Challenges in perturbative QCD

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February 2002

- Where we are now
  - ▷ Successes
  - ▷ Limitations
- Why we need NNLO
  - ▷ progress in two-loop amplitudes
  - ▷ where we are stuck
- Outlook

Framework

- We assume that $SU(3)$ Lagrangian of quarks and gluons can be used to make perturbative predictions of hadronic observables - up to power suppressed effects
- Factorisation
  separates the short distance perturbative effects from the long distance nonperturbative inputs - the pdf's
- Evolution
  DGLAP: how the pdf's perturbatively vary with factorisation scale, or equivalently resumming collinear logarithms
  BFKL: resumming high energy (or low x) logarithms
- Infrared Resummation
  resumming large final state logarithms in semi-inclusive quantities e.g. jet rates
- Parton Shower
  simulates event though radiation from underlying hard process
  resums soft/collinear logarithms through coherent branching
- Hadronisation
  modelled in shower MC
  estimated through integral over gluon off shellness k^2

$$F(Q^2) = \int dk^2 \mathcal{F}(Q^2, k^2) \alpha_s(k^2)$$

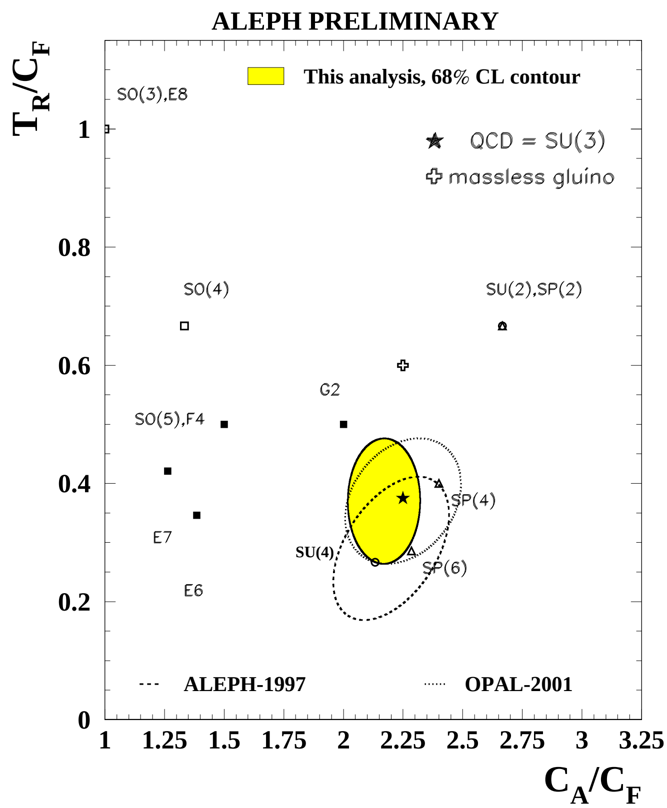
Where are we now? - NLO

- 1990's have been the decade for testing perturbative QCD to next-to-leading order
- DGLAP evolution to NLO used for some time, together with global analyses of DIS, Drell-Yan and jet production at NLO
- many observables computed to NLO - many general purpose Monte Carlo programs exist for $2 \to 2$ scattering processes e.g
  - DISENT, DISASTER++ for $ep \to (2+1)$ jet
  - EVENT, EVENT2 for $e^+e^- \to 3$ jets
  - JETRAD for $p\bar{p} \to 2$ jets
  - DYRAD for $p\bar{p} \to V + 1$ jet
  - etc etc
- some general purpose Monte Carlo programs starting to exist for $2 \to 3$ scattering processes e.g
  - NLOJET++ for $ep \to (3+1)$ jet
  - MENLOPARC, DEBRECEN, EERAD2, MERCUTIO for $e^+e^- \to 4$ jets
  - TRIRAD, NLOJET++ for $p\bar{p} \to 3$ jets
  - MCFM for $p\bar{p} \to V + 2$ jets

$\Longrightarrow$ Many notable successes

Successes

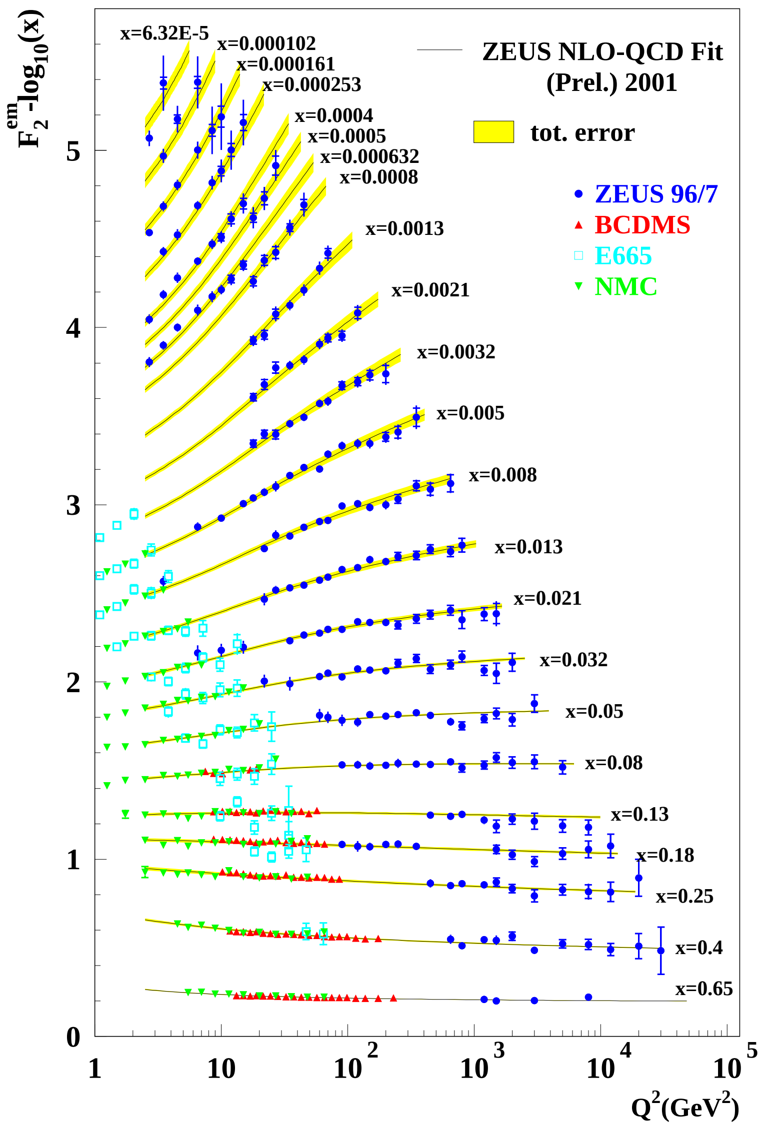
- Determining the quadratic casimirs of QCD in $e^+e^- \to$ 4 jets. Is it really $SU(3)$?



OPAL	C_A	3.02	± 0.25	± 0.49
	C_F	1.34	± 0.13	± 0.22
	$\alpha_s(M_Z)$	0.120	± 0.011	± 0.020
ALEPH	C_A	2.93	± 0.14	± 0.49
	C_F	1.35	± 0.07	± 0.22
	$\alpha_s(M_Z)$	0.119	± 0.006	± 0.022

Successes

- Scaling violations predicted by QCD



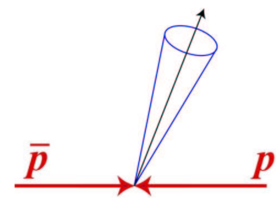
- Good fit over whole range of x and Q^2 - different assumptions for H1/ZEUS and how much fixed target data is compatible.
- Tradeoff between gluon at low x and α_s

$\Longrightarrow$ higher gluon and lower α_s preferred by fixed target

Successes

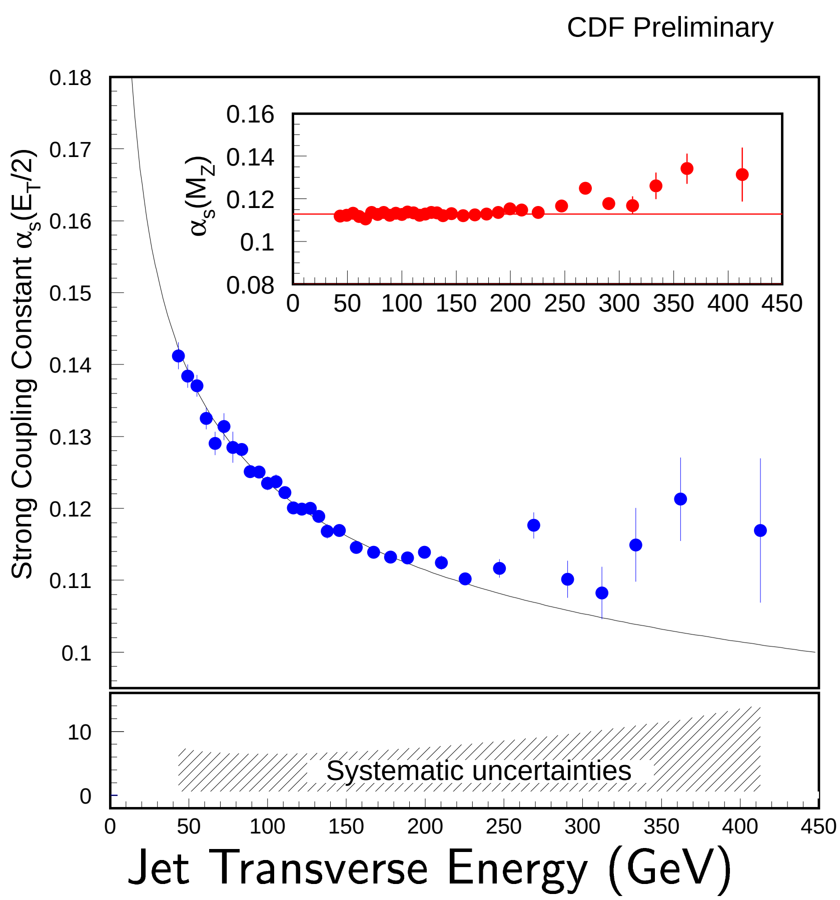
- Evolution of strong coupling constant in $p\bar{p} \to jet$

$$\frac{d\sigma}{dE_T} = A\alpha_s^2(E_T) + B\alpha_s^3(E_T)$$



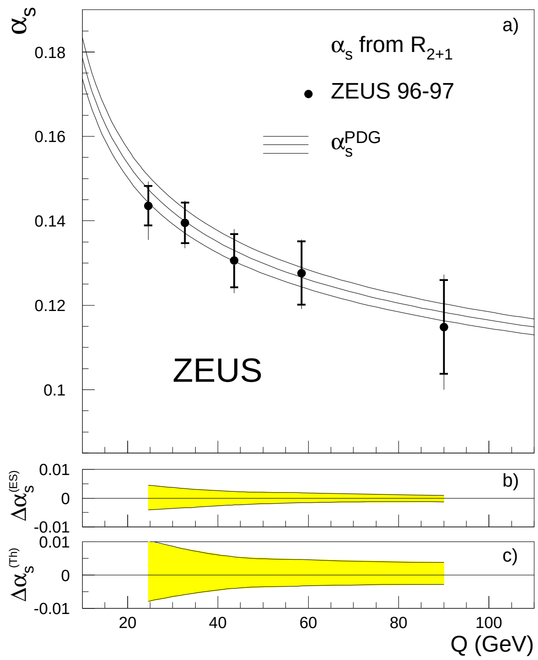
- Extracting α_s at each value of jet energy demonstrates running of coupling constant and gives

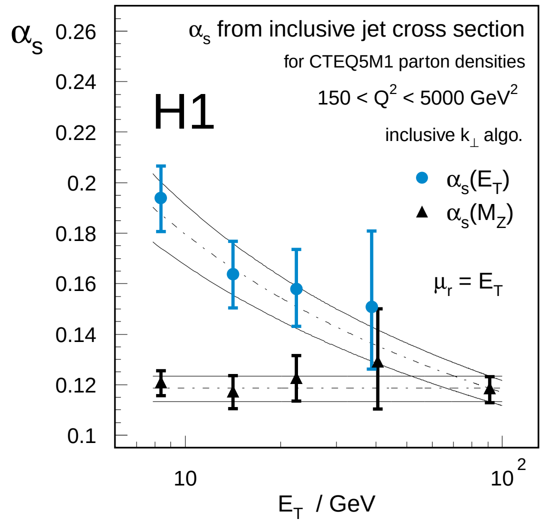
$$\alpha_s(M_Z) = 0.1129 \pm 0.0001(stat)^{+0.0078}_{-0.0089}(exp.syst)$$



Successes

- Determination of α_s from Jets in DIS.



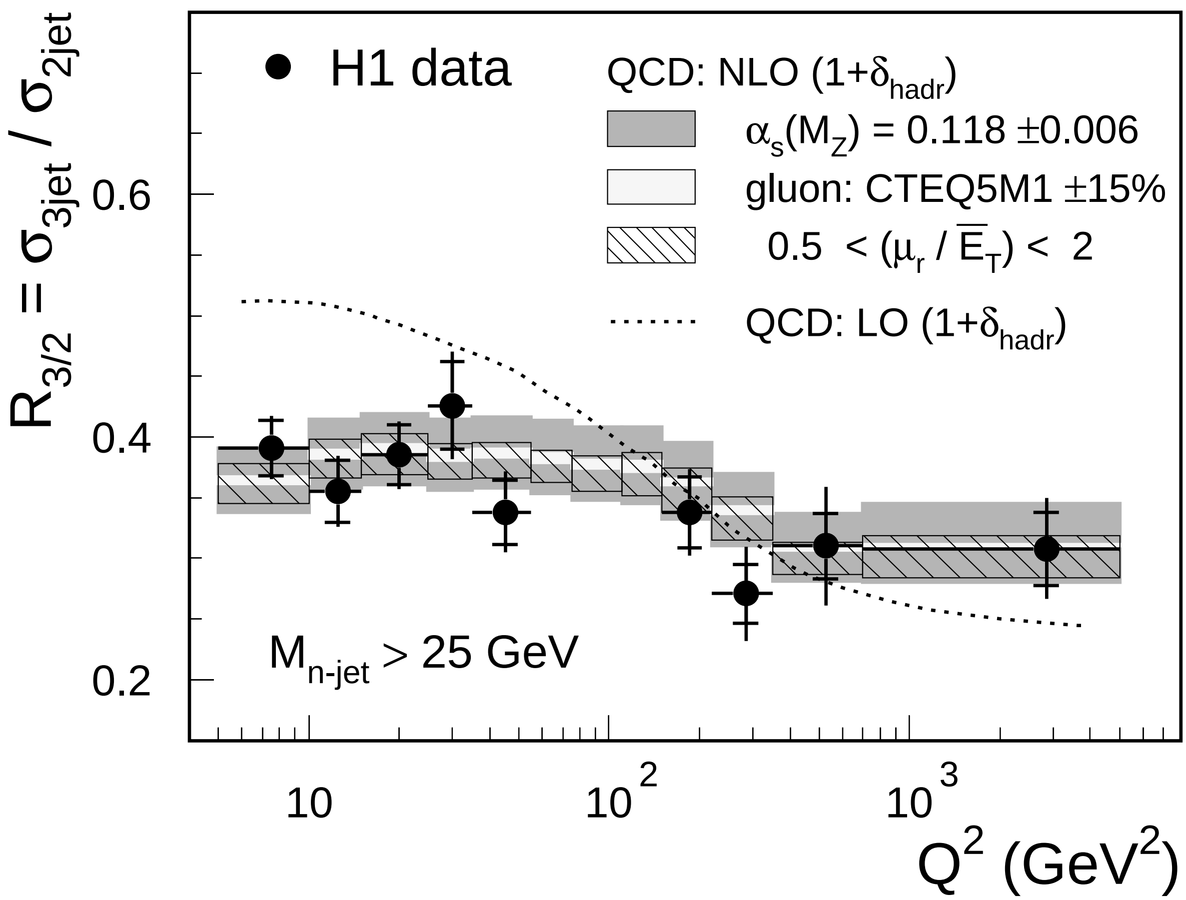


	$\alpha_s(M_Z)$	Stat.	Exp.	Th.	PDF
Zeus	0.1166	± 0.0019	$^{+0.0024}_{-0.0033}$	$^{+0.0057}_{-0.0044}$	
H1	0.1186	± 0.0007	± 0.0030	$^{+0.0039}_{-0.0045}$	$^{+0.0033}_{-0.0023}$

- Large theoretical uncertainties from renormalisation scale dependence and/or pdf's.
- In $R^{2+1} = d\sigma_{2+1}/dQ^2/d\sigma_{tot}/dQ^2$ large part of pdf uncertainty cancels out due to correlations
- Consistency with world average from HERA

Successes

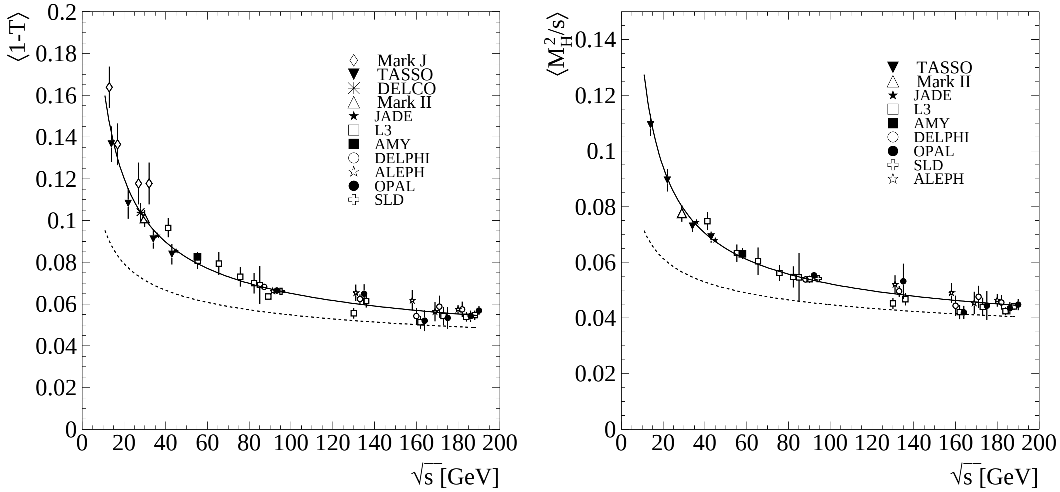
- $ep \to 3+1$ jet rate has large NLO corrections - typical of almost all $2 \to 3$ processes - and hence significant scale dependence.



Nagy and Trocsanyi, hep-ph/0104315

Successes

- Description of event shapes over wide range of center of mass energies.



One non-perturbative parameter

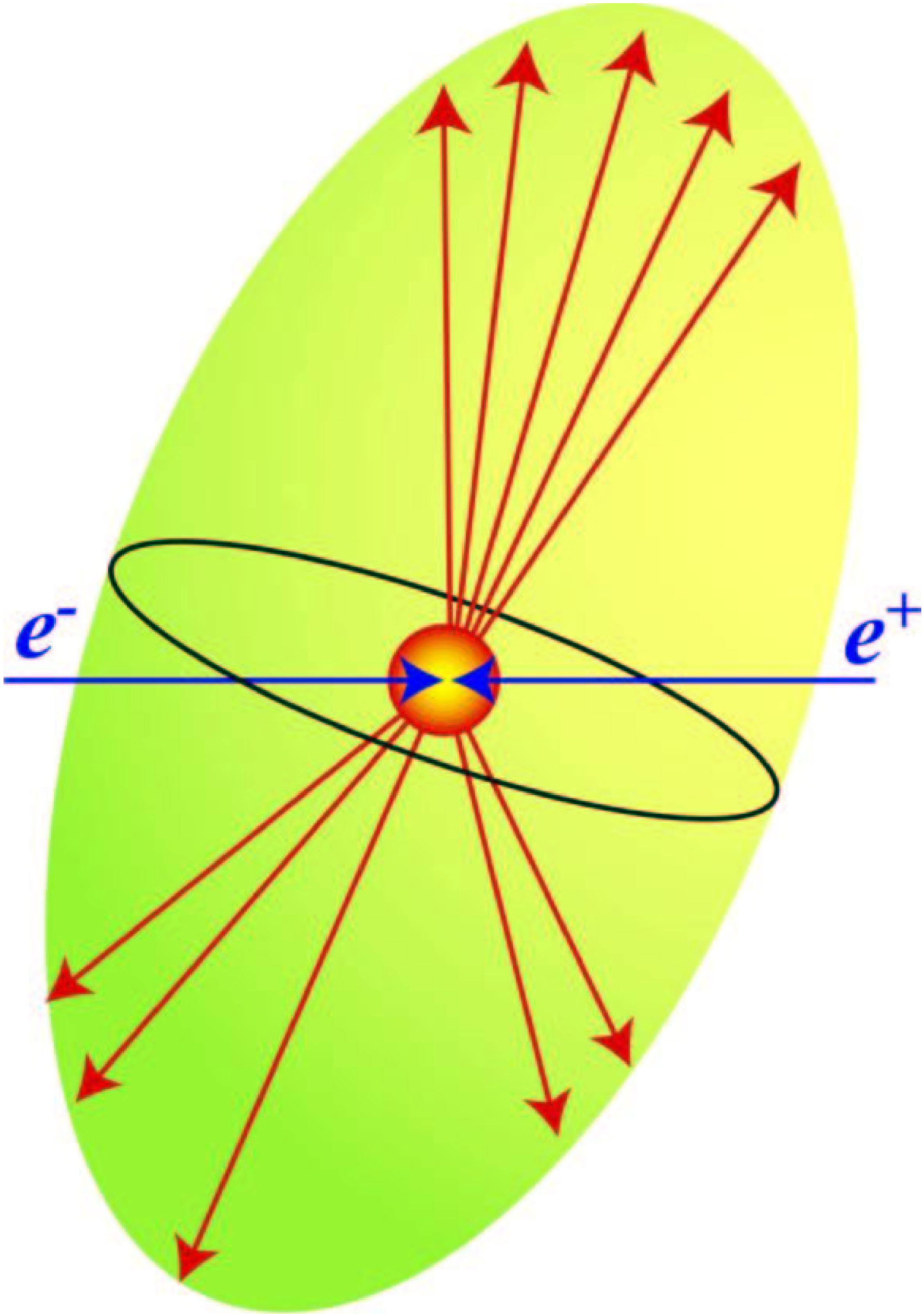
$$\alpha_0(\mu_I) = \frac{1}{\mu_I} \int_0^{\mu_I} \alpha_s(k) dk$$

	$\langle 1 - T \rangle$	$\langle M_H^2 \rangle$
$\alpha_s(M_Z)$	$0.1217^{+0.0065}_{-0.0054}$	$0.1165^{+0.0047}_{-0.0038}$
$\alpha_0(2\ GeV)$	$0.528^{+0.074}_{-0.051}$	$0.663^{+0.111}_{-0.078}$

Similar consistent results for other observables $\langle B_T \rangle$, $\langle B_W \rangle$, $\langle C \rangle$ ...

Movilla Fernandez et al, hep-ex/0105059

Thrust



What we need to achieve in the next five years?

1) NLO predictions for a whole range of multiparticle final states, e.g. $pp \to V + \mathrm{multi\ jets}$

2) NNLO extraction and evolution of pdf's from DIS data
   - α_s and gluon

3) Fully consistent NNLO global fits of pdf's using DIS, Drell Yan and jet data
   - including sensible error estimates

4) NNLO determination of α_s using jet data in e^+e^- and ep (as well as $p\bar{p}$)

5) NLO parton shower Monte Carlo

at the same time there will be significant improvement in

6) Resummation of infrared logarithms for more complicated final states

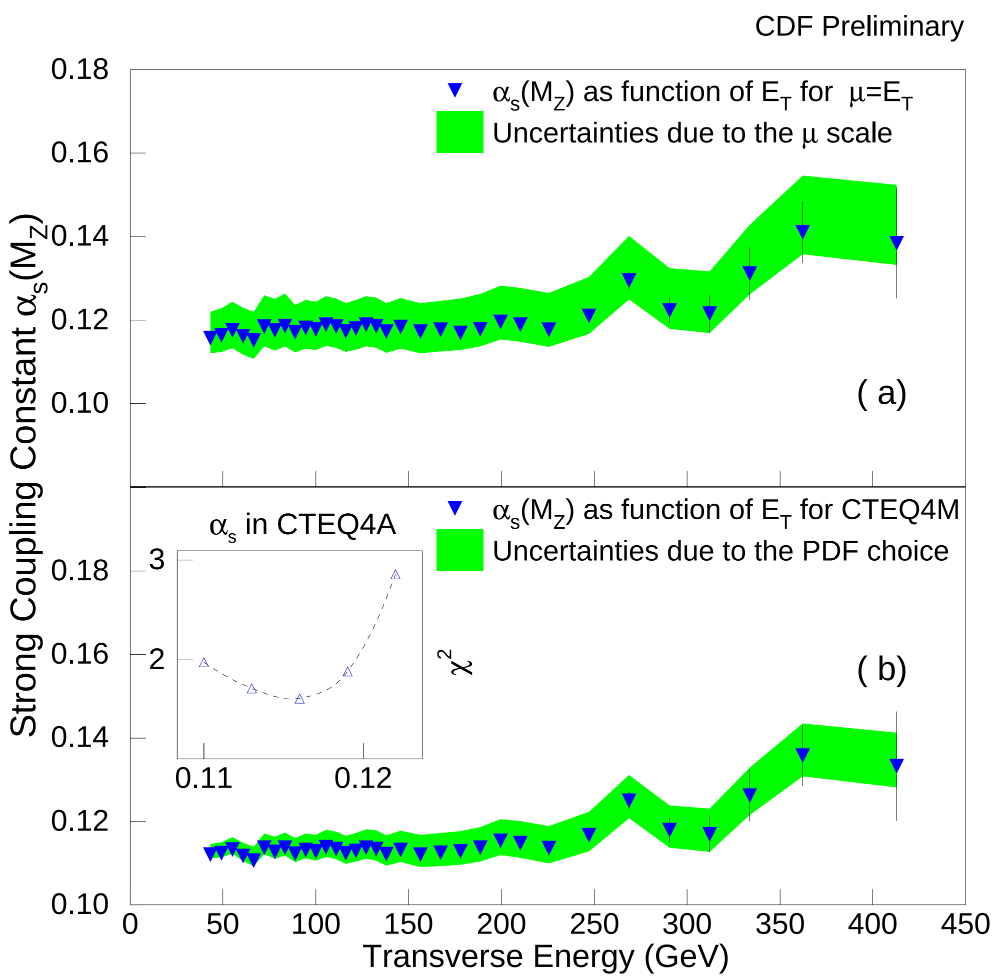
7) Much better understanding of power corrections and how they fit with perturbative calculations

8) . . .

Why go beyond NLO?

In many cases, the uncertainty from the pdf's and from the choice of renormalisation scale give uncertainties that are as big or bigger than the experimental errors.

e.g. theoretical uncertainties in α_s extraction from $p\bar{p} \to$ jet are due to renormalisation scale and pdf's



Why do we vary renormalisation scale?

- The theoretical prediction should be independent of μ_R
- The change due to varying the scale is formally higher order. If an observable $\mathcal{O}bs$ is known to order α_s^N then,

$$\frac{\partial}{\partial \ln(\mu_R^2)} \sum_0^N A_n(\mu_R) \alpha_s^n(\mu_R) = \mathcal{O}\left(\alpha_s^{N+1}\right).$$

- So the uncertainty due to varying the renormalisation scale is way of guessing the uncalculated higher order contribution.

Why do we vary renormalisation scale? - cont

- ... but the variation only produces copies of the lower order terms

$$\mathcal{O}bs = A_0\alpha_s(\mu_R) + \left(A_1 + b_0 A_0 \ln\left(\frac{\mu_R^2}{\mu_0^2}\right)\right)\alpha_s(\mu_R)^2$$

A_1 will contain logarithms and constants that are not present in A_0 and therefore cannot be predicted by varying μ_R.

For example, A_0 may contain infrared logarithms L up to L^2, while A_1 would contain these logarithms up to L^4.

- μ_R variation is only an estimate of higher order terms
- A large variation probably means that predictable higher order terms are large - but doesnt say anything about A_1.

Renormalisation scale dependence

For example, $p\bar{p} \to$ jet, scale dependence is predictable with NLO calculation

$$\begin{aligned} \frac{d\sigma}{dE_T} &= \alpha_s^2(\mu_R) A \\ &+ \alpha_s^3(\mu_R) \left(B + 2b_0 L A\right) \\ &+ \alpha_s^4(\mu_R) \left(C + 3b_0 L B + (3b_0^2 L^2 + 2b_1 L) A\right) \end{aligned}$$

with $L = \log(\mu_R/E_T)$.

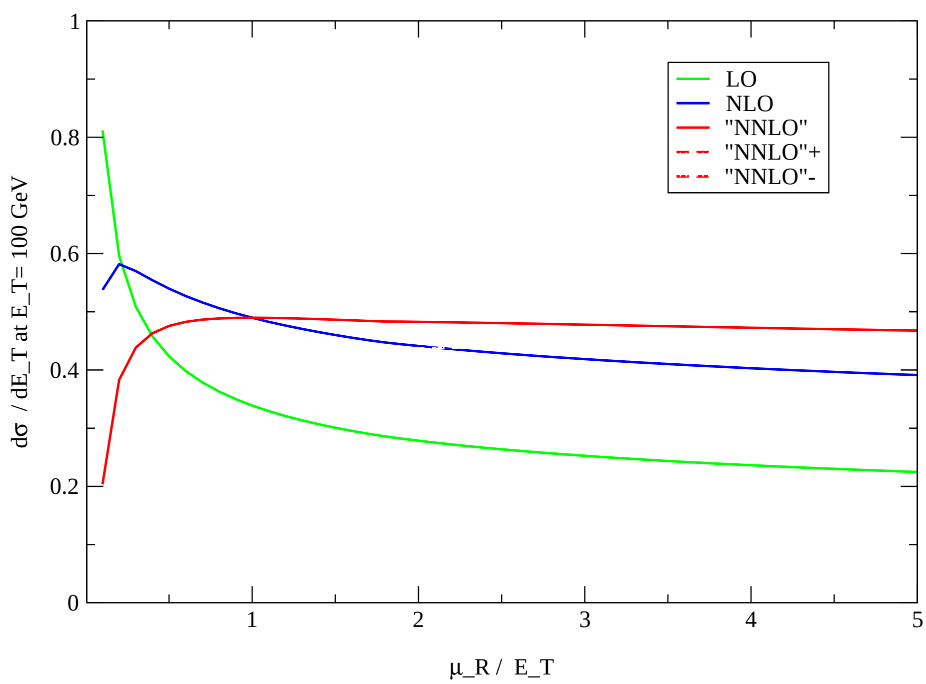


Figure 1: Single jet inclusive distribution at $E_T = 100$ GeV and $0.1 < |\eta| < 0.7$ at $\sqrt{s} = 1800$

The NNLO coefficient C is unknown. The curves show guesses $C = 0$ (solid) and $C = \pm B^2/A$ (dashed). Scale

Renormalisation scale uncertainty

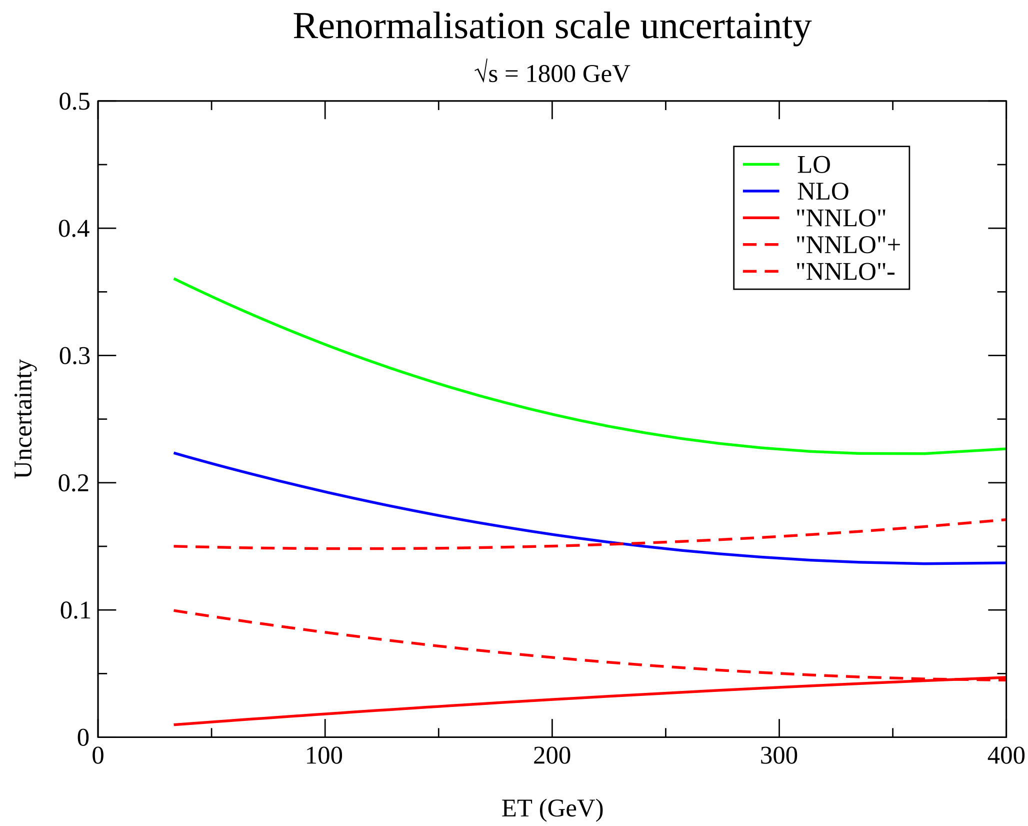
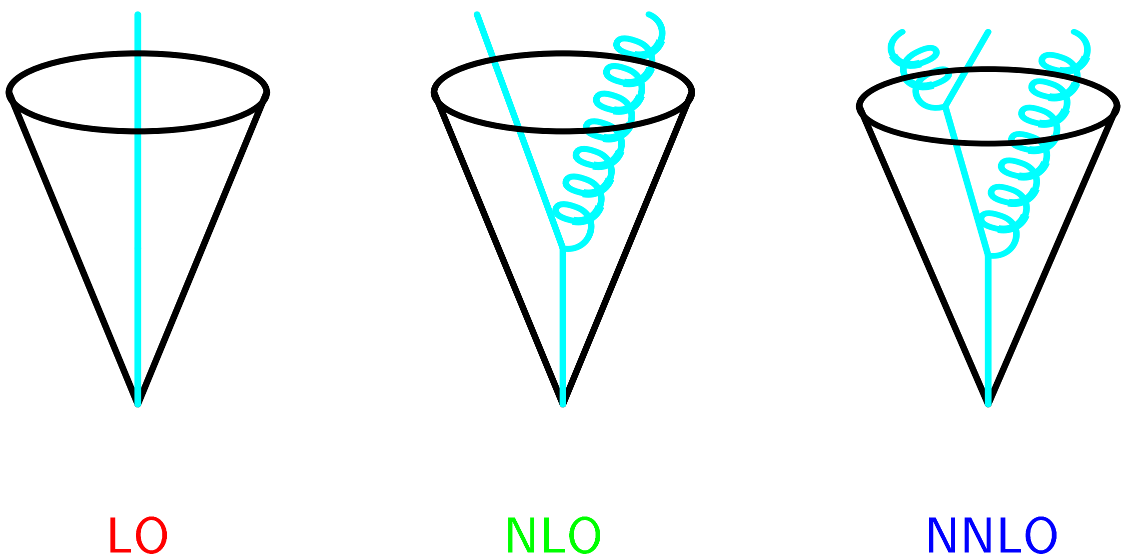


Figure 2: Uncertainty obtained by varying μ_R between $E_T/2$ and $2E_T$ (and keeping $\mu_F = E_T$).

$\Longrightarrow$ Inclusion of NNLO contribution should significantly decrease theoretical renormalisation scale uncertainty for that observable

Jet algorithms

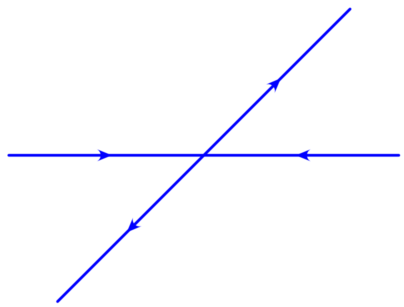
Also there is a mismatch between the number of hadrons and the number of partons in the event. At NLO at most two partons make a jet - while at NNLO three partons can combine to form the jet



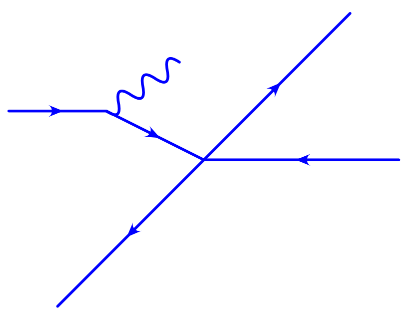
Perturbation theory starts to reconstruct the shower
$\Longrightarrow$ better matching of jet algorithm between theory and experiment
$\Longrightarrow$ need for better jet algorithms

Description of the initial state

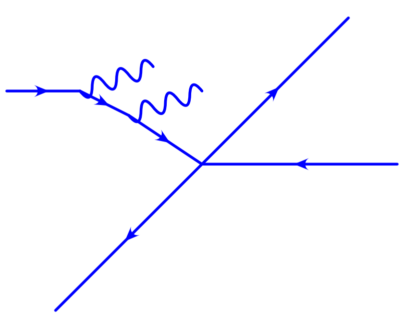
LO At lowest order final state has no transverse momentum

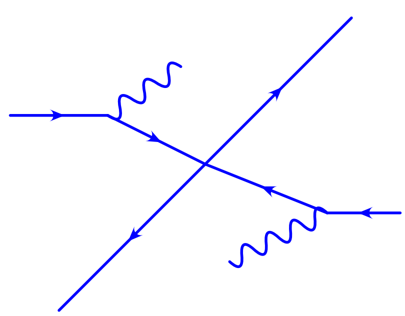


NLO Single hard radiation gives final state transverse momentum, even if no additional jet observed



NNLO Double radiation on one side or single radiation off each incoming particle gives more complicated transverse momentum to final state





Higher orders and power corrections

NLO Phenomenological power corrections match data with coefficient of $1/Q$ extracted from data.

$$\langle 1 - T \rangle \sim 0.33\alpha_s + 1.0\alpha_s^2 + \frac{\lambda}{Q}$$

At NLO, $\lambda \sim 1$ GeV gives a good description of the data.

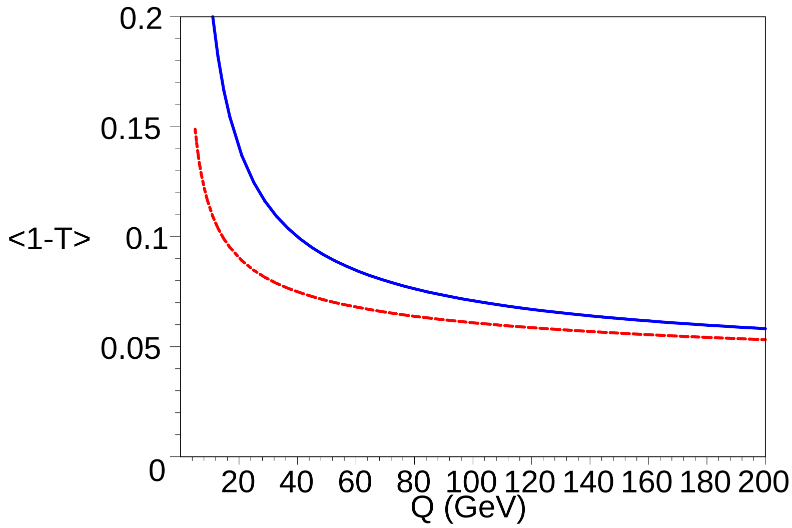


Figure 3: .$\langle 1 - T \rangle$ with NLO and no power correction and NLO with power correction $\lambda = 1$ GeV.

The power correction parameterises the unknown higher orders as well as the genuine non-perturbative correction

Higher orders or power corrections

NNLO Higher orders partially remove need for power correction

$$\langle 1-T\rangle \sim 0.33\alpha_s + 1.0\alpha_s^2 + A\alpha_s^3 + \frac{\lambda\ \mathrm{GeV}}{Q}$$

If we guess $A = 3$, then $\lambda = 0.5$ GeV is good fit.

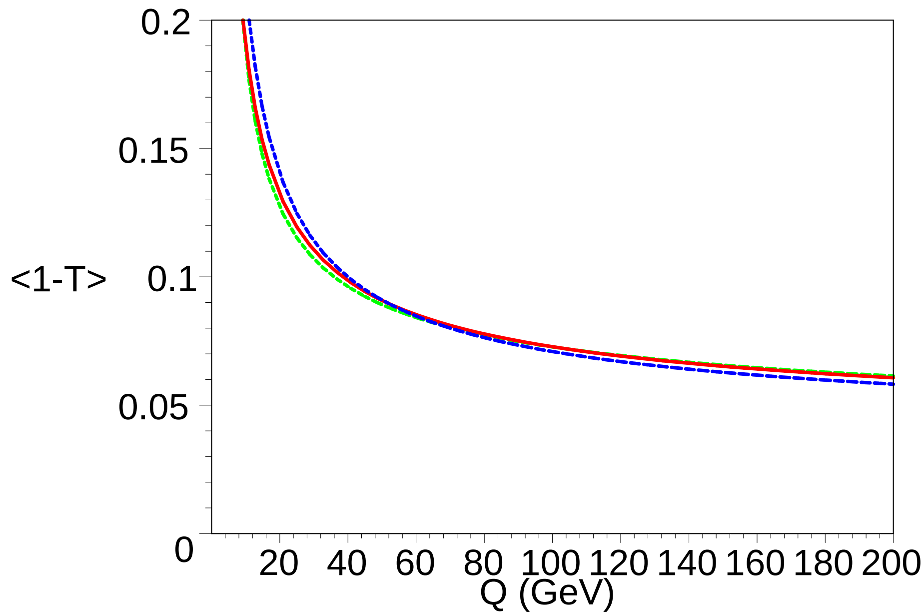


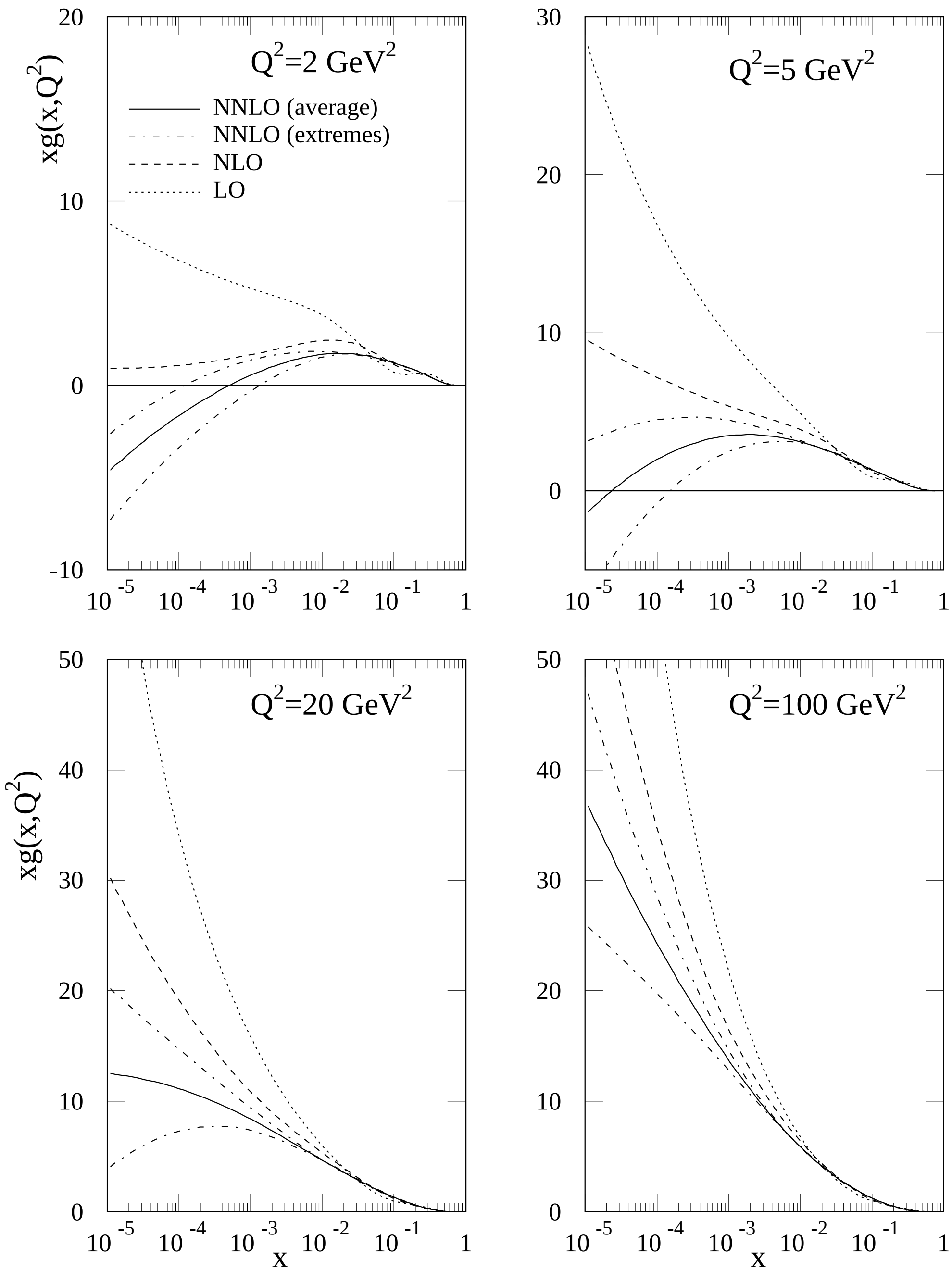
Figure 4: $\langle 1-T\rangle$ with NLO and $\lambda = 1$ GeV, "NNLO" with $\lambda = 0.5$ GeV and "All orders" with no power correction.

At present data not good enough to tell difference between $1/Q$ and $1/\log(Q/\Lambda)^3$.

All orders If higher orders form geometric series, then can avoid power correction altogether!!

Impact of NNLO PDF's

- Recent calculation of moments of the three-loop splitting functions Larin, Nogueira, van Ritbergen, Vermaseren
  Retey, Vermaseren and Gracey
  together with analytic interpolating forms van Neerven, Vogt



Impact of NNLO PDF's

- Give information about scale uncertainty of NNLO pdf's.

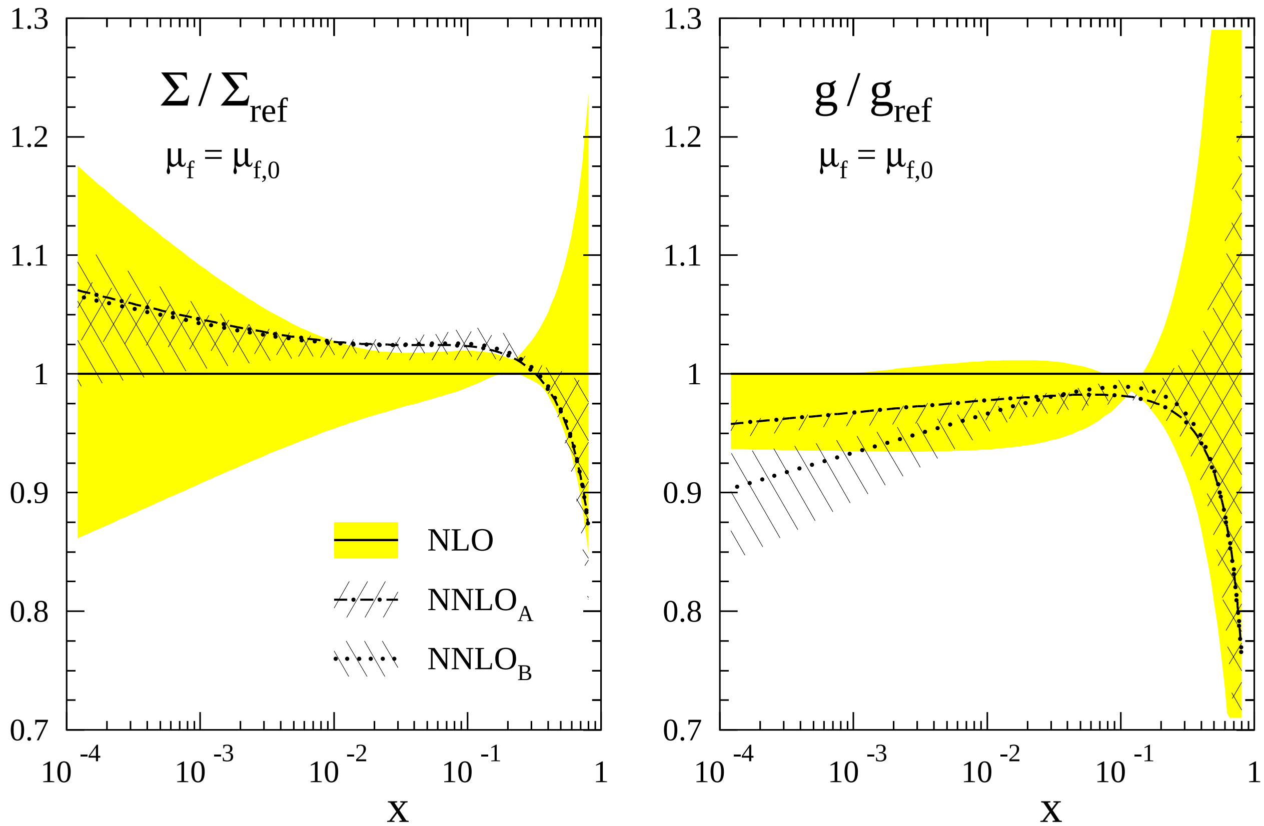
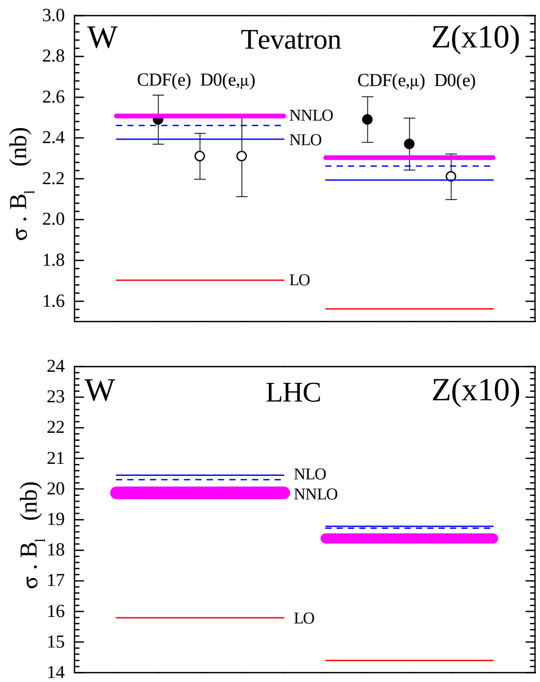


Figure 5: Comparison of the NLO and NNLO singlet quark and gluon densities and scale-uncertainty bands.

- See significant reduction in scale variation Vogt and van Neerven, hep-ph/0006154
- Analytic splitting functions expected anytime now

Impact of NNLO PDF's

- NNLO coefficient functions for DIS available for some time van Neerven and Zijlstra
  and Drell Yan Hamberg, van Neerven, Matsuura
  ⟹ allow NNLO fits to DIS and DY data.
- "approximate" NNLO evolution and partial "NNLO" fits indicate likely impact of effects.



- "NNLO" with NLO α_s and NLO evolution of pdf's
- NNLO with NNLO α_s and "NNLO" evolution of pdf's

⟹ movement between pdf's and coefficient functions

- At present NNLO fits necessarily include some NLO observables e.g. jets

Why go beyond NLO? - summary

We expect a variety of improvements at NNLO

- √ Reduced renormalisation scale dependence
- √ Better matching of parton-level jet algorithm with experimental hadron-level algorithm
- √ Better description of transverse momentum of final state due to double radiation of initial state
- √ Reduced power correction as higher perturbative powers of $1/\ln(Q/\Lambda)$ mimic genuine power corrections like $1/Q$.
- √ Full NNLO global fit of PDF's should also reduce the theoretical uncertainty
- √ ...

These improvements will be necessary if and when the experimental accuracy for a given observable is better than the 10% level.

Progress at NNLO

- One scale processes

  Fully inclusive processes like DY, DIS, Higgs production with simple kinematics can be done analytically

  Harlander, Kilgore

  Anastasiou, Melnikov

- Multi scale processes

  Semi inclusive basic scattering processes like (for example)

  - $pp \to$ jets,
  - $ep \to 2+1$ jets,
  - $e^+e^- \to 3$ jets

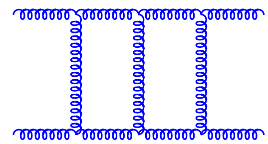
  where the data is copious (and very precise) at LEP, TEVATRON, HERA, LHC and LC. Processes with massless internal propagators and at most one off-shell leg.

Anatomy of NNLO calculation

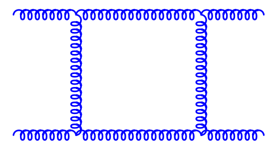
Example, pure gluon ingredients to $pp \to 2$ jets

DOUBLE VIRTUAL:

2 loop, 2 parton final state



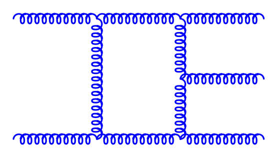
$|$ 1 loop $|^2$, 2 parton final state



VIRTUAL RADIATION:

1 loop, 3 parton final states

or 2+1 parton final state

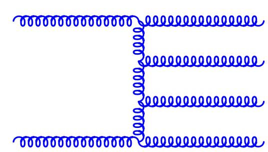


DOUBLE RADIATION:

tree, 4 parton final states

or 3+1 parton final states

or 2+2 parton final state



where n theoretically unresolved soft or collinear partons are indicated as +n

▷ Much more sophisticated infrared cancellation between

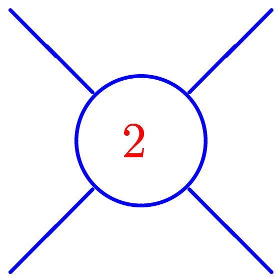
- n and $n+1$ particle contributions when one particle unresolved

  $\Longrightarrow$ soft and collinear limits of one loop amplitudes

- n and $n+2$ particle contributions when two particles unresolved

Structure of Two-loop contribution

- The many (thousands) of tensor integrals appearing in two-loop graphs can be written in terms of a few Master Integrals MI_j

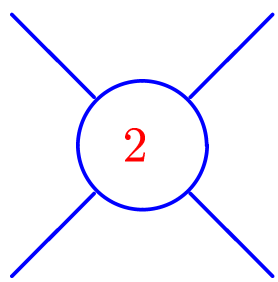


$$= \sum a_j \; \mathrm{MI}_j$$

where the a_j are polynomials in s, t and u and the space-time dimension D. $\Longrightarrow$

- The MI_j can be expanded in $\epsilon = (4-D)/2$ so that

$$\sum_{\text{diagrams}}$$



$$= \sum_{i=1}^{4} \frac{X_i}{\epsilon^i} + X_0$$

▷ The infrared singular terms X_i for $i = 1, \ldots, 4$ are predictable

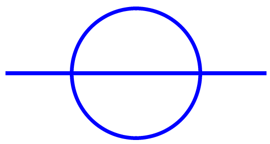
Catani, hep-ph/9802439

and must be analytically cancelled with the contributions from single unresolved $2 \to 3$ and double unresolved $2 \to 4$ processes

▷ The finite remainder X_0 contributes to the NNLO correction

Master Integrals - onshell

- The trivial topologies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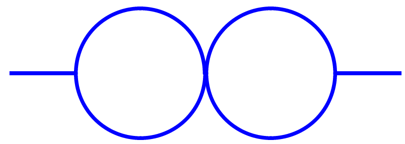
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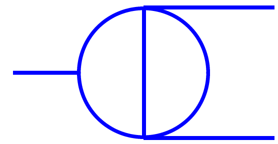
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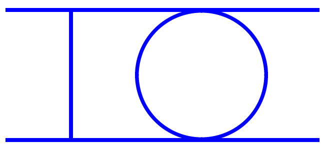
- The less trivial topologies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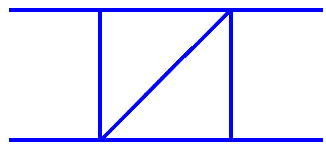
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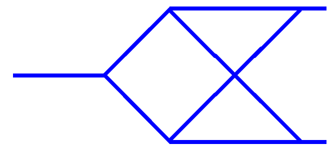
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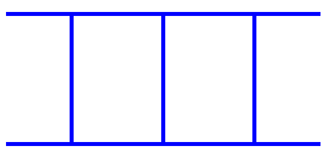
- The non-trivial topologies
  - ▷ The planar boxes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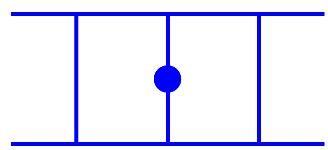
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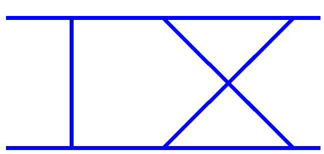
Smirnov, hep-ph/9905323

Smirnov and Veretin, hep-ph/9907385

  - ▷ The non-planar boxes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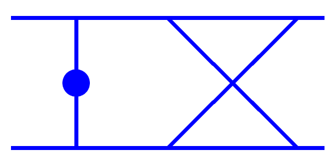
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Tausk, hep-ph/9909506

Anastasiou, Gehrmann, Oleari,Remiddi and Tausk, hep-ph/0003261

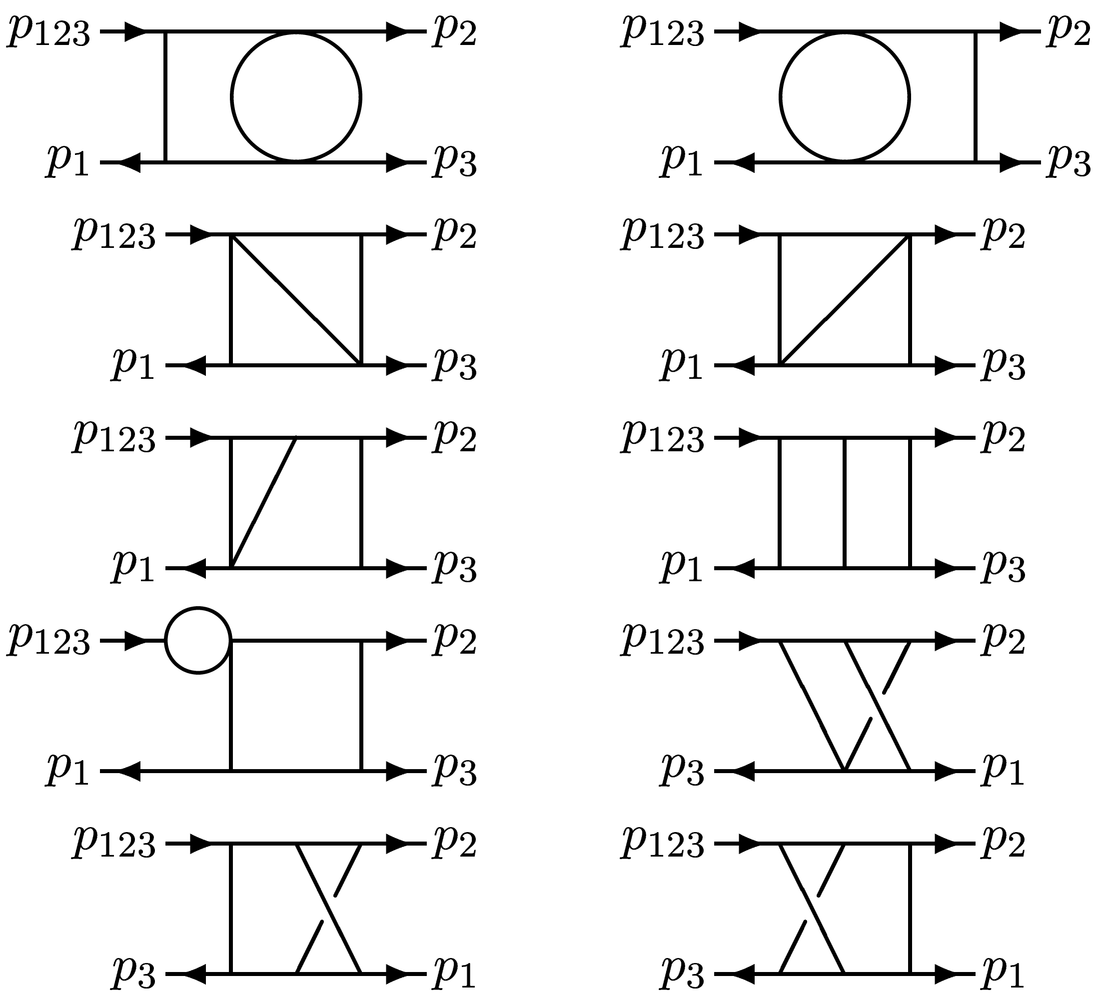
All can be expressed in terms of Nielsen polylogarithms.

Master Integrals - one offshell leg

- Much more complicated and require new (2-dimensional harmonic polylogarithm) functions to describe them
- there are also many more

Gehrmann and Remiddi, hep-ph/0008287, hep-ph/0101124

For example,



plus five extra integrals needed for the tensors.

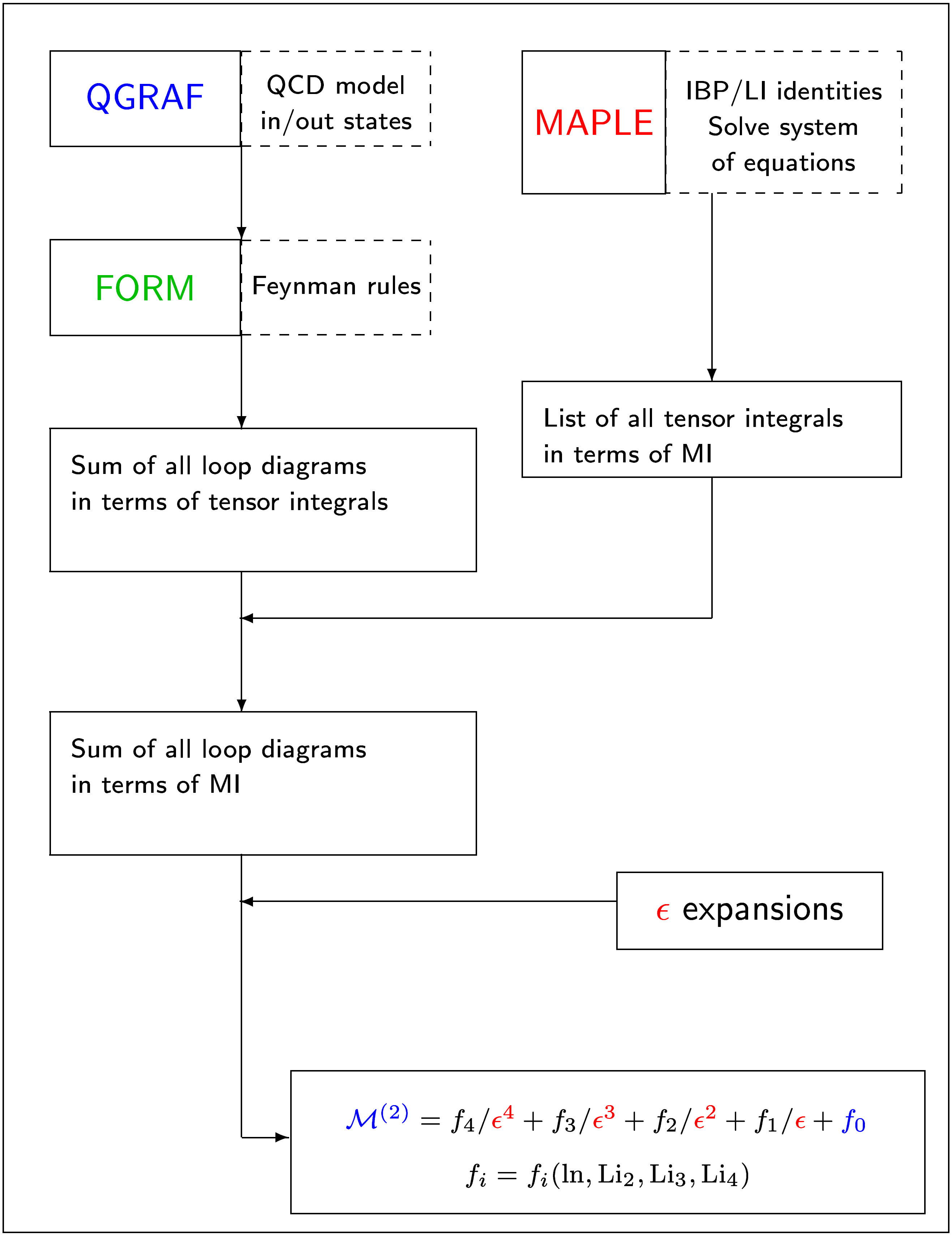
Application to two-loop QCD scattering

$2 \to 2$ Process	Tree	One loop	Two loops
$gg \to gg$	4	81	1771
$q\bar{q} \to gg$	3	30	595
$q\bar{q} \to \bar{q}'q'$	1	10	189

- One loop amplitudes by Ellis and Sexton (1986)
- .. needs automating

Matrix Element calculation

General algorithm



Progress in last couple of years

On-shell Process	Tree × Two-loop	Helicity amplitudes
$e^+e^- \to \mu^+\mu^-(e^+e^-)$	$\surd$(00)	
$q\bar{q} \to q\bar{q}(\bar{q}'q')$	$\surd$(00)	
$q\bar{q} \to gg$	$\surd$(01)	
$gg \to gg$	$\surd$(01)	$\surd$(00), $\overline{\surd}$(02)
$gg \to \gamma\gamma$	—	$\surd$(01)
$\gamma\gamma \to \gamma\gamma$	—	$\surd$(01), $\overline{\surd}$(02)
$q\bar{q} \to g\gamma(\gamma\gamma)$	$\surd$(02)	
Off-shell Process		
$e^+e^- \to q\bar{q}g$	$\surd\surd$(01)	$\surd\surd$(02), $\surd$(02)

$\surd$ Bern, De Freitas, Dixon, Ghinculov, Kosower, Wong

$\surd$ Anastasiou, Binoth, Glover, Marquard, Oleari, Tejeda-Yeomans, van der Bij

$\surd\surd$ Garland, Gehrmann, Glover, Koukoutsakis, Remiddi

$\surd$ Moch, Uwer, Weinzierl

Rapid progress in last two years...since Loops and Legs in Bastei

Finite parts

- The finite remainder is defined as

$$\mathcal{F}inite = 2\,\mathrm{Re}\langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \rangle - \mathcal{P}oles$$

  and is given in terms of real logs and polylogs for each scattering channel and each colour

- e.g., leading colour part of $q_1\bar{q}_1 \to q_2\bar{q}_2$

$$\begin{aligned}
A_s \quad = \quad & \Big[2\,\mathrm{Li}_4(x) + \left(-2\,X - \frac{11}{3}\right)\mathrm{Li}_3(x) + \left(X^2 + \frac{11}{3}\,X - \frac{2}{3}\,\pi^2\right)\mathrm{Li}_2(x) \\
& + \frac{121}{18}\,S^2 + \left(-\frac{11}{3}\,X^2 + 11\,X - \frac{296}{27}\right) S + \frac{1}{6}\,X^4 + \left(\frac{1}{3}\,Y - \frac{49}{18}\right) X^3 \\
& + \left(\frac{11}{6}\,Y - \frac{5}{6}\,\pi^2 + \frac{197}{18}\right) X^2 + \left(-\frac{2}{3}\,Y\,\pi^2 - \frac{47}{18}\,\pi^2 + 6\,\zeta_3 - \frac{95}{24}\right) X \\
& + \left(\frac{11}{24}\,\pi^2 - 7\,\zeta_3 - \frac{409}{216}\right) Y + \frac{113}{720}\,\pi^4 - \frac{7}{6}\,\pi^2 + \frac{197}{36}\,\zeta_3 + \frac{23213}{2592} \Big] \left[\frac{t^2+u^2}{s^2}\right] \\
& + \Big[-3\,\mathrm{Li}_4(y) + 6\,\mathrm{Li}_4(x) - 3\,\mathrm{Li}_4(z) + \left(-2\,X - \frac{7}{2}\right)\mathrm{Li}_3(x) \\
& + 3\,X\,\mathrm{Li}_3(y) + \left(\frac{1}{2}\,X^2 + \frac{7}{2}\,X + \frac{1}{2}\,\pi^2\right)\mathrm{Li}_2(x) + \left(-\frac{11}{6}\,X^2 + \frac{11}{6}\,X\right) S \\
& + \left(\frac{1}{2}\,Y\,\pi^2 - \frac{13}{9}\,\pi^2 - \zeta_3 - \frac{32}{9}\right) X + \left(\frac{7}{4}\,Y - \frac{3}{4}\,\pi^2 + \frac{44}{9}\right) X^2 \\
& + \left(\frac{1}{2}\,Y - \frac{49}{36}\right) X^3 - \frac{7}{120}\,\pi^4 + \frac{47}{36}\,\pi^2 + 2\,\zeta_3 \Big] \left[\frac{t^2-u^2}{s^2}\right] + \left[3\,X^2\right] \frac{t^3}{s^2 u} \\
& + 3\,\mathrm{Li}_4(y) - 3\,\mathrm{Li}_4(x) + 3\,\mathrm{Li}_4(z) - 3\,X\,\mathrm{Li}_3(y) - \frac{5}{2}\,\mathrm{Li}_3(x) \\
& + \left(\frac{5}{2}\,X - \frac{1}{2}\,\pi^2\right)\mathrm{Li}_2(x) - \frac{11}{6}\,X\,S + \frac{1}{8}\,X^4 + \left(-\frac{1}{2}\,Y + \frac{1}{3}\right) X^3 \\
& + \left(\frac{5}{4}\,Y + \frac{1}{4}\,\pi^2 + \frac{1}{6}\right) X^2 + \left(-\frac{1}{2}\,Y\,\pi^2 - \frac{7}{6}\,\pi^2 + 3\,\zeta_3 + \frac{32}{9}\right) X \\
& + \frac{1}{40}\,\pi^4 - \frac{11}{36}\,\pi^2 + 4\,\zeta_3
\end{aligned}$$

$$x = -\tfrac{t}{s}, \quad y = -\tfrac{u}{s}, \quad X = \ln(x), \quad Y = \ln(y), \quad S = \ln\left(\tfrac{s}{\mu^2}\right)$$

Conclusions

DOUBLE VIRTUAL:

largely solved for processes of interest

- ▷ All $2 \to 2$ parton-parton scattering processes calculated at two-loops together with $\gamma^* \to q\bar{q}g$ etc
- √ strong checks with infrared pole structure, high energy limit, QED results
- ▷ heavy quark's? Top and Bottom quark cross section at NNLO?

VIRTUAL RADIATION:

can isolate poles with straightforward extension of NLO Dipole subtraction

DOUBLE RADIATION:

infrared properties of double unresolved tree graphs known, but not how to isolate them

This is the bottleneck

Topic for discussion: how to develop subtraction terms to isolate the infrared singularities

This is the big challenge for 2003.

Where will we be in five years?

I confidently expect that we will, amongst other things, achieve the following

1) NNLO extraction and evolution of pdf's from DIS data
almost there, soon analytic splitting functions and then phenomenology can begin

2) Fully consistent NNLO global fits of pdf's using DIS, Drell Yan and jet data
again almost there, "NNLO" fits already in place, need NNLO jet production calculations to make real global fit

3) NNLO determination of α_s using jet data in e^+e^- and ep (as well as $p\bar{p}$)
A lot of work needs to be done to learn how to combine the infrared divergent parts - the two-loop, one-loop one-unresolved and tree-level double unresolved. NLO Monte Carlos for $2 \to 3$ processes can be used as seeds.

4) NNLO heavy quark production
NLO doesnt work too well. Needs a whole new set of master integrals to evaluate the two loop contribution

5) NLO Monte Carlo programs for multiparticle final states
Needed for backgrounds to multiparticle signatures. Work has started on how to evaluate the loop contribution numerically