Azimuthal Correlation in DIS

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Plan of talk:

- Introduction — 3-jet event shapes
- Azimuthal correlation in DIS
- NLL resummation
- Inclusion of NP effects
- Comparison with EEC in e^+e^-
- Conclusions

Introduction

3-jet events are those involving three hard jets *including* beam jets:

- in e^+e^- the process

$$e^+e^- \to 3 \text{ jets}$$

  is obviously called a 3-jet event.

- in DIS the process

$$\gamma^* P \to \text{remnant} + 2 \text{ jets}$$

  is a 3-jet event. These are also called 'dijet' because they are (2+1).

- in PP collisions,

$$PP \to \gamma^* + 2 \text{ remnants} + \text{ jet}$$

  is similarly a 3-jet event in this terminology, although there is only one hard transverse jet.

All these have in common that soft radiation is emitted from an arrangement of three hard partons. The soft matrix elements are known and factorize.

3-jet shapes are those whose leading contribution comes at order α_s^2 in perturbation theory. They are most easily studied in 3-jet events.

- **Out-of-plane momentum:** 3-jet events are (almost) planar. With a suitable definition of the event plane we can study

$$K_{\text{out}} = \sum_h |p_h^{\text{out}}| \, .$$

  - In e^+e^-: A. Banfi, Yu.L. Dokshitzer, G. Marchesini and G. Zanderighi, JHEP 07 (2000) 002, Phys. Lett. B508 (2001) 269 and JHEP 03 (2001) 007;
  - In DIS: A. Banfi, G. Marchesini, G. Smye and G. Zanderighi, hep-ph/0111157;
  - In PP: A. Banfi, G. Marchesini, G. Smye and G. Zanderighi, JHEP 08 (2001) 047.

- D-parameter in e^+e^-:

  - A. Banfi, Yu.L. Dokshitzer, G. Marchesini and G. Zanderighi, JHEP 05 (2001) 040.

- Azimuthal angular correlations: in processes with a natural axis (Breit axis in DIS, beam axis in PP) we can study observables of the generic type

$$H(\chi) = \sin\chi \sum_{ab} \frac{p_a p_b}{Q^2} \, \delta(\cos\chi + \cos\phi_{ab}) \, .$$

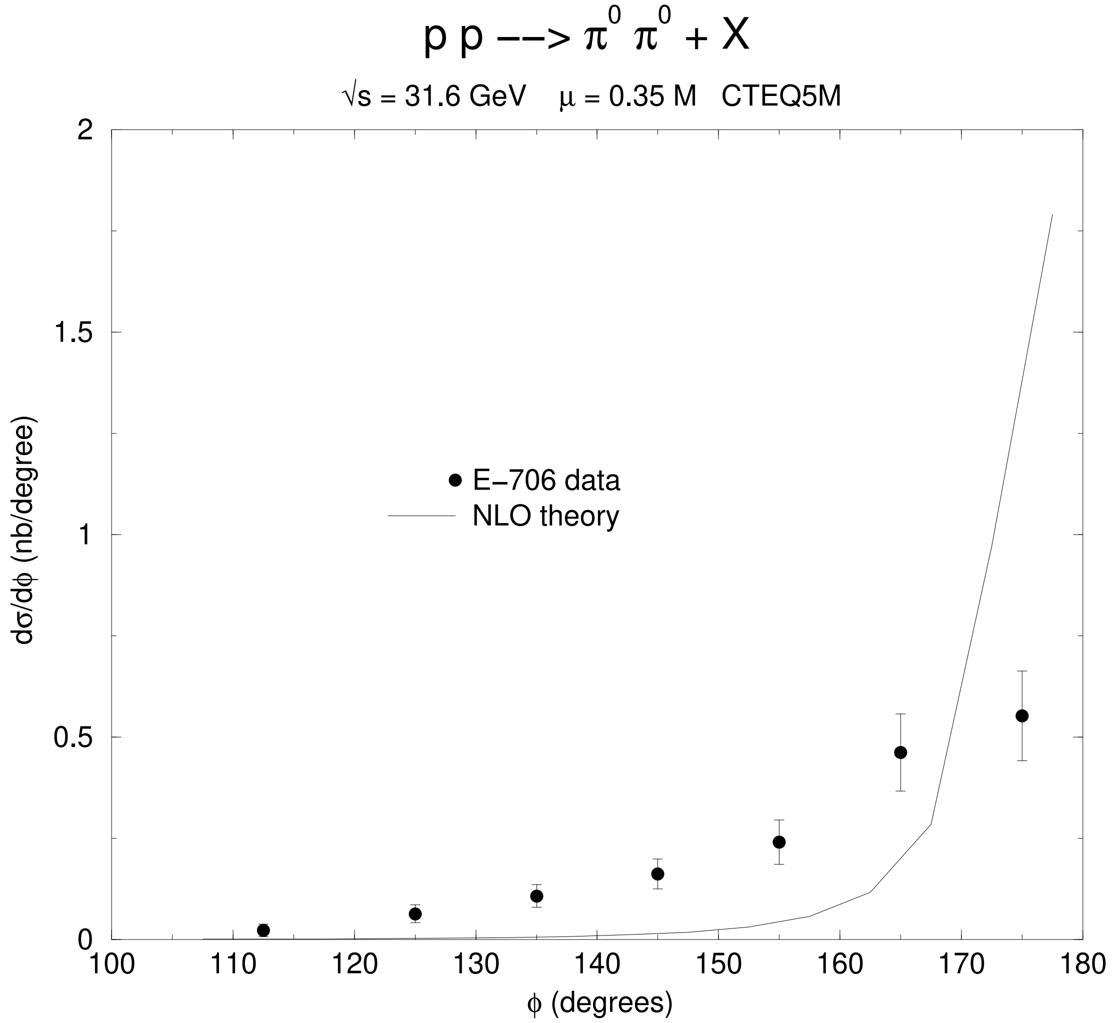
  - In DIS a, b run over all outgoing hadrons: the subject of this talk;
  - In PP a runs over outgoing hadrons, b the vector boson;
  - or in PP a, b running over hadrons is being studied, but is a 4-jet event shape and is more complicated than those considered here.

Experimental results:

NONE YET for azimuthal correlation in DIS

As an example of what type of behaviour we might expect from such an observable, here is the angular distribution of π^0 pairs produced in hadronic collisions:

J.F. Owens, hep-ph/01110036



Azimuthal correlation in DIS

A. Banfi, G. Marchesini and GS, work in progress

Define (inspired by the EEC in e^+e^-)

$$H(\chi) = \sin\chi \sum_{hh'} \frac{p_{th}p_{th'}}{Q^2}\,\delta(\cos\chi + \cos\phi_{hh'})\,, \quad 0 \le \chi \le \pi\,,$$

where the sum runs over all outgoing hadrons, p_{th} is the transverse momentum of hadron h and $\phi_{hh'} = \phi_h - \phi_{h'}$ the azimuthal angle between hadrons h and h' in the Breit frame.

The weighted azimuthal differential cross section at fixed x_{B}, Q^2 and y_c is then

$$\frac{d\sigma(\chi, y_c)}{d\chi\, dx_{\mathrm{B}} dQ^2} = \sum_n \int \frac{d\sigma_n}{dx_{\mathrm{B}} dQ^2}\,\Theta(y_c - y_3)\cdot H(\chi)\,,$$

y_c being upper limit on 3-jet resolution to ensure $E_t \lesssim Q$.

One introduces the normalized correlation

$$\begin{aligned} \frac{d\Sigma(\chi, y_c)}{d\chi} &= \sigma^{-1}(y_c)\frac{d\sigma(\chi, y_c)}{d\chi\, dx_{\mathrm{B}} dQ^2}\,, \\ \sigma(y_c) &= \sum_n \int \frac{d\sigma_n}{dx_{\mathrm{B}} dQ^2}\,\Theta(y_c - y_3)\left(\sum_{h=1}^{n}\frac{p_{th}}{Q}\right)^2\,, \end{aligned}$$

normalized to 1 by taking all χ values up to π.

Near $\chi = 0$ there are large logs that need to be resummed:

$$\Sigma(\chi) = 1 - C_T \frac{\alpha_s}{4\pi} \ln^2 \frac{\chi_0^2}{\chi^2} + \ldots \qquad C_T = 2C_F + C_A \,,$$

(also near $\chi = \pi$ but we have not studied there).

Basic partonic structure:

$$q\, p_1 \to p_2\, p_3\, k_1 \cdots k_n \,, \qquad p_1 = x_1 P \,.$$

We consider the secondary partons to be soft. The outgoing hard partons p_2 and p_3 are almost coplanar, with soft recoil.

The *event plane* is the plane formed by the incoming proton direction $\vec{n}_P$ and the unit vector $\vec{n}_M$ which enters the definition of thrust major

$$T_M = \max_{\vec{n}_M} \frac{1}{Q} \sum_h |\vec{p}_h \cdot \vec{n}_M| \,, \qquad \vec{n}_M \cdot \vec{n}_P = 0 \,.$$

We take $\vec{n}_P$ in the z-direction, and $\vec{n}_M$ in the y-direction.

Then the hard scale for the process is $P_t = \frac{1}{2} T_M Q$ and

$$\begin{aligned} H(\chi) \quad &\simeq \quad \frac{2P_t^2}{Q^2} \left(1 - \sum_i \frac{|k_{yi}|}{P_t} \right) \delta(\chi - \chi_{23}) \\ &\quad + \frac{2P_t^2}{Q^2} \sum_i \frac{k_{ti}}{P_t} [\delta(\chi - \chi_{2i}) + \delta(\chi - \chi_{3i})] \,, \end{aligned}$$

Splitting the expression of $H(\chi)$ into a 'hard' contribution independent of k_i, and a 'soft' contribution proportional to k_i, we can write, to first order in softness,

$$H(\chi) \simeq H_h(\chi) \, + \, H_s(\chi) \, ,$$

where the hard piece is

$$H_h(\chi) = \frac{2P_t^2}{Q^2} \, \delta\left(\chi - \frac{|K_x|}{P_t}\right) \, , \qquad K_x = \sum_i k_{xi}$$

and the soft piece is

$$\begin{aligned} H_s(\chi) \quad &= \quad \frac{2P_t^2}{Q^2} \sum_i \frac{k_{ti}}{P_t} \left[\delta\left(\chi - \left|\bar{\phi}_i - \frac{K_x}{P_t}\right|\right) \right. \\ &\qquad \left. - |\cos\bar{\phi}_i| \delta\left(\chi - \frac{|K_x|}{P_t}\right) \right] . \end{aligned}$$

Here $\bar{\phi}_i$ is the azimuthal angle between k_i and either the hard parton p_2 or p_3:

$$\bar{\phi}_i = \begin{cases} \phi_{i2} \text{ for } k_i \text{ near } p_2, \\ \phi_{3i} \text{ for } k_i \text{ near } p_3. \end{cases}$$

Note the cancellation of the collinear singluarity in $H_s(\chi)$ as $\bar{\phi}_i \to 0$.

NLL resummation

To perform the resummation of double and single logarithms, we need to factorize:

- Matrix elements — well known;
- Observable $H_h(\chi) + H_s(\chi)$ — Fourier transform into 1-dimensional impact parameter space:

$$H_h(\chi) = \frac{2P_t^2}{Q^2} \int_{-\infty}^{\infty} \frac{P_t db}{\pi} \cos(P_t b \chi) \prod_i e^{ibk_{xi}} \,,$$

and similarly for $H_s(\chi)$;

- Phase space $\delta(\frac{x_{\rm B}}{x} - x_1 \prod_{i \in C_1} z_i)$ — Mellin transform as standard.

The resummation follows an established pattern. The result is

$$\frac{d\sigma(\chi, y_c)}{d\chi\, dx_{\rm B}\, dQ^2} = \sum_\rho \int_{x_{\rm B}}^1 \frac{dx}{x} \int_0^{\xi_c} d\xi \left(\frac{d\hat{\sigma}_\rho}{dx\, d\xi\, dQ^2} \right) C_\rho(\alpha_s) \mathcal{I}_\rho(\chi)$$

with the distribution $\mathcal{I}$ given by

$$\mathcal{I}_\rho(\chi) = \frac{4P_t^2}{Q^2} \int_0^{\infty} \frac{P_t db}{\pi} \cos(P_t b \chi) \cdot \mathcal{P}_\rho \left(\frac{x_{\rm B}}{x}, \bar{b}^{-1} \right) \cdot \mathcal{A}_\rho(b) \,.$$

Emissions with $|k_x| < \bar{b}^{-1}$ are included in the parton density function $\mathcal{P}$; those with $|k_x| > \bar{b}^{-1}$ are in the radiation factor $\mathcal{A}$. This gives the factorisation $\mathcal{P} \cdot \mathcal{A}$.

The radiation factor is given by the sum of the hard and the soft contributions $\mathcal{A} = \mathcal{A}^h + \mathcal{A}^s$:

$$\mathcal{A}^h_\delta(b) = e^{-\mathcal{R}_\delta(\bar{b})}\,, \quad \mathcal{A}^s_\delta(b) = e^{-\mathcal{R}_\delta(\bar{b})} \cdot B_\delta(b)\,.$$

with $\bar{b} = e^{\gamma_E}|b|$ and the radiator $\mathcal{R}_\delta$ given by

$$\mathcal{R}_\delta(\bar{b}) = \sum_{a=1}^{3} C_a^{(\delta)} \int_{1/\bar{b}}^{Q} \frac{dk}{k} \frac{2\alpha_s(2k)}{\pi} \ln \frac{Q_a^{(\delta)}}{2k}\,,$$

where the various hard scales $Q_a^{(\delta)}$ relate to the standard 3 dipole antennae, and the colour factors are

$$C_a^{(\delta)} = C_F\,, \ \ (a \neq \delta)\,, \qquad C_\delta^{(\delta)} = C_A\,.$$

The soft contribution does not contribute to this resummation since the factor $B_\delta(b)$ is order α_s without any logarithmic enhancement.

The inversion of the transform is an old problem — the 'single log' expression for $\mathcal{I}$ reads

$$\mathcal{I}_\rho(\chi) = \frac{2P_t^2}{Q^2}\frac{d}{d\chi} \left[e^{-\mathcal{R}(\frac{1}{\chi P_t})} \cdot \frac{e^{-\gamma_E \mathcal{R}'(\frac{1}{\chi P_t})} \sec \frac{\pi}{2}\mathcal{R}'(\frac{1}{\chi P_t})}{\Gamma(1+\mathcal{R}'(\frac{1}{\chi P_t}))} \cdot \mathcal{P}_\rho\left(\frac{x_{\mathrm{B}}}{x}, \chi P_t\right) \right]$$

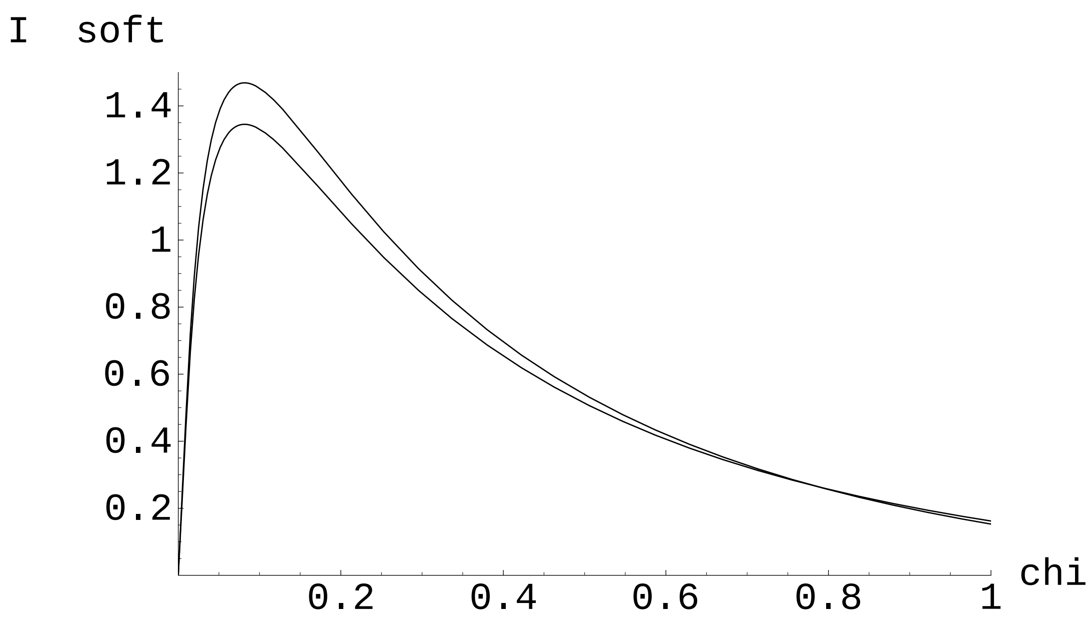
with divergences at $\mathcal{R}' = 1, 3, 5....$ This is because the integral is *not* dominated by contributions around $b = \frac{1}{\chi P_t}$: contributions over the whole b-range are important. We have no choice but to evaluate the b-integral numerically.

To understand the physics, consider the toy model

$$\mathcal{I}^{\text{toy}}(\chi) = \int_0^\infty \frac{2P_t db}{\pi} \cos(P_t b \chi) \cdot e^{-0.215 \ln^2(\bar{b} P_t)}$$

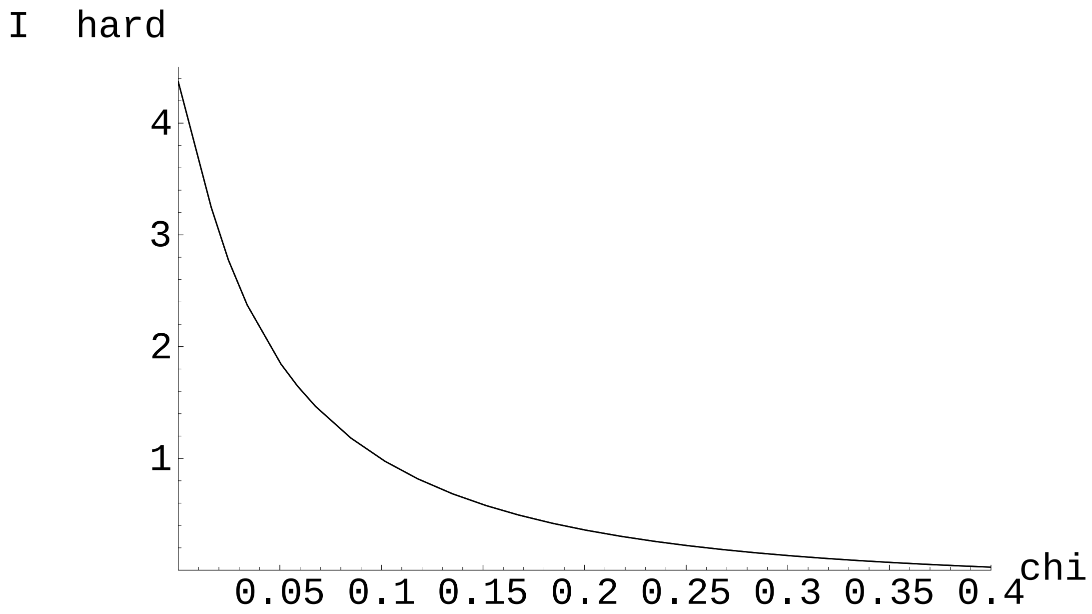
and split the integration region into two pieces:

- $\bar{b} > \frac{1}{\chi P_t}$: the contribution from multiple emissions with $|k_x| < \chi P_t$, whose total is χP_t. We see the Sudakov suppression, and the standard SL formalism is valid:



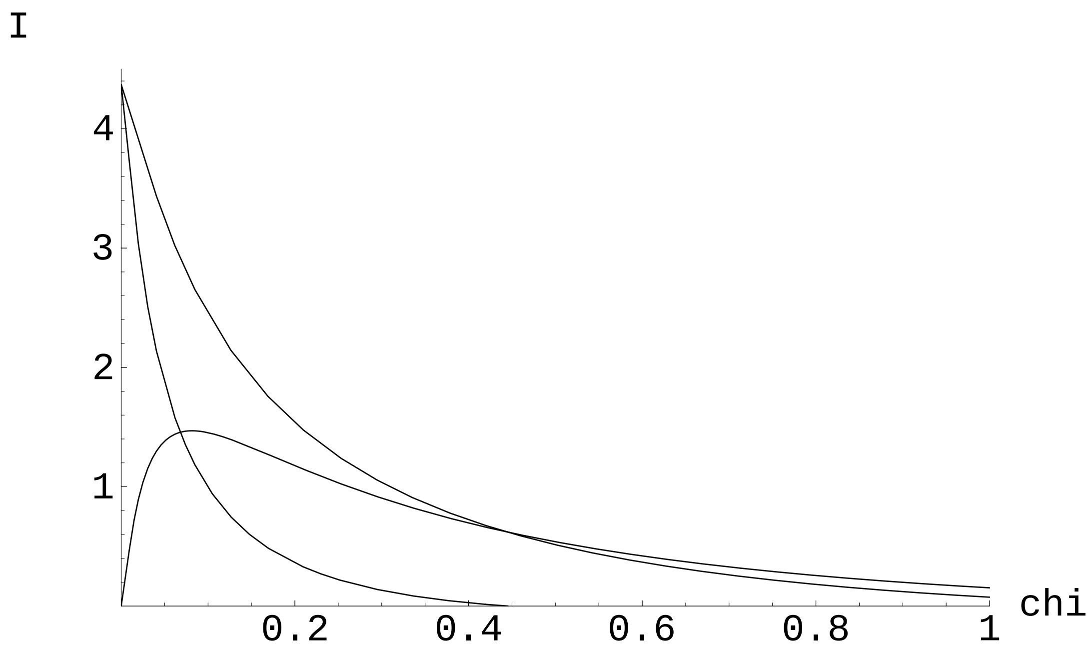
The peak is clear at $\mathcal{R}' \approx 1$, i.e. $\chi = 0.1$.

- $\bar{b} < \frac{1}{\chi P_t}$: the contribution from some hard emissions with $|k_x| > \chi P_t$, but which combine with opposite signs to give a total of χP_t:



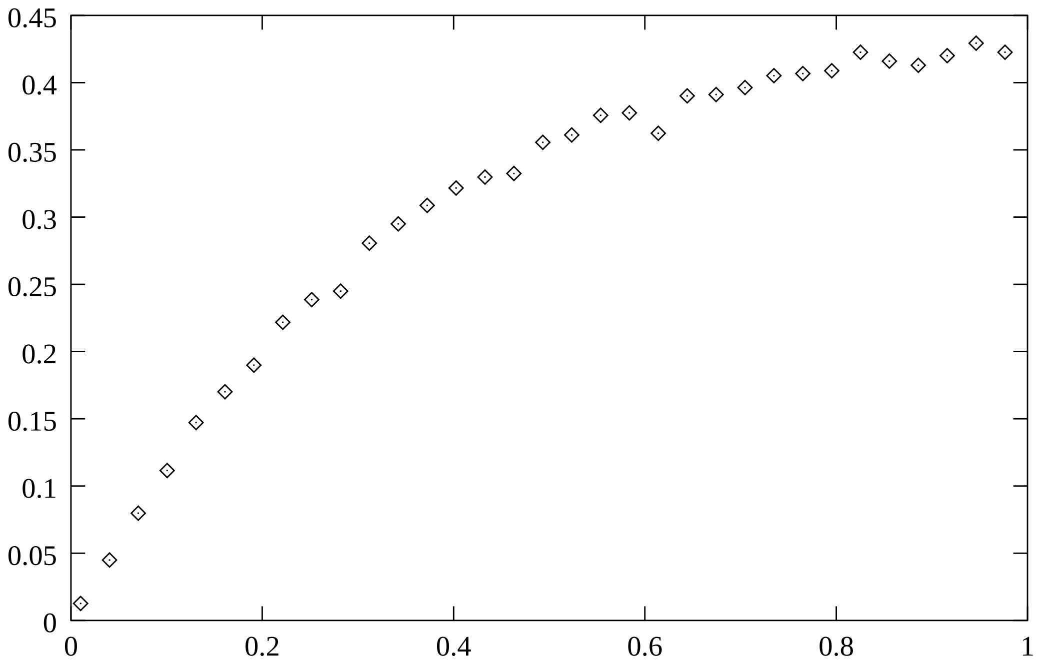
This is a *hard emission contribution*, that dominates the distribution at small χ. It is not correctly treated by the soft approximation, so matching with NLO is required.

- Total: plot showing the two above contributions and their total...



Enough of the toy model... back to reality.

A very preliminary plot.....



This is the integrated distribution $\Sigma(\chi, 2.5)$, with $Q = 30$ GeV, $x_{\mathrm{B}} = 0.1$ and $s = 98400$ GeV^2.

The differential distribution $d\Sigma/d\chi$ is the derivative of this, which rises to a constant at $\chi = 0$.

Better plots will appear shortly.

Including non-perturbative effects

Non-perturbative 'power corrections' to the above perturbative result come from several sources:

- the parton density functions $\mathcal{P}_\rho\left(\frac{x_{\rm B}}{x}, \bar{b}^{-1}\right)$ — leading contribution is $\mathcal{O}(b^2)$;
- the radiator $\mathcal{R}_\delta(\bar{b})$ — again leading contribution is $\mathcal{O}(b^2)$;
- the soft contribution $B_\delta(b)$ — leading contribution is $\mathcal{O}(b)$. This therefore dominates at small b (as in EEC).

The *leading* power correction is calculated as

$$\delta B_\delta(b) = -\left(C_2^{(\delta)} + C_3^{(\delta)}\right) b \cdot \lambda^{\rm NP}\,.$$

This simple form is because the contribution comes from emissions *collinear* to the two outgoing hard partons. The NP parameter $\lambda^{\rm NP}$ is

$$\lambda^{\rm NP} \equiv \mathcal{M}\frac{4}{\pi^2}\mu_I \left\{\alpha_0(\mu_I) - \bar{\alpha}_s - \beta_0\frac{\bar{\alpha}_s^2}{2\pi}\left(\ln\frac{Q}{\mu_I} + \frac{K}{\beta_0} + 1\right)\right\}$$

with $\bar{\alpha}_s = \alpha_s(Q)$, μ_I the IR matching scale and

$$\alpha_0(\mu_I) = \int_0^{\mu_I}\frac{dk}{\mu_I}\alpha_s(k)\,.$$

The final prediction including the leading NP correction is

$$\frac{d\Sigma(\chi, y_c)}{d\chi} = \frac{d\Sigma^{\mathrm{PT}}(\chi, y_c)}{d\chi} \left\{1 - \lambda^{\mathrm{NP}} \langle (C_2 + C_3) \cdot b \rangle \right\},$$

with b (and the outgoing hard parton charges) averaged over the PT distribution.

One can give an estimation of $\langle b \rangle$ by using the one-loop coupling approximation. At $\chi = 0$ we obtain

$$\langle b \rangle_{\chi=0} \sim \frac{\mathrm{const}}{\Lambda_{\mathrm{QCD}}} \left(\frac{\Lambda_{\mathrm{QCD}}}{Q} \right)^{\gamma}, \quad \gamma = \frac{4C_T}{\beta_0} \ln \frac{2 + \frac{4C_T}{\beta_0}}{1 + \frac{4C_T}{\beta_0}} = 0.63\,.$$

Taking into account the two-loop coupling the exponent γ is not a constant but smoothly dependent on Q.

The detailed numerical analysis will shortly be forthcoming.

Comparison with EEC in e^+e^-

Yu. Dokshitzer, G. Marchesini and B. Webber, JHEP 07 (1999) 12

The azimuthal correlation has some interesting similarities with the energy-energy correlation in e^+e^- annihilation.

- Observable definition:

$$\frac{d\Sigma(\chi)}{d\chi} \equiv \frac{d\sigma}{\sigma\, d\chi} = \sum_n \int \frac{d\sigma_n}{\sigma} H(\chi)\,,$$

$$H(\chi) = \sin\chi \sum_{a,b} \frac{E_a\, E_b}{Q^2}\, \delta(\cos\chi + \cos\theta_{ab})\,.$$

  Here $\chi = \pi - \theta_{ab}$ with θ_{ab} the *polar angle* between particles a and b.

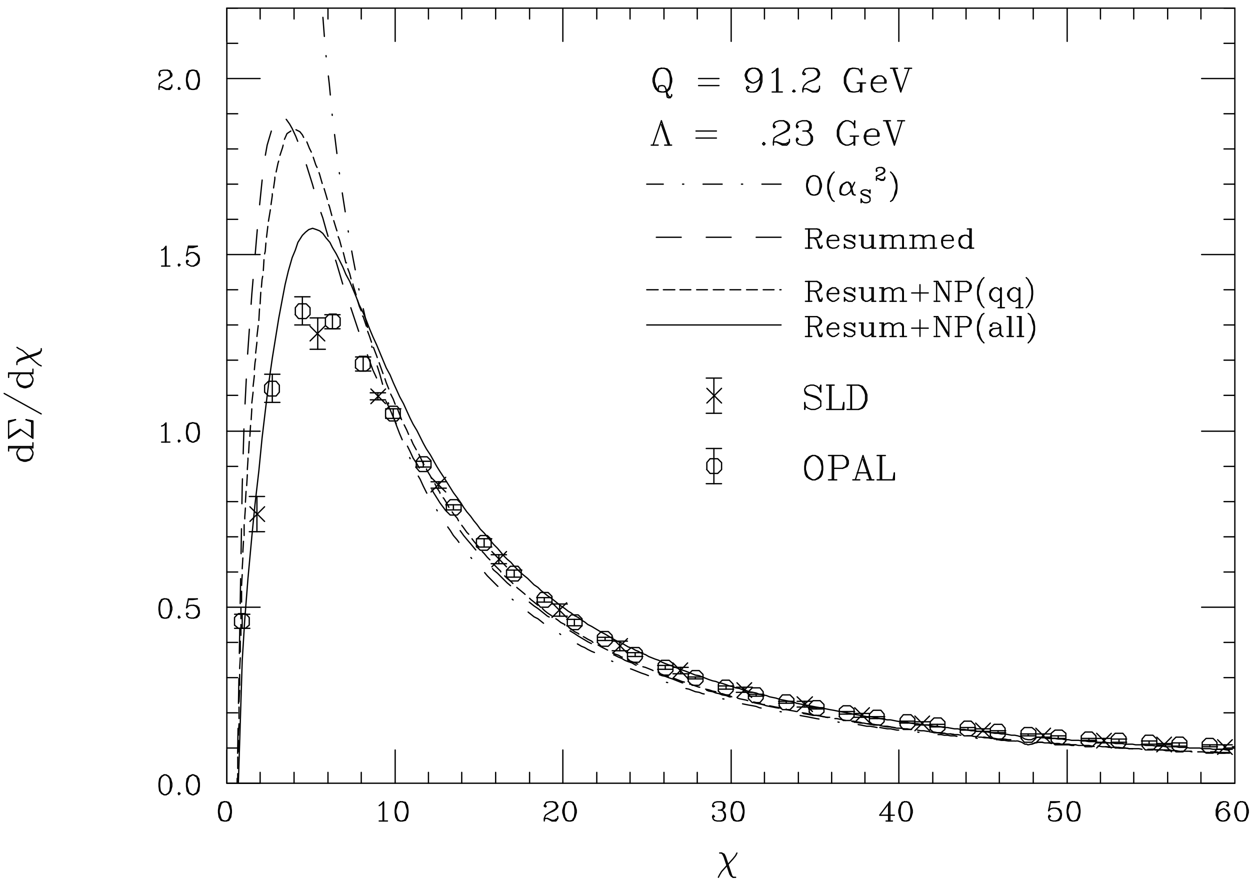
- Resummed PT and NP result: in 2-dimensional impact parameter space

$$\mathcal{I}(\chi) \simeq \frac{Q^2\chi}{8} \int_0^\infty b\, db\, J_0(\tfrac{1}{2}bQ\chi) e^{-\mathcal{R}(b)} \left(1 - 2C_F b\lambda^{\mathrm{NP}}\right)\,.$$

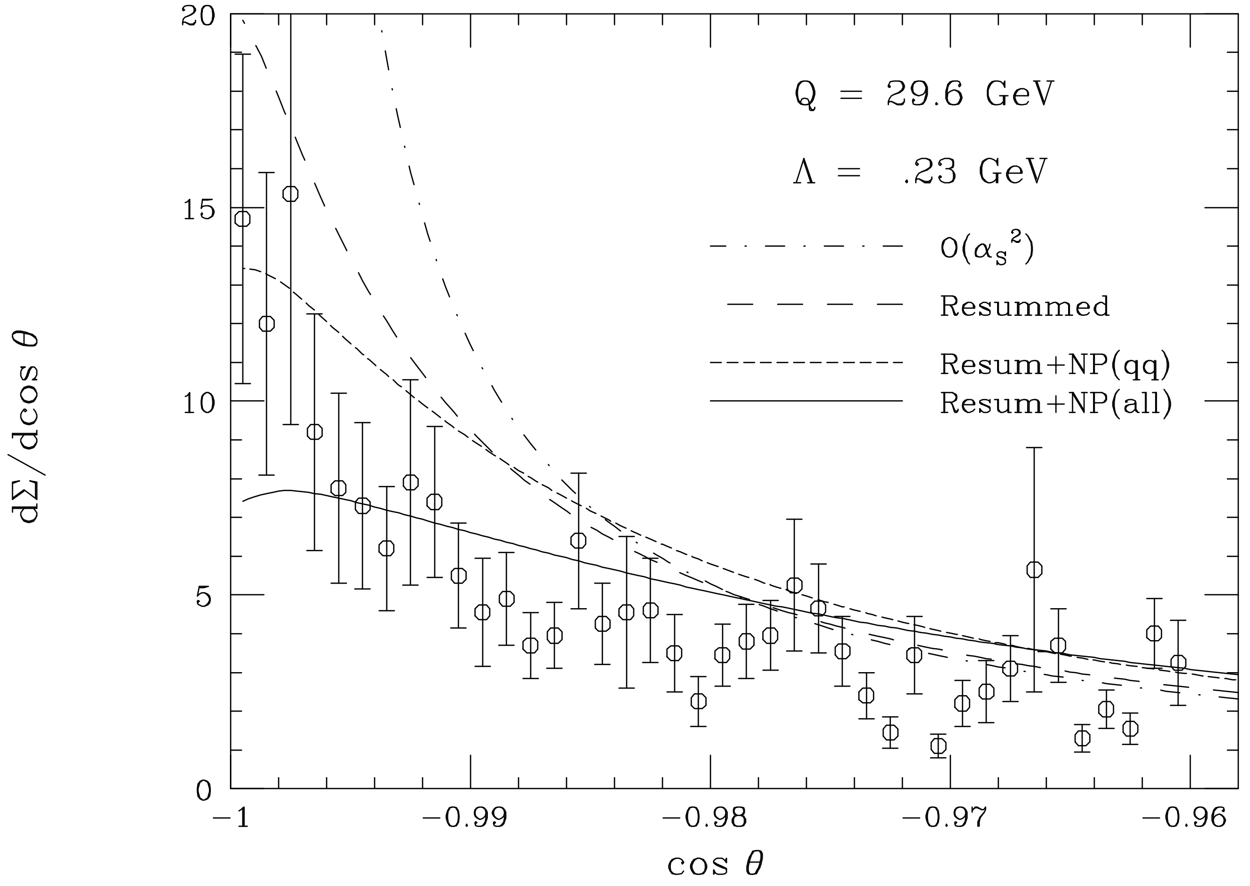
  This gives a distribution $d\Sigma/d\chi$ with a peak at $\mathcal{R}' = 1$ (the SL approximation does not diverge until $\mathcal{R}' = 2$). For small χ it behaves as χ: the distribution $d\Sigma/d\cos\chi$ has a plateau.

- NP contribution to plateau height:

$$\langle b\rangle_{\chi=0} \sim \frac{\mathrm{const}}{\Lambda_{\mathrm{QCD}}} \left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)^{\gamma}, \quad \gamma = \frac{8C_F}{\beta_0} \ln \frac{3 + \frac{8C_F}{\beta_0}}{2 + \frac{8C_F}{\beta_0}} = 0.32\,.$$



Q = 91.2 GeV
Λ = .23 GeV
O(αs²)
Resummed
Resum+NP(qq)
Resum+NP(all)
SLD
OPAL
dΣ/dχ
χ



Q = 29.6 GeV
Λ = .23 GeV
O(αs²)
Resummed
Resum+NP(qq)
Resum+NP(all)
dΣ/dcos θ
cos θ

Conclusions

- The azimuthal correlation in DIS is an interesting new observable that could be studied experimentally.
- It is one of a class of 3-jet observables, some of which have recently been analyzed.
- It also has similarities with the energy-energy correlation in e^+e^- annihilation.
- It is a good observable for the study of soft radiation effects.
- It is a good observable for the study of NP 'power corrections'.
- Our paper (A. Banfi, G. Marchesini, GS) will be available soon.
- We encourage experimental collaborations to publish data for this observable.