Event Shapes, Power Corrections, RGI and the β–function

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Overview:

1) Specify data and corrections
2) Compare to power correction models
3) Introduce and compare to RGI perturbation theory
4) Precise Measurement of the QCD β-Function
5) Summary

Experimental Comments

Talk is mainly based on:
DELPHI Collab., Ralf Reinhardt et al.,
A study of the energy evolution of event shape distributions and their means with the DELPHI detector at LEP,
(to appear in Eur. Phys. J. C)

- Data
  - radiative DELPHI Z events at E_{CM} =45, 66 and 76 GeV
  - DELPHI data from LEP1 and LEP2 (E_{CM} =89 to 202 GeV)
  - low energy data from various experiments

- Observables
  - usual event shapes (distributions and means):
    Thrust, C-Parameter, Major, jet–broadenings, EEC, JCEF
  - jet–masses ($M^2_{h/s}/E^2_{vis}$)
    in alternative definitions (“E-scheme” and “p-scheme”)

Mass Effects for Event Shapes

Hadron masses influence shape observables.
Two types of observables:

- e.g. Thrust: depending on particle momenta; mass–dependence enters via energy conservation.
- e.g. Jet Masses: direct mass–dependence; avoid by choosing E– or p–scheme:

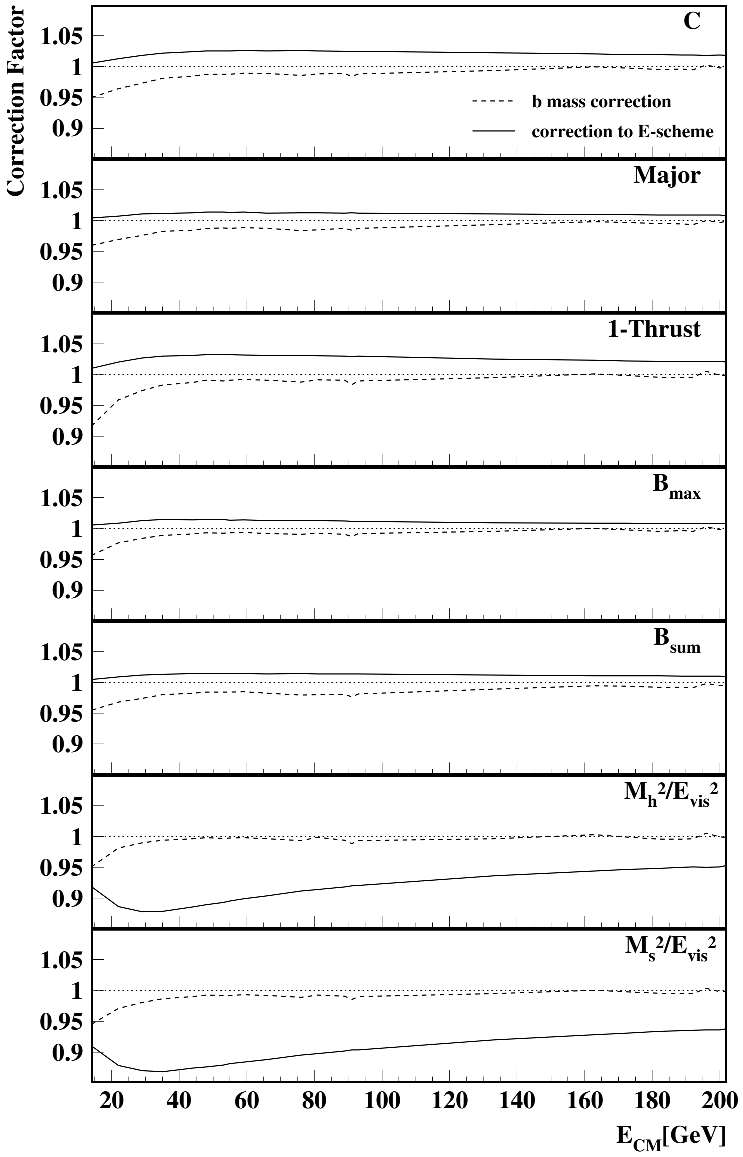
$$p = (\vec{p}, E) \longrightarrow (\vec{p}, |\vec{p}|) \qquad (p\text{-scheme})$$

$$p = (\vec{p}, E) \longrightarrow (\hat{p}E, E) \qquad (E\text{-scheme})$$

Furthermore:

- Transverse momentum from B–decays.

Estimate effects using Monte Carlo models;
b-correction: calculate b+udsc/udsc;
Change to E-scheme, b-corr. for shape means $\Longrightarrow$



Describing Event Shape Spectra

Topology of hadronic events measured by event shapes f (e.g. $f = 1 - \mathrm{Thrust}$).

Distributions described by different calculations:

- $\mathcal{O}(\alpha_s^2)$; renorm. scale factor $x_\mu = \mu/E_{CM}$

$$\frac{1}{\sigma}\frac{d\sigma}{df} = \mathcal{D}(f)_{\mathrm{pert}} =$$

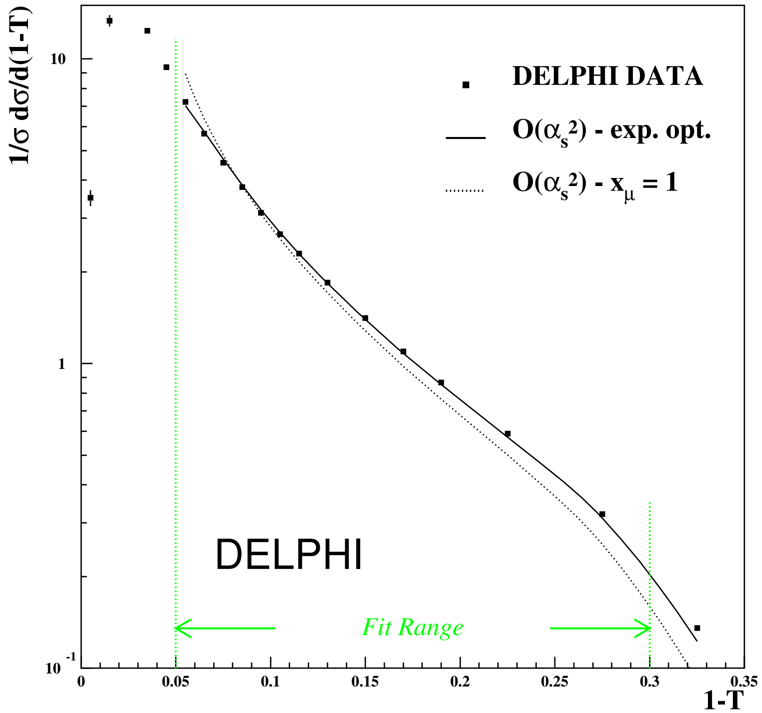
$$A_f \cdot \bar{\alpha}_s(\mu) + (A_f \cdot \beta_0 \log x_\mu + B_f)\,\bar{\alpha}_s(\mu)^2$$

- 2 jet regime: resummed LLA/NLLA
- 2/3 jet regime: matched $\mathcal{O}(\alpha_s^2 | x_\mu = 1)$/NLLA

- apply MC hadronisation corrections

$$C = \frac{\mathcal{D}_{parton}(f)}{\mathcal{D}_{hadron}(f)}$$

- or use "Power Correction" models

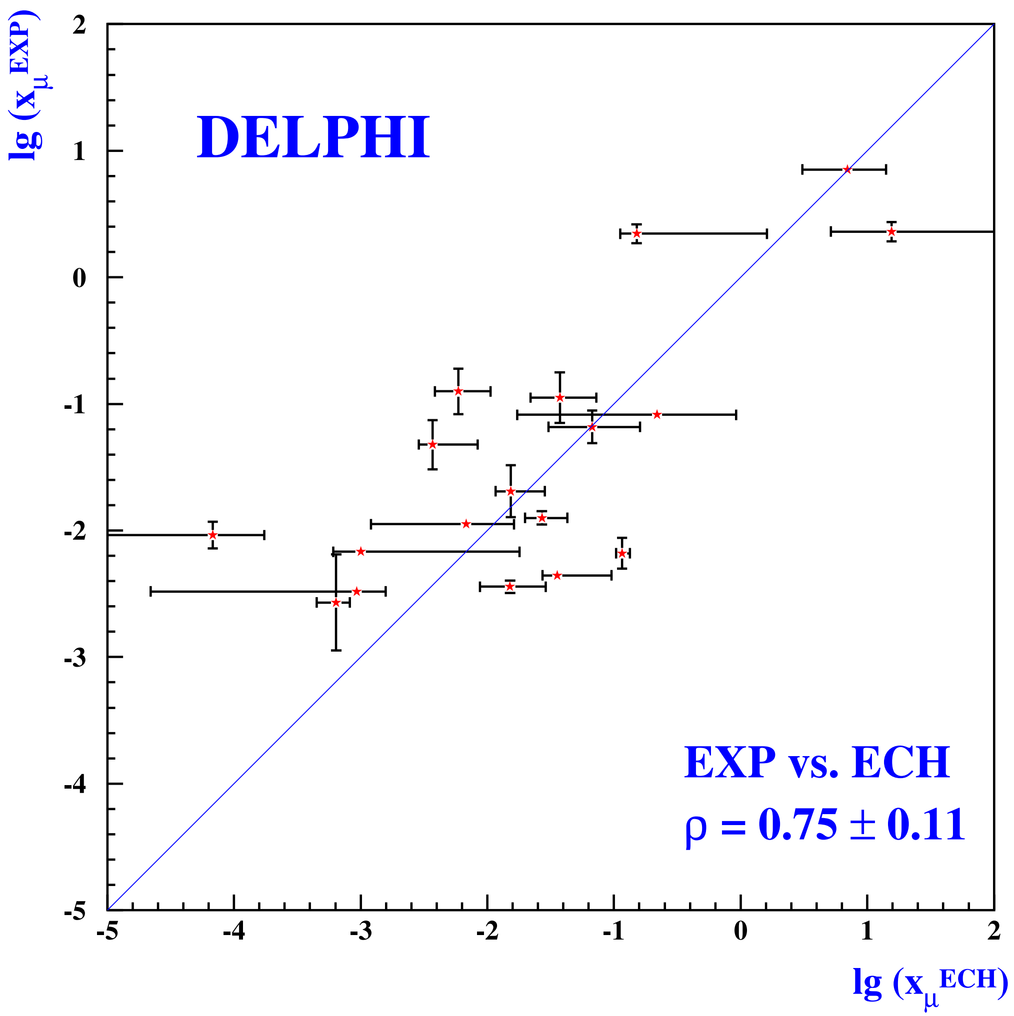


$\mathcal{O}(\alpha_s^2)$ prediction in general inconsistent with data except x_μ is optimised!

Then: $\alpha_s(M_Z)$ values obtained from different observables consistent!

Describing Event Shape Spectra

Observe correlation of exp. optim. scales and theor. ECH/PMS/FAC scale choices



Theoretical scales choices:

- FAC fastest apparent convergence
  truncated $=$ all orders prediction
- PMS principle of minimal sensitivity
  makes x_μ dependence minimal
- ECH effective charge
  resums collinear divergencies using β-function

- values of these scales are similar

Theoreticians dislike scale fixing.
No room for theoretical uncertainty.

Correlation remains to be understood!

Describing Event Shape Spectra

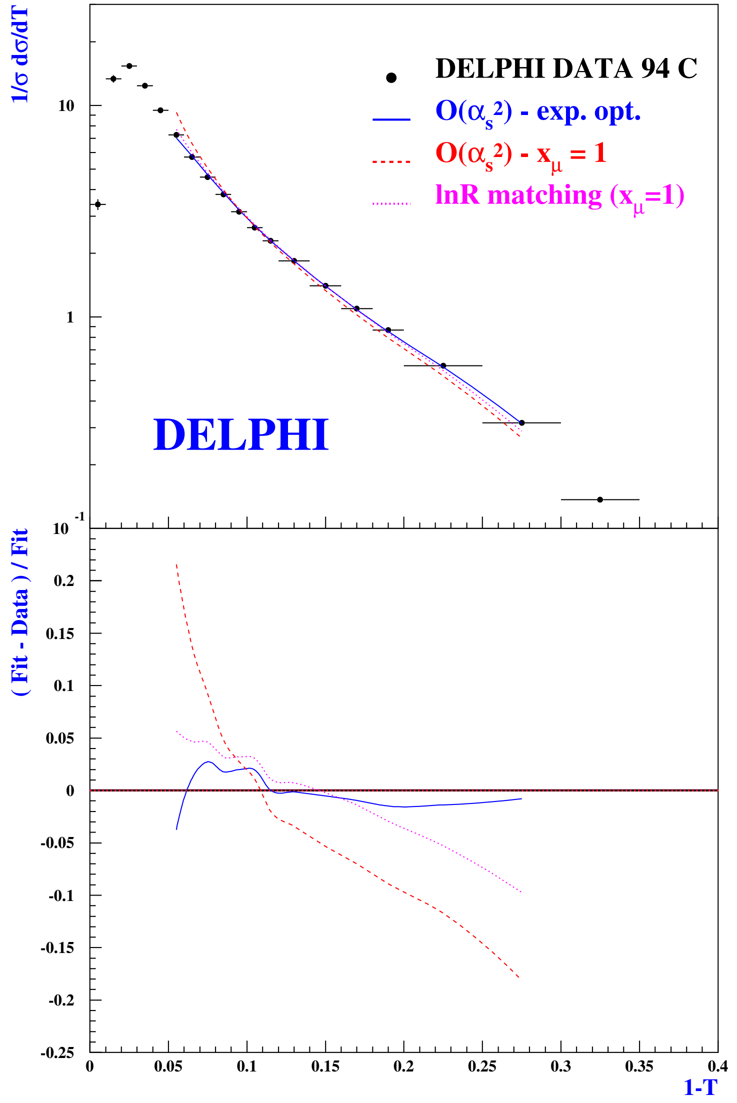
Matching NLLA and $\mathcal{O}(\alpha_s^2|x_\mu = 1)$ predictions:

Expected to improve prescription

Observe residual shortfall à la $\mathcal{O}(\alpha_s^2|x_\mu = 1)$

- Extracted $\alpha_s(M_Z)$ fit range dependend
- ($\uparrow$) Implies bias of $\alpha_s(M_Z)$

Thrust is worst case $\longrightarrow$



Improvements required

$\mathcal{O}(\alpha_s^3)$ predictions?
Matching including optimised scales?
Other more complete calculations.

Power Corrections à la Dokshitzer–Webber

non–perturbative effects are predicted to shift event shape distributions:

$$\mathcal{D}_f(f) = \mathcal{D}_f^{pert}(f - \mathcal{P}c_f)$$

$$\mathcal{P} = \frac{4C_F}{\pi^2}\mathcal{M}\frac{\mu_I}{E_{\mathrm{CM}}}\left[\alpha_0(\mu_I) - \alpha_s(\mu) - \beta_0\frac{\alpha_s^2(\mu)}{2\pi}\left(\log\frac{E_{\mathrm{CM}}}{\mu_I} + \frac{K}{\beta_0} + 1\right)\right]$$

c_f calculated perturbatively. Evidently the mean values get shifted too:

$$\langle f\rangle = \langle f_{pert}\rangle + \langle f_{pow}\rangle \qquad \langle f_{pow}\rangle = c_f \cdot \mathcal{P}$$

Keep in mind:

For distributions need resummed/matched predictions; limited set of observables.
For mean values in general $\mathcal{O}(\alpha_s^2)$ predictions are used.

f_{pow} corresponds to $\overline{MS}$ scheme!

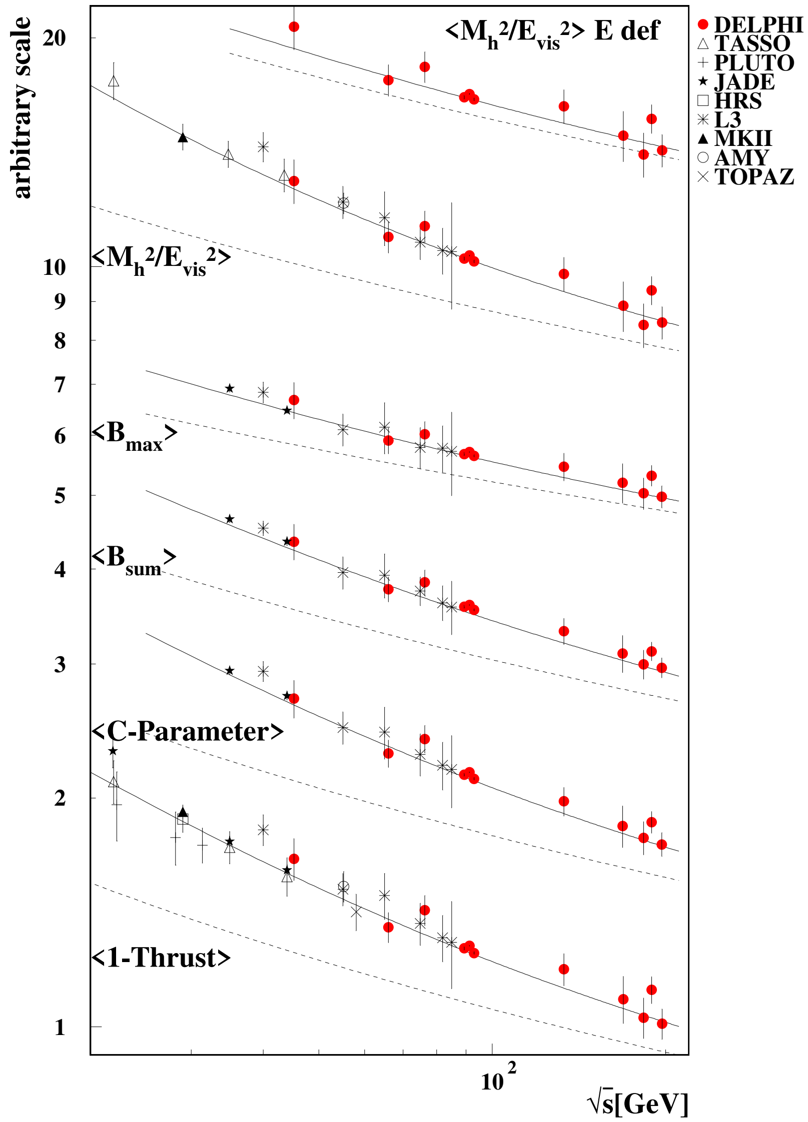
Power Corrections à la Dokshitzer–Webber (Results I)

Mean Values: $\Longrightarrow$

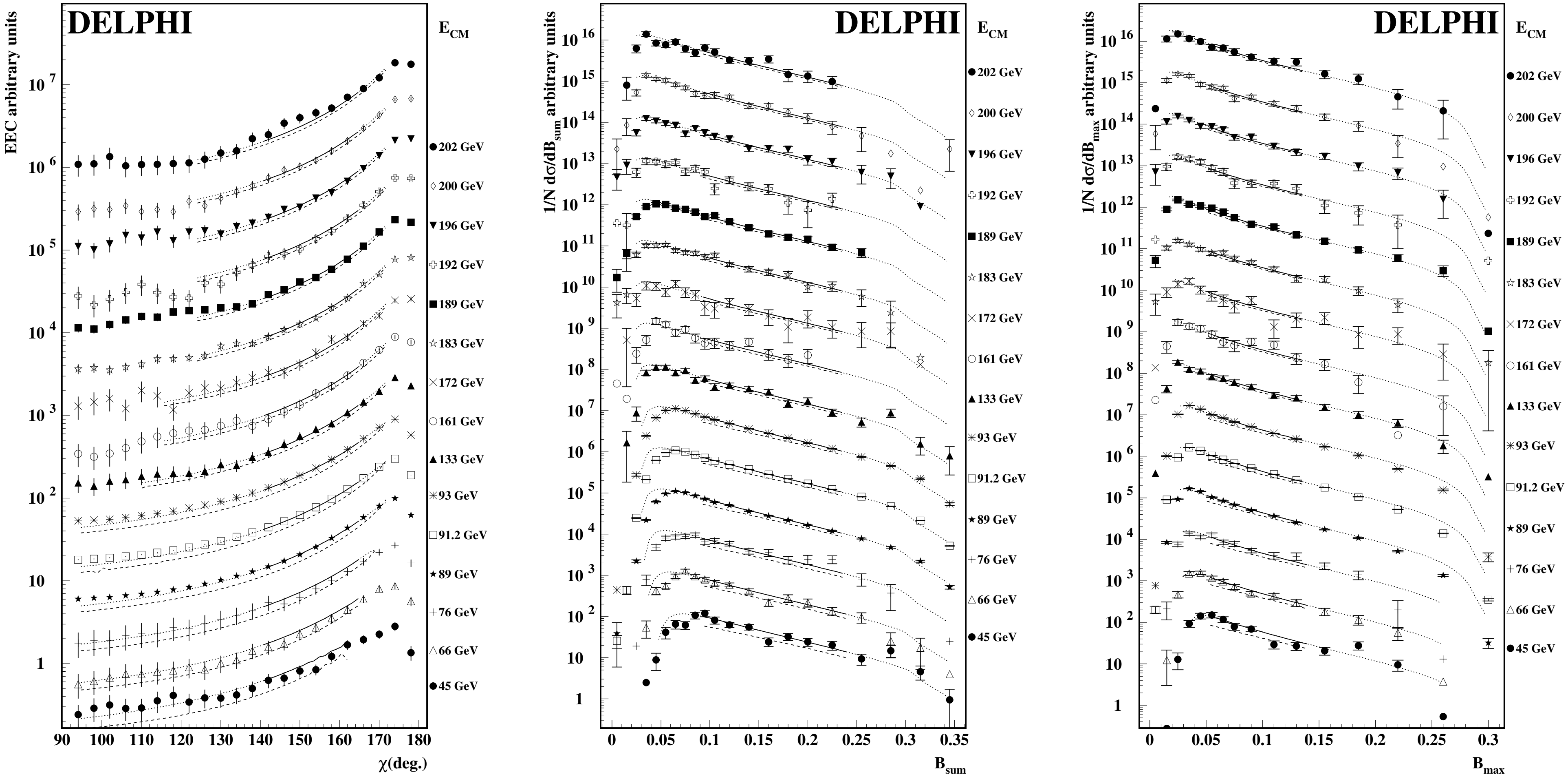
The perturbative calculation
in the $\overline{MS}$ scheme
needs typically a 10% power correction
to fit the data.

Distributions:

Data can be described
over the whole energy range and a
wide range in the observable f
with two free paramters (α_s and α_0).



Power Corrections à la Dokshitzer–Webber (Results II)



EEC $\qquad$ B_{sum} $\qquad$ B_{max}

Power Corrections à la Dokshitzer–Webber (Results III)

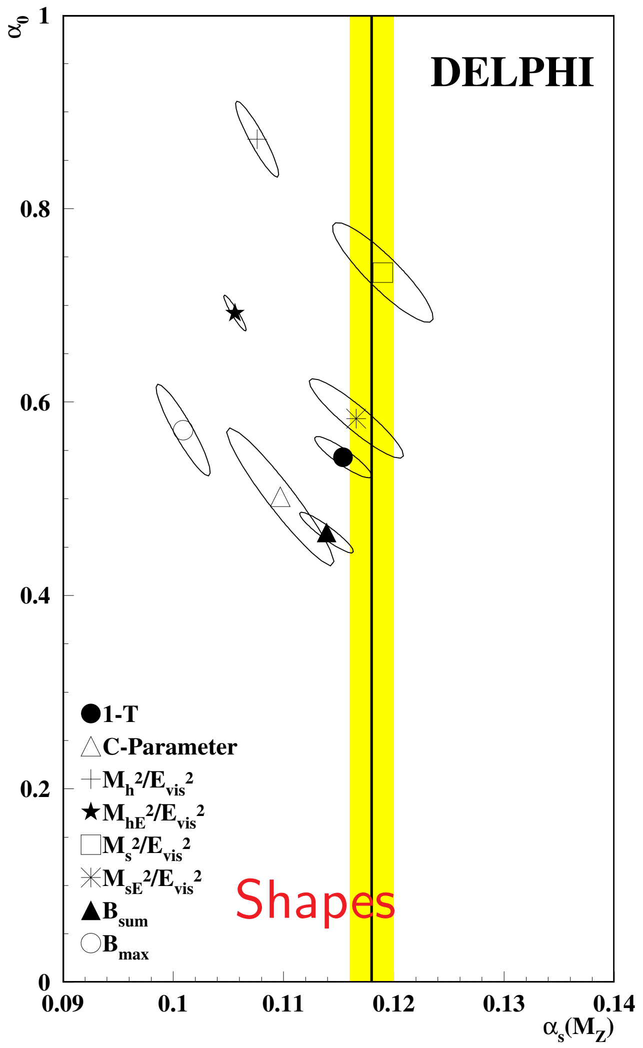
Expect:

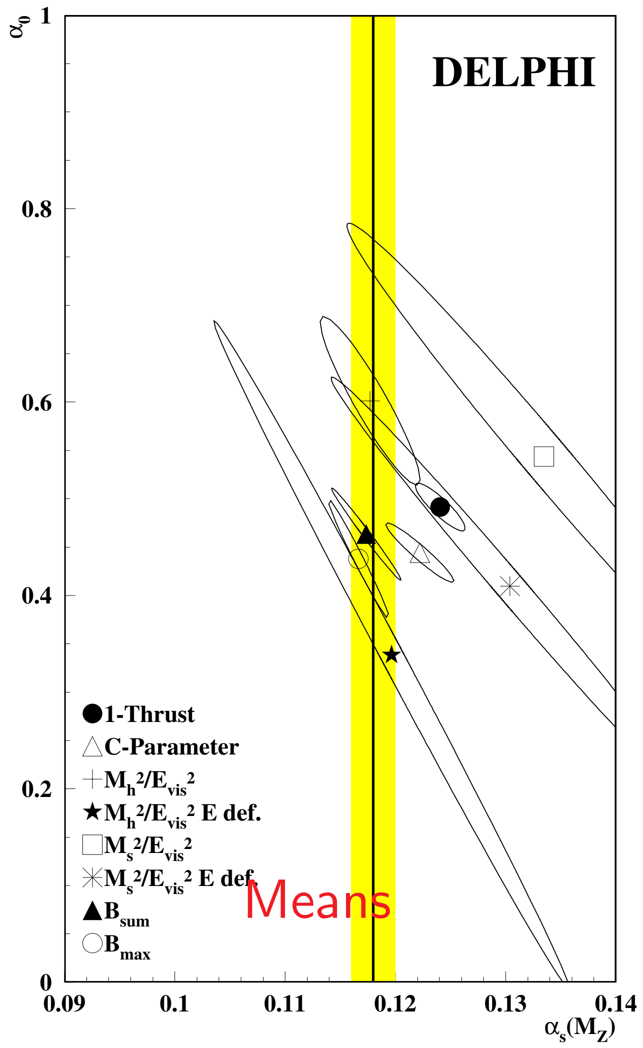
"universal" $\alpha_0 \approx 0.5$

Find:

Consistency
between distributions
and means poor.

Holds even if
conventional
Jet Masses
excluded!





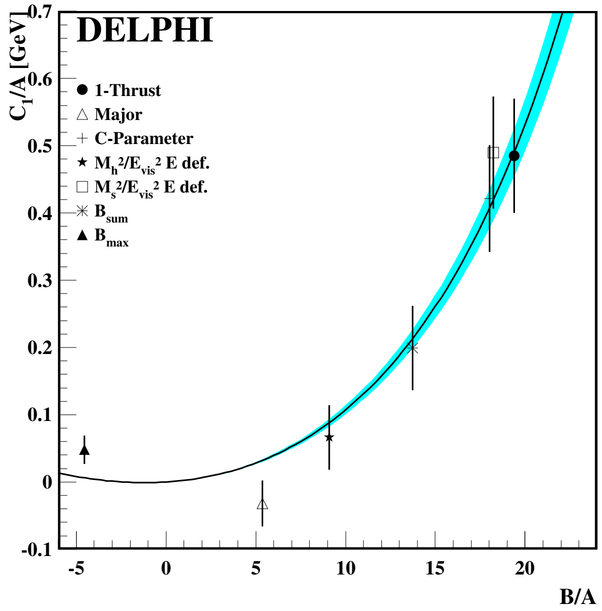
Differences because of $\mathcal{O}(\alpha_s^2)/NLLA$ perturbative calculations?

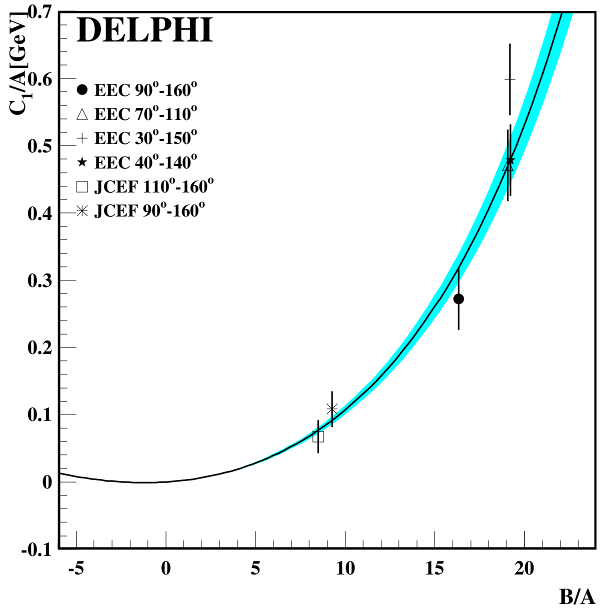
Naive Power Corrections

Assess power terms of any shape observable mean by fitting the simple ansatz:

$$\langle f \rangle = \langle f \rangle_{pert} + \langle f \rangle_{pow}$$

$$\langle f_{pow} \rangle = C_1 / E_{CM}$$





Surprising:
The "non–perturbative" power corrections show a strong correlation with the perturbative coefficients B/A.

$\Rightarrow$ Power corrections predominantly account for higher order perturbative effects?

Renormalisation Group Invariant (RGI) Perturbation Theory

Basic requirement, demand observable $R = \langle f \rangle / A$ to fulfil RGE:

$$Q\frac{dR}{dQ} = -bR^2(1 + \rho_1 R + \rho_2 R^2 + \ldots) = b\rho(R) \quad .$$

b, ρ_i are scheme invariant (hence the name of the method). Solution:

$$b \ln \frac{Q}{\Lambda_R} = \frac{1}{R} - \rho_1 \ln\left(1 + \frac{1}{\rho_1 R}\right) + \underbrace{\int_0^R dx \left(\frac{1}{\rho(x)} + \frac{1}{x^2(1+\rho_1 x)}\right)}_{\text{vanishes in second order}}$$

This very simple form of RGI only applies to observables depending on a single energy scale (e.g. mean values). Numerically RGI=ECH.

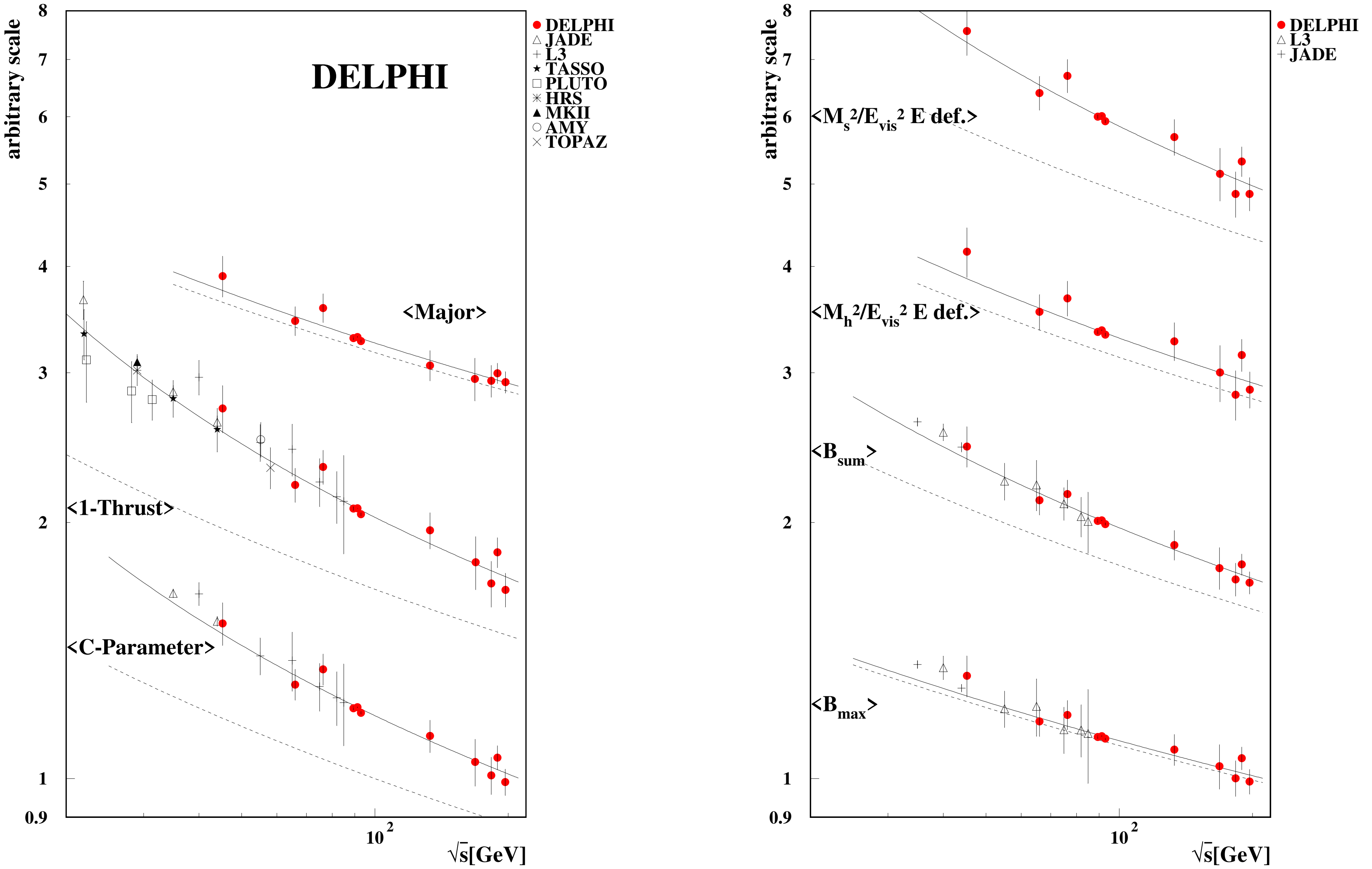
Exact conversion to $\overline{MS}$ reference–scheme.

$$\frac{\Lambda_R}{\Lambda_{\overline{MS}}} = e^{B/2A/b}\left(\frac{2c_1}{b}\right)^{-c_1/b}$$

Power corrections can be included.

$$\rho(x) \to \rho(x) - \frac{K_0}{b} x^{-c_1/b} e^{-1/bx}$$

RGI – Description of Shape observable Means



RGI Needs no Significant Power Corrections

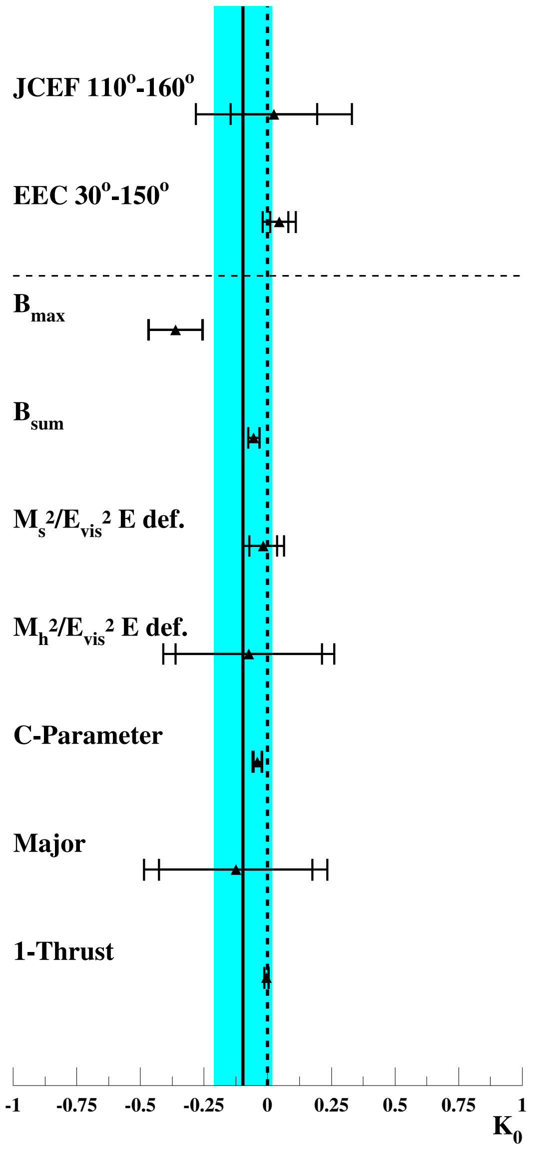
Fitting RGI with power–corrections to a large set of observables:

Yields K_0 compatible with 0!

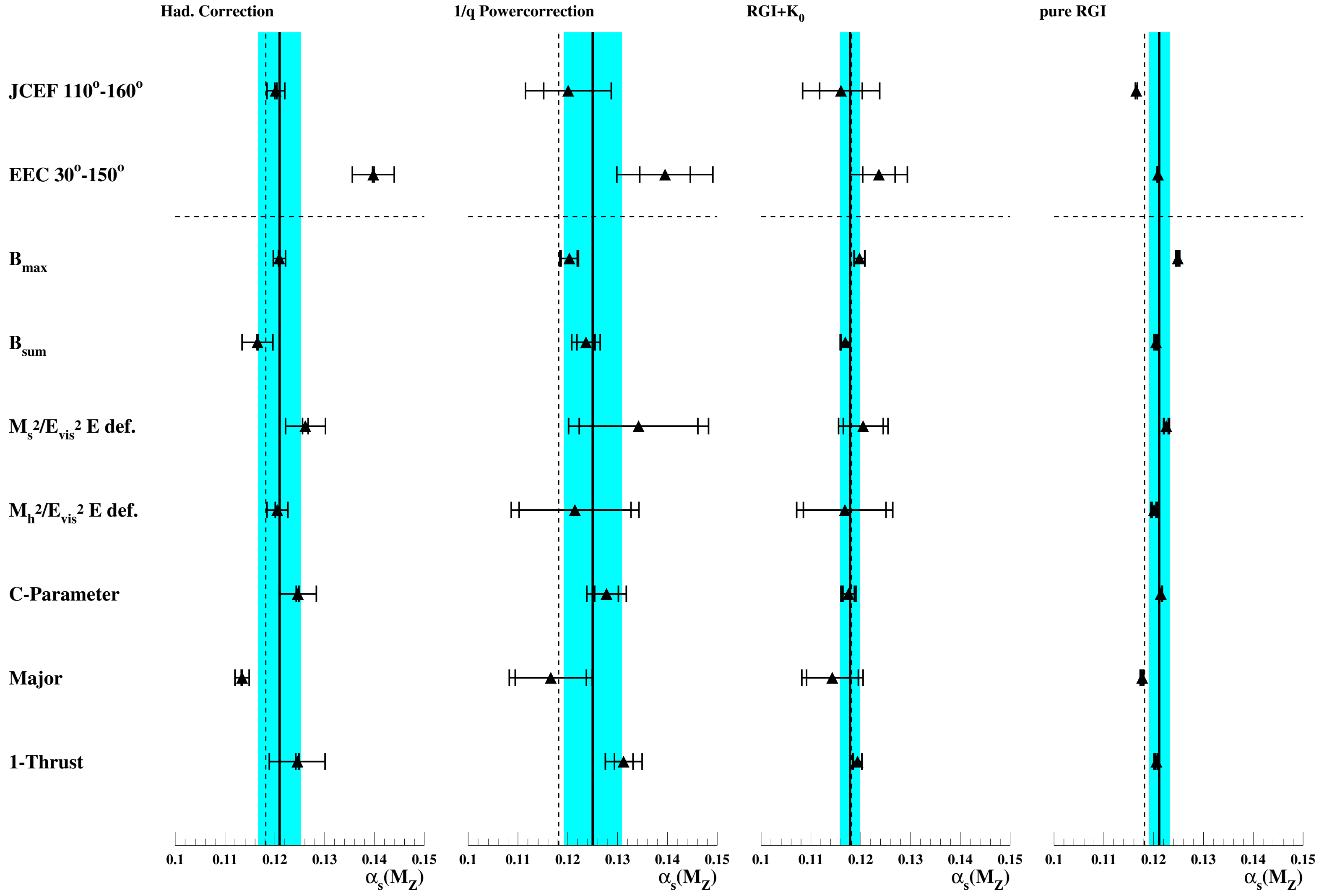
This should be viewed as a virtue of both: RGI and inclusiveness of mean values.

Presence of genuine power suppressed terms for means unclear so far!

Possible contribution:
only $\sim 2\%$ (relative) at Z energies.



Compare α_s from Means Obtained with Various Methods



RGI yields smallest scatter; $\langle \alpha_s(M_Z) \rangle = 0.121 \pm 0.002$

"Predict" Power Terms using RGI

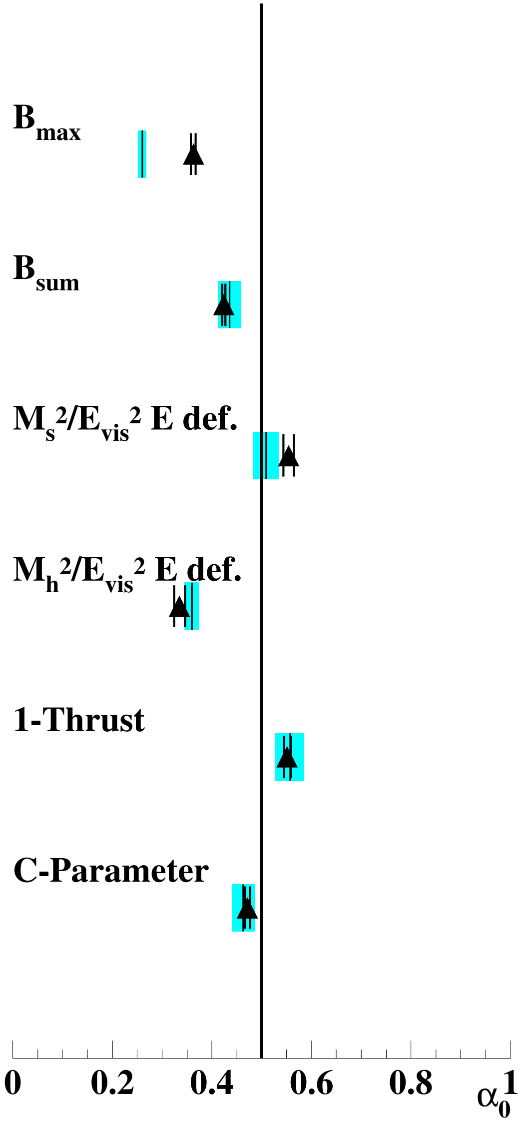
Set RGI and Power Model predictions equal:

$$\langle R \rangle_{RGI} \cdot A = \langle f \rangle_{pert} + \langle f \rangle_{pow}$$

Solve for Power Model parameters α_0 or C_1.

"Predicted" α_0 follows trend of the data.

Agrees better than any presumed universal value ~ 0.5.



β-Function from Energy Evolution of Shape Means

β-function itself is not predicted by RGI! RGI applies individualy for all measurements.
$\Longrightarrow$ Energy evolution of shapes directly measures β-function.

Measurement based on general field–theory; QCD not precondition!?

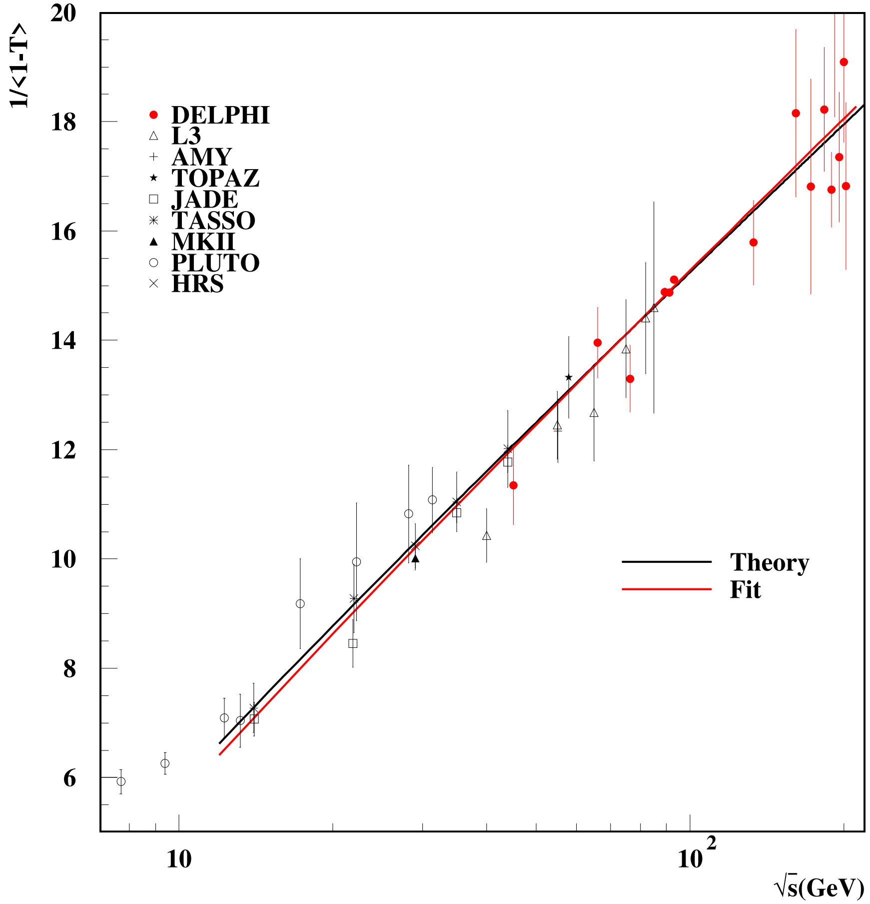
β-function: slope of e.g. $1/\langle 1-T\rangle$ plot
From final data of low E expts + DELPHI

$\beta_0 = 7.86 \pm 0.32$ QCD :7.67

Additional uncertainty due to possible power terms small!
Rel. error $\sim 0.6\cdot$ rel. power contr. at M_Z

assuming QCD: number of active flavours

$n_f = 4.75 \pm 0.44$ QCD :5



Compare: α_s running $(R_\tau, F_2, F_3, R_Z, \dots)$ world data $\beta_0 = 7.76 \pm 0.43$

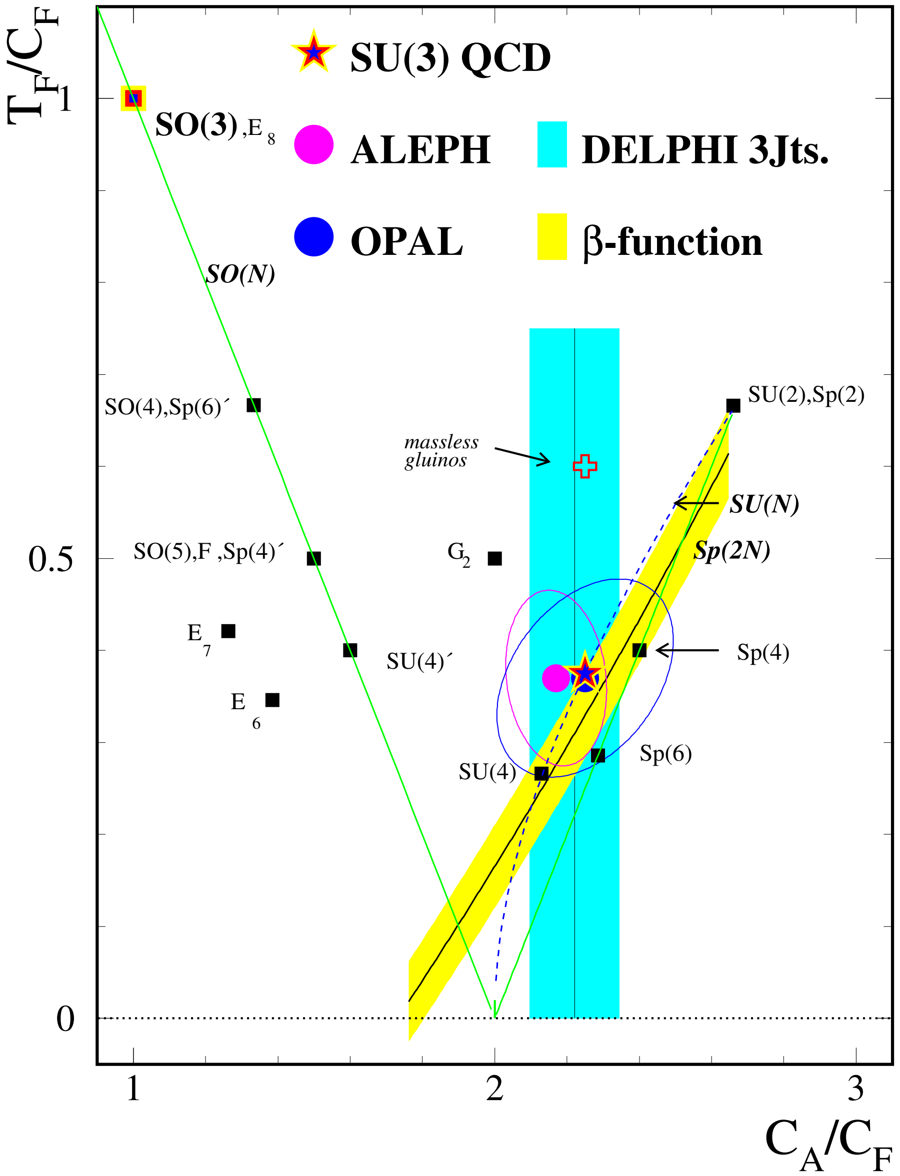
Consequences of β-Function Measurement

Measurement restricts strongly gauge group of the strong interaction.

Compare measurements of colour factors from:

- 4–jet angular distributions.
- Multiplicity of gluon and quark jets.

And implies
model independent limits on new physics
e.g. light gluinos



Summary

- $\mathcal{O}(\alpha_s^2)$ predictions need optimised scales in order to describe the data.
  $\Longrightarrow$ Implications for matched predictions !?
- Event shape distributions can be described by Power Models;
  but consistency of parameters?
- Mean values can be more efficiently described by RGI perterturbation theory.
  Consistent α_s!
- So far no experimental evidence for genuine power terms from mean values!
- Obtain basic and clean measurement of the β–function of strong interaction.