# Coverage of Confidence Intervals Including Systematic Uncertainties

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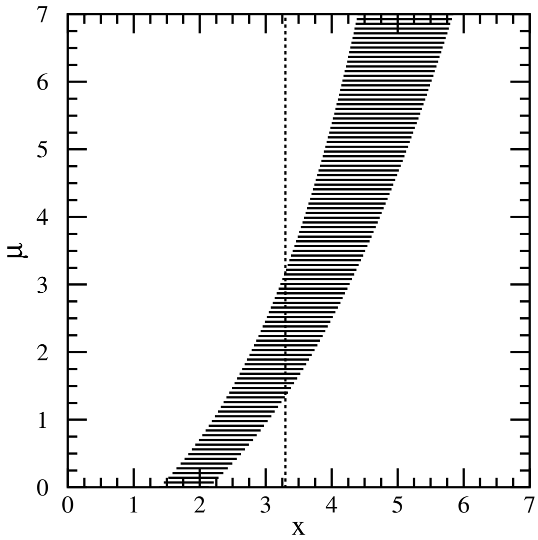

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- The (extended) Highland & Cousins Method.
- **PO**issonian **L**imit **E**stimator
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# Inclusion of Systematic Uncertainties

- Consider Poisson distribution $P(n|s+b)$, b known, in presence of systematic uncertainties.

$\rightarrow$ Fold the Poissonian with a probability density function describing the uncertainties.
$\rightarrow$ apply construction with this new PDF.

- background process uncertainty:

$$q(n|s+b) = \frac{1}{\sqrt{2\pi\sigma_b^2}} \int_0^\infty db'\, P(n|s+b') e^{\frac{-(b-b')^2}{2\sigma_b^2}}$$

- signal eff uncertainty:

$$q(n|...) = \frac{1}{\sqrt{2\pi\sigma_\epsilon^2}} \int_0^\infty d\epsilon'\, P(n|(b+s\epsilon'))\, e^{\frac{-(1-\epsilon')^2}{2\sigma_\epsilon^2}}$$

- in general both, and background efficiency uncertainty, and correlations.

# poissonian limit estimator: `pole`

- Performs Frequentist constructions and uses a MC method to perform the integrals.

- Following types of uncertainties included:
  - Uncertainty in the signal and BG efficiencies. (and correlation)
  - Uncertainty in the background processes.

- Following shapes of uncertainties included:
  - Gaussian (truncated)
  - Flat (symmetric, asymmetric)
  - log - normal (mean/peak centered on 1.)

# pole cont'd

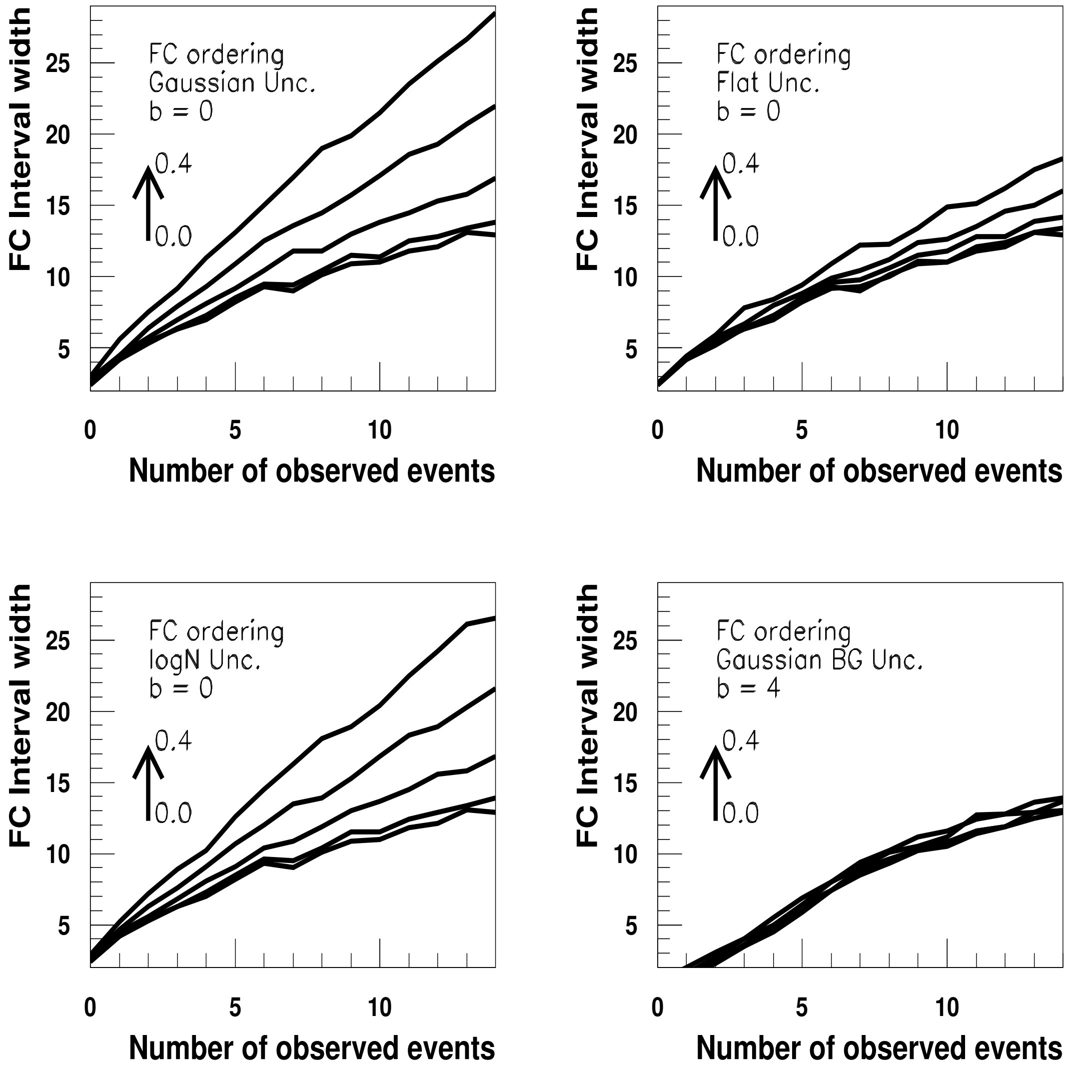
- following ordering schemes:

- Neyman
- Neyman with Conditioning
- Feldman & Cousins (FC)
- FC with Conditioning

- Fortran program
  and some documentation available from:
  http://www3.tsl.uu.se/∼conrad/pole.html
  (J. C., O. Botner, A. Hallgren and C.P. de los Heros, hep-ex/0202013)

NEW: A portable F90 package has been developed
includes also: coverage calculator
(J. C. & R. Andersson, in prep)

# pole results : Width Dependence



## `pole` results: significantly less events observed than expected background

| $n_0$ | $b$ | sig. eff. unc. | FC ordering | FC with cond |
|---|---|---|---|---|
| 6 | 12 | 0 | **0; 1.65** | 0; 4.00 |
| | | 0.2 | **0; 1.5** | 0; 4.20 |
| | | 0.4 | **0; 1.35** | 0; 5.15 |
| | | | | |
| 11 | 12 | 0 | 0; 5.80 | 0; 6.90 |
| | | 0.2 | 0; 5.85 | 0; 7.3 |
| | | 0.4 | 0; 6.65 | 0; 8.95 |

→ Counterintuitive behavior without conditioning !

CAUTION !!

# Coverage: Definition and Calculation

- Definition of *Coverage*

  An algorithm is said to have the correct *coverage* if it provides intervals which contain the "true" value of the tested parameter in a fraction 1- $\alpha$ of indefinitely many **identical** experiments.
  (independent of the tested parameter)

- Calculation of Coverage (*Helene approach):*

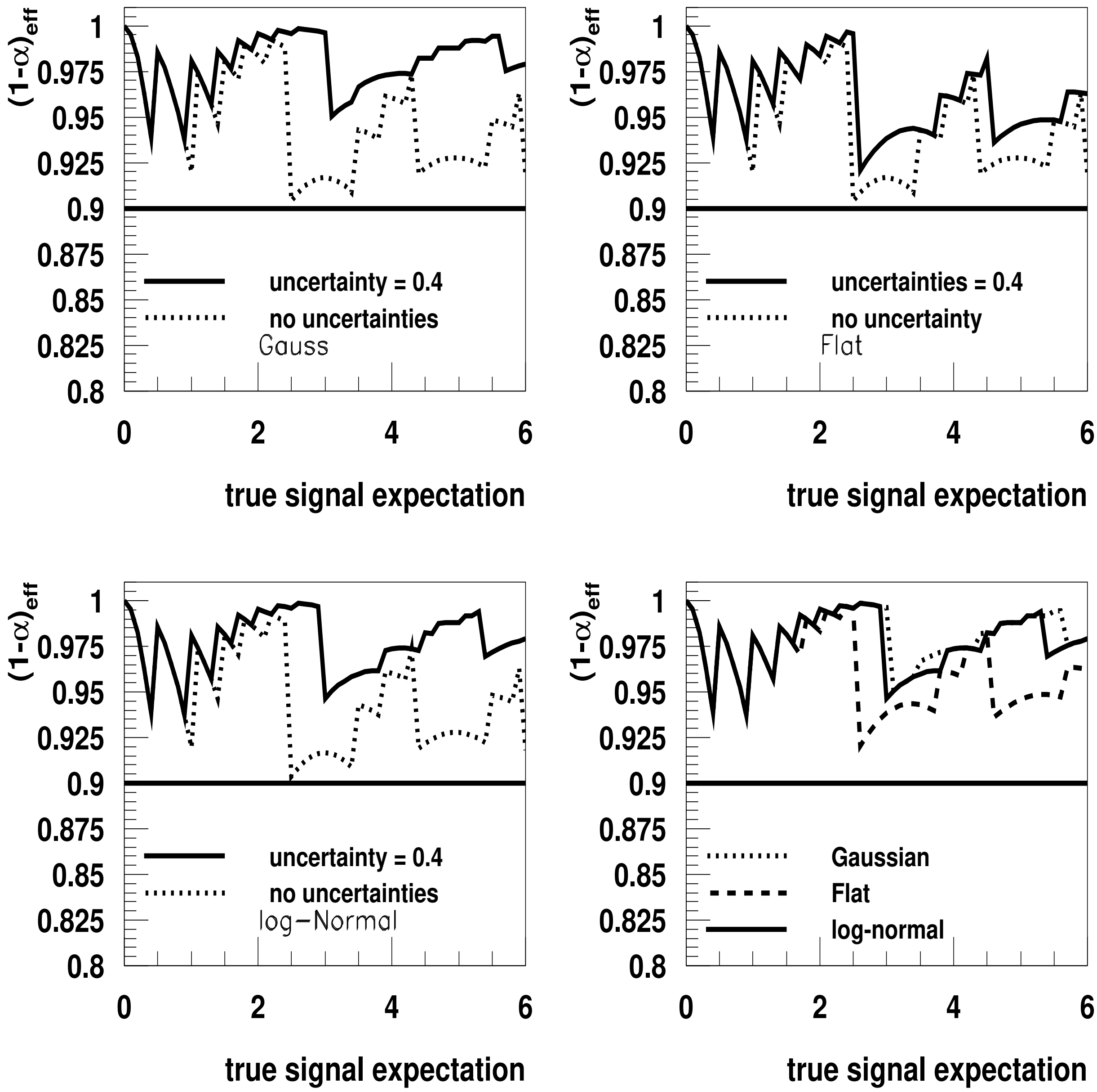
$$(1-\alpha(s))_{eff} = \sum_{n|s\,\epsilon[s_1,s_2]} P(n|s+b)$$

→ calculate intervals.
→ for a given $s$, take sum over all $n$ which

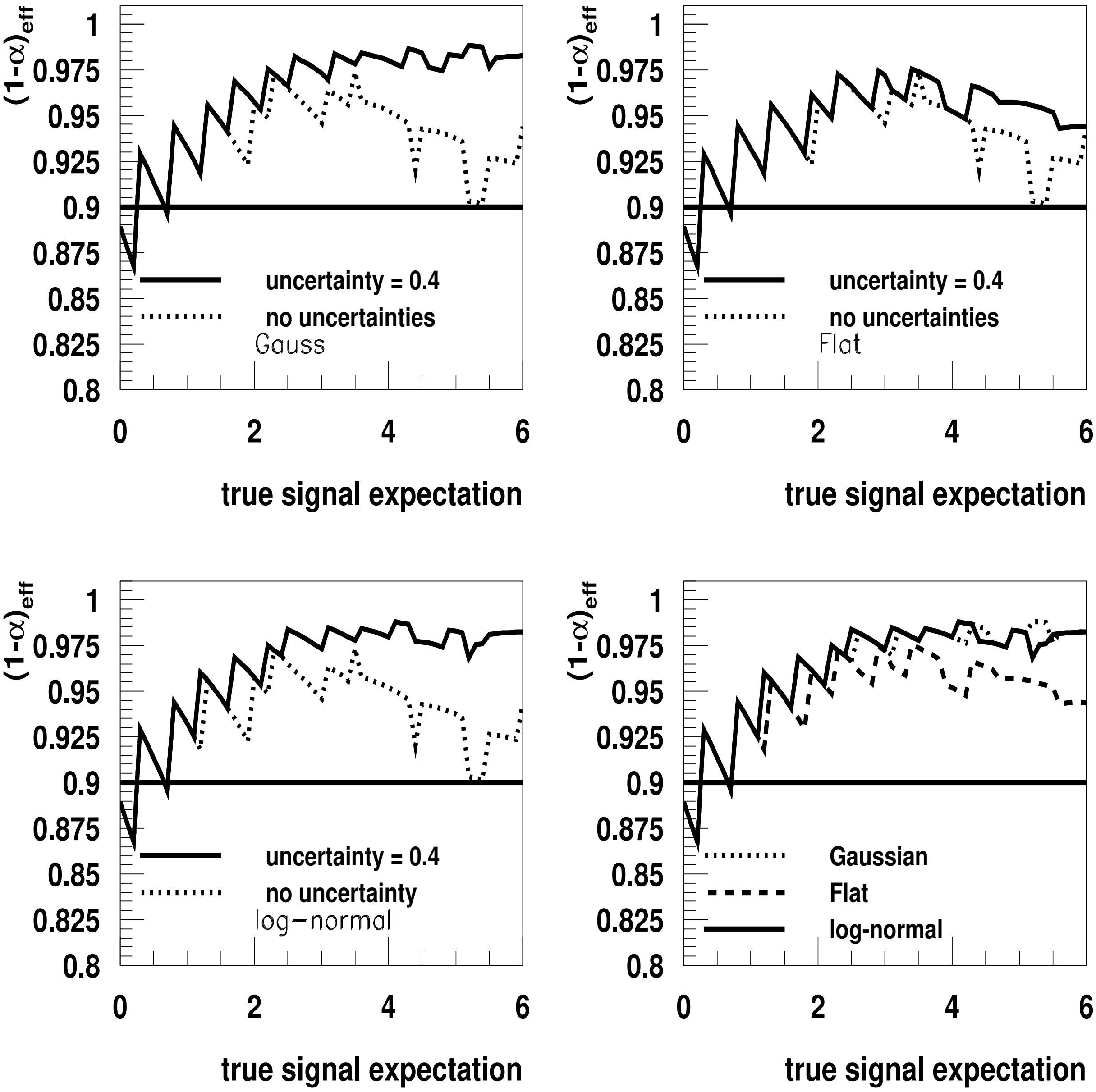
# Overview over Coverage Calculations

| Ordering | sys unc. type | unc. shape | unc. range | s | b |
|---|---|---|---|---|---|
| FC | sig. eff. | Gauss | [0, 0.4] | [0, 6] | 0 |
| | | Flat | [0, 0.4] | [0, 6] | 0 |
| | | log-Normal | [0, 0.4] | [0, 6] | 0 |
| | backg. pred. | Gauss | [0, 0.4] | [0, 6] | 2,4,6 |
| FC | sig. eff. | Gauss | 0.25 | | |
| WIMPs | bck. eff. | | 0.3 | | |
| | bck. pred. | | 0.3 | [0, 9] | 12.0 |
| FC | sig. eff | Gauss | [0, 0.4] | [0, 6] | 4 |
| with | | Flat | [0, 0.4] | [0, 6] | 4 |
| condit. | | log-Normal | [0, 0.4] | [0, 6] | 4 |

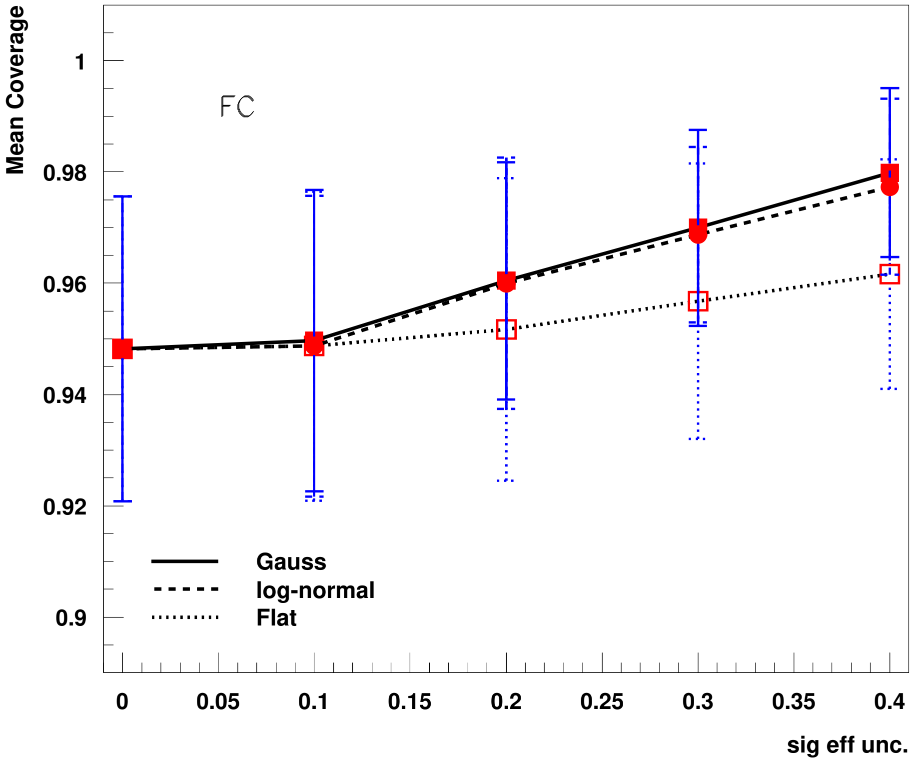
# FC, Signal Efficiency Uncertainty

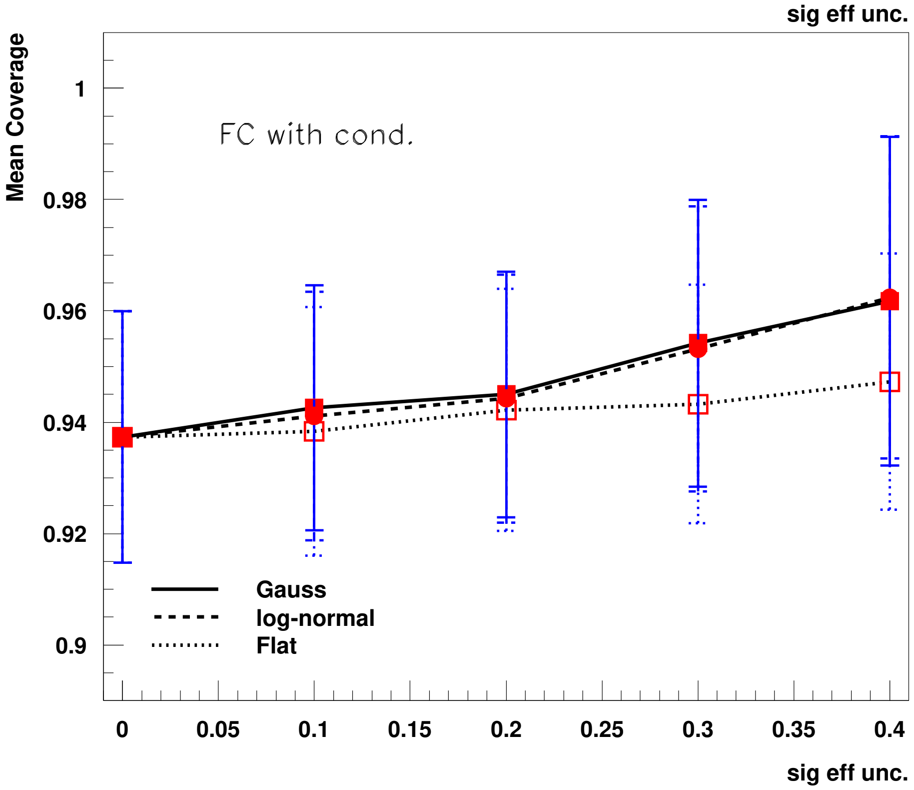


# FC with Cond., Signal Efficiency Uncertainty.

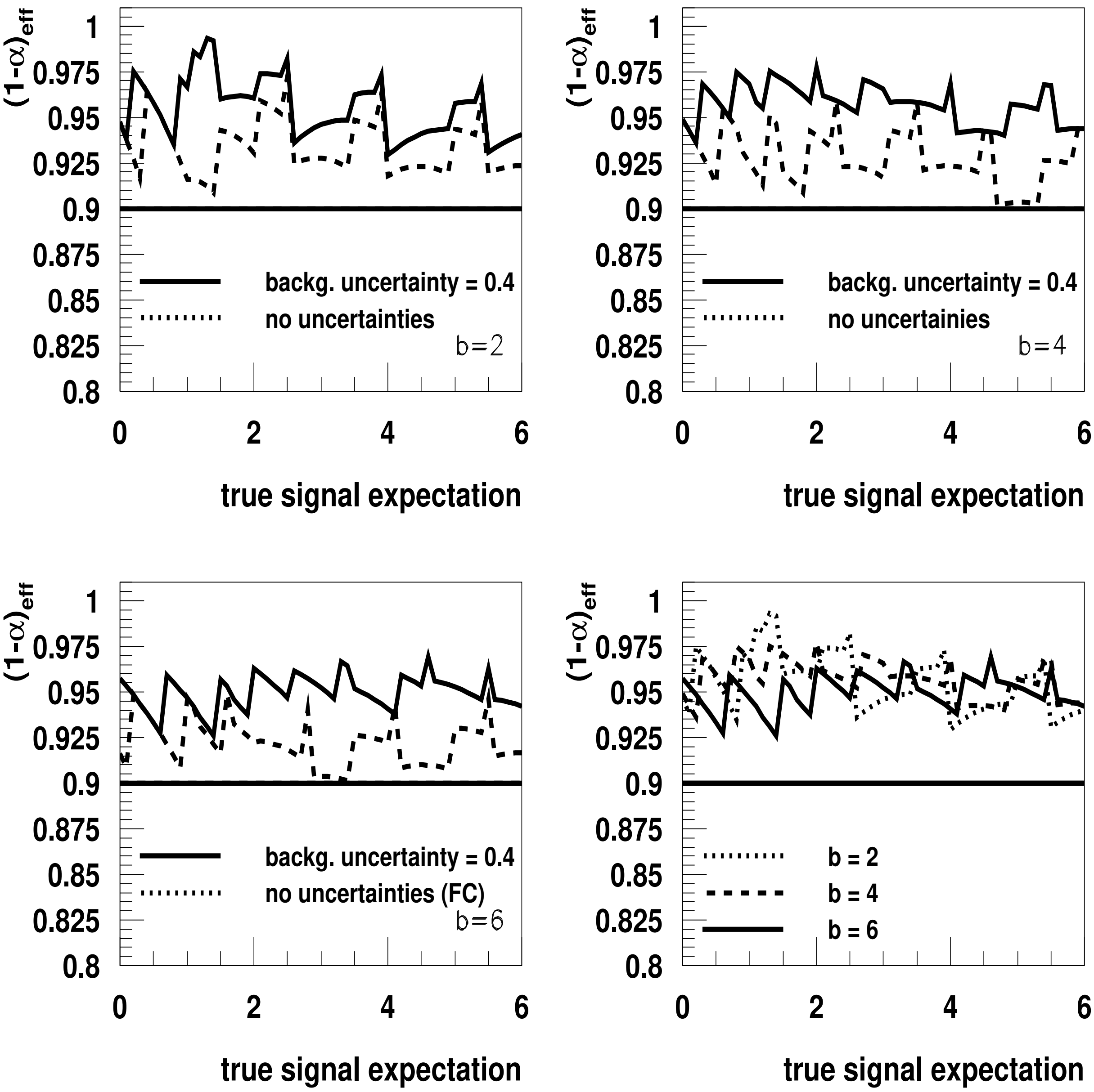


# Mean Coverage: Signal Efficiency Uncertainty



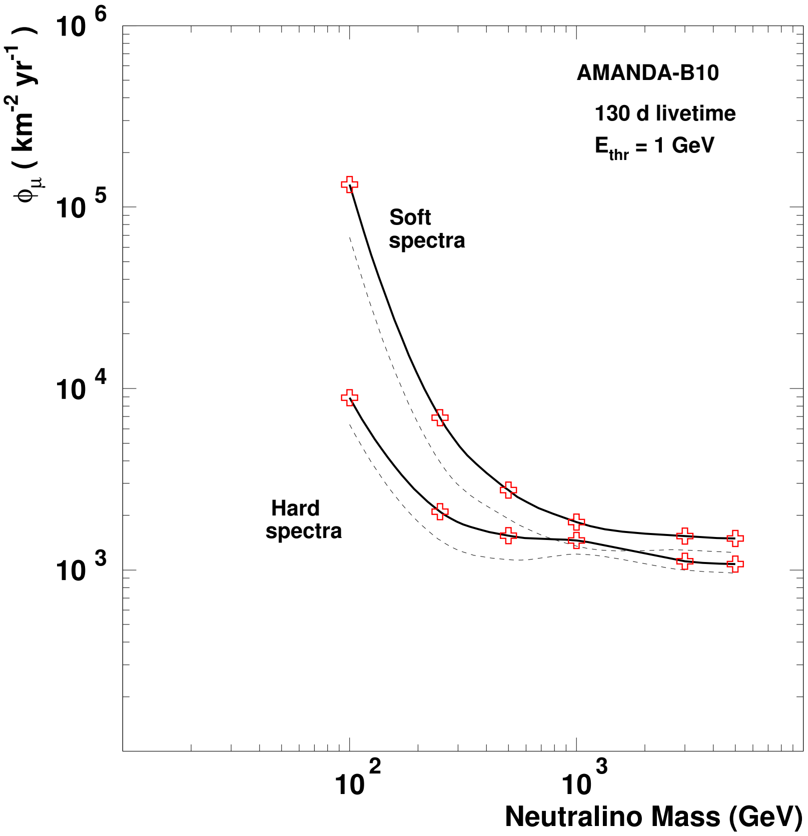



# Gaussian Uncert. in BG pred.



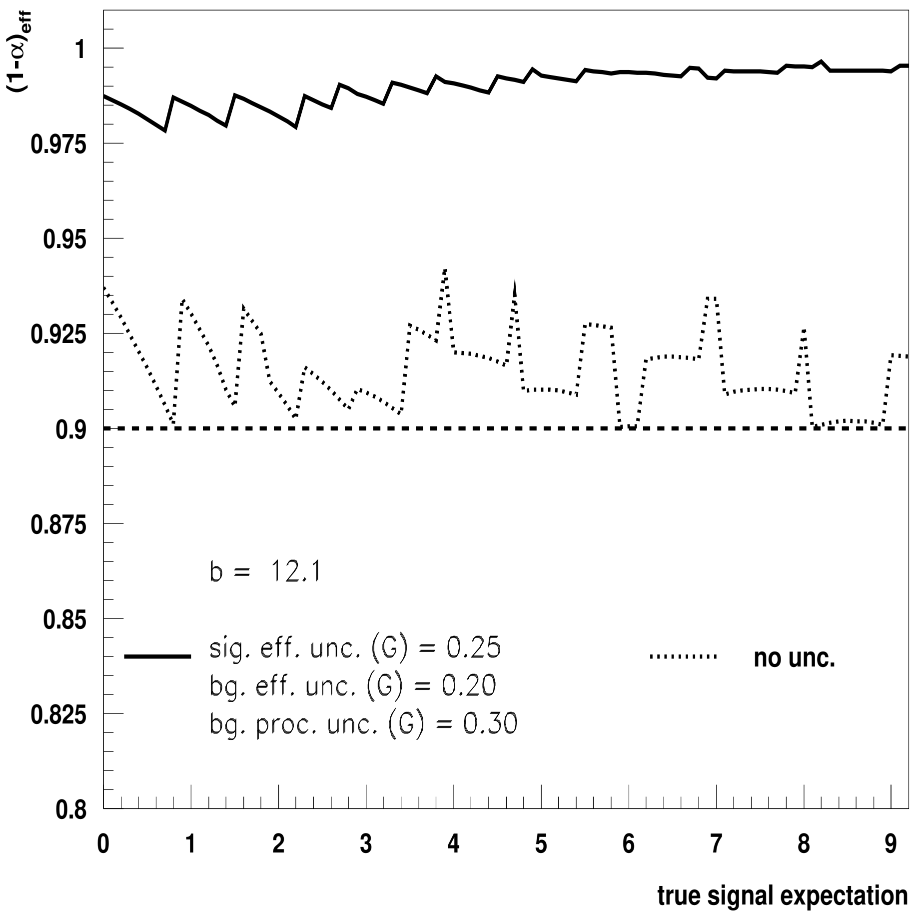
# Real Application: AMANDA WIMP Search I

*preliminary*




(J. Ahrens et. al. (AMANDA-collab), astro-ph/0202370)

# Real Application: AMANDA WIMP Search II.



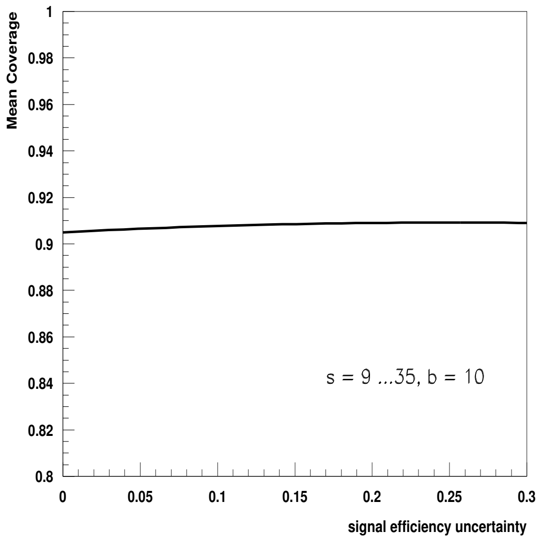
# A word on "Bayesian Coverage"

- Redefinition of *Coverage* (custom-made,sts.)

  An algorithm is said to have the correct *coverage* if it provides intervals which contain the "true" value of the tested parameter in a fraction 1- $\alpha$ of indefinitely many **similar*** experiments (independent of the tested parameter)
  * drawn from the same distribution.

Test:

- MC experiments:
  $\rightarrow$ vary nuisance parameters in each experiment (efficiencies, background prediction) according to PDF describing the uncertainties.



# Summary

- The $FHC^2$ (w/o conditioning) method overcovers:
  - dependent on signal hypothesis (sig.eff).
  - dependent on size of uncertainties.
  - dependent on shape of uncertainties.
  - (dependent on background expectation ?)
- The additional overcoverages seems negligible for small signal and small background (even if unc. of the order of 40 %).
- but CHECK !

# Coverage of FC with Conditioning