HEFIN

Hoge-Energiefysica Instituut Nijmegen

FLUCTUATIONS
and
CORRELATIONS

(INTRODUCTION
and
STATE OF THE ART)

W. KITTEL
(Nijmegen)

I. FORMALISM

INCLUSIVE q-PARTICLE DISTRIBUTION FUNCTIONS:

$$\rho_q(p_1, \ldots, p_q) \equiv \frac{1}{\sigma_{\rm tot}} \frac{{\rm d}\sigma(p_1, \ldots, p_q)}{{\rm d}p_1 \ldots {\rm d}p_q}$$

Integration over $\Omega \longrightarrow$ factorial moments:

$$\int_\Omega \rho_1(p){\rm d}p = \langle n \rangle$$

$$\int_\Omega \int_\Omega \rho_2(p_1, p_2){\rm d}p_1{\rm d}p_2 = \langle n(n-1) \rangle$$

$$\int_\Omega {\rm d}p_1 \ldots \int_\Omega {\rm d}p_q \rho_q(p_1, \ldots, p_q) = \langle n(n-1) \ldots (n-q+1) \rangle$$

(the angular brackets imply the average over the event ensemble)

CUMULANT CORRELATION FUNCTIONS:

$\rho_n\ (p_1, \ldots, p_n)$ contain:

1. interparticle correlations
2. "trivial" contributions from lower order

$\Rightarrow$ cluster expansion (Mueller 1971):

$$
\begin{aligned}
\rho_1(1) &= C_1(1) \\
\rho_2(1,2) &= C_1(1)C_1(2) + C_2(1,2) \\
\rho_3(1,2,3) &= C_1(1)C_1(2)C_1(3) + \\
&\quad + C_1(1)C_2(2,3) + C_1(2)C_2(1,3) + C_1(3)C_2(1,2) + \\
&\quad + C_3(1,2,3)
\end{aligned}
$$

etc.

$C_q(p_1, \ldots, p_q) =$ cumulant correlation functions
vanish whenever one of their arguments becomes statistically independent of the others.

def.: "genuine" correlations of order q if $C_q \neq 0$

from inversion:

$$C_2(1,2) = \rho_2(1,2) - \rho_1(1)\rho_1(2)$$

$$C_3(1,2,3) = \rho_3(1,2,3) - \sum_{(3)} \rho_1(1)\rho_2(2,3) + 2\rho_1(1)\rho_1(2)\rho_1(3)$$

etc.

note: N independent overlapping samples
$\Rightarrow$ C_q are additive

def.: normalized inclusive number densities and correlations

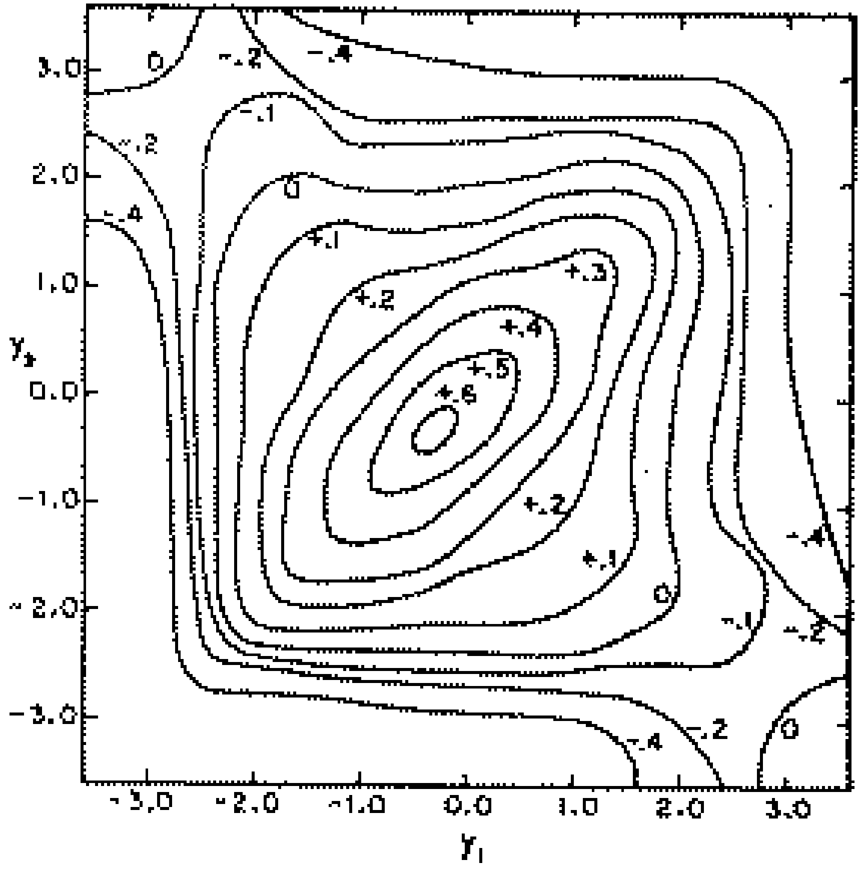
$$R_q(p_1,\ldots,p_y) = \rho_q(p_1,\ldots,p_q)/\rho_1(p_1)\ldots\rho_1(p_q)$$

$$K_q(p_1,\ldots,p_q) = C_q(p_1,\ldots,p_q)/\rho_1(p_1)\ldots\rho_1(p_q)$$

note: $K_q(N) = \frac{1}{N^{(q-1)}} K_q(1)$

in particular: $R_2 = 1 + K_2$
$R_3 = 1 + 3K_2 + K_3$

$q = 2$: extensively studied experimentally

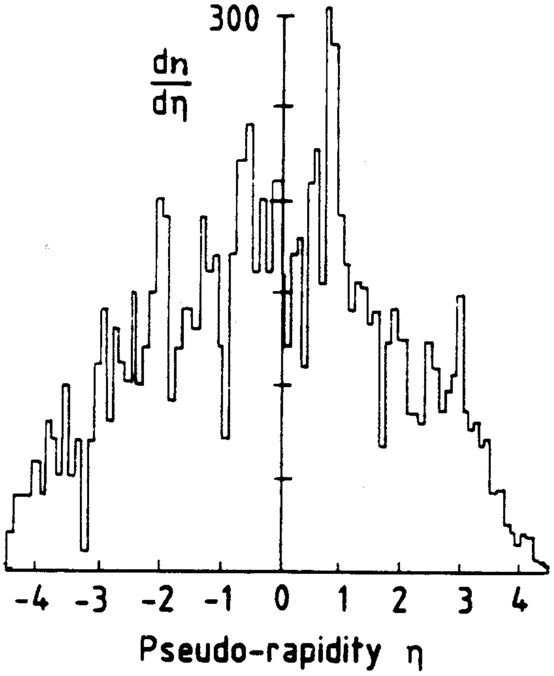


$K_2(y_1, y_2)$

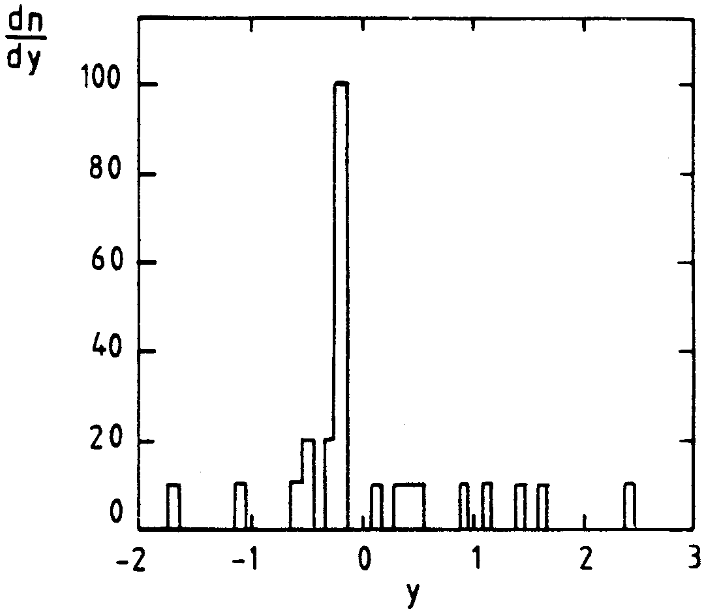
$\Rightarrow$ large for $y_1 \approx y_2$

IS IT WORTH THE EFFORT?
(do higher order correlations exist?)

a) JACEE event



b) NA22 event



(resolution in both cases 0.1 units)

$\mathrm{d}n/\mathrm{d}\eta \approx 300$	$\mathrm{d}n/\mathrm{d}y \approx 100$
$S : B \approx 1 : 1$	60 times average!

YES: spikes exist in y distribution

QUESTION: are they statistical (Poisson) or dynamical?

CELL-AVERAGED FACTORIAL MOMENTS AND CUMULANTS

1. Divide phase space volume into M non-overlapping cells Ω_m (e.g. rapidity intervals of size $\delta y = \Delta y/M$)
2. Integrate over phase space cells Ω_m and average

def. : normalized cell-averaged factorial moments:
(Białas + Peschanski, 1986)

$$F_q(\delta p) \equiv \frac{1}{M}\sum_{m=1}^{M} \frac{\int_{\Omega_m} \rho_q(p_1,\ldots,p_q)\prod_{i=1}^{q}\mathrm{d}p_i}{\left(\int_{\Omega_m}\rho(p)\mathrm{d}p\right)^q}$$

$$\equiv \frac{1}{M}\sum_{m=1}^{M} \frac{\langle n_m(n_m-1)\ldots(n_m-q+1)\rangle}{\langle n_m\rangle^q}$$

$n_m =$ number of particles in cell Ω_m

essential properties:

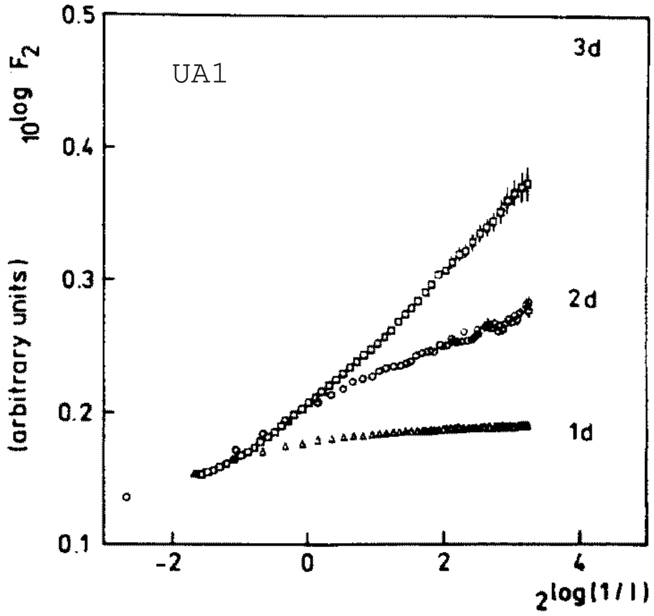
1. Poisson-noise suppression
(not applicable to ordinary moments $\frac{\langle n^q\rangle}{\langle n\rangle^q}$)

2. $F_q = 1$ for Poisson

3. High-order moments act as filter

   $\Rightarrow$ resolve the high-n_m tail of the multiplicity distribution (particularly sensitive to large density fluctuations)

POWER-LAW SCALING (Intermittency)



$$F_q(\delta p) \propto (\delta p)^{-\phi_q}, \ (\delta p \to 0)$$

• scaling law since the ratio of the factorial moments at resolutions L and ℓ

$$F_q(\ell)/F_q(L) = (L/\ell)^{\phi_q}$$

only depends on the ratio L/ℓ, but not on L and ℓ, themselves

• The powers ϕ_q (slopes in a double-log plot) are related to the anomalous dimensions

$$\phi_q = (q-1)d_q, \qquad d_q = D - D_q$$

$$d_q > 0 \quad \Rightarrow \quad \text{fractal structure}$$

Expected from branching or phase transition def.: normalized factorial cumulant moments:

$$K_q(\delta p) \equiv \frac{1}{M}\sum_{m=1}^{M}\frac{\int_{\Omega_m} C_q(p_1,\ldots,p_q)\prod_{i=1}^{q}\mathrm{d}p_i}{\left(\int_{\Omega_m}\rho(p)\mathrm{d}p\right)^q}$$

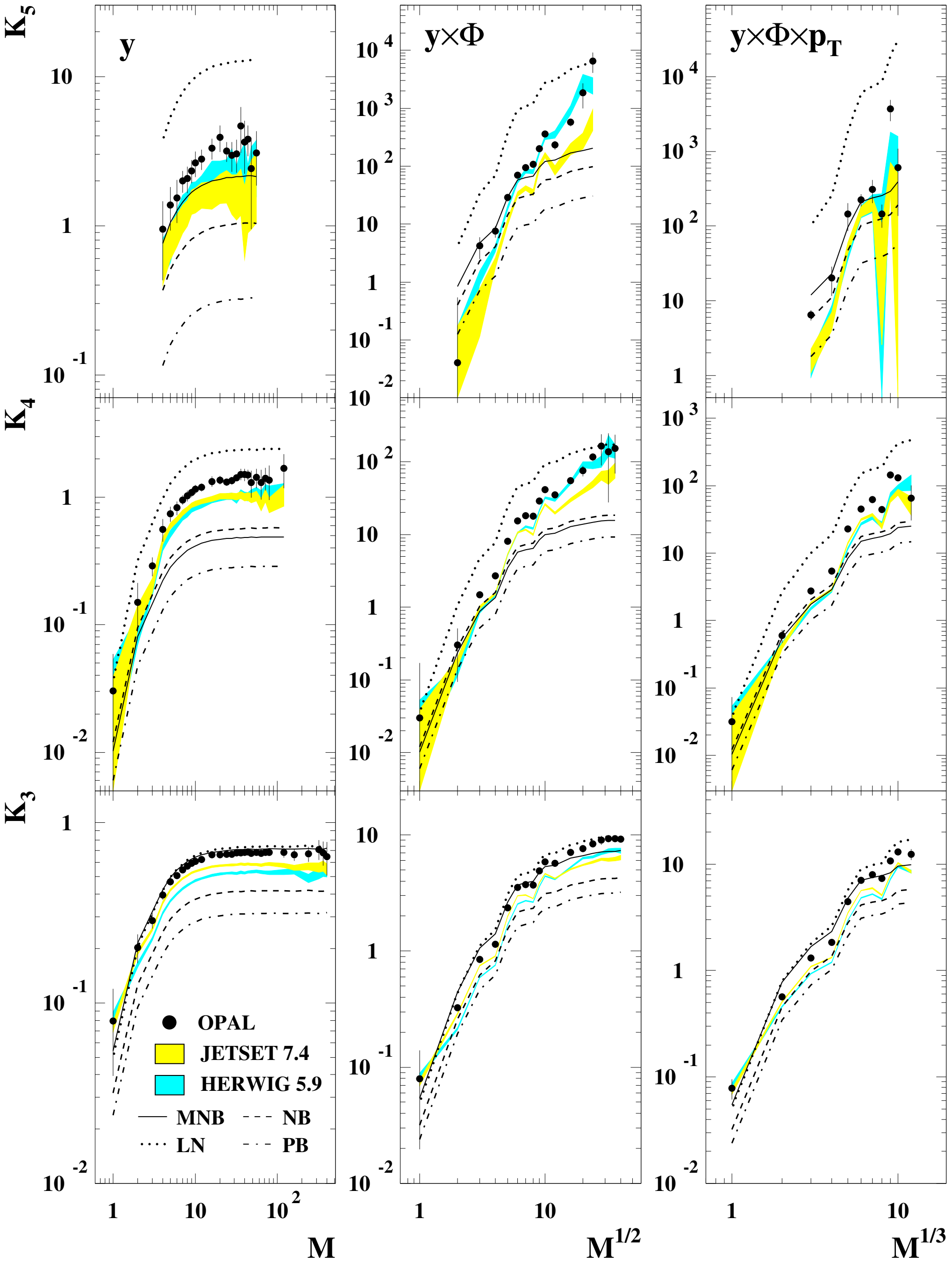
show the genuine correlations

related to the factorial moments
(Carruthers, Eggers, Sarcevic, 1991)

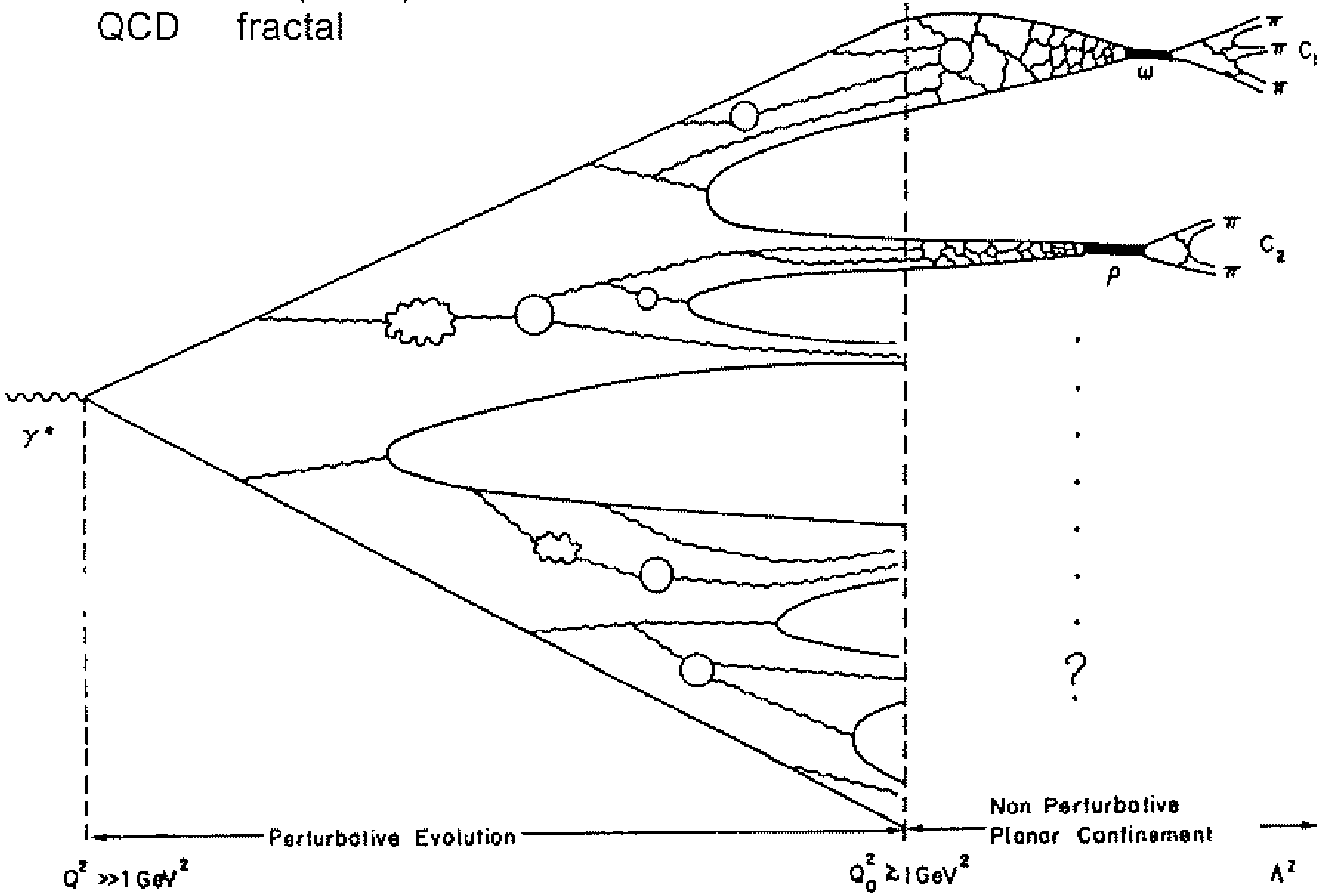
$$\begin{aligned} F_2 &= 1 + K_2 \\ F_3 &= 1 + 3K_2 + K_3 \\ F_4 &= 1 + 6K_2 + 3\overline{K_2^2} + 4K_3 + K_4 \end{aligned}$$

$$(\overline{AB} \equiv \sum_m A_m B_m / M)$$

Sarkisyan (2001), OPAL:



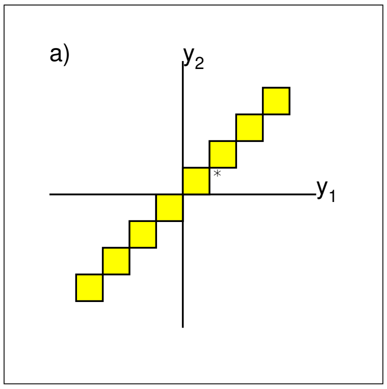
Veneziano (1979!)
QCD fractal

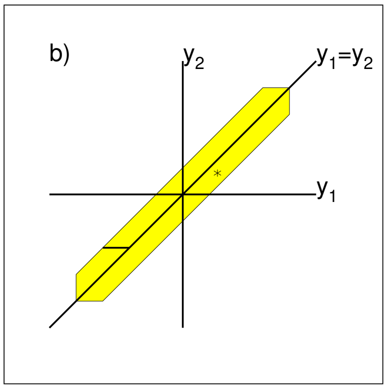


IMPORTANT IMPROVEMENT: CORRELATION (STRIP) INTEGRALS

Hentschel + Procaccia, 1983
Lipa, Carruthers, Eggers, Buschbeck, 1992

$$q = 2$$





$\Omega = \sum_{m=1}^{M} \Omega_m$ $\qquad\qquad$ $\Omega = \Omega_S$

$\Rightarrow$ area doubled and splitting of spikes reduced

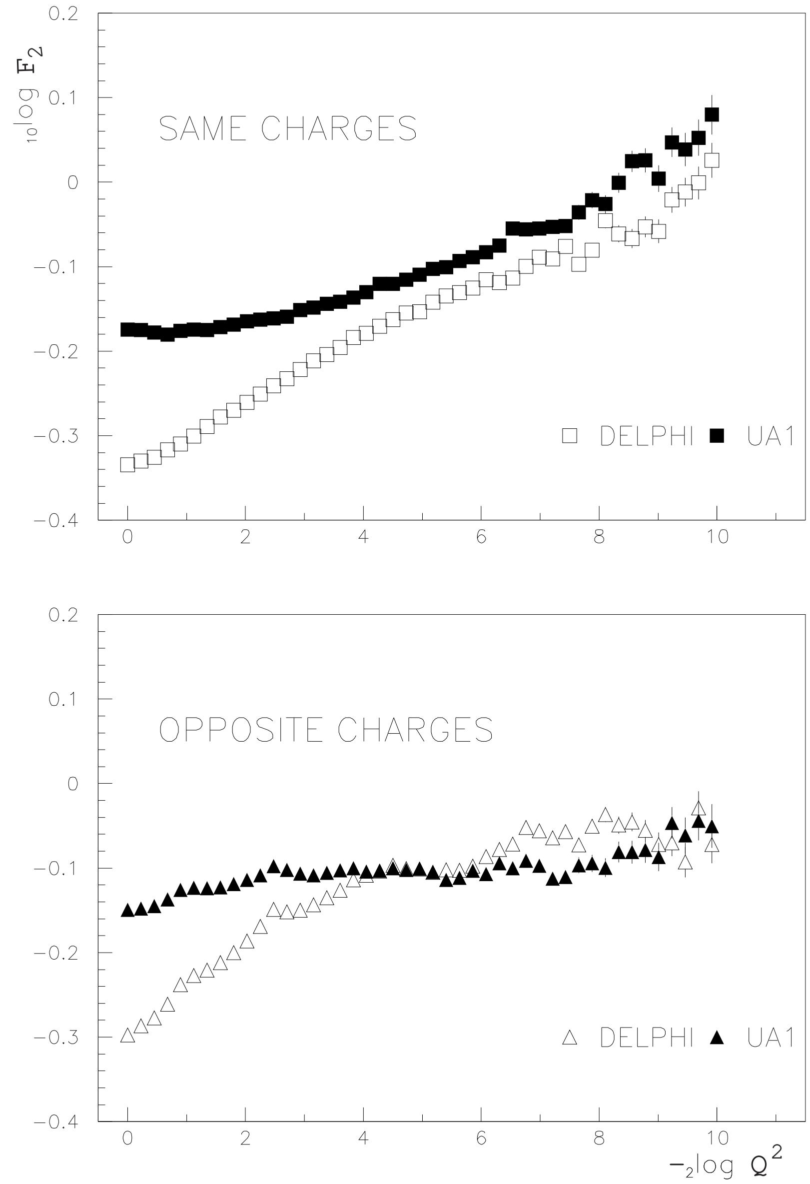
$$F_q^S(\delta y) \equiv \frac{\int_{\Omega_s} \rho_q(y_1 \ldots y_q)\Pi_i dy_i}{\int_{\Omega_s} \rho_1(y_1)\ldots\rho_1(y_q)\Pi_i dy_i}$$

distance measures: $|y_i - y_j|$

$$Q_{ij}^2 = -(p_i - p_j)^2$$

$$F_q^S(Q^2) = \frac{1}{\text{norm}} \left\langle q! \sum_{i_1<\ldots<i_q} \prod_{\substack{all\ \ pairs \\ k_1,k_2}} \Theta(Q^2 - Q^2_{i_{k_1},i_{k_2}}) \right\rangle$$

normalization: build "mixed" events from tracks randomly selected from different events e$^+$e$^-$ versus hadron-hadron collisions



$Q^2 > 0.03$ GeV^2:

- e^+e^- increase faster than hh
- increase similar for both charge combinations

small Q^2:

- e^+e^- approach hh
- increase faster for "same charges"

$\Rightarrow$ two domains: QCD and BE !

II. EXPERIMENTAL OBSERVATIONS on FLUCTUATIONS

1. Fractal behavior in all types of collisions

2. Resonances + BEC: yes, but not enough

3. Multifractal vs. Monofractal

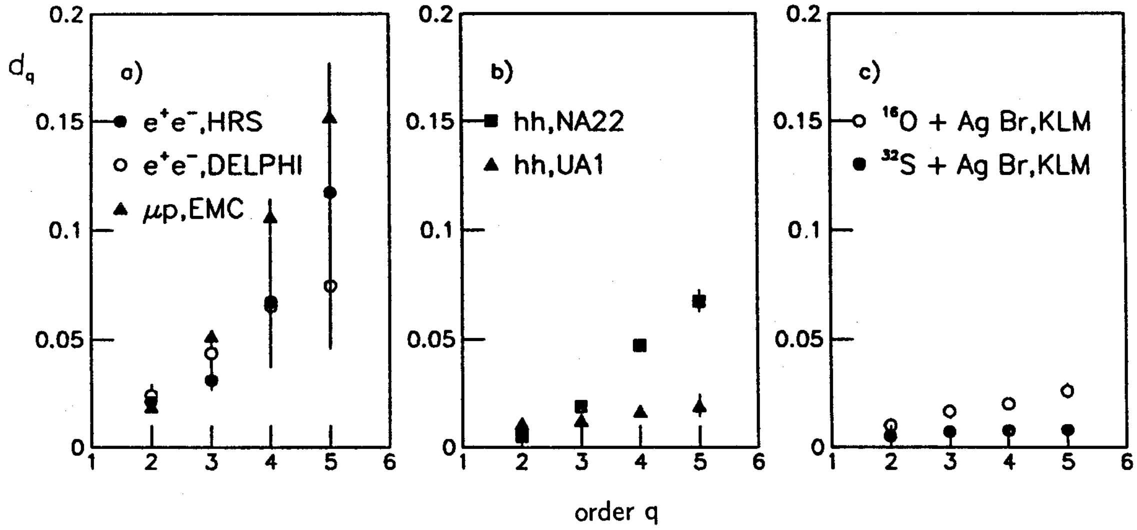
4. Self-Similar vs. Self-Affine Fractals

5. QCD: $H_q \equiv K_q/F_q$ oscillate: need 3 NBD's

6. QCD: $D_q = \frac{q+1}{q}\gamma_0 \quad \Rightarrow$ fractal
   deviations from:
   - α_s running
   - energy-conservation!
   - limits of LPHD crossed

ad 3. MULTIFRACTAL

Białas:



Rényi dimensions $D_q = D - d_q$

W. Ochs: q dependence indicative for the process

$$\frac{\mathrm{d}_q}{\mathrm{d}_2} = \frac{1}{q-1}\frac{q^\mu - q}{2^\mu - 2}$$

Lévy index $\mu = 0 \Rightarrow \frac{\mathrm{d}_q}{\mathrm{d}_2} = 1$ (monofractal, 2^{nd} order phase transition)

$\mu = 2 \Rightarrow \frac{\mathrm{d}_q}{\mathrm{d}_2} = \frac{q}{2}$ (multifractal, multipl. cascade)

$\mu \approx 1.6$ for all types of collisions

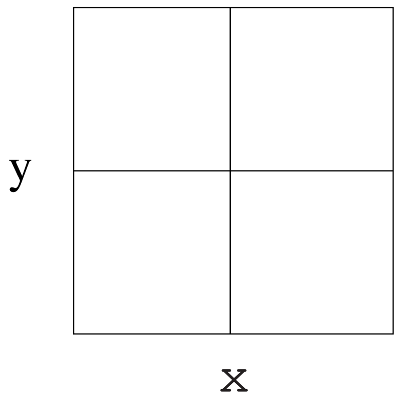
except: NA22 3D ! $\Rightarrow$ not self-similar ?

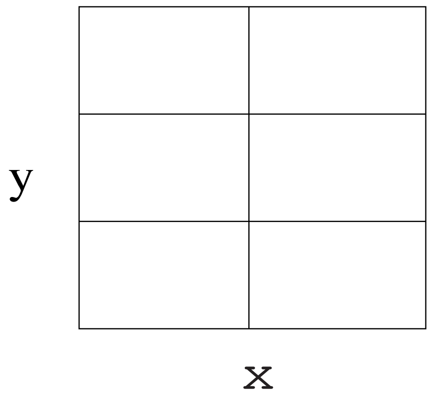
ad 4. SELF-SIMILAR vs. SELF-AFFINE FRACTAL

(Wu Yuanfang + Liu Lianshou et al.)

$D \geq 2$: power law if

$M_1 = M_2 \longrightarrow$ self-similar $\qquad$ $M_1 \neq M_2 \longrightarrow$ self-affine





characterized by Hurst exponents:

$$H_{xy} = \ln M_x / \ln M_y \quad (0 \leq H_{xy} \leq 1)$$

obtained from fit to one-dimensional projections

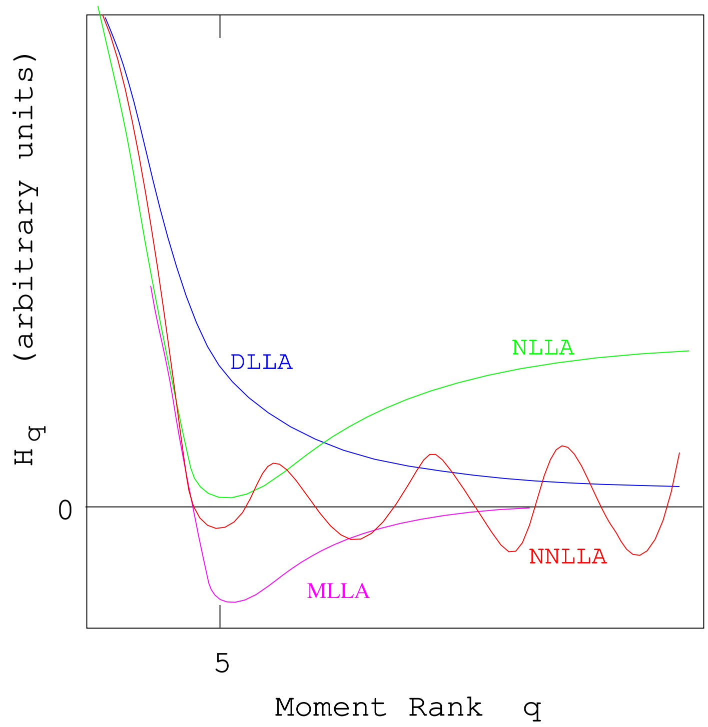
Experiment	H_{yp_T}	$H_{y\varphi}$	$H_{p_\mathrm{T}\varphi}$
NA22	0.48 ± 0.06	0.47 ± 0.06	0.99 ± 0.01
NA27		0.74 ± 0.07 $(H_{\eta\varphi})$	

hh: $\Rightarrow$ longitudinal direction privileged

e^+e^- (L3): no privileged direction

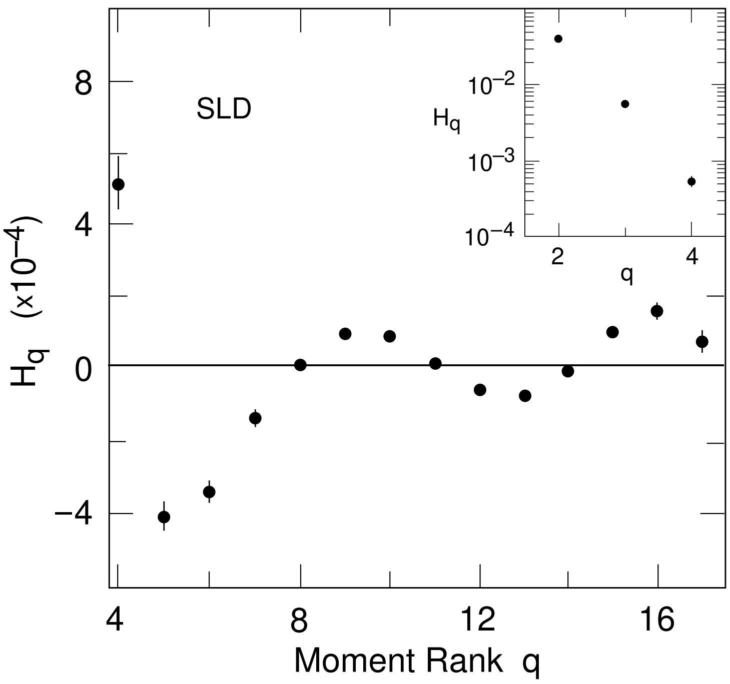
ad 5. H_q OSCILLATIONS

Perturbative QCD predictions for soft gluons
(Dremin et al., 1993,94)



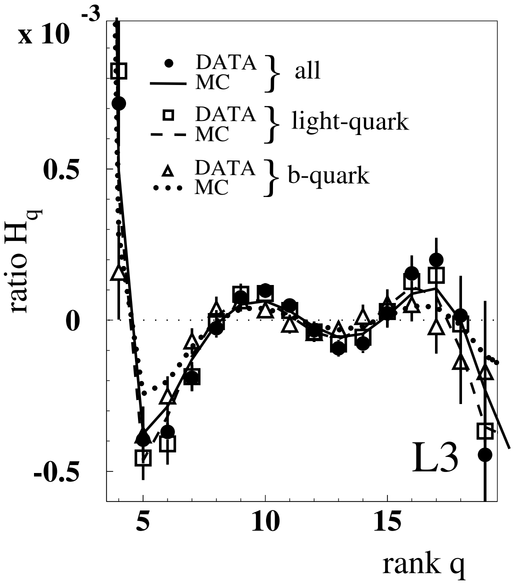
$$H_q = K_q / F_q$$

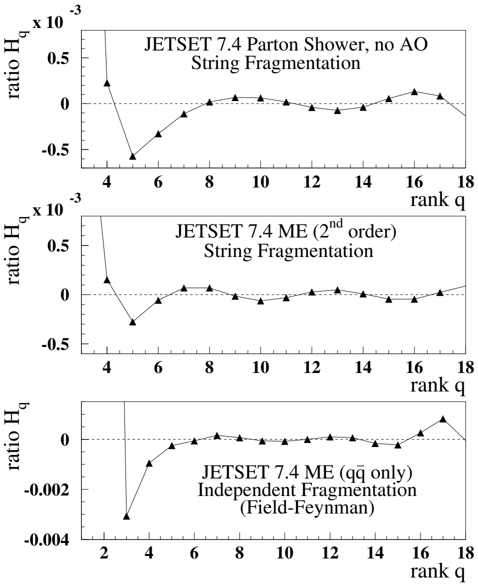
Data on hadrons (assuming LPHD)
Gianini (1993)
SLD collaboration (1996)



but also in other types of collisions (heavy-ion)

But (L3, 2001):



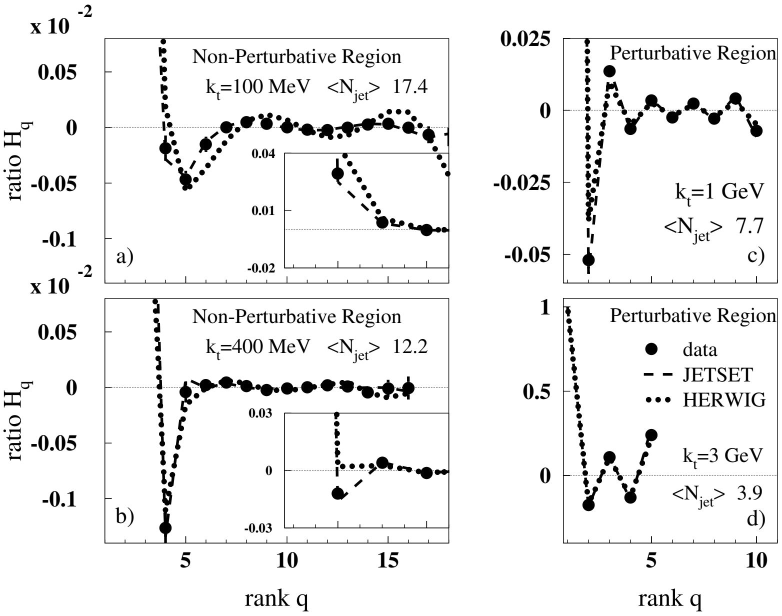


$\Rightarrow$ also JETSET (not NNLA)

also $q\bar{q}$ + independent fragmentation (no QCD branching)

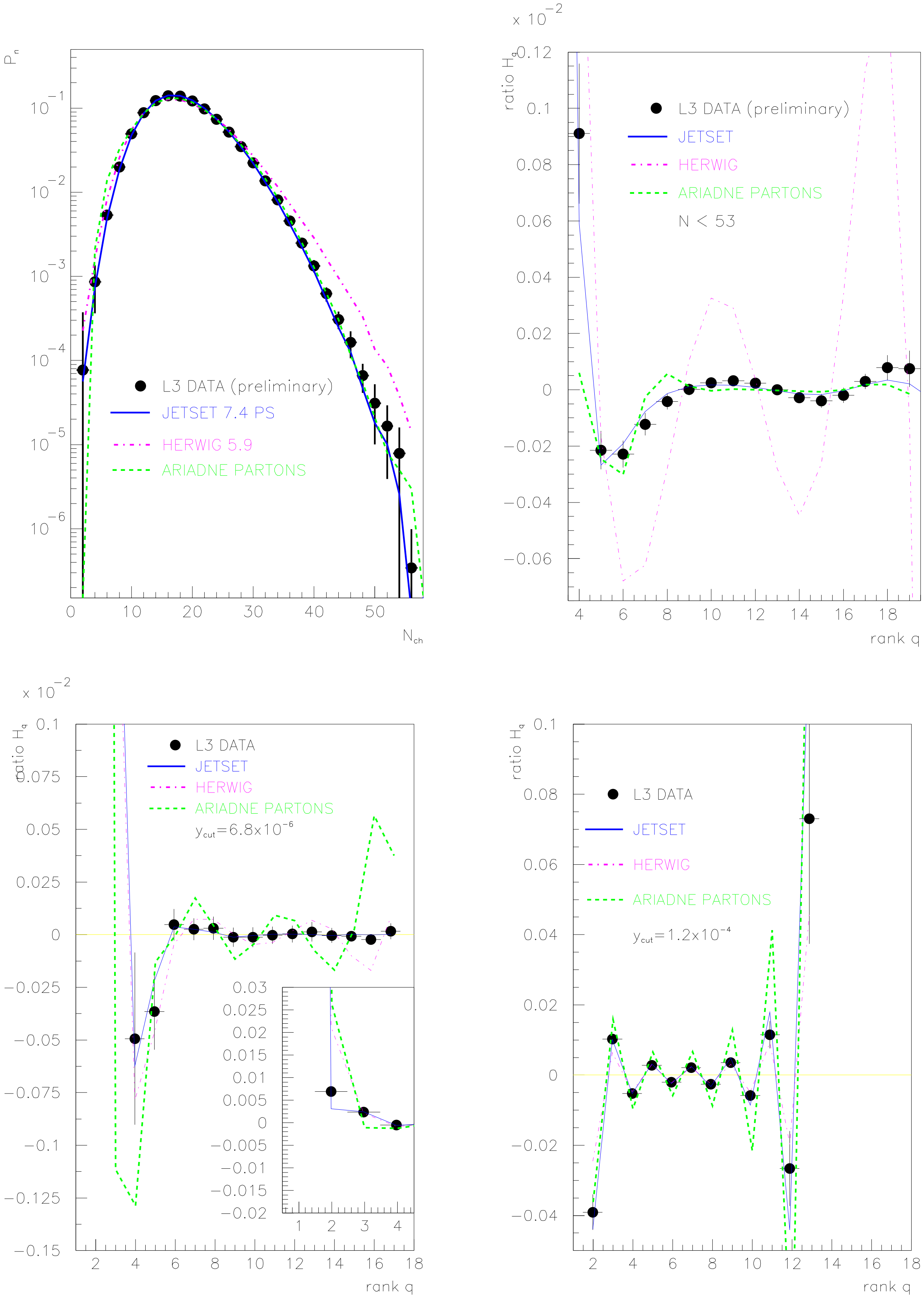
energy-momentum conservation?

jets:



not from perturbative level

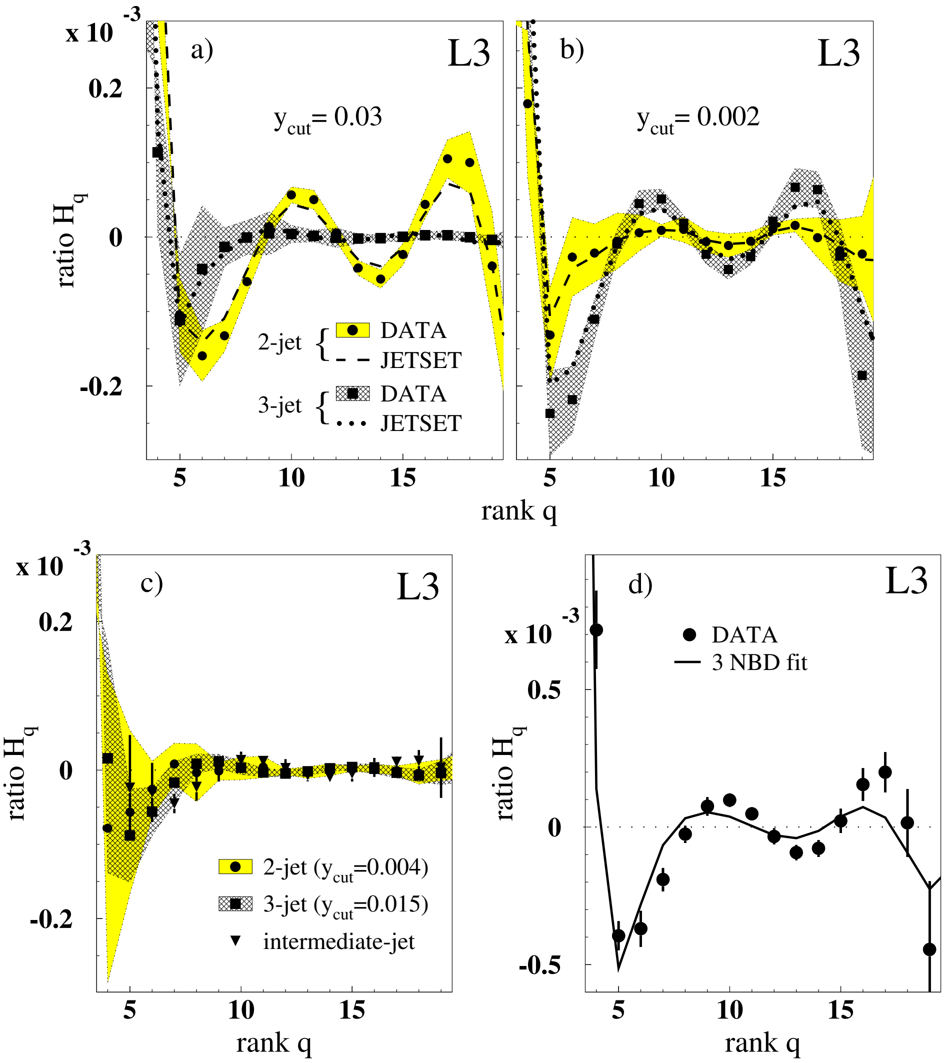
W. Metzger (ISMD99)



Superposition of Negative Binomials

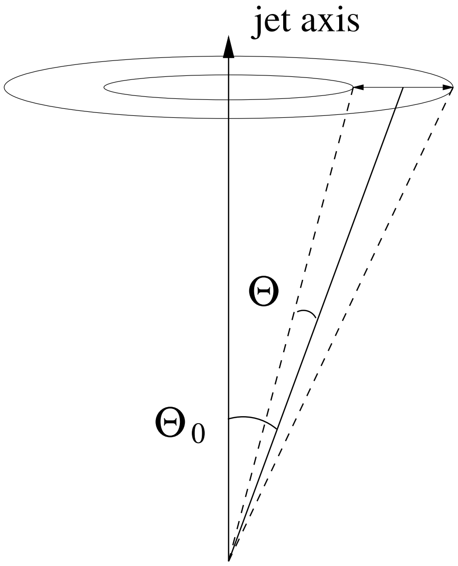
Giovannini, Lupia, Ugoccioni (1996)

L3 (2001):



$\Rightarrow$ not from QCD directly, but related to the jet topology
(interplay hard + soft)

ad 6. ANGULAR INTERMITTENCY



Scaling variable

$$z = \frac{\ln(\Theta_0/\Theta)}{\ln(E\Theta_0/\Lambda)}$$

$$0 < \Theta \leq \Theta_0 \quad \Rightarrow \quad \infty > z > 0$$

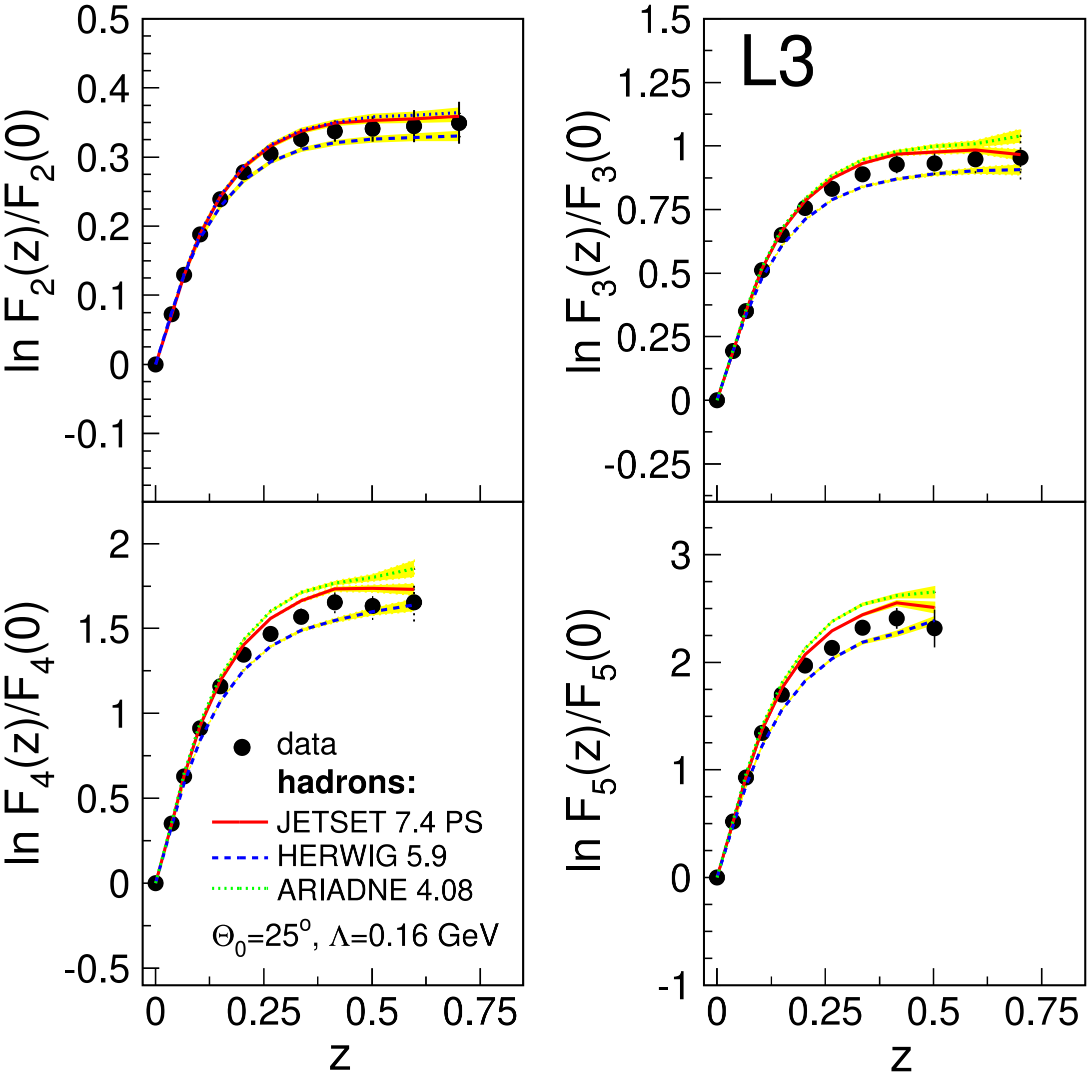
L3 (1998), DELPHI (1999)
ZEUS (2001, see also L. Zawiejski)

$\Rightarrow$ Fluctuations not in agreement with DLLA and MLLA, but can be reproduced by models

Nevertheless:

- only one parameter (Λ)
- predictions are asymptotic
- for partons (not for particles)

$\Rightarrow$ encouraging


L3
ln F2(z)/F2(0)
ln F3(z)/F3(0)
ln F4(z)/F4(0)
ln F5(z)/F5(0)
z
data
hadrons:
JETSET 7.4 PS
HERWIG 5.9
ARIADNE 4.08
Θ0=25°, Λ=0.16 GeV

- Fixed coupling regime in DLLA:

$$D_q = \gamma_0 \frac{q+1}{q},$$

$\gamma_0 = \sqrt{2C_{\rm A}\alpha_s/\pi}$.

- Running coupling regime in DLLA:

Yu.Dokshitzer, I.M.Dremin, Nucl.Phys. B402 (1993) 139

$$D_q \simeq \gamma_0 \frac{q+1}{q}\left(1+\frac{q^2+1}{4q^2}z\right) \qquad (a)$$

- Another DLLA estimation:

Ph.Brax, J.-L.Meunier, R.Peschanski, Z.Phys. C62 (1994) 649

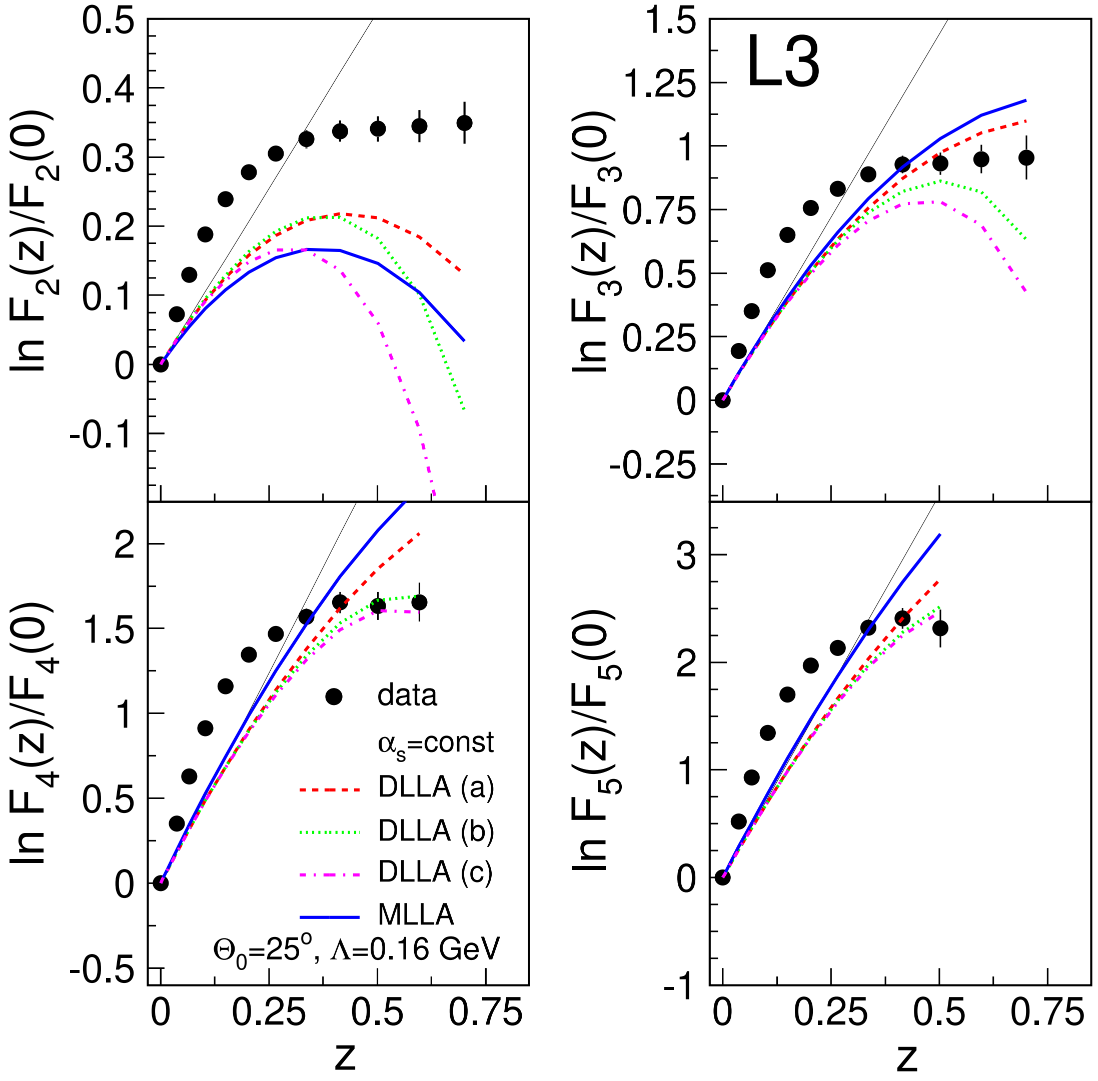
$$D_q \simeq 2\gamma_0 \frac{q+1}{q}\left(\frac{1-\sqrt{1-z}}{z}\right) \qquad (b)$$

- DLLA estimation for cumulant moments:

W.Ochs and J.Wosiek, Phys.Lett. B289 (1992) 159; B304 (1993) 144

$$D_q \simeq 2\gamma_0 \frac{q-w(q,z)}{z(q-1)},$$

$$w(q,z) \quad = \quad q\sqrt{1-z}\left(1-\frac{\ln(1-z)}{2q^2}\right) \ . \qquad (c)$$


L3
ln F2(z)/F2(0)
ln F3(z)/F3(0)
ln F4(z)/F4(0)
ln F5(z)/F5(0)
z
z
data
αs=const
DLLA (a)
DLLA (b)
DLLA (c)
MLLA
Θ0=25°, Λ=0.16 GeV

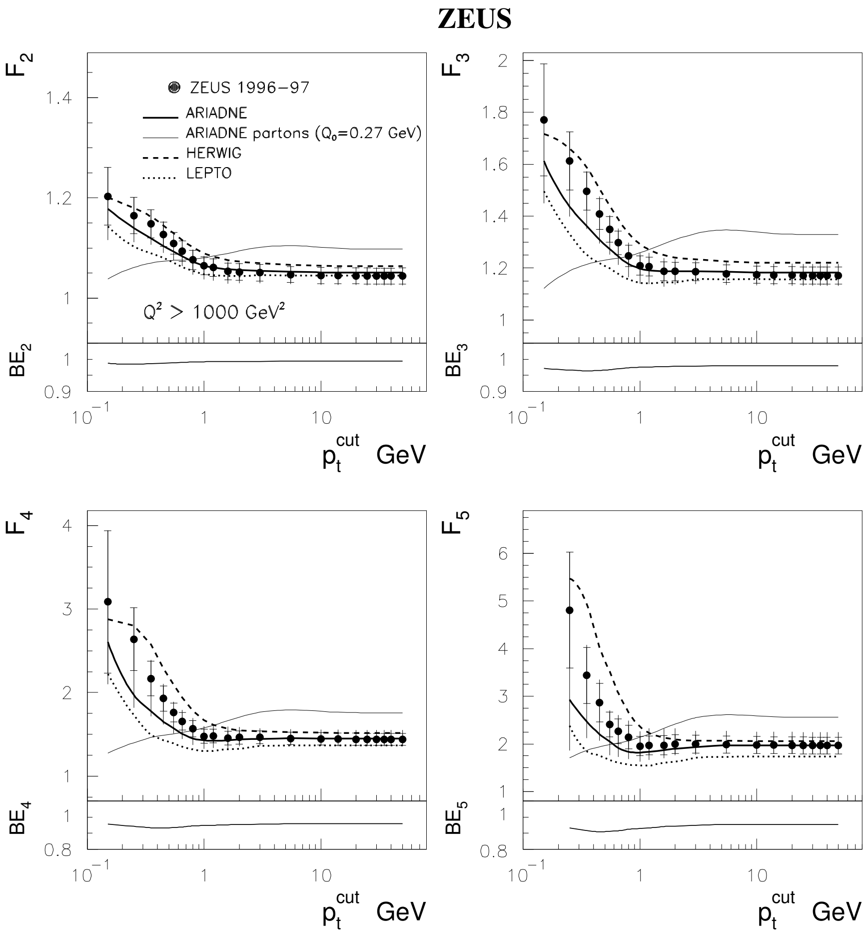
(Transverse-) Momentum Cuts

Lupia, Ochs, Wosiek (1999):

DLA: $$F_q(p_t^{\mathrm{cut}}) \simeq 1 + \frac{q(q-1)}{6}\frac{\ln(p_{\mathrm{t}}^{\mathrm{cut}}/Q_0)}{\ln(P/Q_0)}$$

$$F_q(p^{\mathrm{cut}}) \simeq C(q) > 1$$

LPHD $\Rightarrow$ also for particles



$\Rightarrow$ Limits of LHPD are crossed, i.e., the F_q are sensitive to dynamical details of non-perturbative QCD.

ad 7. EVENT-TO-EVENT FLUCTUATIONS

7.1 Erraticity

Cao + Hwa (95, 96)
Hefei: (Wang Shaoshun, Wung Zhaomin et al.)
Wuhan: (Liu Lianshou, Wu Yuanfang, Fu Jinghua, Liu Fuming ....)

F_q^e for every event (averaged over M bins)

$\Rightarrow$ F_q^e distribution $P(F_q^e)$

$\Rightarrow$ take moments $C_{q,p} = \frac{\langle (F_q^e)^p \rangle}{\langle F_q^e \rangle^p}$

However: dominated by statistical fluctuations
$\Rightarrow$ only high multiplicities

7.2 Void analysis

Hwa + Zhang (2000)
Hwa + Wu Yuanfang (1999)/ Wang Shaoshun, Wu Chang

- consider event with n particles located at y_i
- $y_0 = y_{min}, y_{n+1} = y_{max}, \quad x_i = y_{i+1} - y_i$
- for each event define moments

$$G_q^e = \frac{1}{n+1} \sum_{i=0}^{n} x_i^q$$

$\Rightarrow \quad G_q^e$ distribution $P(G_q^e)$

7.3 Entropy

Białas, Czyż, Wosiek (1999, 2000)
not restricted to large or small multiplicity

- phase space region divided into M bins
- event chatacterized by number m_i of particles in bin i (set $s \equiv \{m_i\}$)
- count the number n_s of realizations of any given set s in the event sample
- total number of coincidences of k configurations:

$$N_k = \sum_s n_s(n_s - 1) \dots (n_s - k + 1)$$

- coincidence probability of k configurations:

$$C_k = \frac{N_k}{N(N-1)\dots(N-k+1)}$$

  with $N = \Sigma n_s$ the total number of events in the sample

- Renyi entropies:

$$H_k \equiv -\frac{\log C_k}{k-1} \quad , \quad k > 1$$

a) can be used to extrapolate to $H_1 \equiv S$
b) H_k themselves contain valuable information about the multiparticle system