Dissipative Dynamics of Inflation

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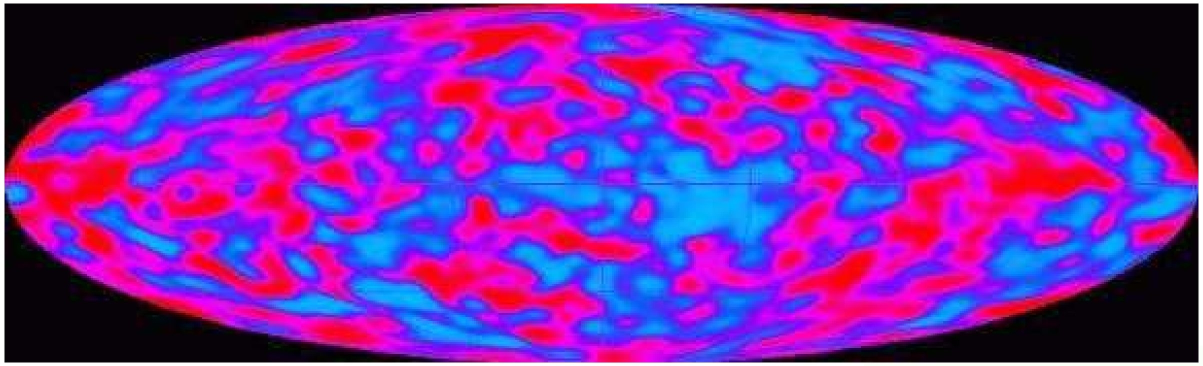
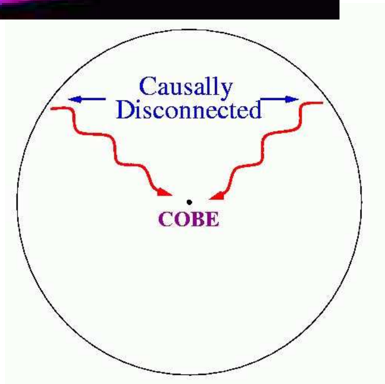
Inflation

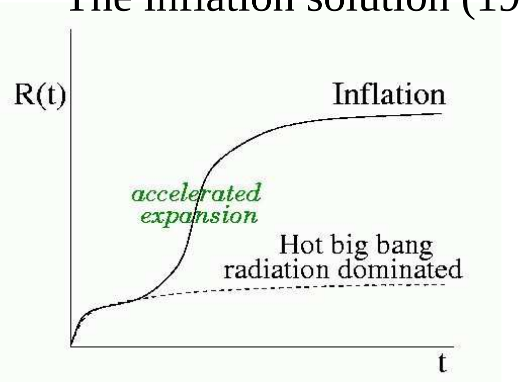


The puzzle of cosmological initial conditions:



$$\frac{\Delta T}{T} < 10^{-5}$$ in CMBR without causal contact

The inflation solution (1981+)

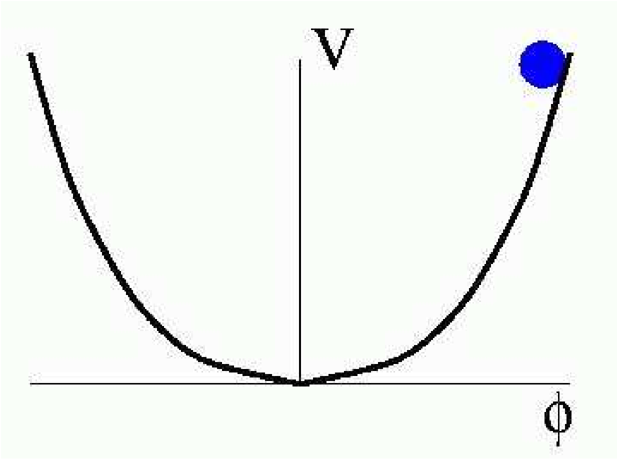


Accelerated expansion $\ddot{R}(t) > 0$

implies $\rho + 3p < 0$

How to get this from high energy physics?

Scalar field ("inflaton") dynamics



$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi) + \frac{(\nabla\phi)^2}{2R^2}$$
$$p = \frac{\dot{\phi}^2}{2} - V(\phi) - \frac{(\nabla\phi)^2}{6R^2}$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

= Supercooled Inflation Just Choose $V(\phi)$

Potential energy dominated $3H\dot{\phi} \gg \ddot{\phi}$, "slow-roll"

But what about particle creation?

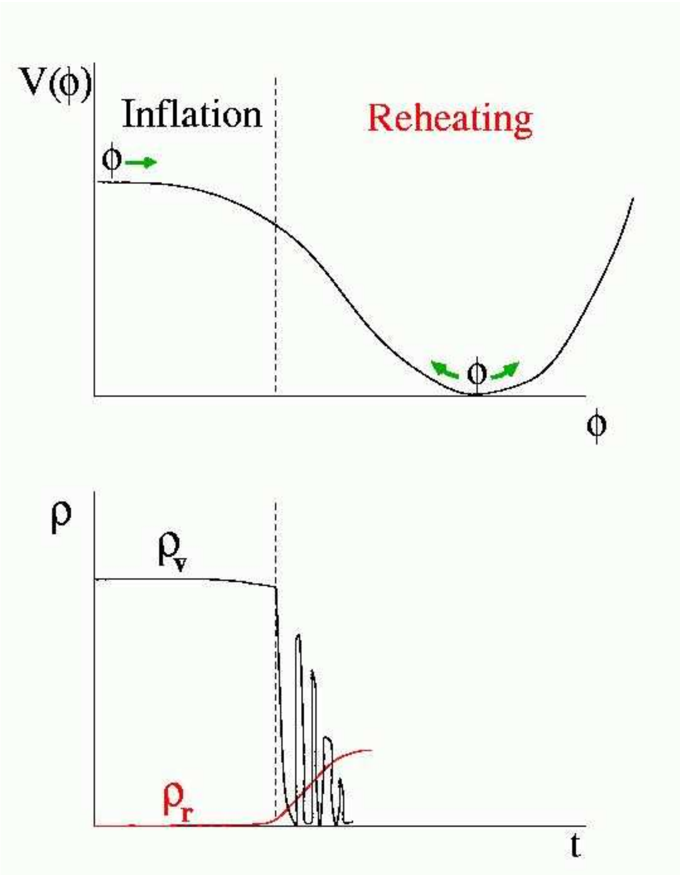
$$\ddot{\phi} + 3H\dot{\phi} + \eta\dot{\phi} + V'(\phi) = 0$$

Warm Inflation: η dominates

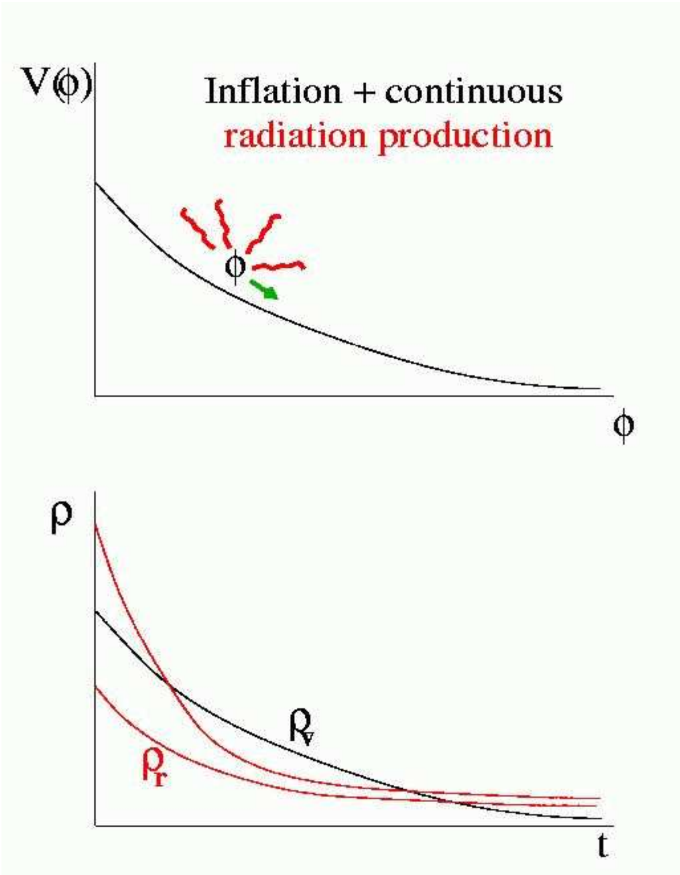
Slow-roll now means $\eta\dot{\phi} \gg 3H\dot{\phi}, \ddot{\phi}$, overdamped

The Warm Inflation Difference

Supercooled Inflation



Warm Inflation



Energetics

Consider GUT scale inflation: $M \sim 10^{15}$GeV, $\rho_v \sim M^4 \sim 10^{60}$GeV

$\Rightarrow H \sim 10^{10}$GeV

$T > H$ requires > 1 part in $\sim 10^{20}$ of vacuum energy $\rightarrow$ radiation
(influences structure formation)

$T > 1$GeV requires > 1 part in $\sim 10^{60}$ $\rho_v \rightarrow \rho_r$
(makes reheating unnecessary)

Supercooled inflation Basic Assumption: Scalar field isolated, will NOT dissipate the above amounts during inflation

Warm inflation (PRL **75**, 3218 (1995)): Expects above, or even more, radiation production during inflation

Equipartition Hypothesis of Statistical Mechanics: would expect the scalar field to distribute its energy evenly amongst all degrees of freedom

Dynamical Question: Will the relevant time scales during inflation prohibit the minute' radiation production given above?

Scalar Field (Φ) Dynamics

$$\mathcal{L}_\Phi = \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{m_\phi^2}{2}\Phi^2 - \frac{\lambda}{4!}\Phi^4$$

$$\mathcal{L}_I = \Phi \sum_{j=1}^{N_\psi} h_j \bar{\psi}_j \psi_j, \frac{1}{2}\sum_{j=1}^{N_\chi} g_j^2 \chi_j^2 \Phi^2$$

$\Phi = \varphi + \phi$

Effective equation of motion (EOM) for $\varphi(t)$:

$$\ddot{\varphi}(t) + m_\phi^2 \varphi(t) + \frac{\lambda}{6}\varphi^3(t) + \frac{\lambda}{2}\varphi(t)\langle\phi^2\rangle + \frac{\lambda}{6}\langle\phi^3\rangle + \varphi(t)\sum_{j=1}^{N_\chi} g_j^2 \langle\chi_j^2\rangle = 0$$

$\langle\phi^2\rangle$, $\langle\phi^3\rangle$ and $\langle\chi_j^2\rangle$ perturbative evaluation
2-loop order

Green's Functions

Satisfy appropriate Schwinger-Dyson equations

$$\left[\Box + m_\phi^2 + \frac{\lambda}{2}\varphi^2\right] G_\phi(x,x') + \int d^4z \Sigma_\phi(x,z) G_\phi(z,x') = i\delta(x,x')$$

$$[i \not{\partial} - m_{\psi_k} - h_k\varphi] \, S_{\psi_k}(x,x') + \int d^4z \Sigma_{\psi_k}(x,z) S_{\psi_k}(z,x') = i\delta(x,x')$$

The momentum-space Fourier transform

$$G_\phi(x,x') = i\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}.(\mathbf{x}-\mathbf{x}')} \begin{pmatrix} G_\phi^{++}(\mathbf{q},t-t') & G_\phi^{+-}(\mathbf{q},t-t') \\ G_\phi^{-+}(\mathbf{q},t-t') & G_\phi^{--}(\mathbf{q},t-t') \end{pmatrix}$$

Bosons

$$\begin{aligned}
G_\phi^{++}(\mathbf{q},t-t') &= G_\phi^{>}(\mathbf{q},t-t')\theta(t-t') + G_\phi^{<}(\mathbf{q},t-t')\theta(t'-t) \\
G_\phi^{--}(\mathbf{q},t-t') &= G_\phi^{>}(\mathbf{q},t-t')\theta(t'-t) + G_\phi^{<}(\mathbf{q},t-t')\theta(t-t') \\
G_\phi^{+-}(\mathbf{q},t-t') &= G_\phi^{<}(\mathbf{q},t-t') \\
G_\phi^{-+}(\mathbf{q},t-t') &= G_\phi^{>}(\mathbf{q},t-t')
\end{aligned}$$

Fermions

$$\begin{aligned}
S_\psi^{++}(\mathbf{q},t-t') &= S_\psi^{>}(\mathbf{q},t-t')\theta(t-t') + S_\psi^{<}(\mathbf{q},t-t')\theta(t'-t) \\
S_\psi^{--}(\mathbf{k},t-t') &= S_\psi^{>}(\mathbf{q},t-t')\theta(t'-t) + S_\psi^{<}(\mathbf{q},t-t')\theta(t-t') \\
S_\psi^{+-}(\mathbf{q},t-t') &= S_\psi^{<}(\mathbf{q},t-t') \\
S_\psi^{-+}(\mathbf{q},t-t') &= S_\psi^{>}(\mathbf{q},t-t')
\end{aligned}$$

Bubble Resummed Approximation

At any temperature $T = 1/\beta$

For Bosons

$$G^>_\phi(\mathbf{q}, t-t') = \frac{1}{2\omega_\phi}\{[1+n_\phi(\omega_\phi - i\Gamma_\phi)]e^{-i(\omega_\phi - i\Gamma_\phi)(t-t')} + n_\phi(\omega_\phi + i\Gamma_\phi)e^{i(\omega_\phi+i\Gamma_\phi)(t-t')}\}$$
$$\theta(t-t') + \frac{1}{2\omega_\phi}\{[1+n_\phi(\omega_\phi + i\Gamma_\phi)]e^{-i(\omega_\phi+i\Gamma_\phi)(t-t')} + n_\phi(\omega_\phi - i\Gamma_\phi)$$
$$e^{i(\omega_\phi - i\Gamma_\phi)(t-t')}\}\theta(t'-t)\ ,$$
$$G^<_\phi(\mathbf{q}, t-t') = G^>_\phi(\mathbf{q}, t'-t)\ ,$$

n_ϕ is the Bose distribution function,

For Fermions

$$S^>_\psi(\mathbf{q}, t-t') = -\frac{1}{2\omega_\psi}\left\{e^{-i(\omega_\psi - i\Gamma_\psi)(t-t')}\left[(\omega_\psi - i\Gamma_\psi)\gamma_0 - \not{q} + m_{\psi,r} - i\frac{\Gamma_\psi(\omega_\psi - i\Gamma_\psi)}{m_{\psi,r}}\right]\right.$$
$$\times[1-n_\psi(\omega_\psi - i\Gamma_\psi)]$$
$$\left.-e^{i(\omega_\psi+i\Gamma_\psi)(t-t')}\left[-(\omega_\psi + i\Gamma_\psi)\gamma_0 - \not{q} + m_{\psi,r} + i\frac{\Gamma_\psi(\omega_\psi + i\Gamma_\psi)}{m_{\psi,r}}\right] n_\psi(\omega_\psi + i\Gamma_\psi)\right\}\theta(t-t')$$
$$-\frac{1}{2\omega_\psi}\left\{e^{-i(\omega_\psi + i\Gamma_\psi)(t-t')}\left[(\omega_\psi + i\Gamma_\psi)\gamma_0 - \not{q} + m_{\psi,r} + i\frac{\Gamma_\psi(\omega_\psi + i\Gamma_\psi)}{m_{\psi,r}}\right]\right.$$
$$\times[1-n_\psi(\omega_\psi + i\Gamma_\psi)]$$
$$\left.-e^{i(\omega_\psi-i\Gamma_\psi)(t-t')}\left[-(\omega_\psi - i\Gamma_\psi)\gamma_0 - \not{q} + m_{\psi,r} - i\frac{\Gamma_\psi(\omega_\psi - i\Gamma_\psi)}{m_{\psi,r}}\right] n_\psi(\omega_\psi - i\Gamma_\psi)\right\}\theta(t'-t)$$

and

$$S^<_\psi(\mathbf{q}, t-t') = \frac{1}{2\omega_\psi}\left\{e^{i(\omega_\psi + i\Gamma_\psi)(t-t')}\left[-(\omega_\psi + i\Gamma_\psi)\gamma_0 - \not{q} + m_{\psi,r} + i\frac{\Gamma_\psi(\omega_\psi + i\Gamma_\psi)}{m_{\psi,r}}\right]\right.$$
$$\times[1-n_\psi(\omega_\psi + i\Gamma_\psi)]$$
$$\left.-e^{-i(\omega_\psi-i\Gamma_\psi)(t-t')}\left[(\omega_\psi - i\Gamma_\psi)\gamma_0 - \not{q} + m_{\psi,r} - i\frac{\Gamma_\psi(\omega_\psi - i\Gamma_\psi)}{m_{\psi,r}}\right] n_\psi(\omega_\psi - i\Gamma_\psi)\right\}\theta(t-t')$$
$$+\frac{1}{2\omega_\psi}\left\{e^{i(\omega_\psi - i\Gamma_\psi)(t-t')}\left[-(\omega_\psi - i\Gamma_\psi)\gamma_0 - \not{q} + m_{\psi,r} - i\frac{\Gamma_\psi(\omega_\psi - i\Gamma_\psi)}{m_{\psi,r}}\right]\right.$$
$$\times[1-n_\psi(\omega_\psi - i\Gamma_\psi)]$$
$$\left.-e^{-i(\omega_\psi+i\Gamma_\psi)(t-t')}\left[(\omega_\psi + i\Gamma_\psi)\gamma_0 - \not{q} + m_{\psi,r} + i\frac{\Gamma_\psi(\omega_\psi + i\Gamma_\psi)}{m_{\psi,r}}\right] n_\psi(\omega_\psi + i\Gamma_\psi)\right\}\theta(t'-t)\ ,$$

φ Effective Equation of Motion

$$
\begin{aligned}
&\ddot{\varphi}(t)+\left(m_\phi^2+\frac{\lambda}{2}\langle\phi^2\rangle_0\right)\varphi(t)+\frac{\lambda}{6}\varphi^3(t)\\
&+\lambda\varphi(t)\int_{-\infty}^{t}dt'\,\frac{\lambda}{2}\varphi^2(t')\int\frac{d^3\mathbf{q}}{(2\pi)^3}\mathrm{Im}\left[G_\phi^{++}(\mathbf{q},t-t')\right]^2\\
&+\frac{\lambda^2}{3}\int_{-\infty}^{t}dt'\varphi(t')\int\frac{d^3\mathbf{q}_1}{(2\pi)^3}\frac{d^3\mathbf{q}_2}{(2\pi)^3}\mathrm{Im}\left[G_\phi^{++}(\mathbf{q}_1,t-t')G_\phi^{++}(\mathbf{q}_2,t-t')G_\phi^{++}(\mathbf{q}_1+\mathbf{q}_2,t-t')\right]\\
&-4\sum_{j=1}^{N_\psi}h_{j,\phi}\int_{-\infty}^{t}dt'\;h_j\varphi(t')\int\frac{d^3q}{(2\pi)^3}\mathrm{Im}\left[S_{\alpha\beta}^{++}(\mathbf{q},t-t')S^{++\,\beta\alpha}(\mathbf{q},t'-t)\right]=0
\end{aligned}
$$

$\langle\ldots\rangle_0$ is vacuum expectation value at zero background field

Upon insertion of the Green's functions, basic equation:

$$
\begin{aligned}
&\ddot{\varphi}(t)+\bar{m}_\phi^2\;\varphi(t)+\frac{\bar{\lambda}}{6}\varphi^3(t)\\
&+\lambda^2\varphi(t)\int_{-\infty}^{t}dt'\;\varphi(t')\dot{\varphi}(t')\int\frac{d^3q}{(2\pi)^3}\frac{\left[\omega_\phi\cos(2\omega_\phi|t-t'|)+\Gamma_\phi\sin(2\omega_\phi|t-t'|)\right]}{8\omega_\phi^2\left(\Gamma_\phi^2+\omega_\phi^2\right)}e^{-2\Gamma_\phi(q)|t-t'|}\\
&+\frac{\lambda^2}{3}\int_{-\infty}^{t}dt'\;\dot{\varphi}(t')\int\frac{d^3\mathbf{q}_1}{(2\pi)^3}\frac{d^3\mathbf{q}_2}{(2\pi)^3}\left\{\left[\omega_\phi(\mathbf{q}_1)+\omega_\phi(\mathbf{q}_2)+\omega_\phi(\mathbf{q}_1+\mathbf{q}_2)\right]\right.\\
&\times\cos[(\omega_\phi(\mathbf{q}_1)+\omega_\phi(\mathbf{q}_2)+\omega_\phi(\mathbf{q}_1+\mathbf{q}_2))|t-t'|]\\
&\left.+\left[\Gamma_\phi(\mathbf{q}_1)+\Gamma_\phi(\mathbf{q}_2)+\Gamma_\phi(\mathbf{q}_1+\mathbf{q}_2)\right]\sin[(\omega_\phi(\mathbf{q}_1)+\omega_\phi(\mathbf{q}_2)+\omega_\phi(\mathbf{q}_1+\mathbf{q}_2))|t-t'|]\right\}\\
&\times\frac{e^{-[\Gamma_\phi(\mathbf{q}_1)+\Gamma_\phi(\mathbf{q}_2)+\Gamma_\phi(\mathbf{q}_1+\mathbf{q}_2)]|t-t'|}}{8\omega_\phi(\mathbf{q}_1)\omega_\phi(\mathbf{q}_2)\omega_\phi(\mathbf{q}_1+\mathbf{q}_2)}\\
&\times\frac{1}{\left[\Gamma_\phi(\mathbf{q}_1)+\Gamma_\phi(\mathbf{q}_2)+\Gamma_\phi(\mathbf{q}_1+\mathbf{q}_2)\right]^2+\left[\omega_\phi(\mathbf{q}_1)+\omega_\phi(\mathbf{q}_2)+\omega_\phi(\mathbf{q}_1+\mathbf{q}_2)\right]^2}\\
&+\sum_{j=1}^{N_\psi}h_j^2\;\int_{-\infty}^{t}dt'\;\dot{\varphi}(t')\int\frac{d^3q}{(2\pi)^3}\frac{e^{-2\Gamma_{\psi_j}(q)|t-t'|}}{\omega_{\psi_j}^2m_{\psi,r}^2\left(\Gamma_{\psi_j}^2+\omega_{\psi_j}^2\right)}\\
&\times\left\{\left[\Gamma_{\psi_j}^4\omega_{\psi_j}+2\omega_{\psi_j}^3m_{\psi,r}^2-2m_{\psi,r}^4\omega_{\psi_j}+\Gamma_{\psi_j}^2\omega_{\psi_j}(\omega_{\psi_j}^2+m_{\psi,r}^2)\right]\cos(2\omega_{\psi_j}|t-t'|)\right.\\
&\left.+\left[\Gamma_{\psi_j}(2m_{\psi_j}^2-\Gamma_{\psi_j}^2)(\omega_{\psi_j}^2-m_{\psi,r}^2)-\Gamma_{\psi_j}^5\right]\sin(2\omega_{\psi_j}|t-t'|)\right\}=0
\end{aligned}
$$

Adiabatic-Markovian approximation

Derivative expansion for the temporally nonlocal terms

Effective Equation of Motion:

$$\ddot{\varphi}(t) + m_\phi^2\, \varphi(t) + \frac{\lambda}{6}\varphi^3(t) + \eta(\varphi)\dot{\varphi}(t) = 0$$

where

$$\eta(\varphi) = \varphi^2(t) \frac{\lambda^2 \alpha_{\phi,\psi}^2}{128\pi\, \sqrt{m_\phi^4 + \alpha_{\phi,\psi}^4}\, \left(2\sqrt{m_\phi^4 + \alpha_{\phi,\psi}^4} + 2m_\phi^2\right)^{\frac{1}{2}}}$$
$$+ \frac{\lambda^2 \alpha_{\phi,\psi}^2}{4} \int \frac{d^3\mathbf{q}_1}{(2\pi)^3} \frac{d^3\mathbf{q}_2}{(2\pi)^3} \left\{ \frac{1}{\omega_\phi(\mathbf{q}_1)^2 \omega_\phi(\mathbf{q}_2) \omega_\phi(\mathbf{q}_1 + \mathbf{q}_2) \left[\omega_\phi(\mathbf{q}_1) + \omega_\phi(\mathbf{q}_2) + \omega_\phi(\mathbf{q}_1 + \mathbf{q}_2)\right]^3} \right.$$
$$\left. + \mathcal{O}\left(\frac{\Gamma_\phi^2}{\omega_\phi^2}\right) \right\}$$

Examine applicability of approximation

Dissipative kernels $K(t,t')$ must be peaked at $t - t' \approx 0$

$$K(t,t') = \lambda^2 \int \frac{d^3q}{(2\pi)^3} \frac{\left[\omega_\phi \cos(2\omega_\phi |t-t'|) + \Gamma_\phi \sin(2\omega_\phi |t-t'|)\right]}{8\omega_\phi^2 \left(\Gamma_\phi^2 + \omega_\phi^2\right)} e^{-2\Gamma_\phi(q)|t-t'|}$$

ROBUST DISSIPATIVE MECHANISM

(Berera and Ramos, Phys. Rev. D **63**, 103509 (2001))

Focus on Lagrangian:

$$\mathcal{L}[\Phi, \chi_j, \bar{\psi}_k, \psi_k] = \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{m_\phi^2}{2}\Phi^2 - \frac{\lambda}{4!}\Phi^4$$
$$+ \sum_{j=1}^{N_\chi} \left\{ \frac{1}{2}(\partial_\mu \chi_j)^2 - \frac{m_{\chi_j}^2}{2}\chi_j^2 - \frac{f_j}{4!}\chi_j^4 - \frac{g_j^2}{2}\Phi^2\chi_j^2 \right\}$$
$$+ \sum_{i=1}^{N_\psi} \bar{\psi}_i \left[i \not{\partial} - m_{\psi_i} - \sum_{j=1}^{N_\chi} h_{ij,\chi}\chi_j \right] \psi_i \, , \tag{1}$$

Implies reaction: $\phi \to \chi \to \psi$

Regime $m_\chi > 2m_\psi > m_\phi$: copious radiation production
e.g. GUT scale $V^{1/4}(\phi) \sim 10^{15}\text{GeV} \Longrightarrow T \sim 10^{14}\text{GeV}$

PHYSICAL PICTURE

-The evolution of $\varphi(t)$ changes χ-mass $\Longrightarrow$ χ-particle production from mixing $+/-$ frequency modes

(If left intact, quantum correlations of χ-field persist)

-Coupling χ to lighter ψ-field $\Longrightarrow$ χ-particles decay to ψ-particles, decohere χ-field

-Entire process backreacts on $\varphi \Longrightarrow$ dissipative effects

CANONICAL APPROACH

Fields:

$$\chi(x) = \int \frac{d^3k}{(2\pi)^{3/2}} \bar{\chi}_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}} \qquad \pi^{\chi}(x) = \int \frac{d^3k}{(2\pi)^{3/2}} \bar{\pi}^{\chi}_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}}$$

Creation/Annihilation Operators:

$$a_{\mathbf{k}}(t) = (2\omega_{\mathbf{k}}(t))^{-1/2}[\omega_{\mathbf{k}}(t)\bar{\chi}_{\mathbf{k}}(t) + i\bar{\pi}^{\chi}_{-\mathbf{k}}(t)]$$
$$a^{\dagger}_{\mathbf{k}}(t) = (2\omega_{\mathbf{k}}(t))^{-1/2}[\omega_{\mathbf{k}}(t)\bar{\chi}_{-\mathbf{k}}(t) - i\bar{\pi}^{\chi}_{\mathbf{k}}(t)]$$

$$\omega_{\mathbf{k}}(t) \equiv \sqrt{\mathbf{k}^2 + m_{\chi}^2 + g^2\varphi^2(t)} + \mathrm{Im}(\omega_{\mathbf{k}})$$

Bogoliubov Transformation: $[a_{\mathbf{k}}(t), a^{\dagger}_{\mathbf{k}}(t)] \leftrightarrow [a_{\mathbf{k}}(0), a^{\dagger}_{\mathbf{k}}(0)]$

Free χ-Hamiltonian

$$H(t) = \frac{1}{2}\int \frac{d^3k}{(2\pi)^{3/2}} \omega_{\mathbf{k}}(t)[a_{\mathbf{k}}(t)a^{\dagger}_{\mathbf{k}}(t) + a^{\dagger}_{\mathbf{k}}(t)a_{\mathbf{k}}(t)]$$

Observables

$$x_{\mathbf{k}}(t) = \langle a^{\dagger}_{\mathbf{k}}(t)a_{\mathbf{k}}(t)\rangle \qquad y_{\mathbf{k}}(t) = \langle a_{\mathbf{k}}(t)a_{-\mathbf{k}}(t)\rangle$$

$x_{\mathbf{k}}(t)$ - number density of created χ-particles
$y_{\mathbf{k}}(t)$ - correlation amongst produced particles

$$\langle \chi^2(t)\rangle = \int \frac{d^3k}{(2\pi)^3 2\omega_{\mathbf{k}}(t)}[2\mathrm{Re}y_{\mathbf{k}}(t) + 2x_{\mathbf{k}}(t) + 1]$$

Equations of Motion:

$$\dot{x}_{\mathbf{k}}(t) = \frac{\dot{\omega}_{\mathbf{k}}(t)}{\omega_{\mathbf{k}}(t)}\mathrm{Re}(y_{\mathbf{k}}(t))$$
$$\dot{y}_{\mathbf{k}}(t) = \frac{\dot{\omega}_{\mathbf{k}}(t)}{\omega_{\mathbf{k}}(t)}\left(x_{\mathbf{k}}(t) + \frac{1}{2}\right) - 2i\omega_{\mathbf{k}}(t)y_{\mathbf{k}}(t)$$

Energy Conservation: vacuum energy loss rate $\eta\dot{\varphi}$ = energy generation rate into created particles $\int d^3k/(2\pi)^3 \dot{x}_{\mathbf{k}}(t)\omega_{\mathbf{k}}(t)$

RESULTS: Estimates of Dissipation from Quantum Field Theory Dyanmics

Treat inflationary conditions: Overdamped, ultraflat potential

Direct Decay Models $\phi \rightarrow \bar{\psi}\psi, \chi\chi$ $(m_\phi > 2m_{\psi,\chi})$

$T \sim (\rho_r/g^*)^{1/4} \sim \lambda^{1/2} m_\phi/10$

$\rho_v^{1/4} \sim 10^{15-16}$GeV (GUT scale)

$\lambda \sim 10^{-(10-16)}$

$m_\phi \sim 10^{(10-13)}$GeV

$\Rightarrow T \sim 10 - 10^7$GeV

Sets lower bound on radiation production from generic QFT models consistent with inflation

$\Rightarrow$ Reheating redundant

No gravitino overproduction in SUSY versions

Indirect Decay Models $\phi \rightarrow \chi \rightarrow \bar{\psi}\psi$ $(m_\phi < 2m_\psi < m_\chi)$

For same parameter regime, T as large as 10^{14}GeV
- Copious radiation production

Very interesting regime, needs more study