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# Higher-order non-factorizable corrections

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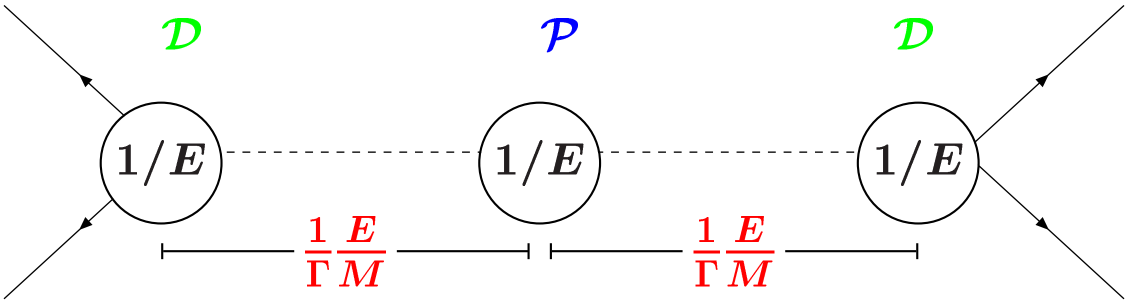
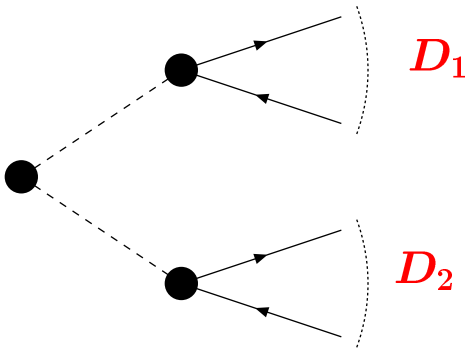
1. **Introduction, 1-loop**
2. **QED**
3. **Effective field theory (QED $\rightarrow$ QCD)**
4. **Applications and Conclusions**

## 1. Introduction

- **Double pole approximation (DPA)**




$$= \mathcal{P}(M, M)\, \frac{\mathcal{D}(M)}{D_1}\, \frac{\mathcal{D}(M)}{D_2} = \mathcal{M}_0$$

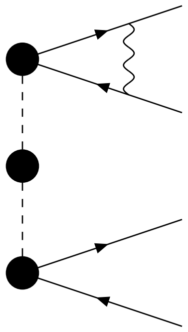
$$D = P^2 - M^2 + i\Gamma M \sim \Gamma M$$

- ∘ DPA = leading $\Gamma/M$

# 1. Introduction

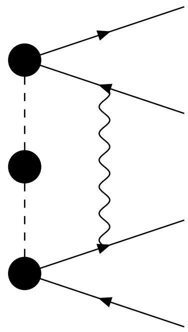
## • 1-loop in DPA

∘ factorizable

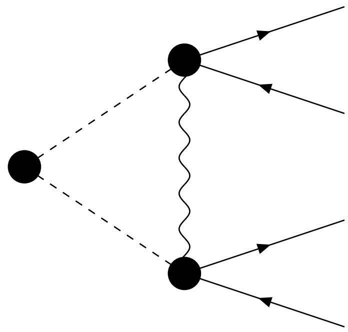


$$+ \quad \ldots \quad \sim \ \mathcal{M}_1 = \alpha \ \ \delta(M,M) \ \ \mathcal{M}_0.$$

∘ non-factorizable



$$+ \quad \ldots \quad =$$



$$\sim \ \mathcal{M}_0 \int_l \mathcal{J}_1 \mathcal{J}_2$$

$$\boldsymbol{\mathcal{J}^{\mu} = e\left[\frac{k}{lk} - \frac{k'}{lk'}\right]^{\mu} \frac{D}{D + 2lp}}$$

| | | |
|---|---|---|
| $\sim \alpha \quad \cdot \Gamma^2 M^2/E^4$ | hard $\boldsymbol{\gamma}$'s | $l \sim E$ |
| $\sim \alpha$ | soft $\boldsymbol{\gamma}$'s | $l \sim \Gamma M/E$ |

## 1. Introduction

- **Distributions**

∘ inv. mass
∘ real radiation
∘ **decay angles**

- **1-loop non-factorizable**

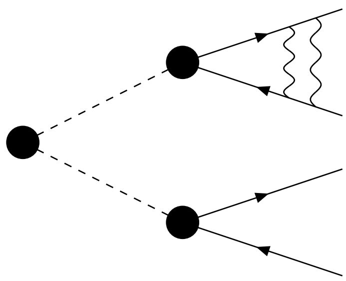
∘ exclusive $\rightarrow$ known
∘ + (real photon) + (inv.mass) $\rightarrow \sim \alpha\Gamma/M$
∘ **+ (decay angles) $\rightarrow \sim \alpha M^4/E^4$**

- **$N$-loop non-factorizable**

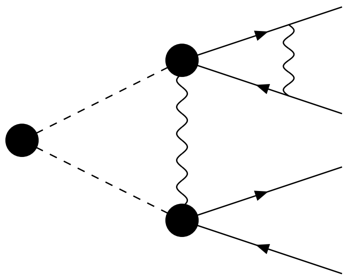
∘ exclusive $\rightarrow$ not known
∘ + (real photon) + (inv.mass) $\rightarrow \sim \alpha^N\Gamma/M$
∘ **+ (decay angles) $\rightarrow$ this talk $\rightarrow$ power suppressed**

## 2. QED

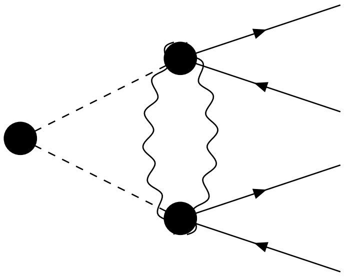
- **2-loop**



$$+ \quad \dots \quad = \mathcal{M}_2$$

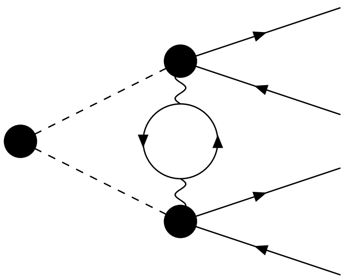


$$+ \quad \dots \quad = \mathcal{M}_1 \int_l \mathcal{J}_1 \mathcal{J}_2$$



$$\sim \frac{1}{2} \mathcal{M}_0 \int_{l_1 l_2} (\mathcal{J}_1 \mathcal{J}_2)(l_1) * (\mathcal{J}_1 \mathcal{J}_2)(l_2)$$

$$\mathcal{J}^\mu(l_1) * \mathcal{J}^\nu(l_2) = e^2 \left( \frac{k}{l_1 k} - \frac{k'}{l_1 k'} \right)^\mu \left( \frac{k}{l_2 k} - \frac{k'}{l_2 k'} \right)^\nu \frac{D}{D + 2 l_1 p + 2 l_2 p}$$



$$\sim \mathcal{M}_0 \int_{l_1, l_2} (\mathcal{J}_1 \mathcal{J}_2) \; e^2 \frac{l_1^2}{l_2^2} \mathrm{Tr} \left( \frac{1}{\not{l}_1} \frac{1}{\not{l}_2 - \not{l}_1} \right)$$

$$\boldsymbol{\alpha}^2 = \boldsymbol{\alpha}_{\text{fact}}^2 + \boldsymbol{\alpha}_{\text{fact}} \, \boldsymbol{\alpha}_{\text{nf}} + \boldsymbol{\alpha}_{\text{nf}}^2 \qquad \boldsymbol{\alpha}_{\text{nf}}^2 = \boldsymbol{\alpha}_{\text{dipole}}^2 + \boldsymbol{\alpha}_{\text{dipole}} \, \boldsymbol{\alpha}_{\text{propag}}$$

- **Integrating over decay products**

$$\boldsymbol{I} = \int_{k\ k'} \left(\frac{\boldsymbol{k}}{\boldsymbol{k}\boldsymbol{l}_1} - \frac{\boldsymbol{k}'}{\boldsymbol{k}'\boldsymbol{l}_1}\right)^{\alpha_1} \cdots \left(\frac{\boldsymbol{k}}{\boldsymbol{k}\boldsymbol{l}_N} - \frac{\boldsymbol{k}'}{\boldsymbol{k}'\boldsymbol{l}_N}\right)^{\alpha_N}$$

$$k \sim k' \sim p \sim E, \qquad k - k' \sim M, \qquad l \sim \Gamma M/E$$

$\boldsymbol{N}$ – odd $\qquad \boldsymbol{\delta}_{\text{dipole}} = \boldsymbol{\alpha}^{\boldsymbol{N}} \cdot \mathbf{0} \qquad \rightarrow \quad \boldsymbol{\alpha}^{\boldsymbol{N}} \left(\frac{\boldsymbol{M}}{\boldsymbol{E}}\right)^{2(\boldsymbol{N}+1)}$

$\boldsymbol{N}$ – even $\qquad \boldsymbol{\delta}_{\text{dipole}} \sim \boldsymbol{\alpha}^{\boldsymbol{N}} \left(\frac{\boldsymbol{M}}{\boldsymbol{E}}\right)^{2\boldsymbol{N}}$

- **Energy scales**

$$\Lambda_1 = E, \quad \Lambda_2 = \Gamma M/E$$

- **Offshellness**

$$D = p^2 - M^2 \quad \sim \quad 0, \quad \Lambda_2^2, \quad \Lambda_1\Lambda_2, \quad \Lambda_1^2,$$

- **Modes**

$$\begin{array}{r|l} P \uparrow & \\ & \text{hard} \\ \Lambda_1 - & \text{resonant} \\ & \text{external} \\ & \text{resonant} \\ \Lambda_2 - & \text{soft} \\ 0 - & \end{array} \qquad\qquad \begin{array}{r|ll} D \uparrow & & \\ \Lambda_1^2 - & \Phi(P) & \text{hard} \\ \Lambda_1\Lambda_2 - & \Phi(P_{\mathrm{M}}, p) & \text{resonant} \\ \Lambda_2^2 - & \Phi(p) & \text{soft} \\ 0 - & \Phi(P_{\mathrm{M}}) & \text{external} \end{array}$$

$$\mathcal{L}_{\mathrm{eff}} = \sum_n \frac{c_n}{\Lambda^n}\, \mathcal{O}_n(\phi).$$

- **Underlying theory**

$$\mathcal{L} = \phi(p^2 - M^2)\phi + \bar{\psi}\ \not{\mathbf{p}}\psi + F^2 + \mathcal{P}X\phi\phi + \phi\bar{\psi}\mathcal{D}\psi$$

- **Integrating out hard modes, $\boldsymbol{\Phi(P)}$**

$$\mathcal{L}_{\text{eff}} = \phi\ (pP_{\text{M}} + iM\Gamma)\ \phi\ +\ \bar{\psi}\ (\not{P}_0 + \not{\mathbf{p}})\ \psi + F^2 \\ + c_{\mathcal{P}}\ \phi\ X\mathcal{P}(P_{\text{M}})\ \phi + c_{\mathcal{D}}\ \phi\ \bar{\psi}\ \mathcal{D}(P_{\text{M}})\ \psi$$

◦ **equivalent to DPA**
$c_{\mathcal{P}},\ c_{\mathcal{D}}$ = factorizable
dynamical = non-factorizable

- $\boxed{\textbf{separation factorizable / non-factorizable}}$

- $\boxed{\mathcal{O}(\Lambda_2/\Lambda_1)\ \textbf{beyond DPA}}$

- **Integrating out resonant modes, $\Phi(P_{\mathrm{M}}, p)$**

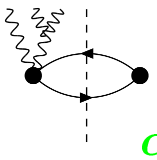
$$\mathcal{L}_{\rm eff} \;=\; F^2 \;+\; \bar\psi_s \, \not{\mathbf{p}} \, \psi_s \;+\; \bar\psi \, \not{P}_0 \, \psi$$

$$+\;\; \boldsymbol{\Lambda(x,y,z)} \;\cdot\; (\bar{\boldsymbol\psi} \boldsymbol{\mathcal{D}}_{\rm int} \boldsymbol\psi)(\boldsymbol x) \;\cdot\; \boldsymbol{\mathcal{P}(y)} \;\cdot\; (\bar{\boldsymbol\psi} \boldsymbol{\mathcal{D}}_{\rm int} \boldsymbol\psi)(\boldsymbol z)$$

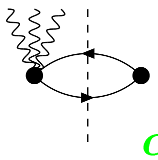
$$\boldsymbol{\sim \frac{1}{(\Lambda_1 \Lambda_2)^2}} \qquad\qquad \boldsymbol{\mathcal{D}\big(1 + F + F^2 + \ldots\big)}$$

$$(\bar{\boldsymbol\psi} \boldsymbol{\mathcal{D}}_{\rm int} \boldsymbol\psi)\Big|_{\boldsymbol F} = \int \Sigma_{\mu\nu} \;\cdot\; \bar\psi \; \mathcal{D} \; \psi \;\cdot\; F_{\mu\nu}$$

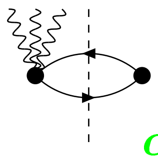
$$(\bar{\boldsymbol\psi} \boldsymbol{\mathcal{D}}_{\rm int} \boldsymbol\psi)\Big|_{\boldsymbol{F^2}} = \int \Sigma_{\mu\nu\sigma\rho} \;\cdot\; \bar\psi \; \mathcal{D} \; \psi \;\cdot\; \mathrm{Tr}\Big(F_{\mu\nu} \; U \; F_{\sigma\rho} \; U\Big)$$



$\boldsymbol\alpha^{\mathbf 2}_{\rm dipole} \boldsymbol\alpha_{\rm propag}$



$\boldsymbol\alpha^{\mathbf 2}_{\rm dipole} \boldsymbol\alpha_{\rm propag}$



$\boldsymbol\alpha^{\mathbf 3}_{\rm dipole}$

◦ $\boxed{\textbf{separation dipole / propagation correction}}$

- **Scaling and estimates**

- **Selection rule**

∘ $\sim eP_0^\mu/(pP_0) \sim e/\Lambda_2 \quad \rightarrow \quad eF/\Lambda_2^2 \quad \rightarrow \quad (eF/\Lambda_2^2)^n$

$$\left(\bar{\boldsymbol{\psi}}\boldsymbol{\mathcal{D}}_{\rm int}\boldsymbol{\psi}\right)\Big|_{\boldsymbol{F^n}} \sim \frac{P_{\rm M}^{2n+m}p^m}{(\Lambda_1\Lambda_2)^{2n+m}}\,(eF)^n\;\;(\bar{\psi}\mathcal{D}\psi)$$

∘ $n=1\ m=1$

$$\left(\bar{\boldsymbol{\psi}}\boldsymbol{\mathcal{D}}_{\rm int}\boldsymbol{\psi}\right)\Big|_{\boldsymbol{F}} \sim \frac{P_{\rm M}^\mu\,p^\nu\,P_{\rm M}^2}{(\Lambda_1\Lambda_2)^3}\;\;eF_{\mu\nu}\;\;(\bar{\psi}\mathcal{D}\psi)$$

∘ estimate

$$\boldsymbol{\delta_1 \sim}\frac{\boldsymbol{\Lambda_2^4}}{\boldsymbol{\Lambda_2^2}}\cdot\frac{\boldsymbol{e}}{\boldsymbol{\Lambda_2}}\frac{\boldsymbol{P^2}}{\boldsymbol{\Lambda_1^2}}\cdot\frac{\boldsymbol{e}}{\boldsymbol{\Lambda_2}}\frac{\boldsymbol{P^2}}{\boldsymbol{\Lambda_1^2}}\boldsymbol{\sim e^2\frac{M^4}{E^4}}$$

$$\boldsymbol{n} - \text{odd} \qquad \boldsymbol{\delta_n \sim \alpha^n\left(\frac{M}{E}\right)^{2(n+1)}}$$

$$\boldsymbol{n} - \text{even} \qquad \boldsymbol{\delta_n \sim \alpha^n\left(\frac{M}{E}\right)^{2n}}$$

## 4. Application and Conclusions

- **Applications**

◦ $W$ and $Z$ bosons

- **Conclusions**

- **3 effects**

◦ factorizable
◦ **non-factorizable due to dipole interaction**
◦ non-factorizable due to propagation

- **Beyond DPA: $\mathcal{O}(\Gamma/M)$-correction**

- **Estimates of non-factorizable corrections**

◦ **suppressed at $E \gg M$**

- **Unsolved**

◦ $\mathcal{O}(\Gamma/M)$-correction
◦ Production-decay nf-correction for charged particles
Similar estimates are valid after real radiation added