Forward Jets and Forward W Production at Hadron Colliders

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Outline of the talk

- Short introduction to BFKL at Hadron Colliders: Leading Logs and its Problems with Momentum Conservation.

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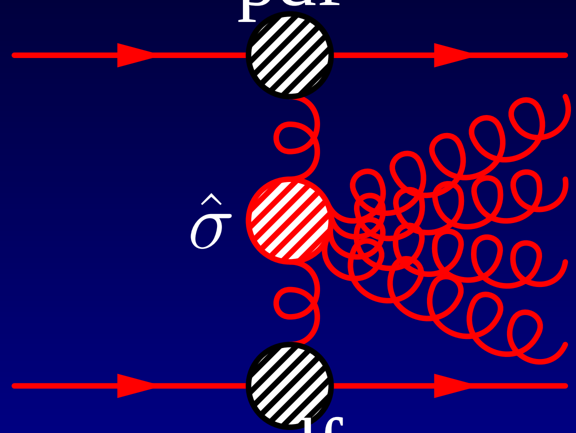
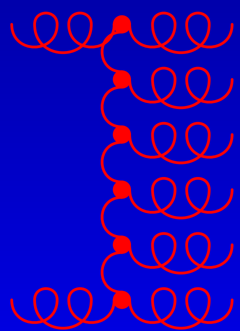
- Construction of a BFKL MC
  Momentum conserved, running coupling effects, experimental cuts
- Application of the MC (and fixed order QCD) to a dijet study: dijet and W + 2 jet production at large rapidity separation

Introduction to BFKL at Hadron Colliders



- High energy limit: $\frac{\hat{s}}{|\hat{t}|} \to \infty$
- Multi particle final states dominates the cross section.



$$\hat{s} \sim p_T^2 e^{\Delta y}$$

$$|\hat{t}| \sim p_T^2$$

$$\ln \frac{\hat{s}}{|\hat{t}|} \sim \Delta y$$

BFKL resums to all orders terms in the perturbative expansion of the form

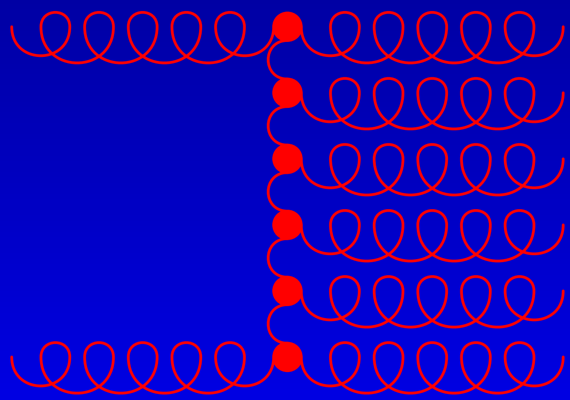
$$\left(\alpha_s \ln \frac{\hat{s}}{|\hat{t}|}\right)^n \sim (\alpha_s \Delta y)^n$$

BFKL at LL

gluon-gluon scattering:

$$\frac{d\hat{\sigma}_{gg}}{d^2\vec{p}_{T_a} d^2\vec{p}_{T_b}} = \underbrace{\left[\frac{C_A\alpha_s}{p_{T_a}^2}\right]}_{\text{QCD IF}} \underbrace{f(\vec{q}_a, \vec{q}_b, \Delta y)}_{\text{BFKL effects}} \underbrace{\left[\frac{C_A\alpha_s}{p_{T_b}^2}\right]}_{\text{QCD IF}}$$

$$\vec{q}_a = \vec{p}_{T_a}, \vec{q}_b = -\vec{p}_{T_b}$$



Resum leading logarithms in $\frac{\hat{s}}{|\hat{t}|} \sim \Delta y$ contributing to $f(\vec{q}_a, \vec{q}_b, \Delta y)$.

Take a good look at f!

The BFKL Equation

$$\omega\, \tilde{f}(\vec{q}_a, \vec{q}_b, \omega) = \frac{1}{2}\,\delta(\vec{q}_a - \vec{q}_b) + \frac{C_A \alpha_s}{\pi^2} \int \frac{d^2\vec{k}}{\vec{k}^2} K(\vec{q}_a, \vec{q}_b, \vec{k}),$$

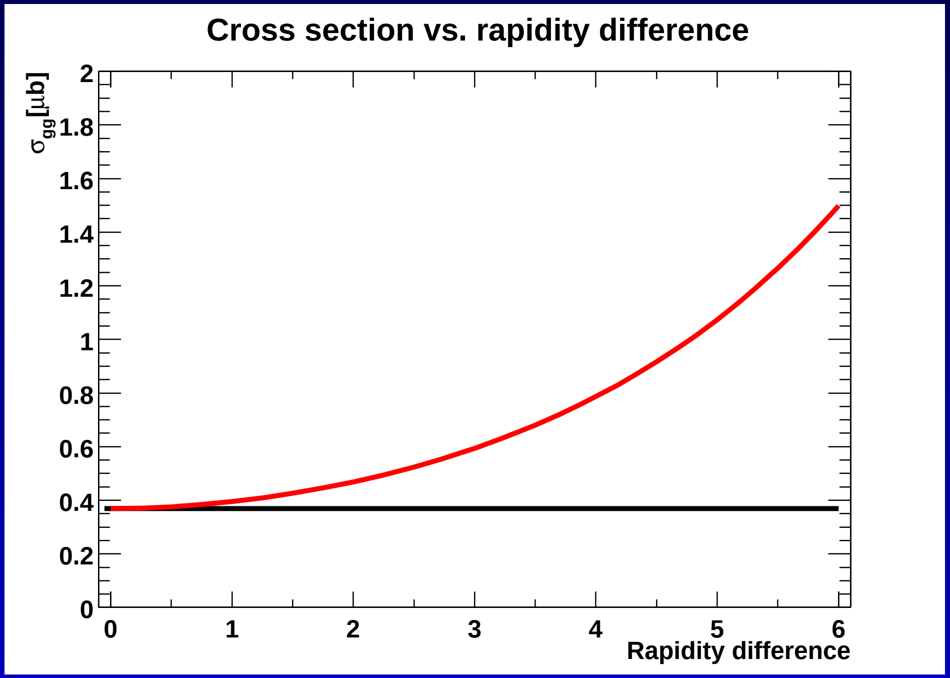
where the kernel K is given by

$$K(\vec{q}_a, \vec{q}_b, \vec{k}) = \tilde{f}(\vec{q}_a + \vec{k}, \vec{q}_b, \omega) - \frac{\vec{q}_a^{\,2}}{\vec{k}^2 + (\vec{q}_a + \vec{k})^2}\, \tilde{f}(\vec{q}_a, \vec{q}_b, \omega)\,.$$

IR divergences from real and virtual gluon radiation cancel.

The BFKL Equation

Solve the BFKL equation analytically by integrating over the full $\vec{k}$ phase space for gluon emission and allowing any number of gluons to radiate.



$$\hat{\sigma}_{gg} = \frac{\pi C_A^2 \alpha_s^2}{2P_{T,\min}^2} \frac{e^{\lambda \Delta y}}{\sqrt{\pi B \Delta y}}, B = 14\zeta(3)\bar{\alpha_s},$$

$$\lambda = \frac{\alpha_s C_A}{\pi} 4 \ln 2 \approx 0.45$$

Energy & Mom. conservation

Problem of including pdf's to evaluate hadronic cross section. So far we have $\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Delta y}$, need $\frac{\mathrm{d}\sigma}{\mathrm{d}\Delta y}$, where

$$\sigma = \int \mathrm{d}x_a \mathrm{d}x_b \, f(x_a) f(x_b) \hat{\sigma}$$

$$x_a = \frac{P_{T_a}}{\sqrt{s}} e^{y_a} + \frac{P_{T_b}}{\sqrt{s}} e^{y_b} + \sum_{i=1}^{n} \frac{k_{i\perp}}{\sqrt{s}} e^{y_i}$$

$$x_b = \frac{P_{T_a}}{\sqrt{s}} e^{-y_a} + \frac{P_{T_b}}{\sqrt{s}} e^{-y_b} + \sum_{i=1}^{n} \frac{k_{i\perp}}{\sqrt{s}} e^{-y_i}$$

These terms are inaccessible in the standard BFKL approach:

BFKL gluons are emitted at no cost in energy!

. . . not just energy conservation

Look for observables that are independent of the PDF's, e.g. angular decorrelation of jets (back to back at LO).
Need better understanding of event topology.
Experimental cuts on BFKL gluon radiation?
Wishes for a MC:

1. Energy/momentum conservation
2. Handle on gluon radiation; possibility of applying exp. cuts

It will turn out we can also handle effects from the running of the coupling

(which scale should the coupling be evaluated at in the BFKL ladder?)

Building a BFKL MC

$$\omega\,\tilde{f}(\vec{q}_a,\vec{q}_b,\omega) = \frac{1}{2}\,\delta(\vec{q}_a-\vec{q}_b) + \frac{\bar{\alpha}_s}{\pi}\int\frac{d^2\vec{k}}{\vec{k}^2}\,K(\vec{q}_a,\vec{q}_b,\vec{k}),$$

$$K(\vec{q}_a,\vec{q}_b,\vec{k}) = \tilde{f}(\vec{q}_a+\vec{k},\vec{q}_b,\omega) - \frac{\vec{q}_a^{\,2}}{\vec{k}^2+(\vec{q}_a+\vec{k})^2}\,\tilde{f}(\vec{q}_a,\vec{q}_b,\omega)\,.$$

Want to solve by iteration to "unfold" the gluon contribution. Need to maintain the cancellation of IR div. Introduce "resolution scale" $\mu \ll P_{T,\min}$. Rewrite the BFKL equation

$$\omega\tilde{f}(\vec{q}_{a\perp},\vec{q}_{b\perp},\omega) = \frac{1}{2}\delta(\vec{q}_{a\perp}-\vec{q}_{b\perp}) + \frac{\bar{\alpha}_s}{\pi}\int_{k_\perp^2>\mu^2}\frac{d^2\vec{k}_\perp}{k_\perp^2}\tilde{f}(\vec{q}_{a\perp}+\vec{k}_\perp,\vec{q}_{b\perp},\omega)$$

$$+\frac{\bar{\alpha}_s}{\pi}\int\frac{d^2\vec{k}_\perp}{k_\perp^2}\left[\tilde{f}(\vec{q}_{a\perp}+\vec{k}_\perp,\vec{q}_{b\perp},\omega)\theta(\mu^2-k_\perp^2) - \frac{q_{a\perp}^2\tilde{f}(\vec{q}_{a\perp},\vec{q}_{b\perp},\omega)}{k_\perp^2+(\vec{q}_{a\perp}+\vec{k}_\perp)^2}\right]$$

μ independent!

Since in the last integral $k_\perp^2 \le \mu^2 \ll q_{a\perp}^2, q_{b\perp}^2$ we will neglect $\vec{k}_\perp$ in $\tilde{f}$...

(small μ dependence)

Building a BFKL MC

...and rewrite the BFKL eq. as

$$(\omega - \omega_0)\,\tilde{f}(\vec{q}_{a\perp}, \vec{q}_{b\perp}, \omega) = \frac{1}{2}\,\delta(\vec{q}_{a\perp} - \vec{q}_{b\perp}) + \frac{\bar{\alpha}_s}{\pi} \int_{k_\perp^2 > \mu^2} \frac{d^2\vec{k}_\perp}{k_\perp^2}\, \tilde{f}(\vec{q}_{a\perp} + \vec{k}_\perp, \vec{q}_{b\perp}, \omega)$$

$$\omega_0 = \frac{\bar{\alpha}_s}{\pi} \int \frac{d^2\vec{k}_\perp}{k_\perp^2} \left[\theta(\mu^2 - k_\perp^2) - \frac{q_{a\perp}^2}{\vec{k}_\perp^2 + (\vec{q}_{a\perp} + \vec{k}_\perp)^2} \right] = \bar{\alpha}_s \ln\left(\frac{\mu^2}{q_{a\perp}^2} \right)$$

ω_0 describes the net effect from unresolved & virtual gluon emission, and we are left with an integral over resolved gluons.

Solve by iteration and perform the inverse Mellin transform to get...

Building a BFKL MC

$$f(\vec{q}_{a\perp}, \vec{q}_{b\perp}, \Delta y) = \sum_{n=0}^{\infty} f^{(n)}(\vec{q}_{a\perp}, \vec{q}_{b\perp}, \Delta y)$$

$$f^{(0)}(\vec{q}_{a\perp}, \vec{q}_{b\perp}, \Delta y) = \left[\frac{\mu^2}{q_{a\perp}^2}\right]^{\bar{\alpha}_s \Delta y} \frac{1}{2}\delta(\vec{q}_{a\perp} - \vec{q}_{b\perp}),$$

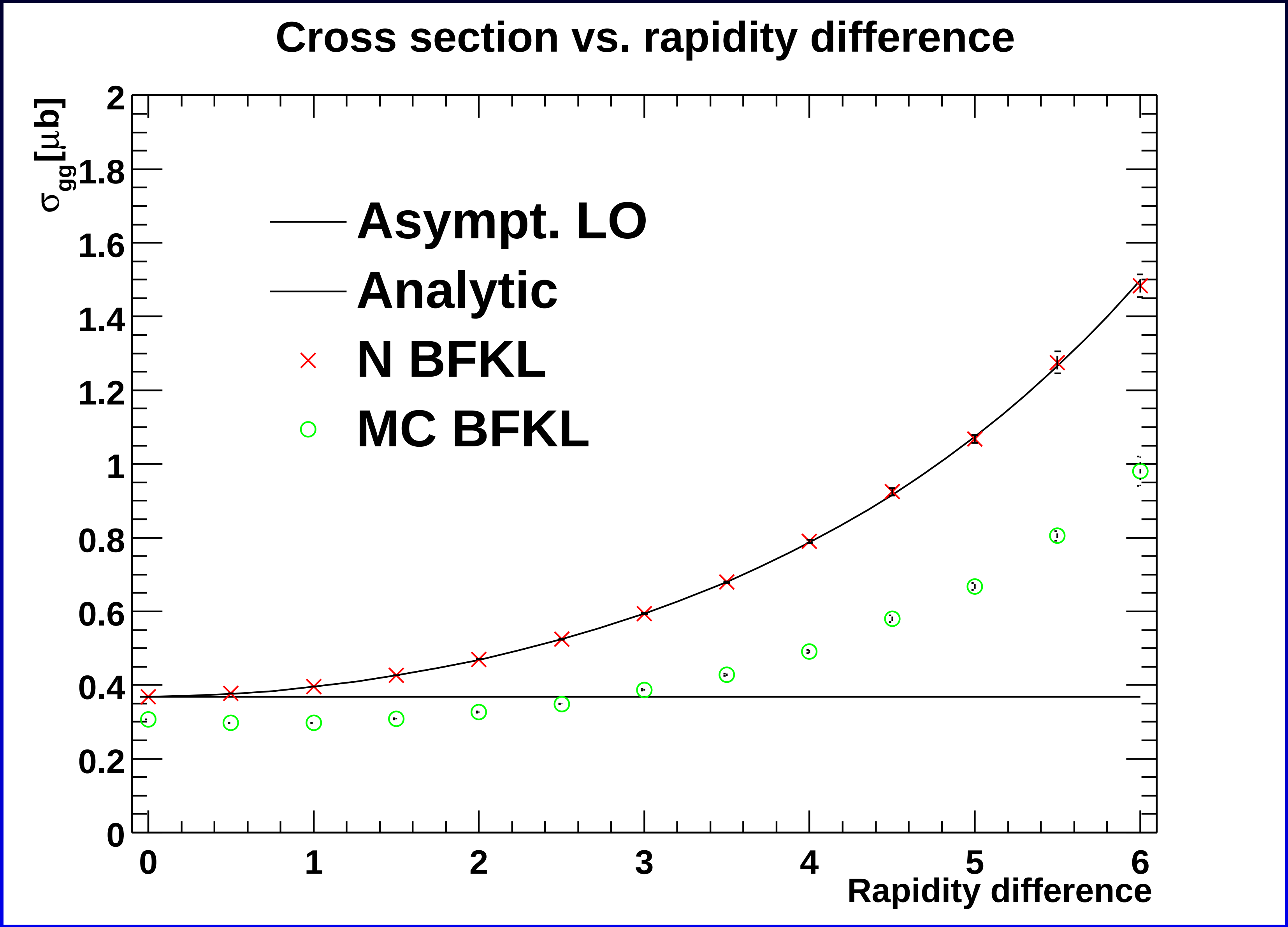
$$f^{(n\geq 1)}(\vec{q}_{a\perp}, \vec{q}_{b\perp}, \Delta y)$$

$$= \left[\frac{\mu^2}{q_{a\perp}^2}\right]^{\bar{\alpha}_s \Delta y} \left\{\prod_{i=1}^{n} \int d^2\vec{k}_{i\perp} dy_i \mathcal{F}_i\right\} \frac{1}{2}\delta(\vec{q}_{a\perp} - \vec{q}_{b\perp} - \sum_{i=1}^{n} \vec{k}_{i\perp})$$

$$\mathcal{F}_i = \frac{\bar{\alpha}_s}{\pi k_{i\perp}^2} \theta(k_{i\perp}^2 - \mu^2) \theta(y_{i-1} - y_i) \left[\frac{(\vec{q}_{a\perp} + \sum_{j=1}^{i-1} \vec{k}_{j\perp})^2}{(\vec{q}_{a\perp} + \sum_{j=1}^{i} \vec{k}_{j\perp})^2}\right]^{\bar{\alpha}_s y_i}$$

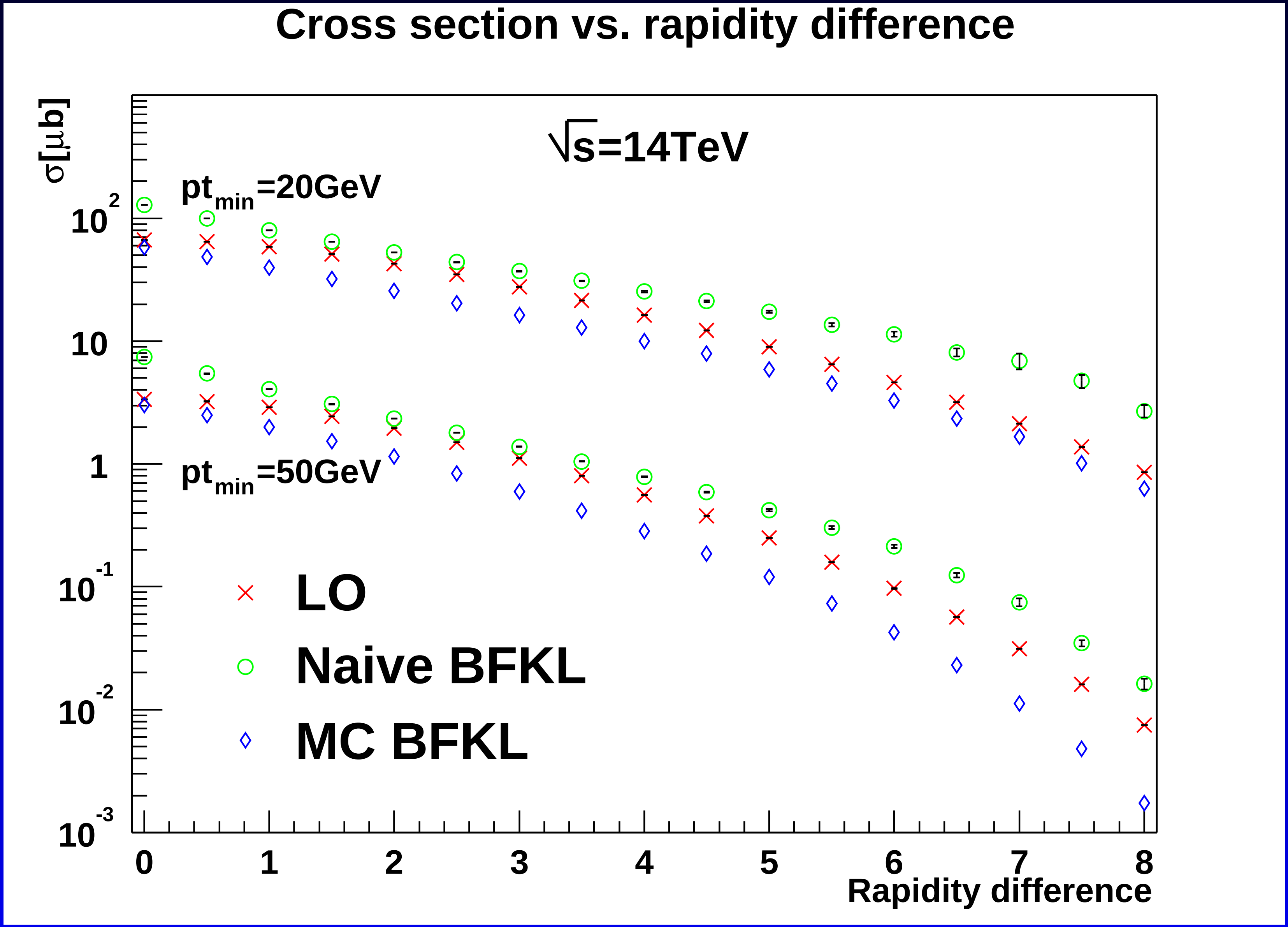
BFKL eq. recast in terms of integrals over resolved gluon phase space. Form factors describes the unresolved radiation. The full sum is still μ–independent. Suitable for MC integration!

Does it work?



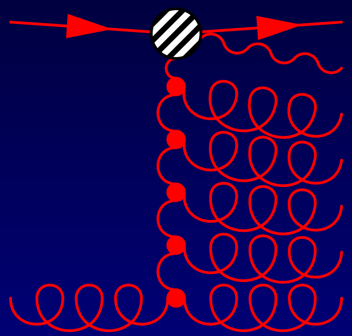
Cross section vs. rapidity difference
σ_gg[μb]
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
Rapidity difference
Asympt. LO
Analytic
N BFKL
MC BFKL

Hadron Cross Sections



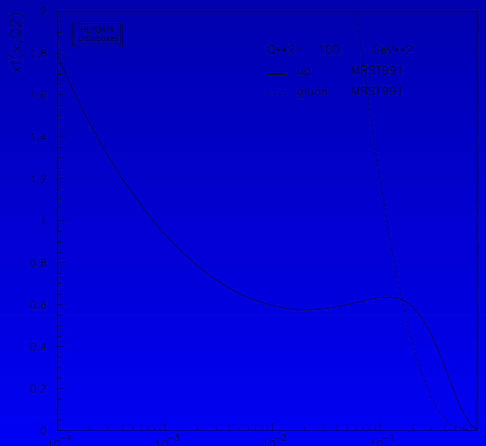
$W + 2$-jet

New setup:

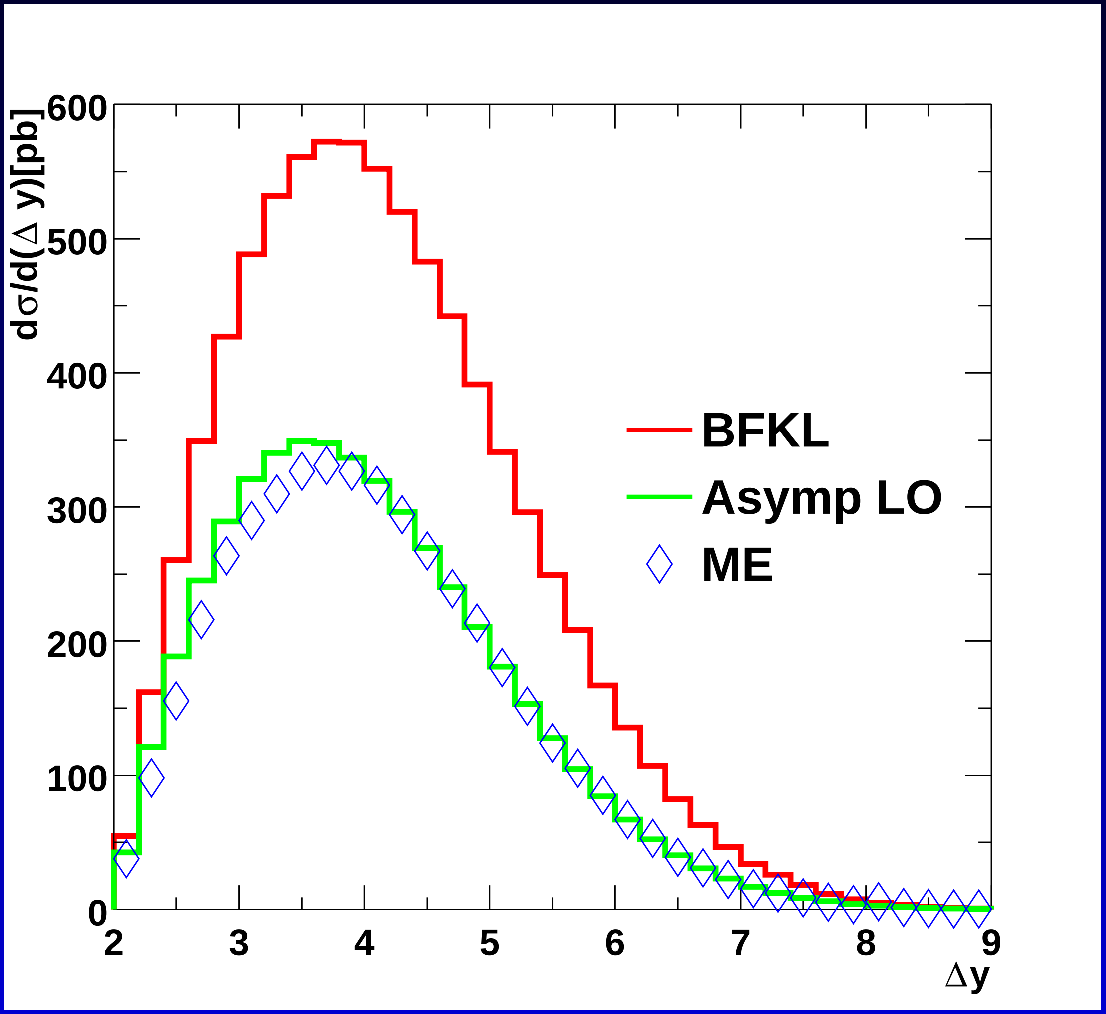


- New Impact Factor
- Same BFKL chain

But now the cross section depends on the quark **and** the gluon pdf The quark pdf is less steep $\longrightarrow$ less pdf suppression $\longrightarrow$ BFKL *enhancement* instead of suppression.



$W + 2$-jet



$$\Delta y = y_{j_2} - y_{j_1}, y_W, y_{j_2} \geq 1, y_{j_1} \leq -1$$

Summary

The Monte Carlo method to study BFKL physics works!

1. It can reproduce (Naive) BFKL
2. It respects energy & momentum conservation
3. It can even include subleading logs!
4. It proved very efficient in incorporating experimental cuts to the momenta of the scattered particles...
5. ... and allows for a further study of the radiation from the BFKL ladder