Factorisation

CP Violation in the SM

- quark mass-eigenstates mix under weak interactions
- mixing described by Cabibbo-Kobayashi-Maskawa (CKM) matrix V (3×3 , unitary)
- parametrized in terms of 3 rotation angles and

one complex phase of $V \rightarrow$ unique source of CP violation in SM



Picture unitarity relation

$$\sum V_{dj} V_{jb}^* = 0$$

as triangle in the complex plane:

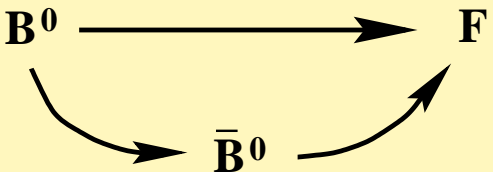
unitarity triangle (UT)

(There are of course 6 triangles: 4 are pretty much squashed, the other 2 nearly identical.)

Goal: test SM by overconstraining the unitarity triangle!

CP Violation in $B_{d,s}^0$ Decays into CP Eigenstates

Measure time-dependent **CP-asymmetry**: ($\text{CP}|F\rangle = n_F|F\rangle$):

$$\begin{aligned}
 \mathcal{A}_{\text{CP}} &= \frac{\Gamma(B_q^0(t) \rightarrow F) - \Gamma(\bar{B}_q^0(t) \rightarrow F)}{\Gamma(B_q^0(t) \rightarrow F) + \Gamma(\bar{B}_q^0(t) \rightarrow F)} \\
 &= \mathcal{A}_{\text{CP}}^{\text{dir}}(B_q \rightarrow F) \cos(\Delta M_q t) + \mathcal{A}_{\text{CP}}^{\text{mix}}(B_q \rightarrow F) \sin(\Delta M_q t)
 \end{aligned}$$


$\mathcal{A}_{\text{CP}}^{\text{dir,mix}}$

depend on

$$\xi_F^{(q)} = -n_F e^{-i\phi_q} \frac{\langle F | \mathcal{H}_{\text{eff}}^{\text{weak}} | \bar{B}^0 \rangle}{\langle F | \mathcal{H}_{\text{eff}}^{\text{weak}} | B^0 \rangle}$$

B^0 - \bar{B}^0 mixing phase

$$\phi_q = \arg M_{12}^{(q)} = \begin{cases} +2\beta & (q = d) \\ \approx 0 & (q = s) \end{cases}$$

$$\mathcal{H}_{\text{eff}}^{\text{weak}}(b \rightarrow r) = \sum_{j=u,c} V_{jr}^* V_{jb} Q^{jr} + \text{c.c.} \rightsquigarrow \text{sum over weak amplitudes}$$

Special case: **dominance of one single amplitude**

\rightsquigarrow hadronic MEs cancel:

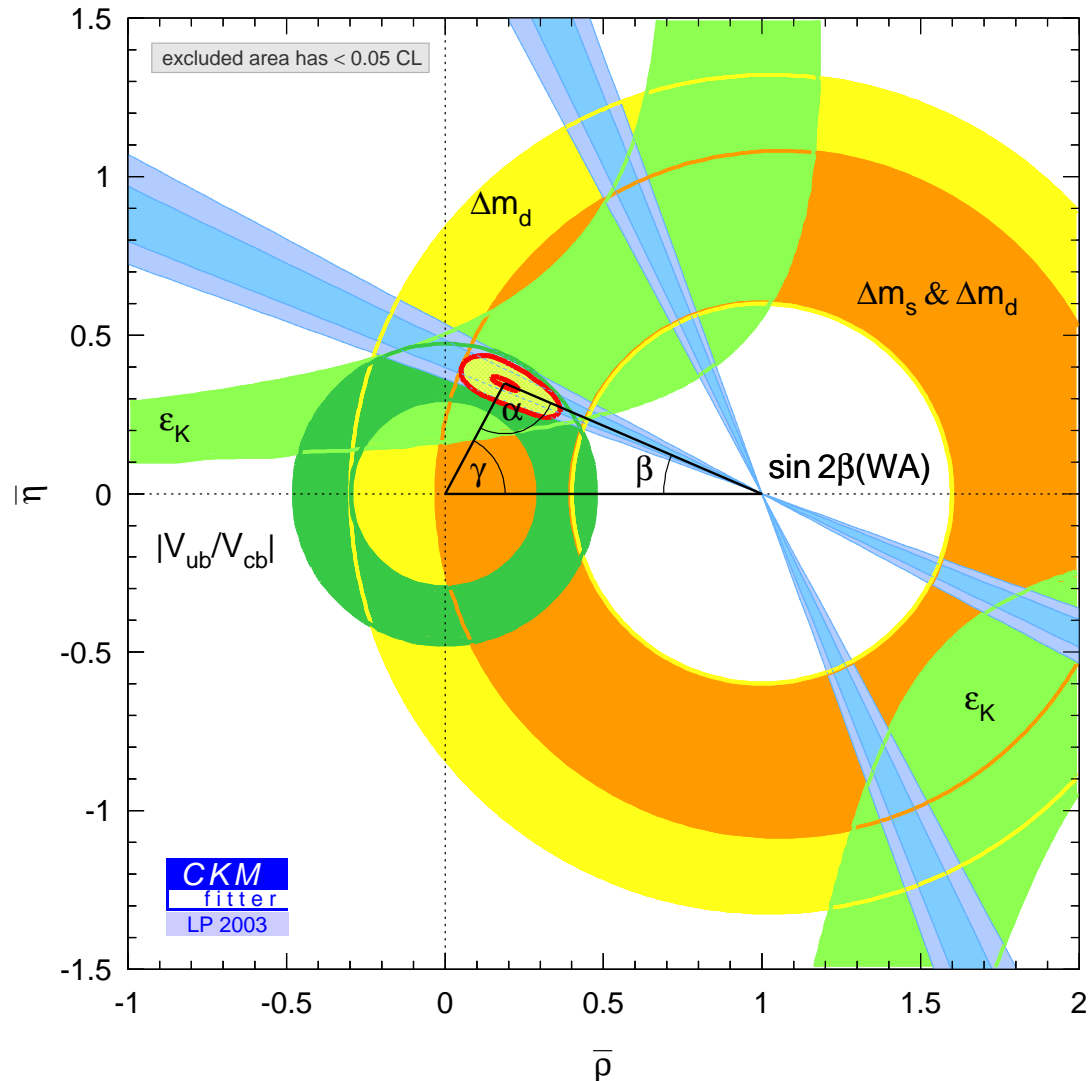
$$\xi_F^{(q)} = -n_F e^{-i(\phi_q - \phi_D^{(F)})}$$

weak decay phase

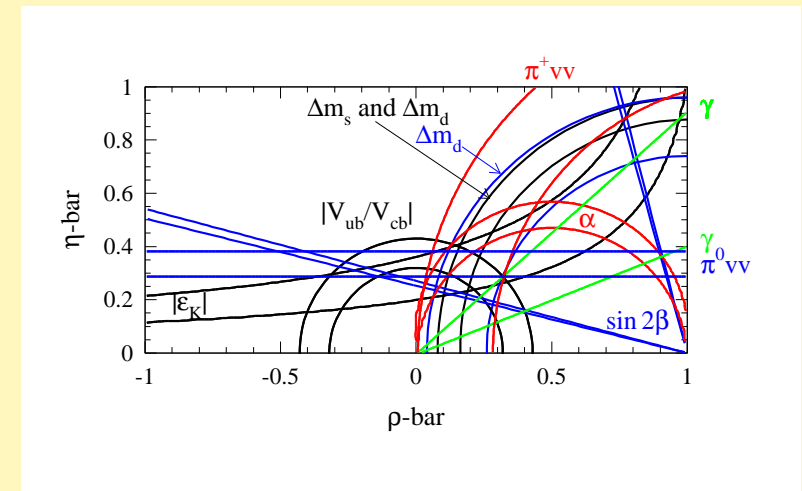
$\phi_D^{(F)} = 0$ for dominant $b \rightarrow ccr$

\Rightarrow **“Gold-plated” decays, e.g. $B \rightarrow J/\psi K_S \rightsquigarrow \beta$**

Present Status of UT



Prophecy for 2010:



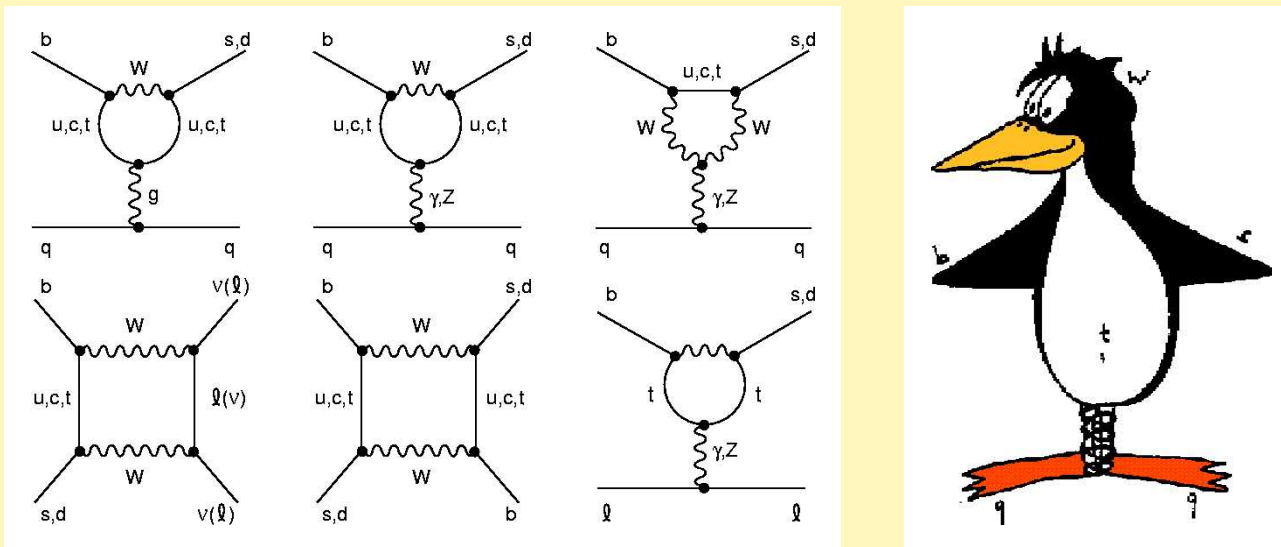
by courtesy of CKM-fitter group,
<http://ckmfitter.in2p3.fr/>

Expect measurements of ΔM_s , α , γ ; improvement on β ; in addition constraints from rare K decays: $K \rightarrow \pi \nu \bar{\nu}$.

The General Case: Penguin Pollution: e.g. $B \rightarrow \pi\pi$

Interference of **two weak amplitudes** $A_{1,2}$, one (often) generated by penguin diagrams:

Flavour-changing neutral current (FCNC) $b \rightarrow s, d$ transitions:



- **loop- and** (in principle) **GIM-suppressed** in SM
- **GIM-suppression** largely **relaxed** because $m_t \gg m_W$
 \rightsquigarrow experimentally accessible BRs $\sim 10^{-5}$, to be compared with FCNC K or D decays: BRs $\sim 10^{-10}$

The General Case: Penguin Pollution: e.g. $B \rightarrow \pi\pi$

Interference of **two weak amplitudes** $A_{1,2}$, one (often) generated by penguin diagrams:

$$A_F = |A_1|e^{i(\phi_{A_1} + \delta_1)} + |A_2|e^{i(\phi_{A_2} + \delta_2)}, \quad \xi_F = -n_F e^{-2i\phi_1} \frac{1 + r e^{i(\Delta - \phi_2 + \phi_1)}}{1 + r e^{i(\Delta + \phi_2 - \phi_1)}}$$

with $\phi_{1,2} = \phi_{A_{1,2}} - \frac{1}{2}\phi_d$: **weak phase**, $\Delta = \delta_2 - \delta_1$: **strong phase**, $r = |A_2/A_1|$.

Assume r small : $\mathcal{A}_{\text{CP}}^{\text{dir}} \approx 2r \sin(\phi_1 - \phi_2) \sin \Delta$

$$\mathcal{A}_{\text{CP}}^{\text{mix}} \approx n_F [\sin 2\phi_1 - 2r \cos 2\phi_1 \sin(\phi_1 - \phi_2) \cos \Delta]$$

No clean measurement of ϕ_1, ϕ_2 !

r, Δ **not** (yet?) accessible to calculation **from first principles**; try to exploit symmetries ($\text{SU}_F(3)$) and limits.

Or use **QCD factorisation**. . .

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Factorization à la BBNS

Beneke/Buchalla/ Neubert/Sachrajda, PRL 83 (1999) 1914

Generic amplitude for heavy-to-light transitions:

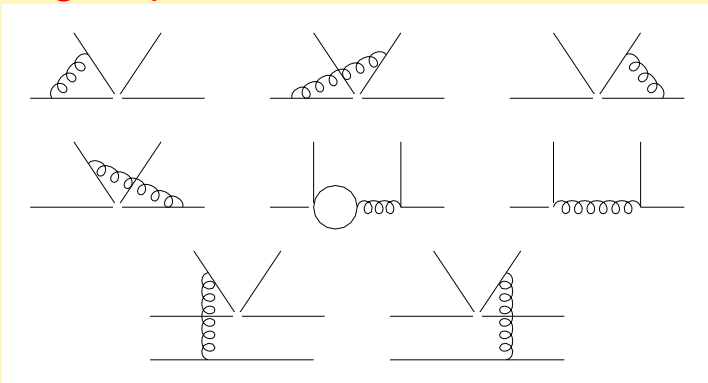
$$\begin{aligned}
 A(B \rightarrow \pi\pi) &= f_+^{B \rightarrow \pi}(0) \int_0^1 dx T^I(x) \phi_\pi(x) + \int_0^1 d\xi dx dy T^{II}(\xi, x, y) \phi_B(\xi) \phi_\pi(x) \phi_\pi(y) \\
 &= A(B \rightarrow \pi\pi)_{\text{fact}} \times (1 + \mathcal{O}(\alpha_s) + \mathcal{O}(\Lambda_{\text{QCD}}/m_b))
 \end{aligned}$$

$f_+^{B \rightarrow \pi}$: weak decay form factor

- shown to be valid at 1-loop in QCD

- naive factorization works up to (calculable) radiative corrections and (uncalculable) power-suppressed terms

$T^{I,II}$: process-dependent **hard scattering amplitudes**



$\phi_{B,\pi}(x)$: universal **light-cone distribution amplitudes**

- describe collinear momentum-distribution of quarks in meson
- obtained from Bethe-Salpeter WFs by integration over transverse momenta
- well-studied for light mesons (e.g. π EM form factor)

Résumé

- **primary objective** of B physics: obtain info on quark mixing (i.e. scalar sector), bounds & constraints on new physics (\mathcal{CP} , rare decays)
- **experimental results** extremely impressive already, and always improving. . .
- **primary obstacle**: contamination by (nonperturbative) QCD; for the moment, th. QCD uncertainties on par (or below) exp. ones; situation bound to change (very) soon. . .
- **era of precision fits**: reminds of LEP: electroweak p.f. \rightsquigarrow info on m_t , m_H , yields bounds on SUSY etc.; main difference: presently, cannot match th. and exp. accuracy to arbitrary precision
- **prospects**: developments in npQCD: lattice: unquenched B calculations? Further development of pQCD methods? \rightarrow need $1/m_b!$