

Impact of electroweak and forward measurements on **P**arton **D**istribution **F**unctions (**PDFs**)

Graeme Watt
(CERN PH-TH)

479.WE-Heraeus-Seminar on “Physics at LHCb”
28th April 2011, Physikzentrum Bad Honnef, Germany

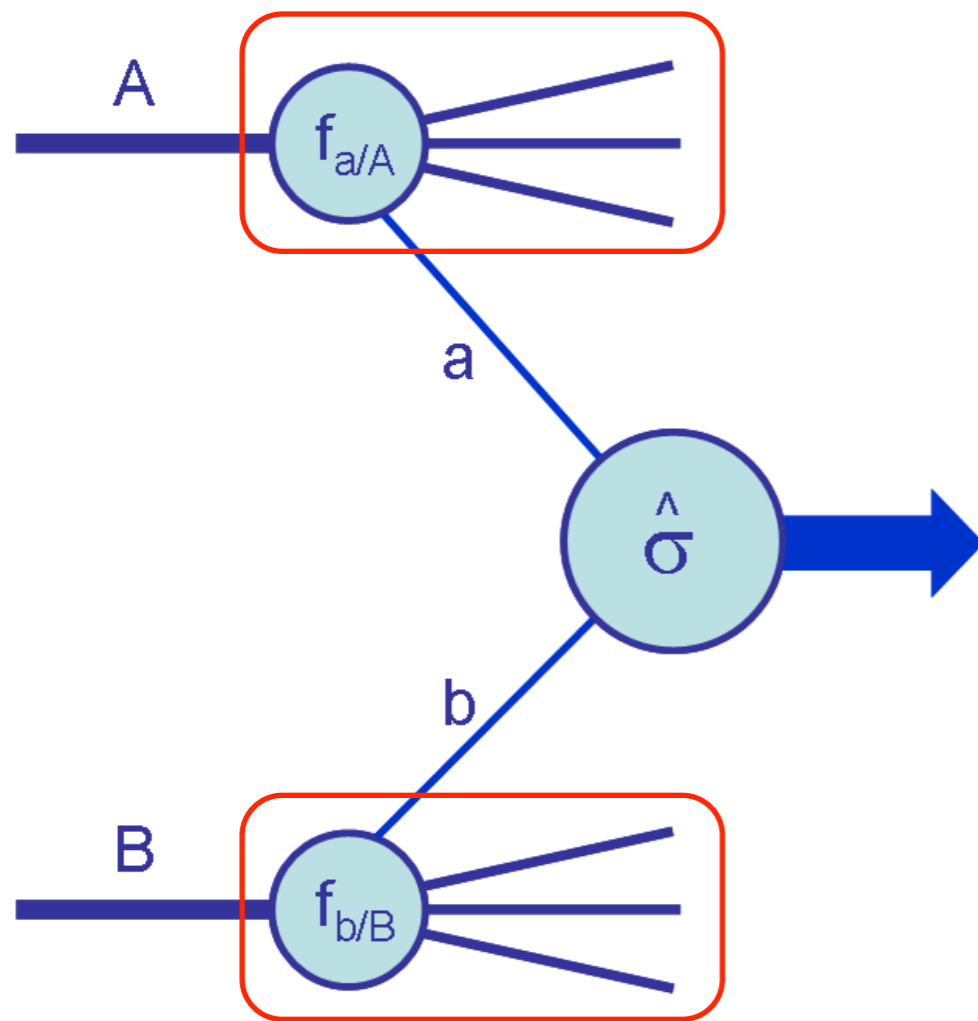
Reference (slightly old, but still relevant):

R. S. Thorne, A. D. Martin, W. J. Stirling and G. Watt,
“Parton Distributions and QCD at LHCb”,
in the proceedings of DIS 2008 [[arXiv:0808.1847](https://arxiv.org/abs/0808.1847)].

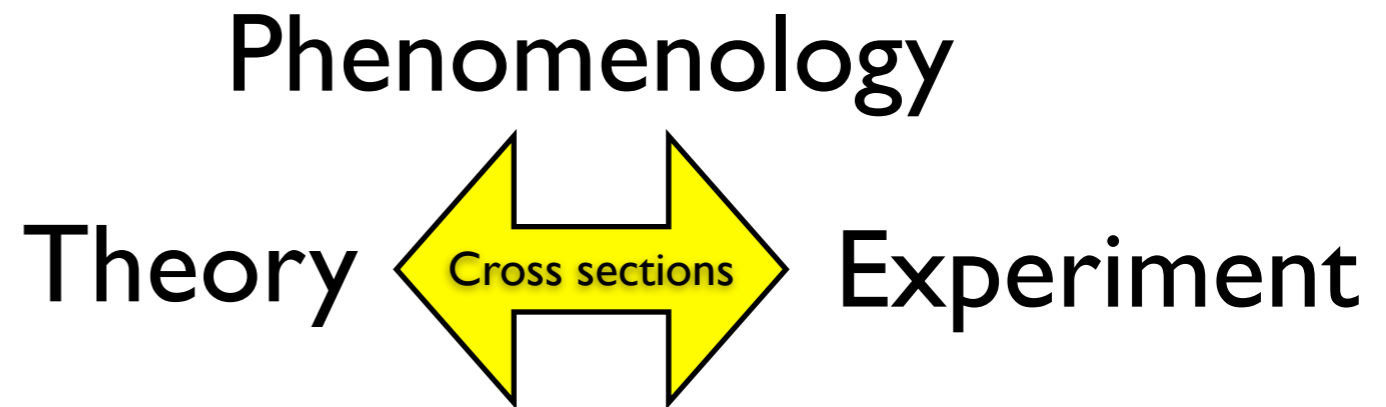
Outline of talk

- Introduction to proton PDFs and global fits.
- Rôle of LHCb electroweak data:
 - W^+ , W^- , Z cross sections (and their ratios).
 - $W^\pm \rightarrow l^\pm \nu$ charge asymmetry.
 - Z rapidity distribution.
- Forward low-mass Drell-Yan production and the theoretical motivation for such a measurement.

$$\text{Event rate} = \text{Cross section } (\sigma) \times \text{Luminosity } (L)$$



Parton Distribution Functions (PDFs)



- Theoretical calculations: Feynman diagrams have initial quarks and gluons.
- **Problem:** the LHC collides **protons**. Need to know density of quarks and gluons (**partons**) inside the proton.

$$\sigma = f_{a/A} \otimes f_{b/B} \otimes \hat{\sigma}_{ab}$$

No predictions for signal or background cross sections without knowledge of PDFs!

QCD factorisation

QCD = Quantum Chromodynamics

(describes interactions between quarks and gluons)

- $f_{a/A}(x, Q^2)$ gives *number density* of partons a in hadron A with momentum fraction x at a hard scale $Q^2 \gg \Lambda_{\text{QCD}}^2$.

$$\sigma_{AB} = \sum_{a,b=q,g} \int_0^1 dx_a \int_0^1 dx_b f_{a/A}(x_a, Q^2) f_{b/B}(x_b, Q^2) \hat{\sigma}_{ab}$$

Perturbative expansion: $\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{\text{LO}} + \alpha_S \hat{\sigma}_{ab}^{\text{NLO}} + \alpha_S^2 \hat{\sigma}_{ab}^{\text{NNLO}} + \dots$

PDF evolution: (DGLAP equation)

$$\frac{\partial f_{a/A}}{\partial \ln Q^2} = \frac{\alpha_S}{2\pi} \sum_{a'=q,g} [P_{aa'}^{\text{LO}} + \alpha_S P_{aa'}^{\text{NLO}} + \dots] \otimes f_{a'/A}$$

α_S evolution:

$$\frac{\partial \alpha_S}{\partial \ln Q^2} = -\beta^{\text{LO}} \alpha_S^2 - \beta^{\text{NLO}} \alpha_S^3 - \dots$$

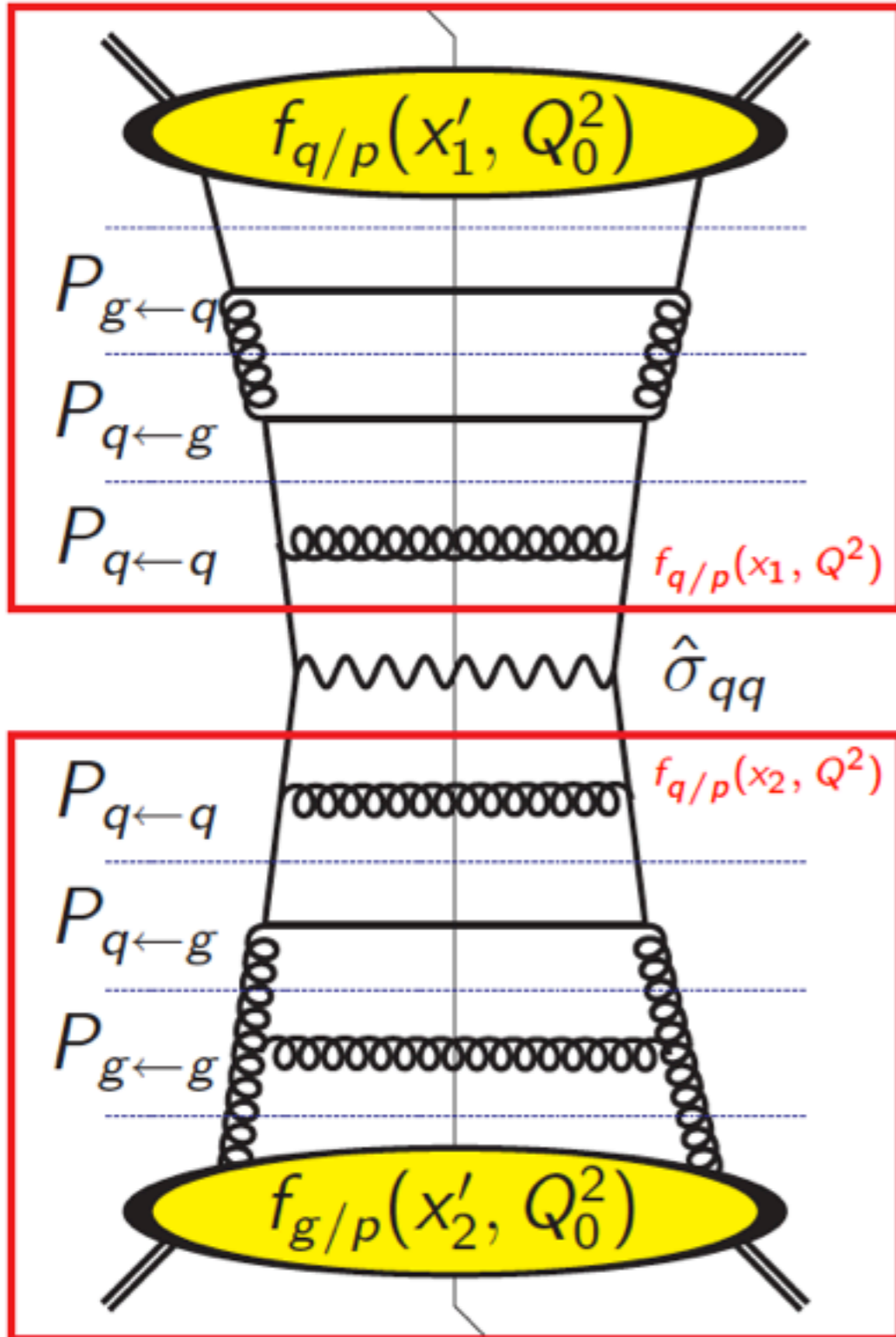
- Need to extract input values $f_{a/A}(x, Q_0^2)$ and $\alpha_S(M_Z^2)$ from data.

NLO is standard.

NNLO
P_{aa'} (2004)
and 2→1.

Lattice
QCD?

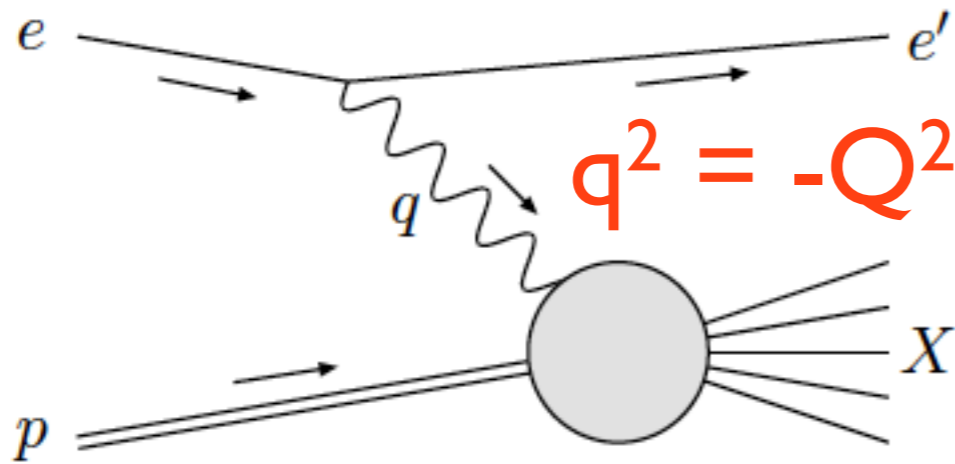
Diagrammatic interpretation



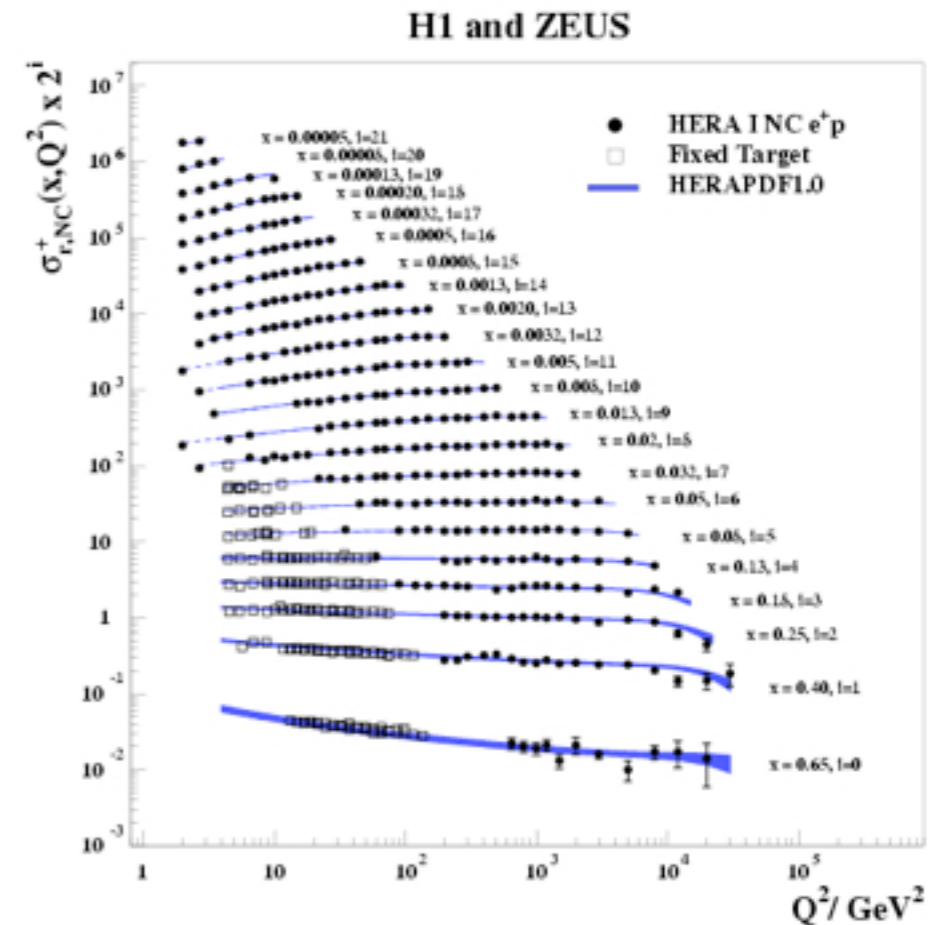
- Drell–Yan production at LO:
 $q\bar{q} \rightarrow V = W/Z/\gamma^*$
- Cut diagram: $|\mathcal{M}|^2 = \mathcal{M}\mathcal{M}^*$
- Large logarithm from collinear gluon emission:
 $\int_{k_0^2}^{Q^2} (dk_T^2/k_T^2) \frac{\alpha_S}{2\pi} P_{q←q}(z)$
- Similar collinear logs from other parton splittings.
- **DGLAP** evolution equation:
$$\frac{\partial f_{a/p}}{\partial \ln Q^2} = \frac{\alpha_S}{2\pi} \sum_{a'=q,g} P_{a←a'} \otimes f_{a'/p}$$
- $f_{a/p}(x, Q_0^2) \Rightarrow f_{a/p}(x, Q^2)$

Deep-inelastic scattering (DIS)

$$F_i(x, Q^2) = \sum_{a=q,g} C_{i,a} \otimes f_{a/A}, \quad C_{i,a} = C_{i,a}^{\text{LO}} + \alpha_S C_{i,a}^{\text{NLO}} + \dots$$

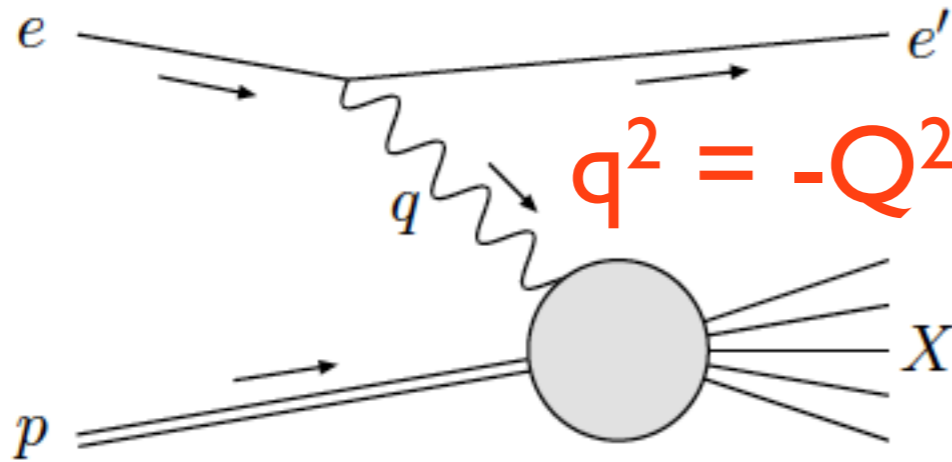


$$F_2^{\text{LO}}(x, Q^2) = \sum_q e_q^2 x f_{q/p}$$



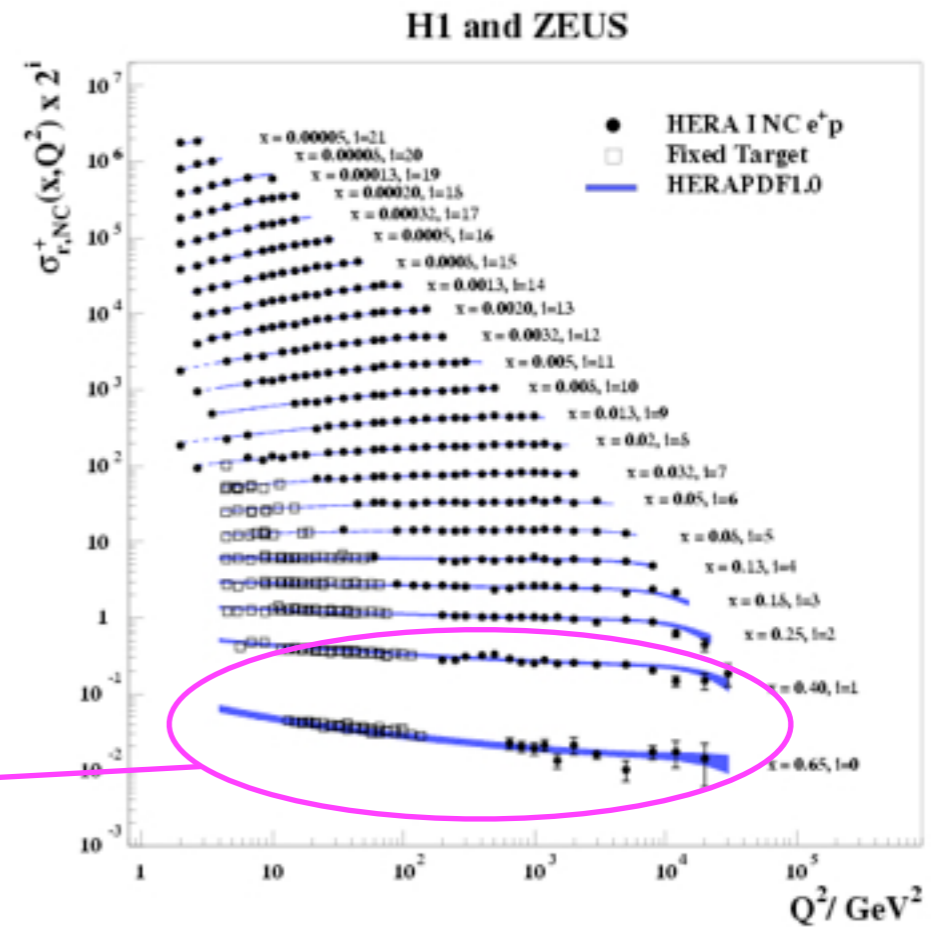
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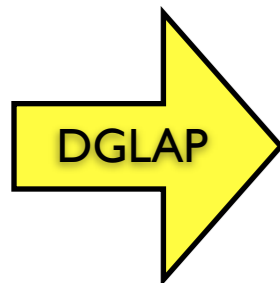


$$F_2^{\text{LO}}(x, Q^2) = \sum_q e_q^2 x f_{q/p}$$

At large x ,
 F_2 falls with Q^2



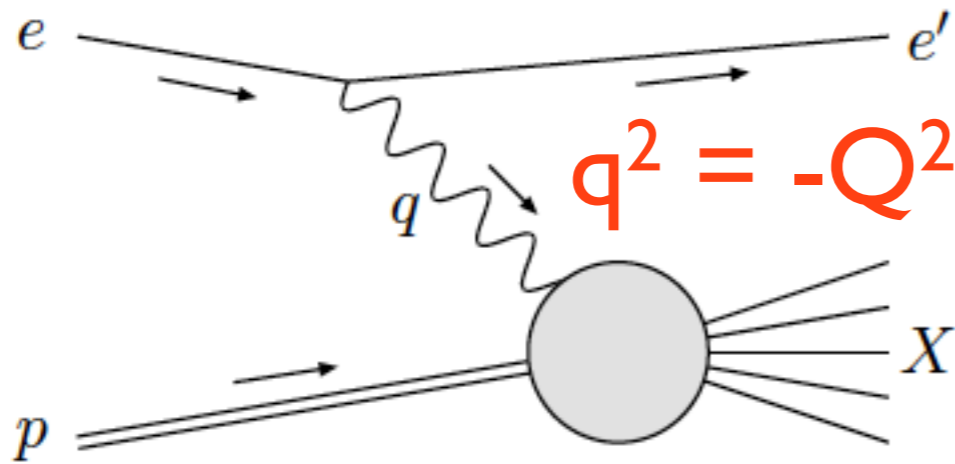
QCD scaling violations



$$\frac{\partial F_2^{\text{LO}}(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_S}{2\pi} \left[P_{q \leftarrow q}^{\text{LO}} \otimes F_2^{\text{LO}} + \sum_q e_q^2 P_{q \leftarrow g}^{\text{LO}} \otimes f_{g/p} \right]$$

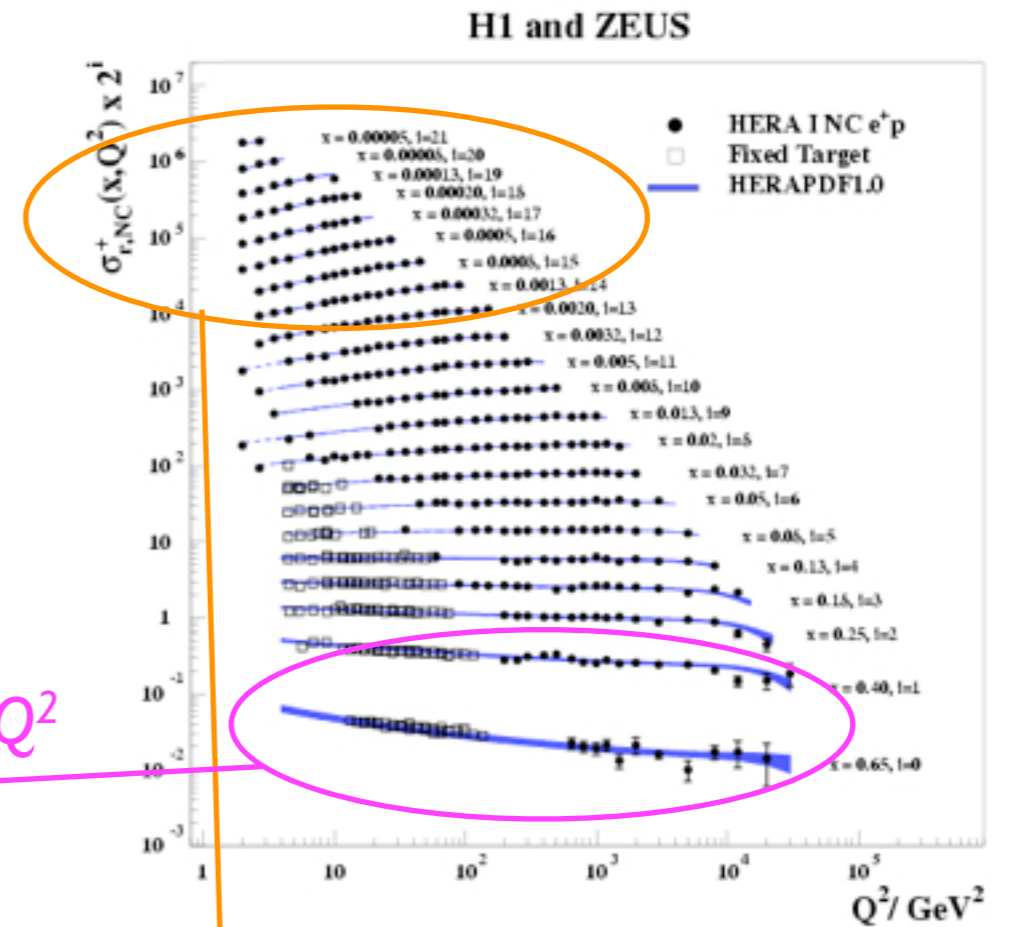
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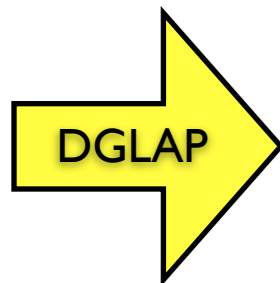


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QCD scaling violations

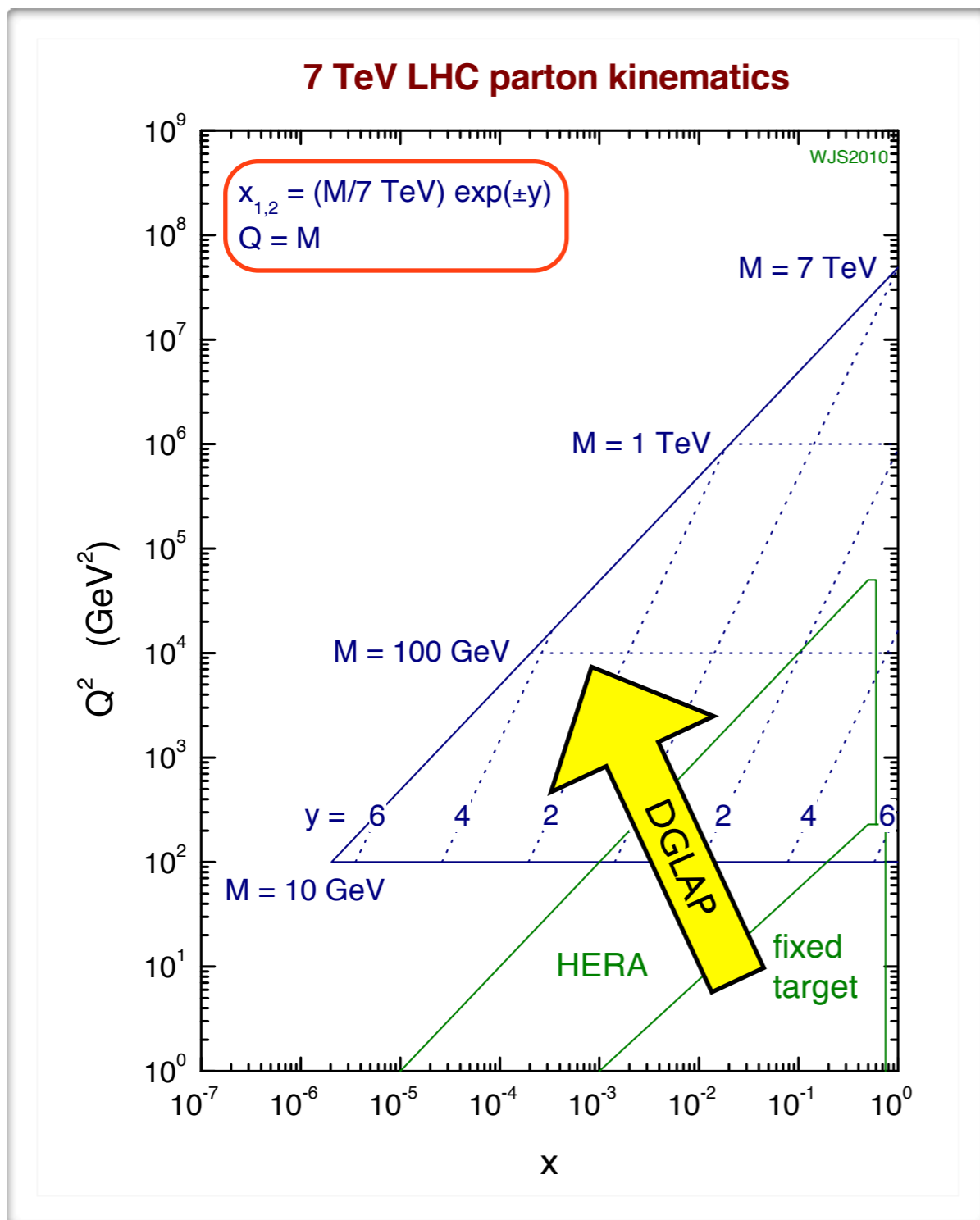


$$\frac{\partial F_2^{\text{LO}}(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_S}{2\pi} \left[P_{q \leftarrow q}^{\text{LO}} \otimes F_2^{\text{LO}} + \sum_q e_q^2 P_{q \leftarrow g}^{\text{LO}} \otimes f_{g/p} \right]$$

At small x ,
 F_2 rises with Q^2

LHC kinematic plane

Produce a final state of mass $M = \sqrt{x_1 x_2 s}$ and rapidity $y = 0.5 \log(x_1/x_2)$ at LHC ($\sqrt{s} = 7 \text{ TeV}$) from two partons with momentum fractions x_1 and x_2 .



- PDFs are **universal**.
- Fit existing data from **HERA** and **fixed-target** experiments, together with **Tevatron** data.
- **HERA** ep (H1, ZEUS).
- **Fixed-target** experiments:
 lp, ld
 (BCDMS, NMC, E665, SLAC),
 νN
 (CCFR, NuTeV, CHORUS),
 pp, pd (E866/NuSea).
- **Tevatron** $p\bar{p}$ (CDF, DØ).
- **DGLAP** evolution gives PDFs at higher Q^2 for LHC.

Which processes constrain different PDFs?

Processes included in MSTW08 global analysis [[arXiv:0901.0002](https://arxiv.org/abs/0901.0002)]:

	Process	Subprocess	Partons	x range
Fixed target	$l^\pm \{p, n\} \rightarrow l^\pm X$	$\gamma^* q \rightarrow q$	q, \bar{q}, g	$x \gtrsim 0.01$
	$l^\pm n/p \rightarrow l^\pm X$	$\gamma^* d/u \rightarrow d/u$	d/u	$x \lesssim 0.01$
	$pp \rightarrow \mu^+ \mu^- X$	$u\bar{u}, d\bar{d} \rightarrow \gamma^*$	\bar{q}	$0.015 \lesssim x \lesssim 0.35$
	$pn/pp \rightarrow \mu^+ \mu^- X$	$(u\bar{d})/(u\bar{u}) \rightarrow \gamma^*$	\bar{d}/\bar{u}	$0.015 \lesssim x \lesssim 0.35$
	$\nu(\bar{\nu}) N \rightarrow \mu^-(\mu^+) X$	$W^* q \rightarrow q'$	q, \bar{q}	$0.01 \lesssim x \lesssim 0.5$
	$\nu N \rightarrow \mu^- \mu^+ X$	$W^* s \rightarrow c$	s	$0.01 \lesssim x \lesssim 0.2$
	$\bar{\nu} N \rightarrow \mu^+ \mu^- X$	$W^* \bar{s} \rightarrow \bar{c}$	\bar{s}	$0.01 \lesssim x \lesssim 0.2$
HERA	$e^\pm p \rightarrow e^\pm X$	$\gamma^* q \rightarrow q$	g, q, \bar{q}	$0.0001 \lesssim x \lesssim 0.1$
	$e^+ p \rightarrow \bar{\nu} X$	$W^+ \{d, s\} \rightarrow \{u, c\}$	d, s	$x \gtrsim 0.01$
	$e^\pm p \rightarrow e^\pm c\bar{c} X$	$\gamma^* c \rightarrow c, \gamma^* g \rightarrow c\bar{c}$	c, g	$0.0001 \lesssim x \lesssim 0.01$
	$e^\pm p \rightarrow \text{jet} + X$	$\gamma^* g \rightarrow q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
Tevatron	$p\bar{p} \rightarrow \text{jet} + X$	$gg, qg, qq \rightarrow 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$
	$p\bar{p} \rightarrow (W^\pm \rightarrow l^\pm \nu) X$	$ud \rightarrow W, \bar{u}\bar{d} \rightarrow W$	u, d, \bar{u}, \bar{d}	$x \gtrsim 0.05$
	$p\bar{p} \rightarrow (Z \rightarrow l^+ l^-) X$	$uu, dd \rightarrow Z$	d	$x \gtrsim 0.05$

Need many processes from different experiments
 \Rightarrow **global** analysis needed for flavour separation.

Paradigm for PDF determination

- 1 **Parameterise** the x dependence for each flavour $a = q, g$ at the input scale $Q_0^2 \sim 1 \text{ GeV}^2$ in some flexible form, e.g.

$$xf_{a/p}(x, Q_0^2) = A_a x^{\Delta_a} (1 - x)^{\eta_a} (1 + \epsilon_a \sqrt{x} + \gamma_a x),$$

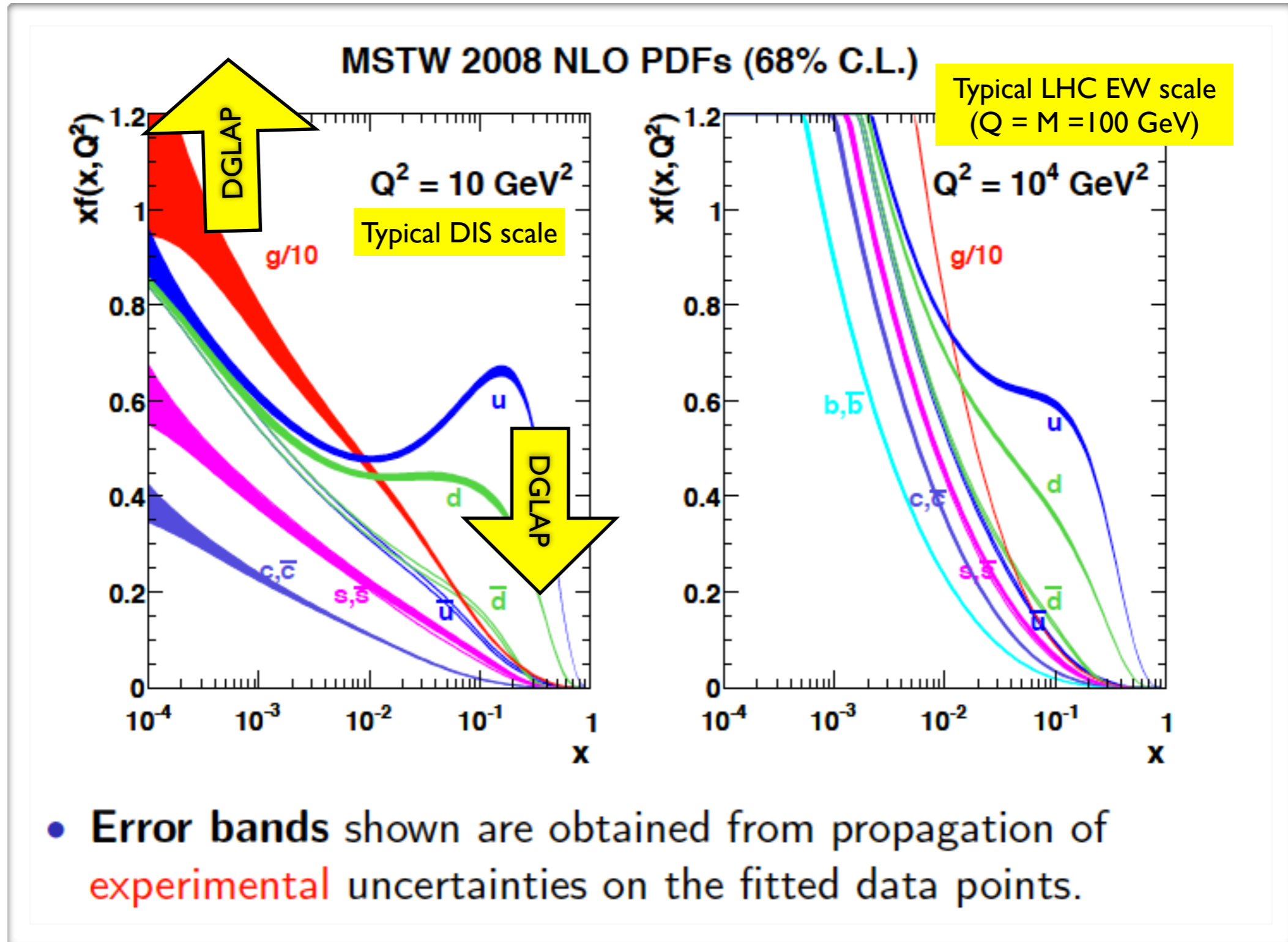
subject to number- and momentum-sum rule constraints.

- 2 **Evolve** the PDFs to higher scales $Q^2 > Q_0^2$ using the DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi) evolution equations.
- 3 **Convolute** the evolved PDFs with $C_{i,a}$ and $\hat{\sigma}_{ab}$ to calculate theory predictions corresponding to a wide variety of data.
- 4 **Vary** the input parameters $\{A_a, \Delta_a, \eta_a, \epsilon_a, \gamma_a, \dots\}$ to minimise

$$\chi^2 = \sum_{i=1}^{N_{\text{pts.}}} \left(\frac{\text{Data}_i - \text{Theory}_i}{\text{Error}_i} \right)^2$$

or generalisations to account for *correlated* systematic errors.

Example of PDFs from global analysis



$$\int_0^1 dx [f_{q/p}(x, Q^2) - f_{\bar{q}/p}(x, Q^2)] = n_q \quad (n_u = 2, n_d = 1, n_s = 0),$$

Number sum rules

$$\sum_{a=q, \bar{q}, g} \int_0^1 dx x f_{a/p}(x, Q^2) = 1$$

Momentum sum rule

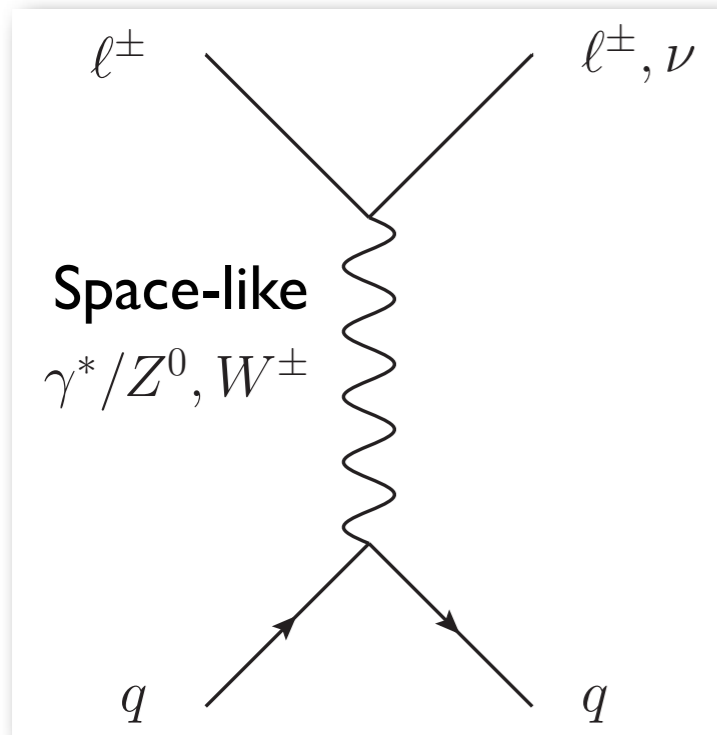
Most recent public PDF fits

Name of fit	Who?	Global?	NNLO?
MSTW08	A. D. M artin, W. J. S tirling, R. S. T horne, G. W att	✓	✓
CT10	CTEQ Collaboration: H.-L. Lai, M. Guzzi, J. Huston, Z. Li, P. Nadolsky, J. Pumplin, C.-P. Yuan	✓	✗
NNPDF2.1	R. Ball, V. Bertone, F. Cerruti, L. Del Debbio, S. Forte, A. Guffanti, J. Latorre, J. Rojo, M. Ubiali	✓	✗
HERAPDF1.0	H1 and ZEUS Collaborations	✗	✓
ABKM09	S. A lekhin, J. B lümlein, S. K lein, S. M och	✗	✓
GJR08/JR09	M. G lück, P. J imenez-Delgado, E. R eya	✗	✓

- “Global” \equiv DIS + fixed-target Drell-Yan + Tevatron W, Z, jets.
- **NNPDF**: parameterise input PDFs by a **N**eural **N**etwork.
- GRV/GJR: more restrictive “dynamical” parameterisation.
- *Three groups with **NLO** global fits, but only one at **NNLO**.*

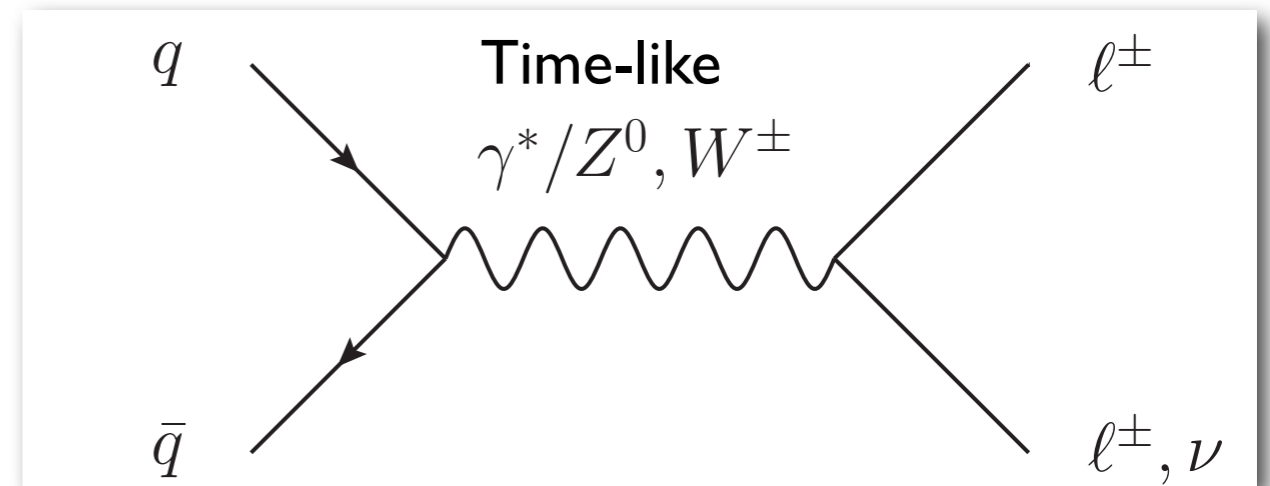
Drell-Yan production at LHC

DIS



Rotate 90°

Drell-Yan



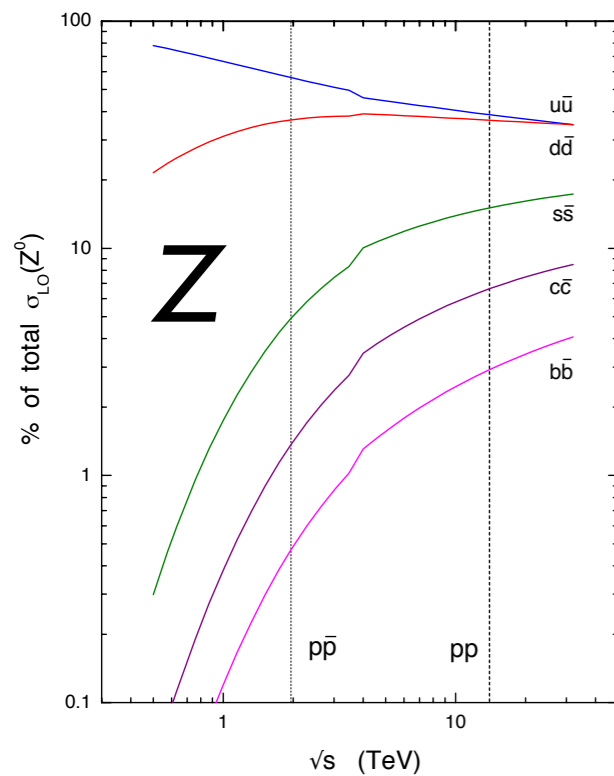
- Analogous process to DIS at a hadron collider.

- Fixed-order calculations known to NNLO:
 - Total cross section [[Hamberg, van Neerven, Matsuura, 1991 \(Erratum, 2002\)](#)].
 - Vector boson rapidity distributions:
[Anastasiou, Dixon, Melnikov, Petriello, hep-ph/0312266](#), [Vrap](#) code.
 - “Fully exclusive” allowing cuts on leptonic decay products:
[Melnikov, Petriello, hep-ph/0609070](#), [FEWZ](#) code;
[Catani, Cieri, Ferrera, de Florian, Grazzini, arXiv:0903.2120](#), [DYNNLO](#) code.

Boson p_T distributions
not described by
fixed-order theory.

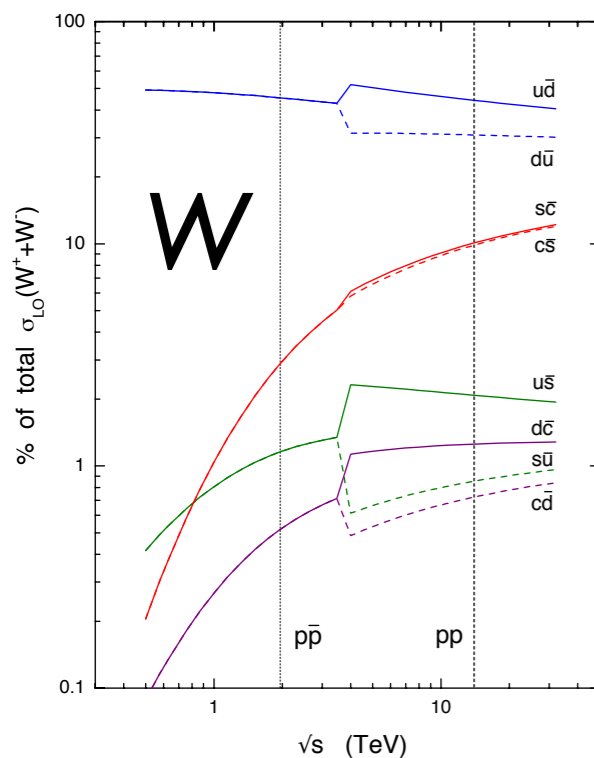
DY flavour decomposition

flavour decomposition of Z^0 cross sections



Plots by J. Stirling

flavour decomposition of W cross sections



$$\sigma_{W^+} \sim u(x_1)\bar{d}(x_2)$$

$$\sigma_{W^-} \sim d(x_1)\bar{u}(x_2)$$

$$\sigma_{Z^0} \sim 0.29 u(x_1)\bar{u}(x_2) + 0.37 d(x_1)\bar{d}(x_2)$$

$$\sigma_{\gamma^*} \sim 0.44 u(x_1)\bar{u}(x_2) + 0.11 d(x_1)\bar{d}(x_2)$$

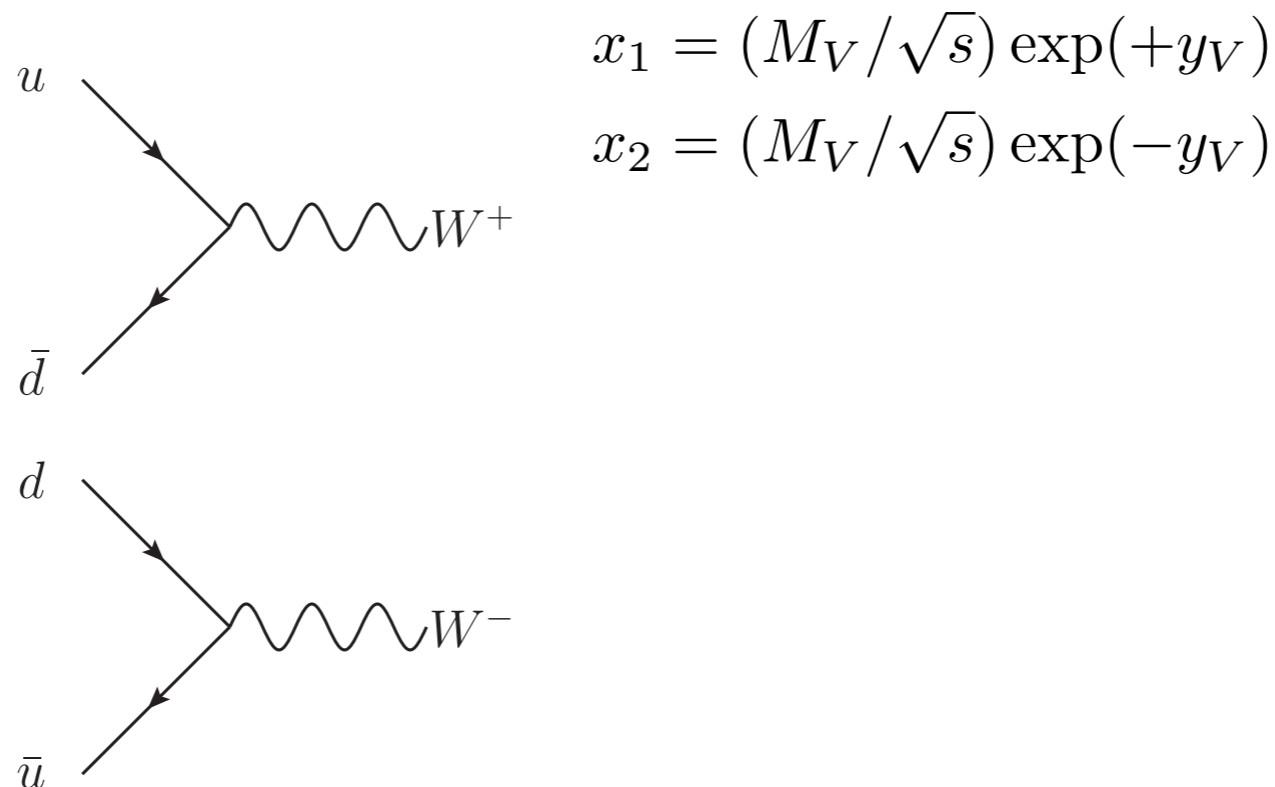
Scale $Q = M_W$.

$$\frac{\sigma_{W^+} + \sigma_{W^-}}{\sigma_{Z^0}} \sim \frac{u(x_1) + d(x_1)}{0.29 u(\tilde{x}_1) + 0.37 d(\tilde{x}_1)}$$

Almost insensitive to PDFs.

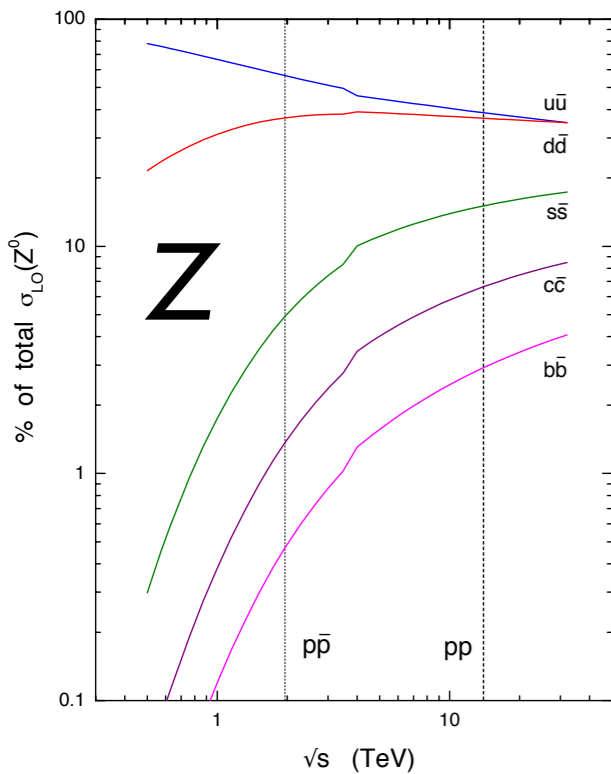
$$\frac{\sigma_{W^+}}{\sigma_{W^-}} \sim \frac{u(x_1)}{d(x_1)}$$

Sensitive to u/d ratio.



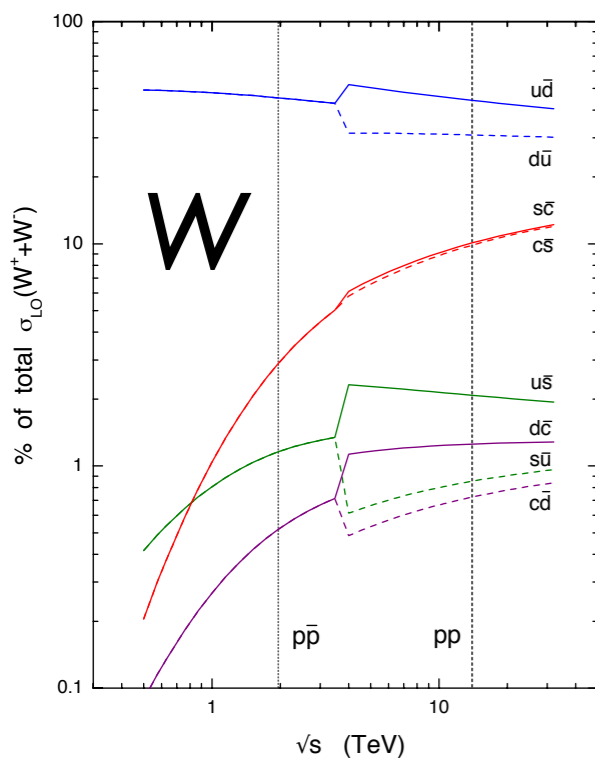
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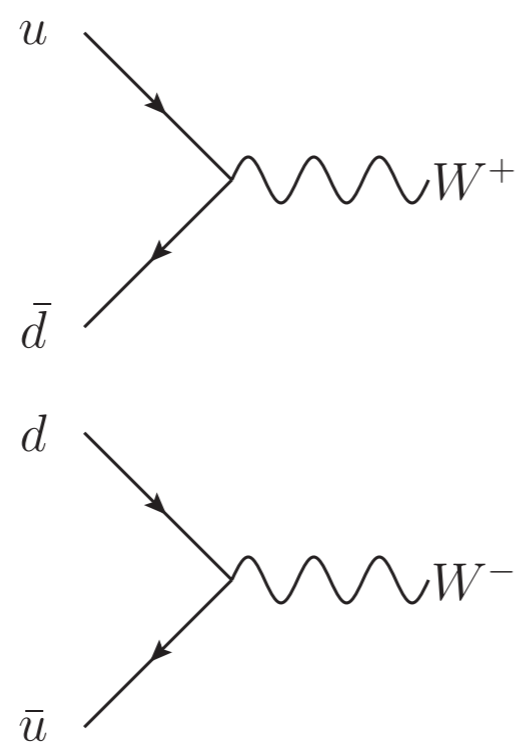
Almost insensitive to PDFs.

Sensitive to u/d ratio.

$$x_1 = (M_V / \sqrt{s}) \exp(+y_V)$$

$$x_2 = (M_V / \sqrt{s}) \exp(-y_V)$$

Structure functions need nuclear corrections.



Neutral-current DIS in charged-lepton-nucleon scattering

$$F_2^{\ell^\pm p} = x \left[\frac{4}{9} (u + \bar{u} + \dots) + \frac{1}{9} (d + \bar{d} + \dots) \right]$$

$$F_2^{\ell^\pm d} = (F_2^{\ell^\pm p} + F_2^{\ell^\pm n})/2 = \frac{5}{18} x (u + \bar{u} + d + \bar{d}) + \dots$$

Charged-current DIS in neutrino-nucleus scattering

$$F_2 \equiv (F_2^{\nu N} + F_2^{\bar{\nu} N})/2 = x (u + \bar{u} + d + \bar{d} + \dots)$$

$$xF_3 \equiv (xF_3^{\nu N} + xF_3^{\bar{\nu} N})/2 = x (u - \bar{u} + d - \bar{d} + \dots)$$

NLO partonic luminosities

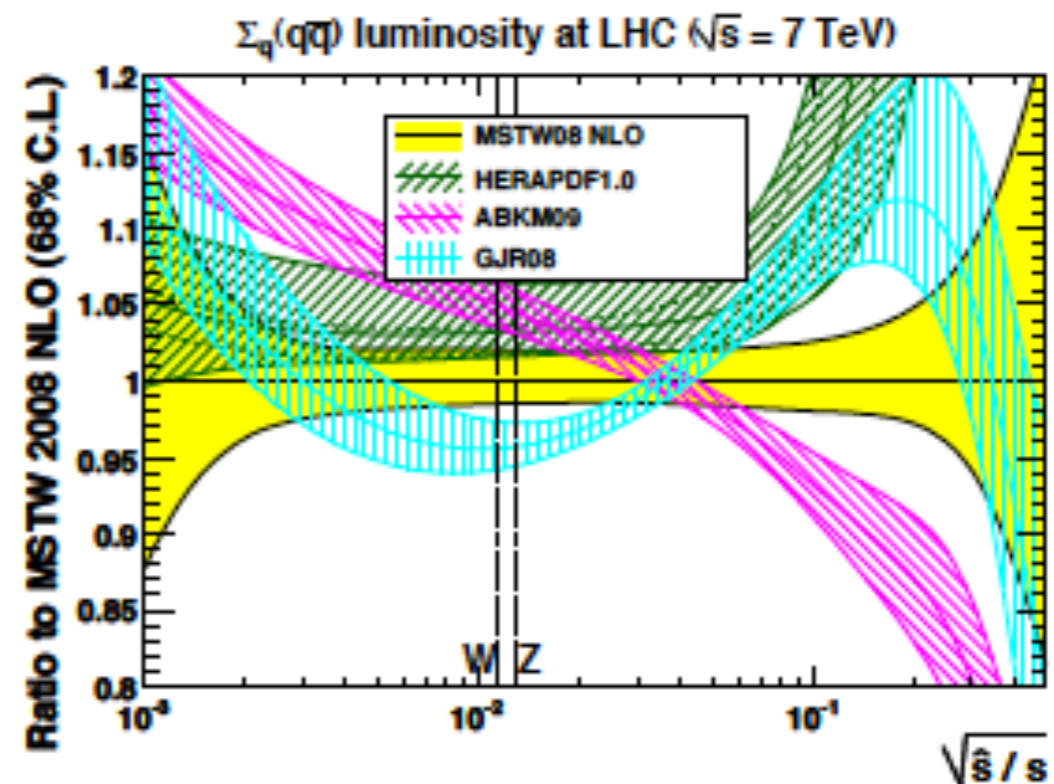
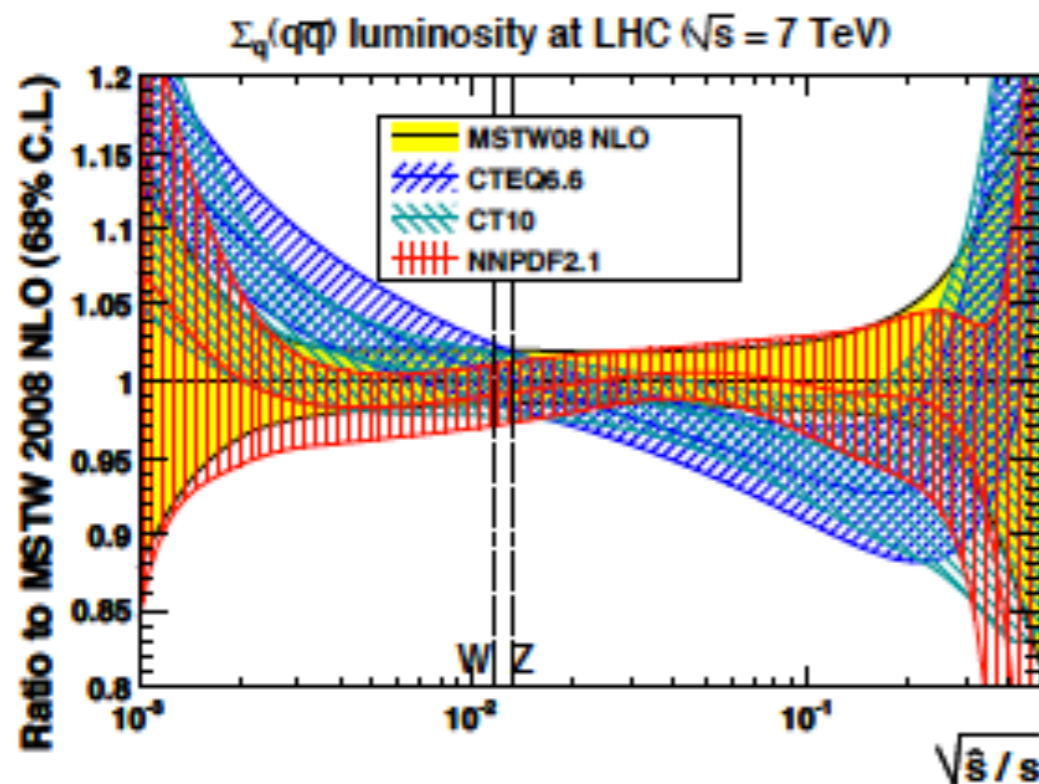
$$\frac{\partial \mathcal{L}_{\Sigma_q(q\bar{q})}}{\partial \tau} = \int_0^{y_{\max}} dy \sum_{q=d,u,s,c,b} [f_q(x_1, \hat{s}) f_{\bar{q}}(x_2, \hat{s}) + (q \leftrightarrow \bar{q})]$$

$$\tau \equiv \sqrt{\hat{s}/s}$$

$$y_{\max} = \ln(1/\sqrt{\tau})$$

$$x_1 = \sqrt{\tau} e^y$$

$$x_2 = \sqrt{\tau} e^{-y}$$



- Relevant values of $\sqrt{\hat{s}} = M_{W,Z}$ are indicated: good agreement for global fits (*left*), but more variation for other sets (*right*).

NNLO partonic luminosities

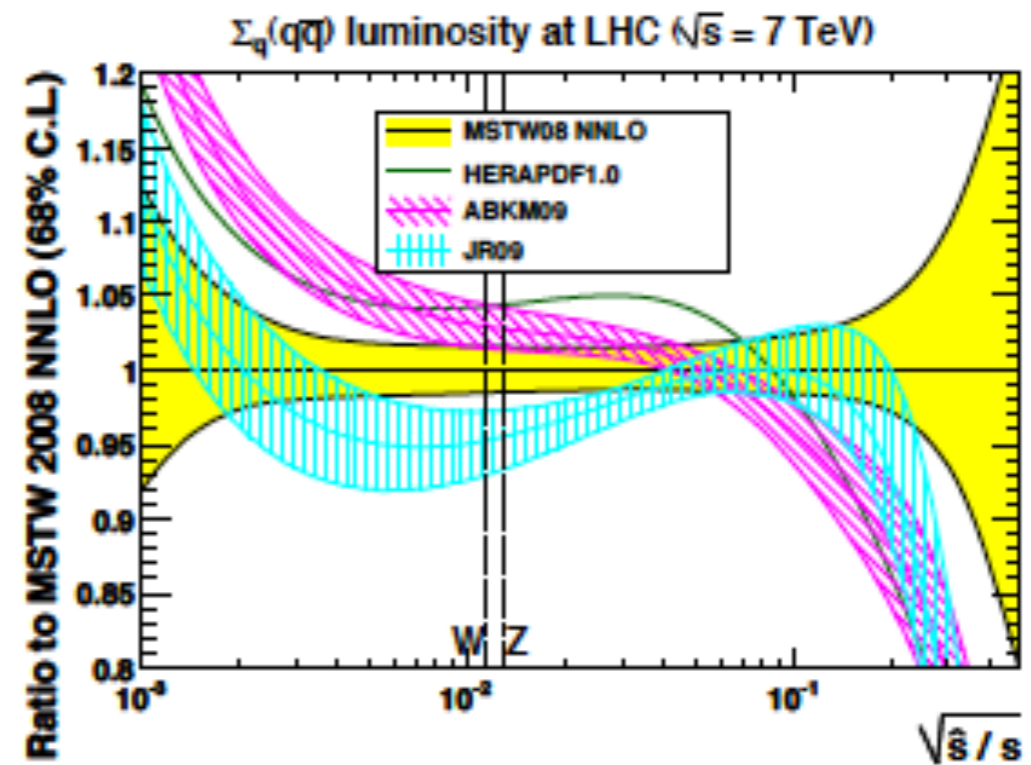
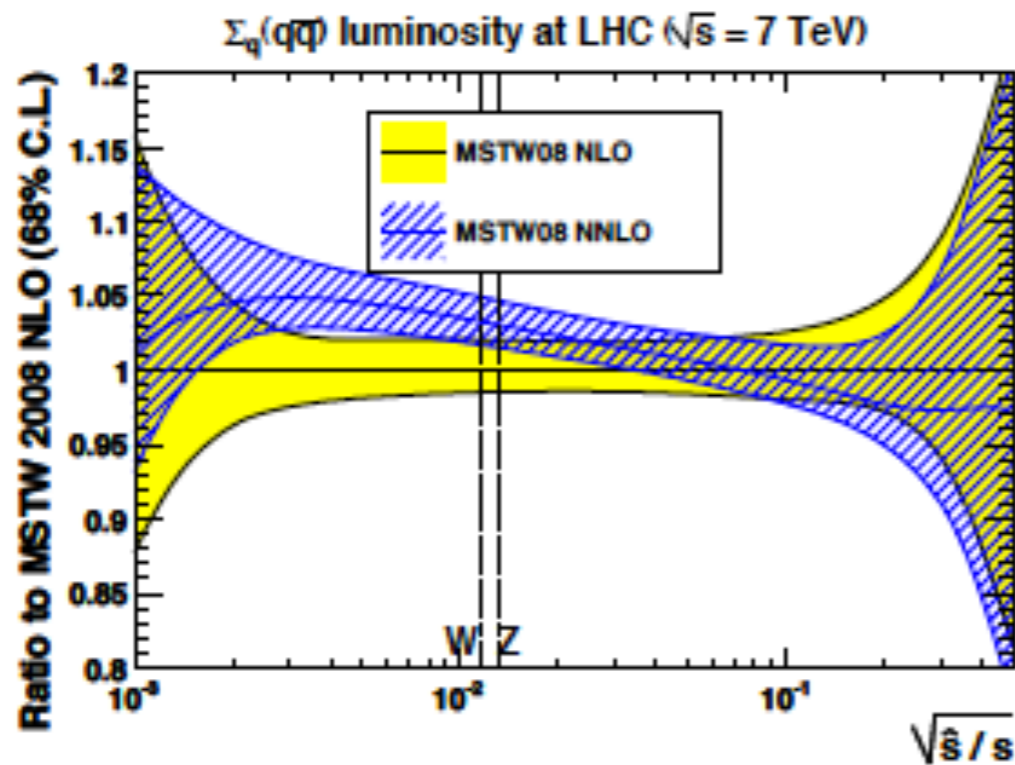
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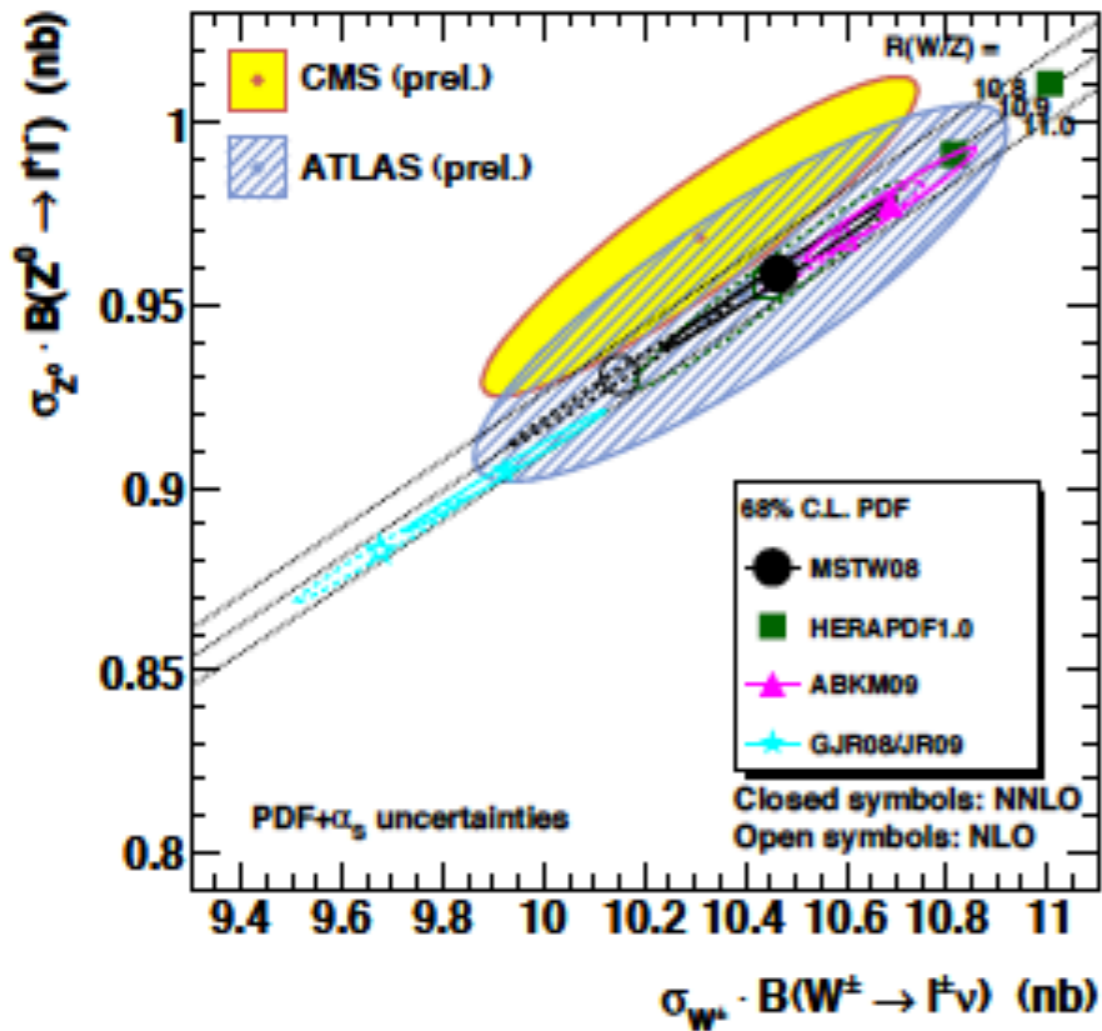


- **Left plot:** compare MSTW NLO and NNLO luminosities.
- **Right plot:** NNLO trend between groups similar to NLO.

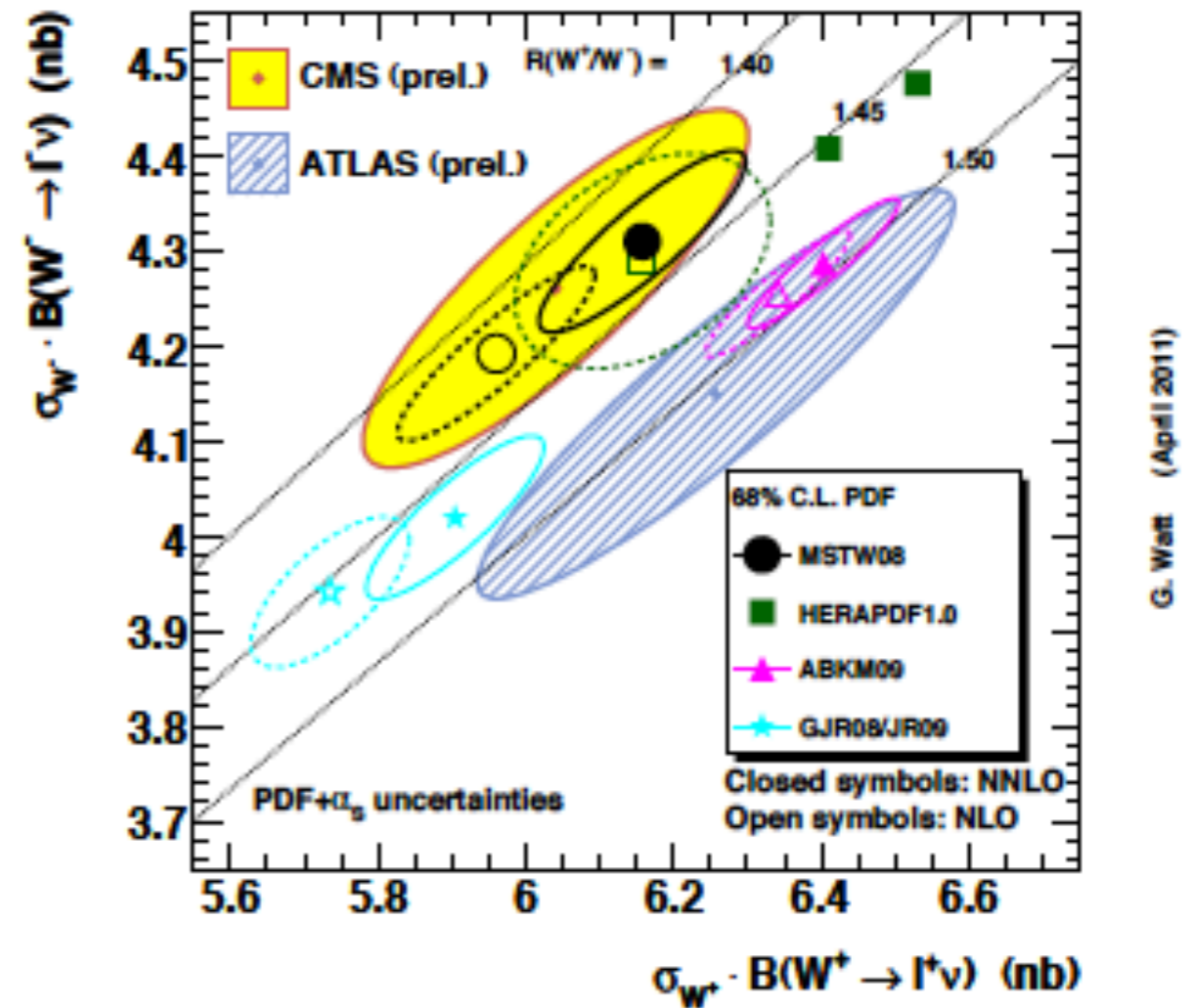
NNLO total cross sections

- Correlation of ellipse \Leftrightarrow uncertainty in ratio of cross sections.

NNLO W and Z cross sections at the LHC ($\sqrt{s} = 7$ TeV)



NNLO W^+ and W^- cross sections at the LHC ($\sqrt{s} = 7$ TeV)



- Apparent discrepancy between CMS and ATLAS for W^+/W^- . CMS ratio favours **MSTW08**, ATLAS favours **ABKM09** (?).

Acceptance: a cautionary tale

- Only about half total cross sections inside ATLAS/CMS acceptance.

- CMS [[arXiv:1012.2466](#), CMS PAS EWK-10-005] use POWHEG (with CTEQ6.6/CT10, MSTW08NLO, NNPDF2.0), check with FEWZ.

$$\Rightarrow \frac{\sigma_{W^+}^{\text{tot}}}{\sigma_{W^-}^{\text{tot}}} = \begin{cases} 1.418 \pm 0.008(\text{stat.}) \pm 0.022(\text{syst.}) \pm 0.029(\text{th.}) & (e) \\ 1.423 \pm 0.008(\text{stat.}) \pm 0.019(\text{syst.}) \pm 0.031(\text{th.}) & (\mu) \\ \boxed{1.421} \pm 0.006(\text{stat.}) \pm 0.014(\text{syst.}) \pm 0.030(\text{th.}) & (e + \mu) \end{cases}$$

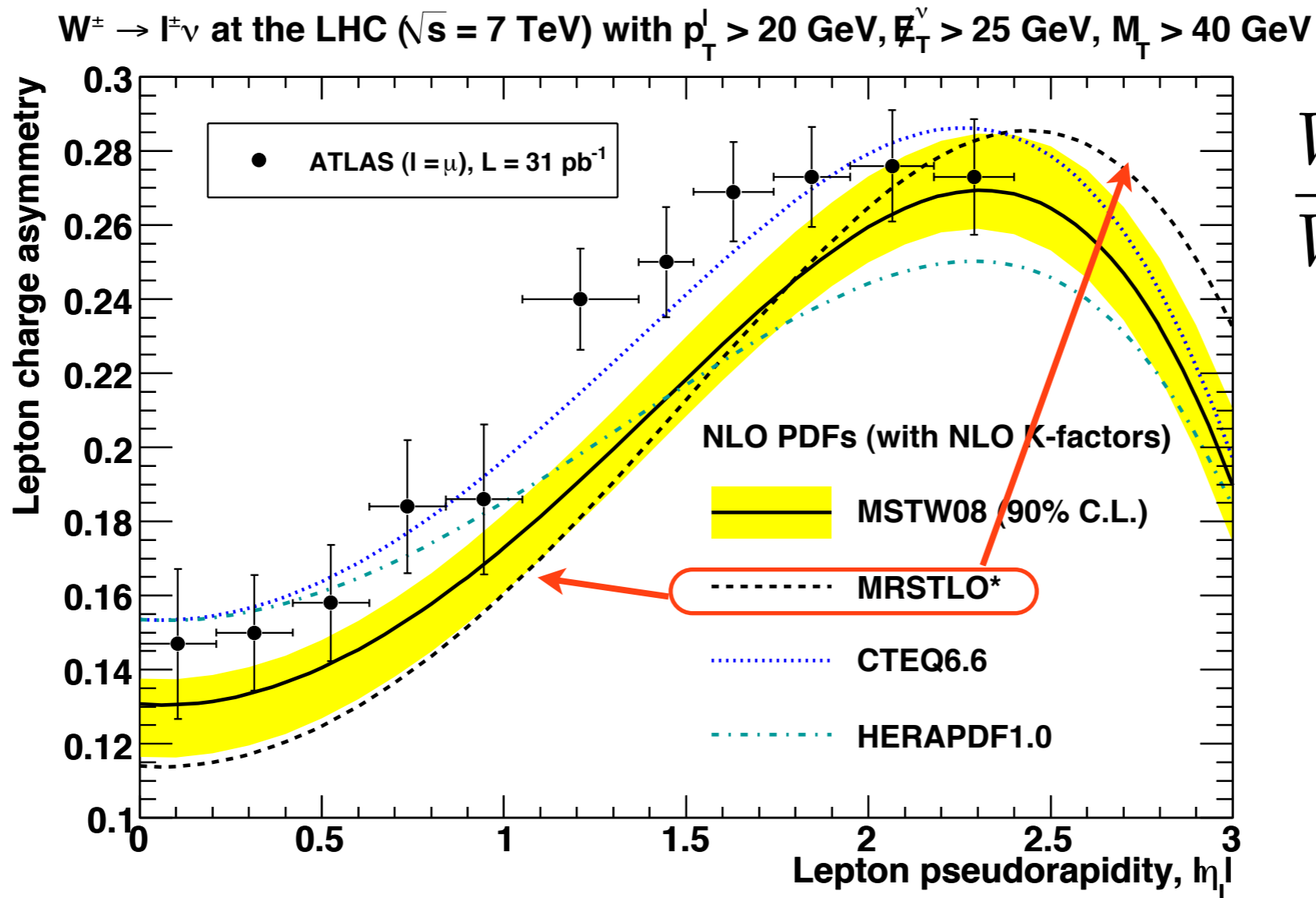
- ATLAS [[arXiv:1010.2130](#), ATLAS-CONF-2011-041] use PYTHIA (with MRSTLO*) $\Rightarrow \sigma_{W^+}^{\text{tot}}/\sigma_{W^-}^{\text{tot}} = \boxed{1.51} \pm 0.03 (e + \mu)$ [from ATLAS ellipse].

MC+PDF	A_{W^+}/A_{W^-}		$\sigma_{W^+}^{\text{tot}}/\sigma_{W^-}^{\text{tot}}$	
	e	μ	e	μ
PYTHIA+MRSTLO*	1.02	1.02	1.50	1.51
PYTHIA+CTEQ6.6	1.05	1.05	1.46	1.47
PYTHIA+HERAPDF1.0	1.03	1.04	1.48	1.49
MC@NLO+HERAPDF1.0	1.05	1.05	1.46	1.47
MC@NLO+CTEQ6.6	1.06	1.06	1.45	1.46

$$\frac{\sigma_{W^+}^{\text{tot}}}{\sigma_{W^-}^{\text{tot}}} = \frac{\sigma_{W^+}^{\text{fid}}}{\sigma_{W^-}^{\text{fid}}} \cdot \frac{A_{W^-}}{A_{W^+}}$$

Alternative acceptance calculations move ATLAS $\sigma_{W^+}^{\text{tot}}/\sigma_{W^-}^{\text{tot}}$ much closer to CMS.

Acceptance for ATLAS



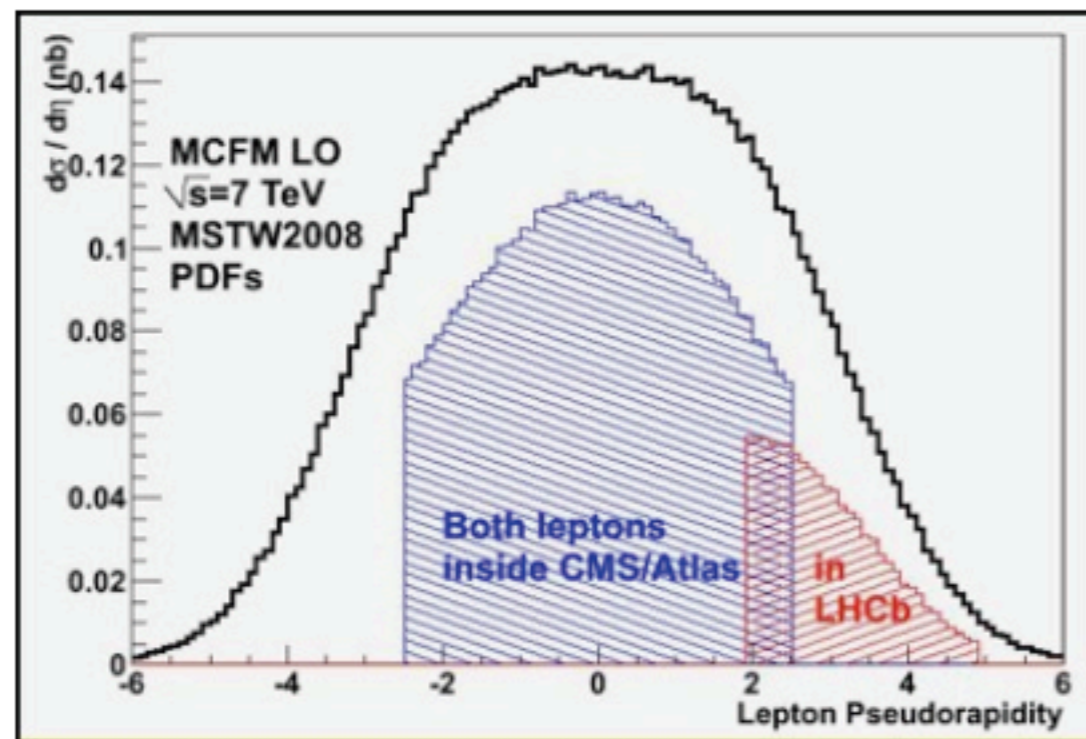
$$\frac{W^+ / W^- - 1}{W^+ / W^- + 1}$$

G. Watt (April 2011)

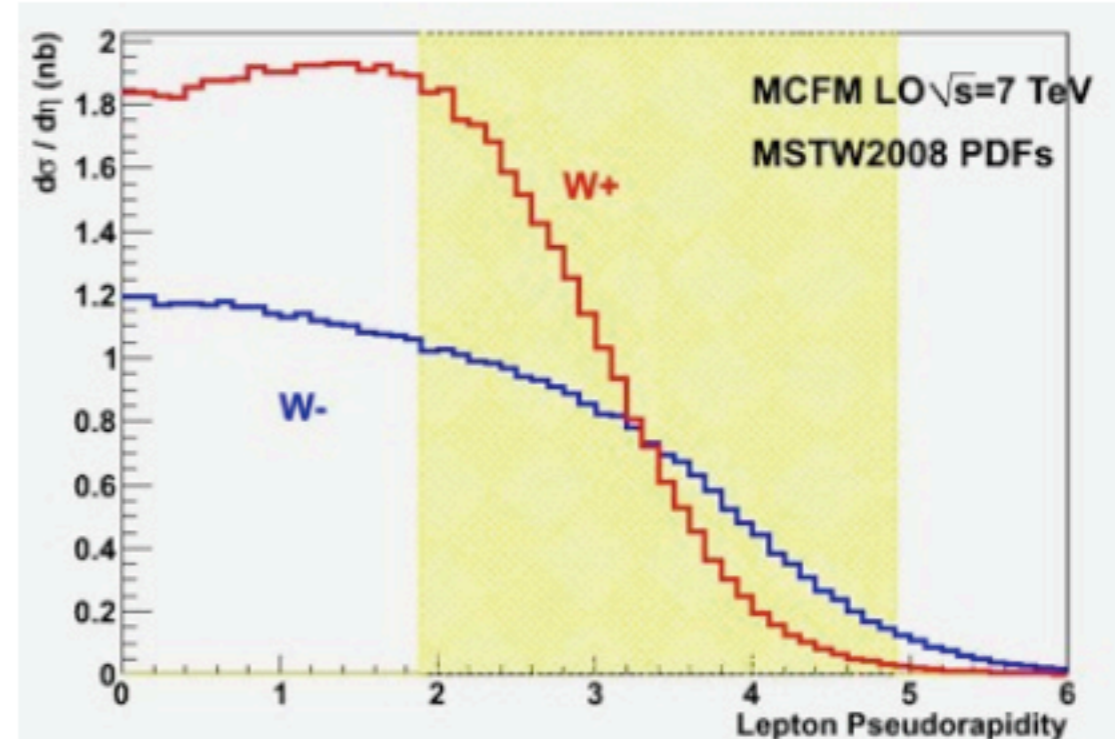
Acceptance = inside/(inside+outside)

- **MRSTLO*** lies well *below* almost all ATLAS data points, then is *above* other theory predictions at larger η_l .
 \Rightarrow Close to worst possible PDF choice for acceptance.

Acceptance for LHCb



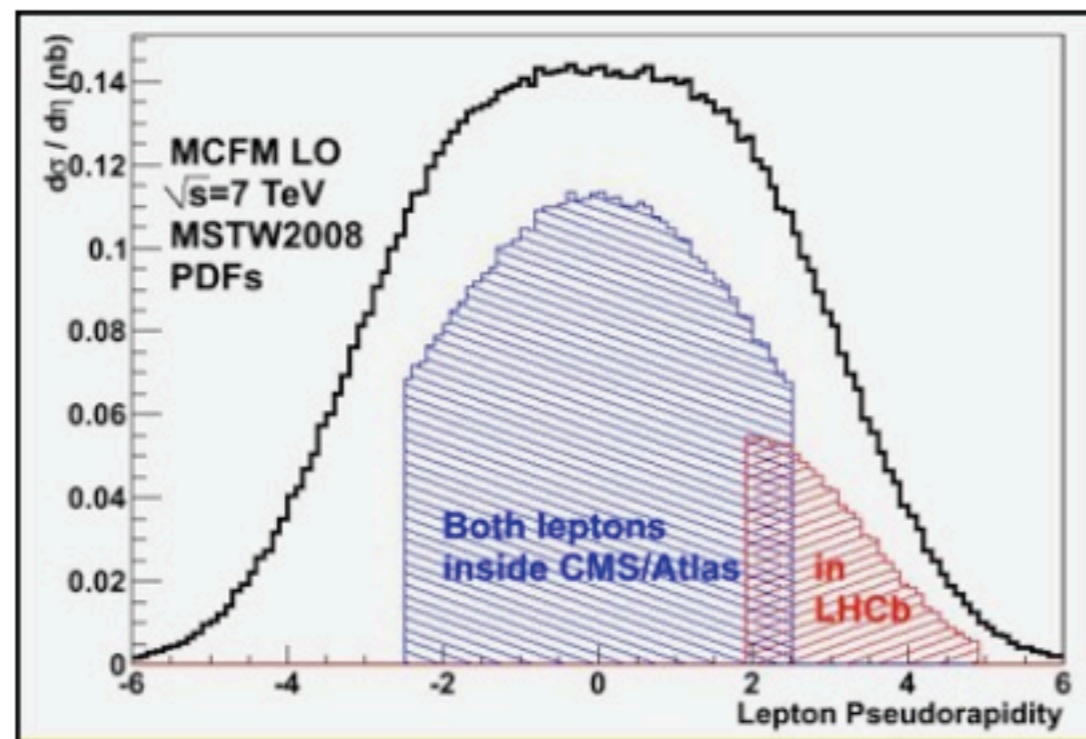
8% of Z within
LHCb acceptance



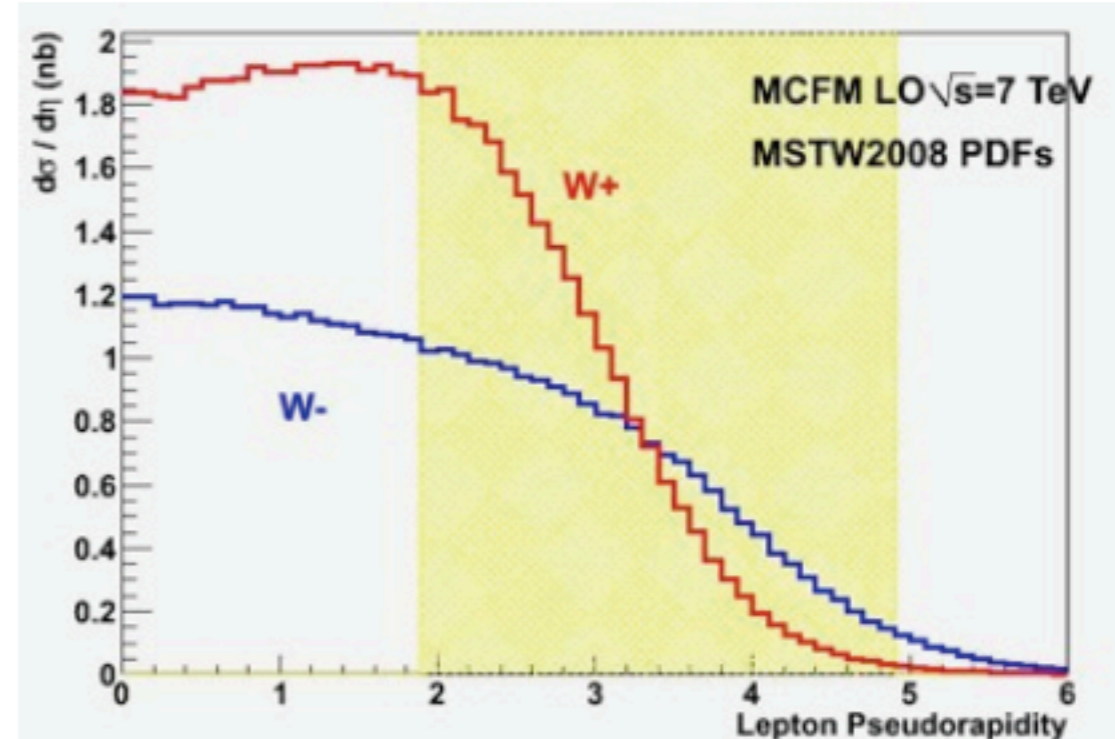
17% (16%) of W^+
(W^-) within LHCb
acceptance

- Q. Should LHCb “measure” a total cross section by extrapolating over all pseudorapidity?

Acceptance for LHCb



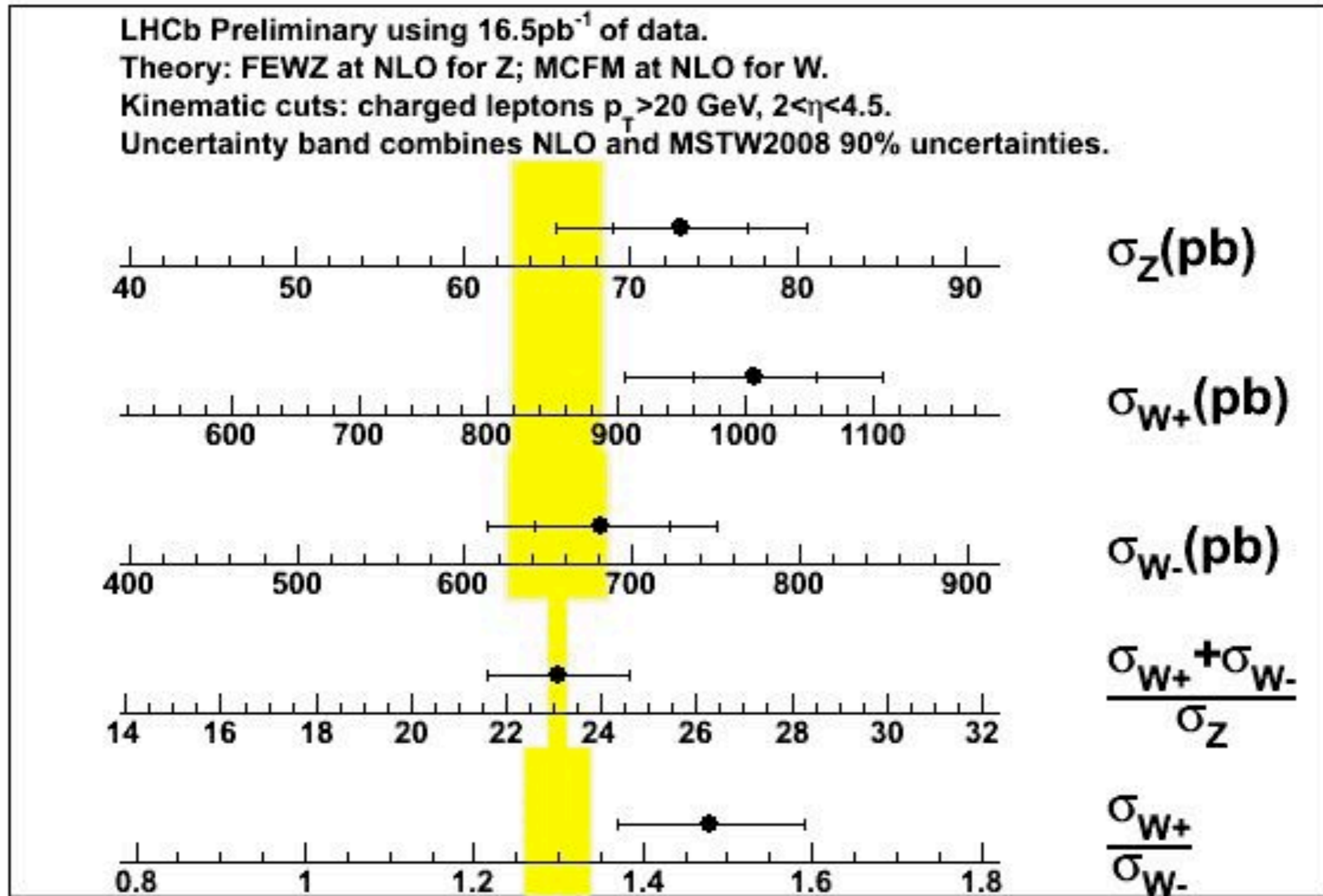
8% of Z within
LHCb acceptance



17% (16%) of W^+
(W^-) within LHCb
acceptance

- Q. Should LHCb “measure” a total cross section by extrapolating over all pseudorapidity?
- A. Don't even think about it!

LHCb Preliminary results



- Compare to theory only inside kinematic cuts. ✓

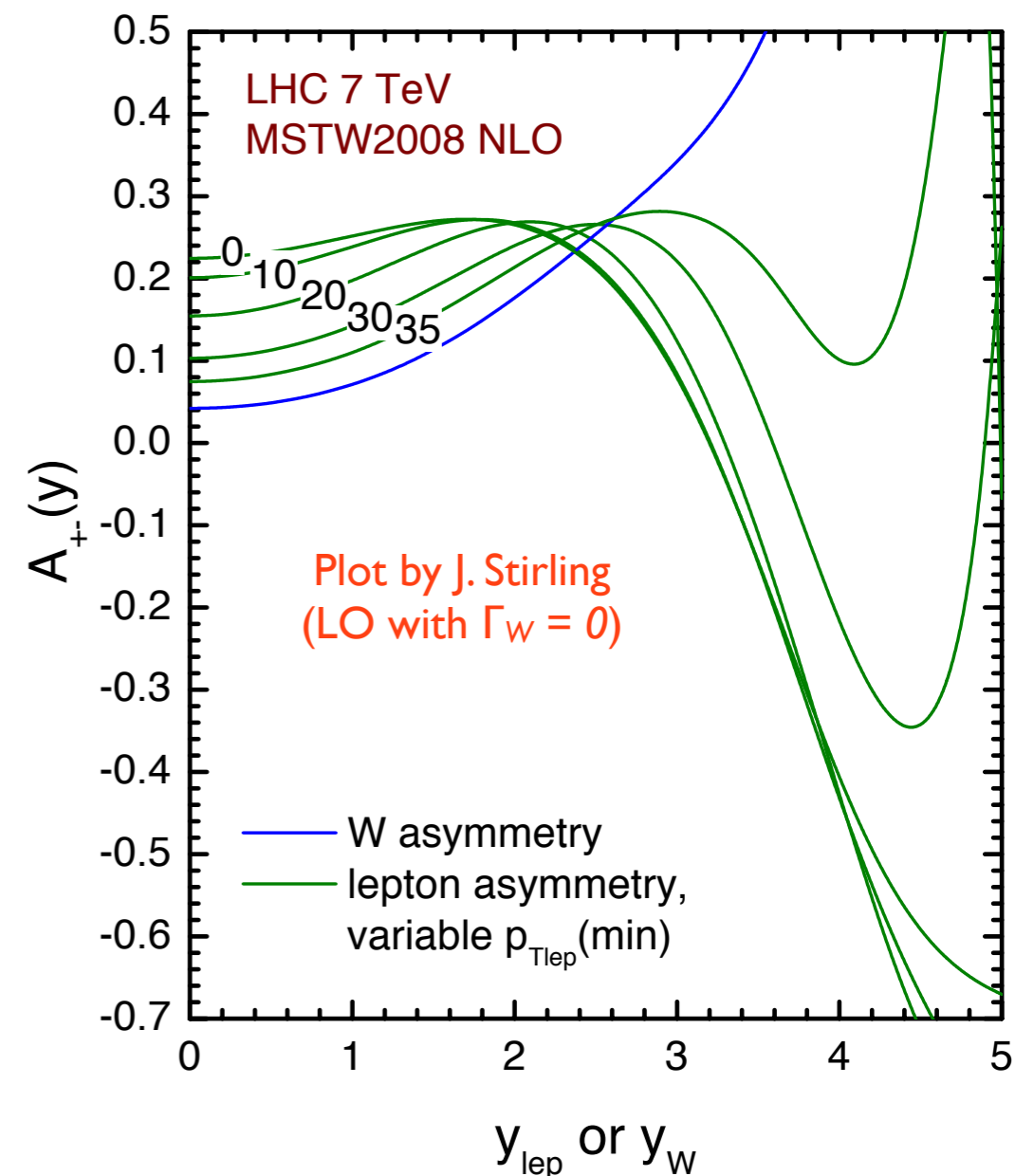
$W^\pm \rightarrow l^\pm \nu$ charge asymmetry

$$A_W(y_W) = \frac{d\sigma(W^+)/dy_W - d\sigma(W^-)/dy_W}{d\sigma(W^+)/dy_W + d\sigma(W^-)/dy_W} \approx \frac{u_v(x_1) - d_v(x_1)}{u(x_1) + d(x_1)}$$

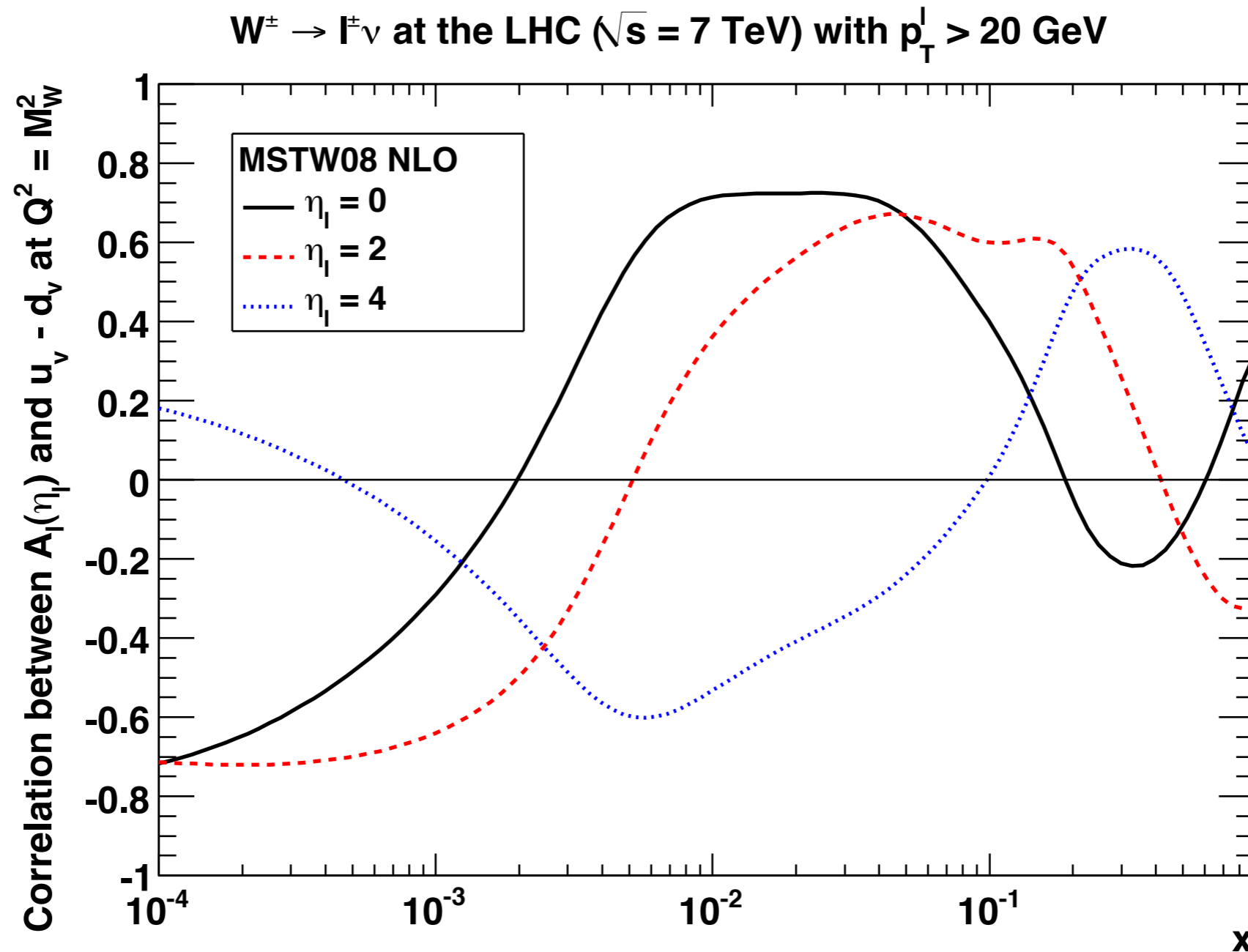
- But can't measure longitudinal momentum of neutrino. ☹️
- Lepton charge asymmetry:

$$A_\ell(\eta_\ell) = \frac{d\sigma(l^+)/d\eta_\ell - d\sigma(l^-)/d\eta_\ell}{d\sigma(l^+)/d\eta_\ell + d\sigma(l^-)/d\eta_\ell} = A_W(y_W) \otimes (W^\pm \rightarrow l^\pm \nu)$$

- Complex interplay of PDF and V-A effects from $W^\pm \rightarrow l^\pm \nu$ decay.



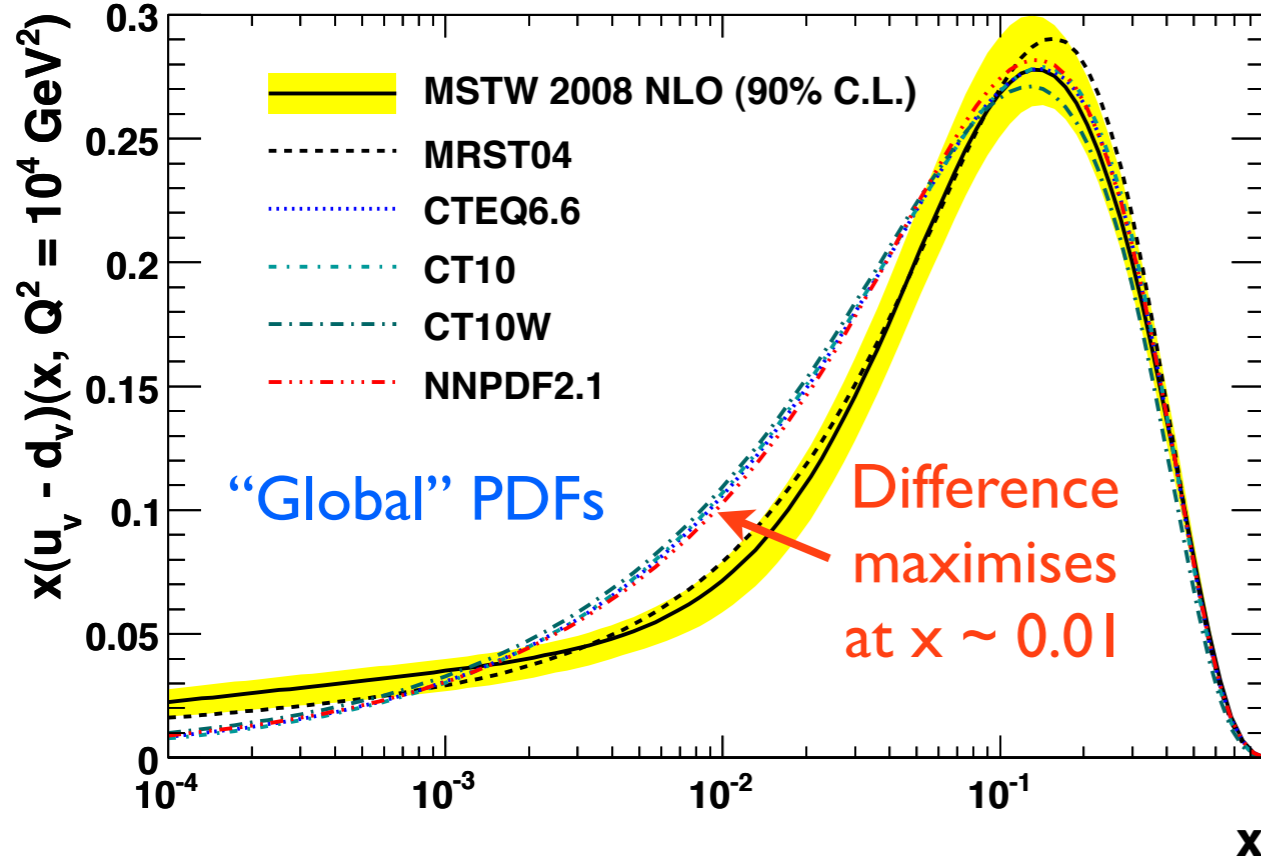
PDF correlation between asymmetry and u_v-d_v versus x



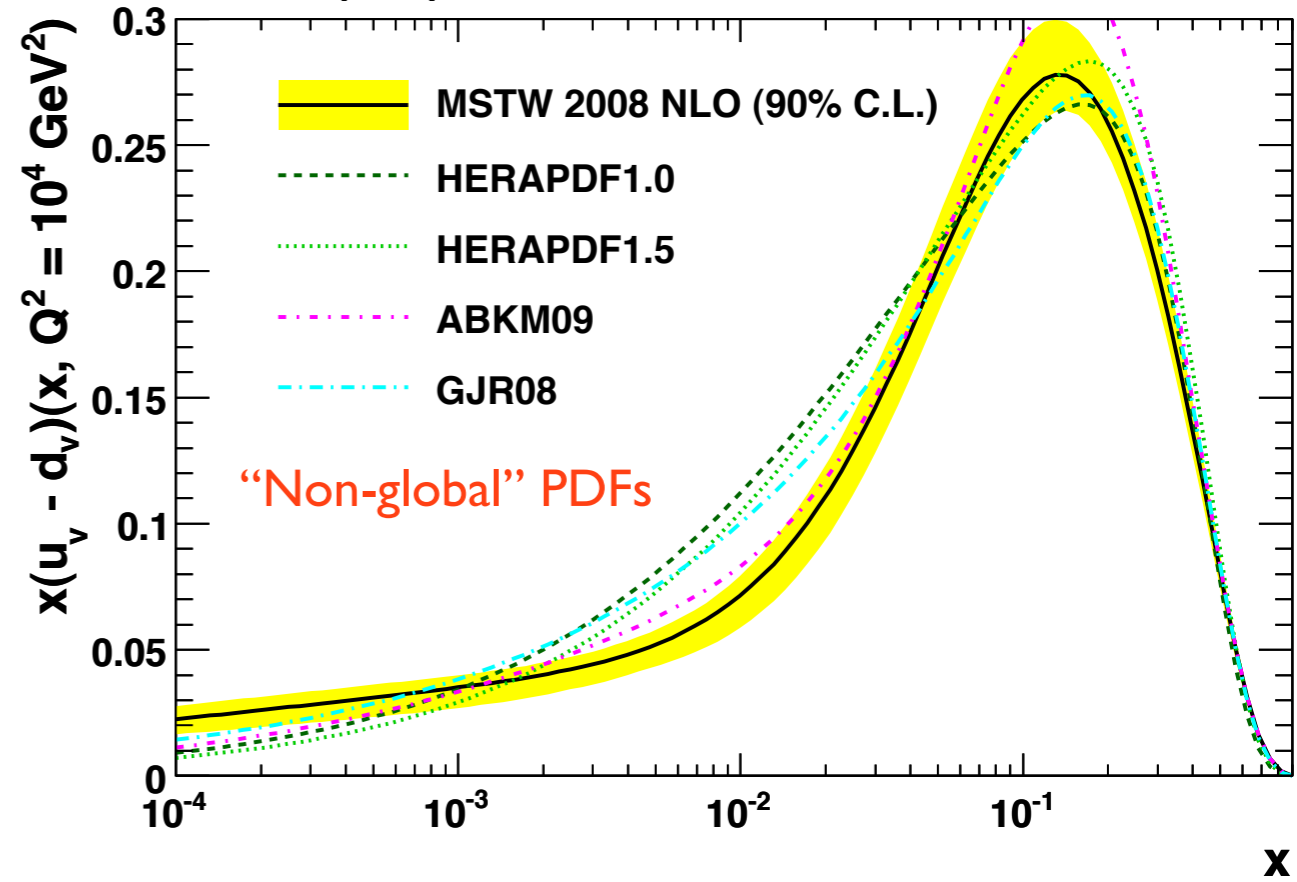
- Indirect (anti)correlation at small x from sum rule: $\int_0^1 dx (u_v - d_v) = 1$

$u_v - d_v$ for different NLO PDFs

$u_v - d_v$ distribution at $Q^2 = 10^4 \text{ GeV}^2$

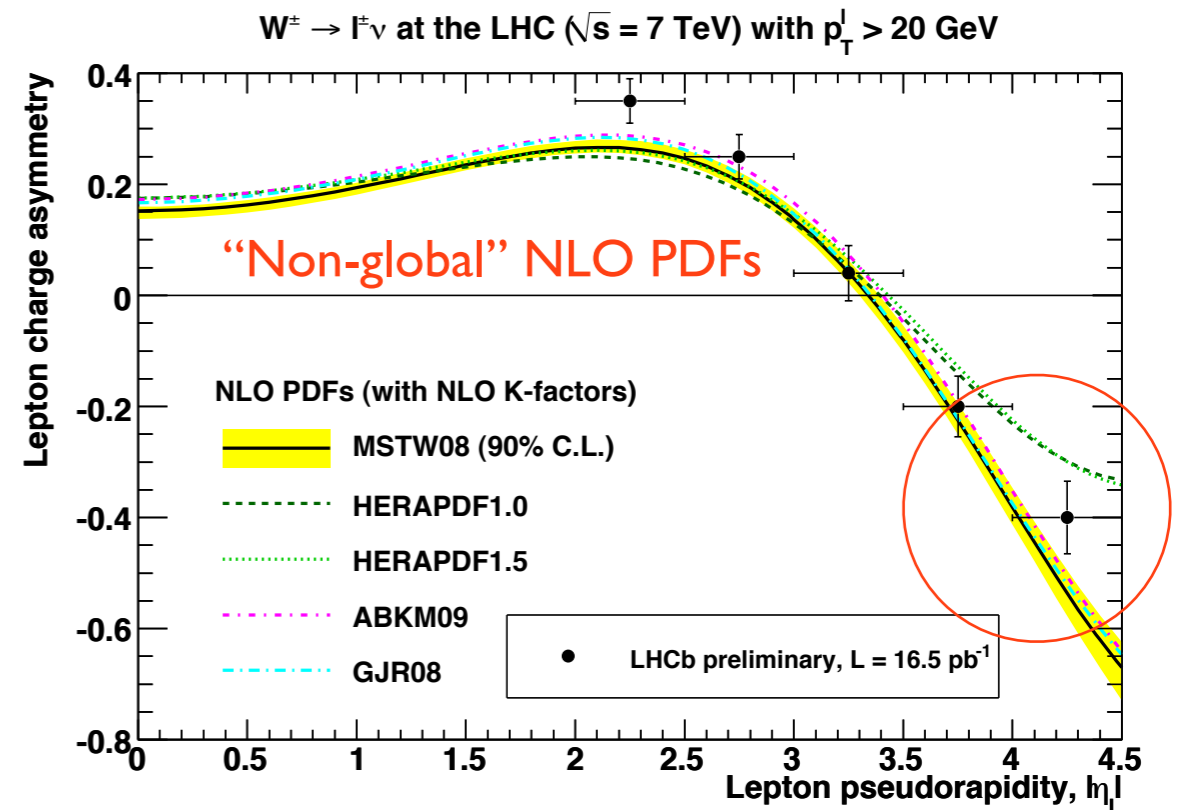
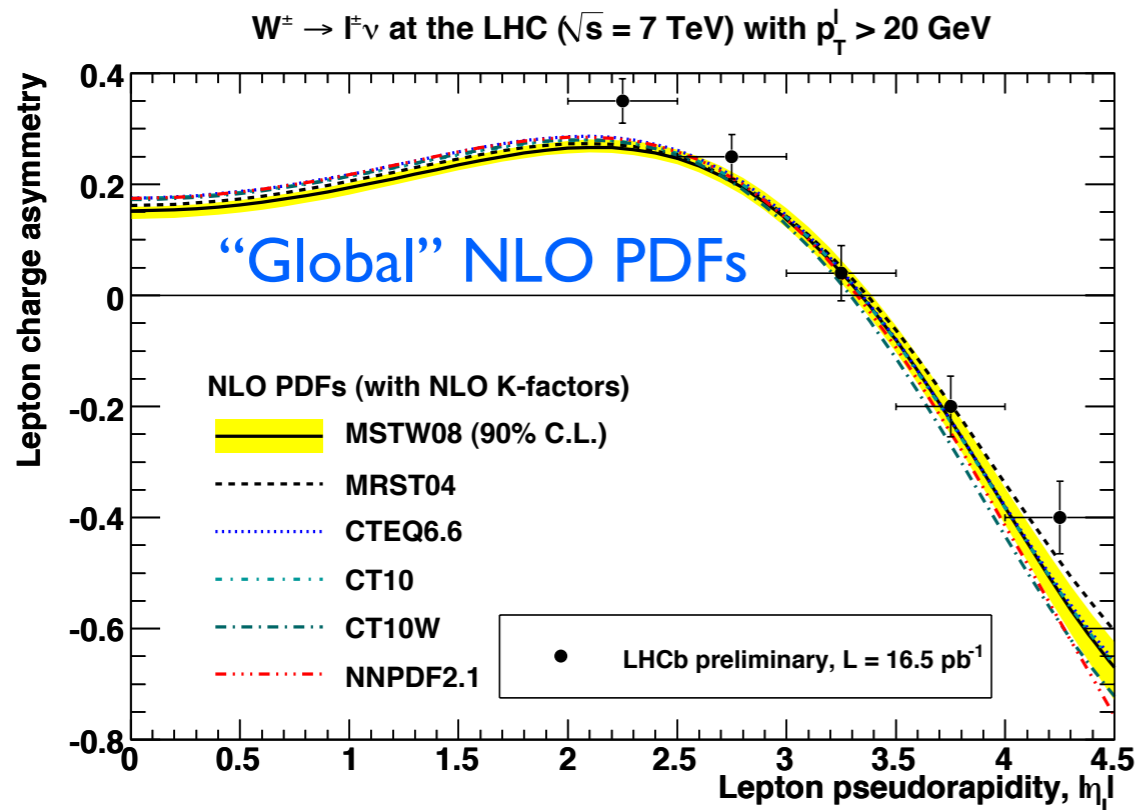


$u_v - d_v$ distribution at $Q^2 = 10^4 \text{ GeV}^2$

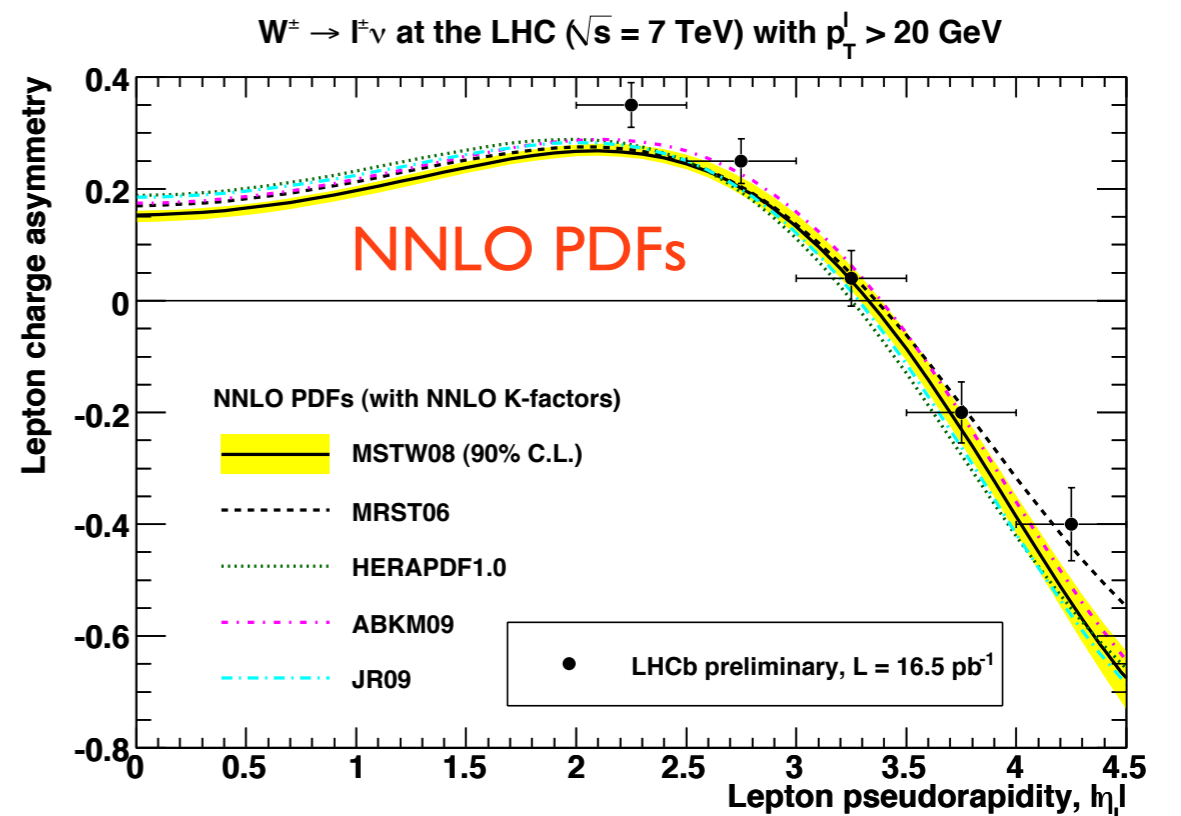


- MSTW08** has input $xu_v \propto x^{0.29 \pm 0.02}$ and $xd_v \propto x^{0.97 \pm 0.11}$.
 Many other groups **assume** equal powers \Rightarrow parameterisation bias.
 (NNPDF: restricted range of small- x preprocessing exponents.)

Description of LHCb data

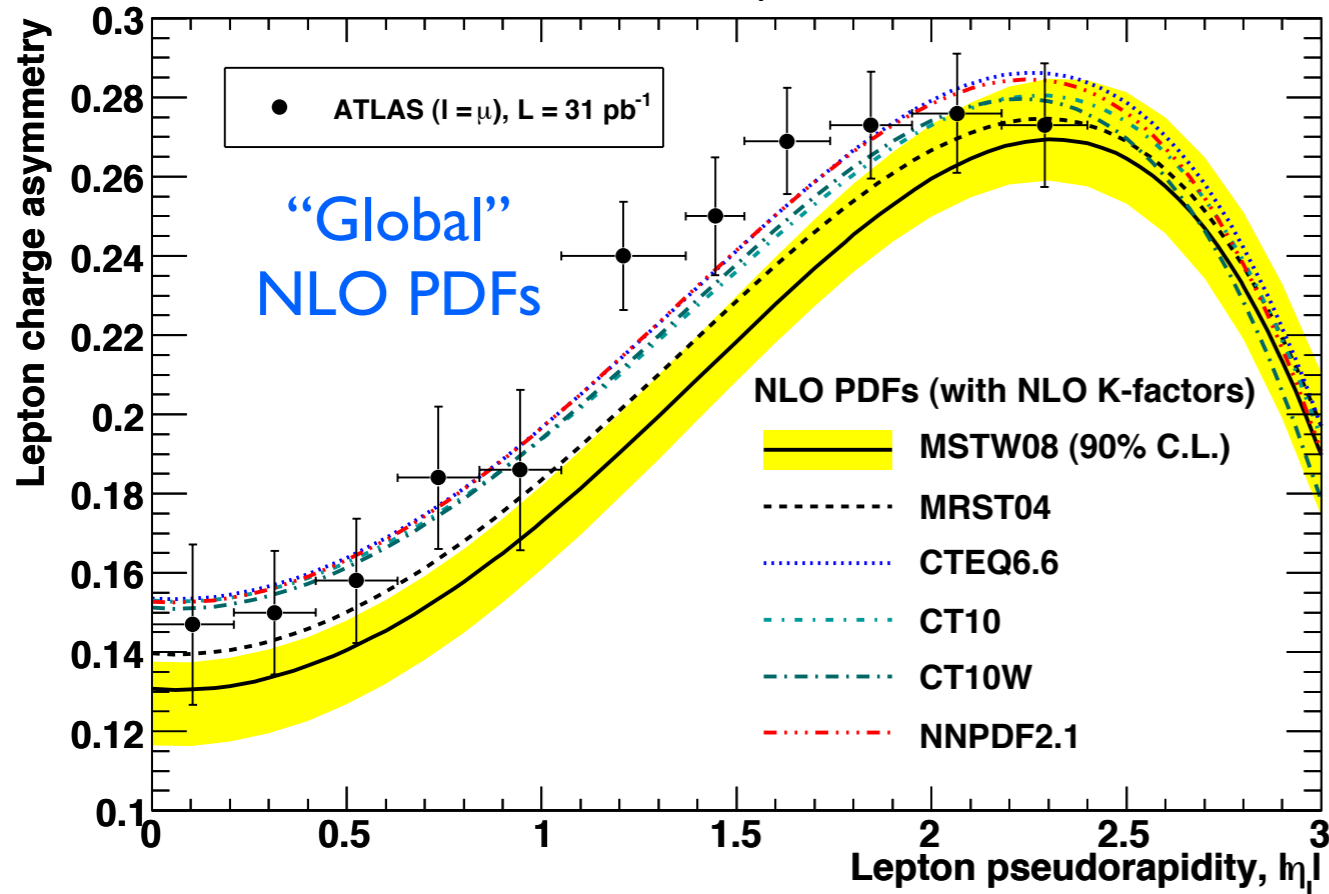


- All PDFs give good description.
- First LHCb data point higher than theory (and ATLAS/CMS).
- HERAPDF different at high η .
- Preliminary LHCb data not corrected for QED FSR.

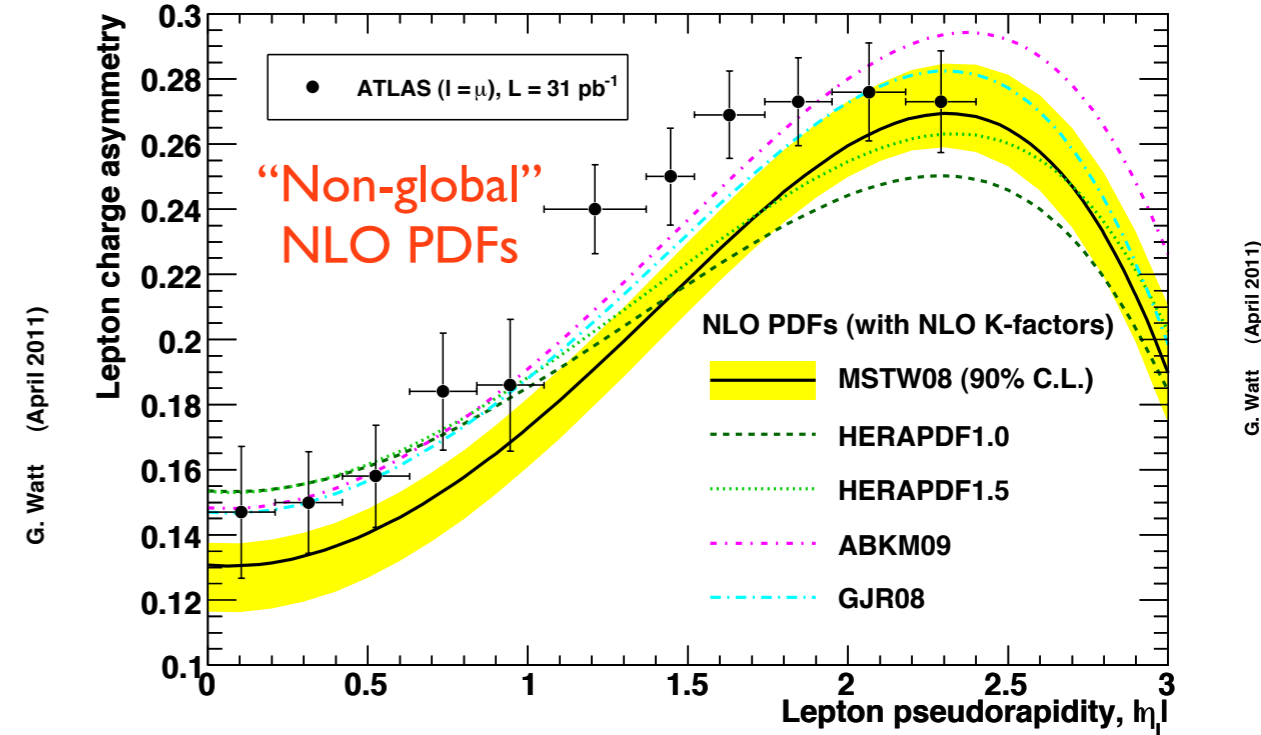


Description of ATLAS data

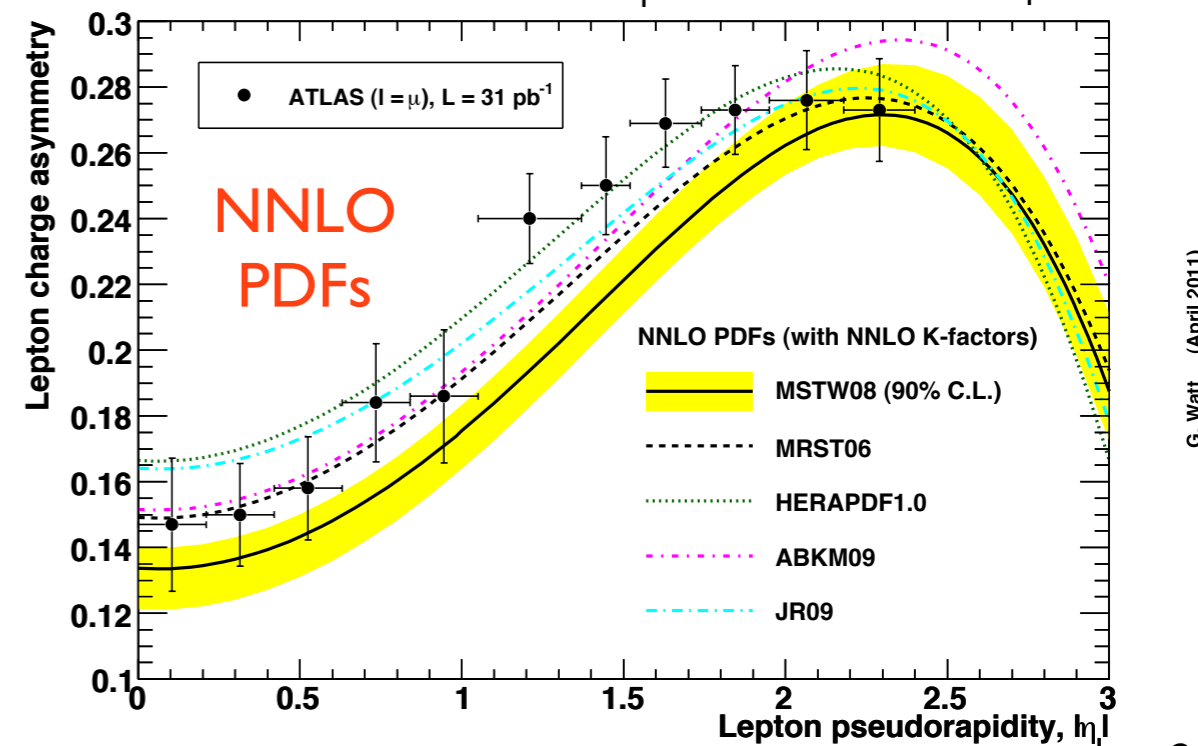
$W^\pm \rightarrow l^\pm \nu$ at the LHC ($\sqrt{s} = 7$ TeV) with $p_T^l > 20$ GeV, $E_T^\nu > 25$ GeV, $M_T > 40$ GeV



$W^\pm \rightarrow l^\pm \nu$ at the LHC ($\sqrt{s} = 7$ TeV) with $p_T^l > 20$ GeV, $E_T^\nu > 25$ GeV, $M_T > 40$ GeV

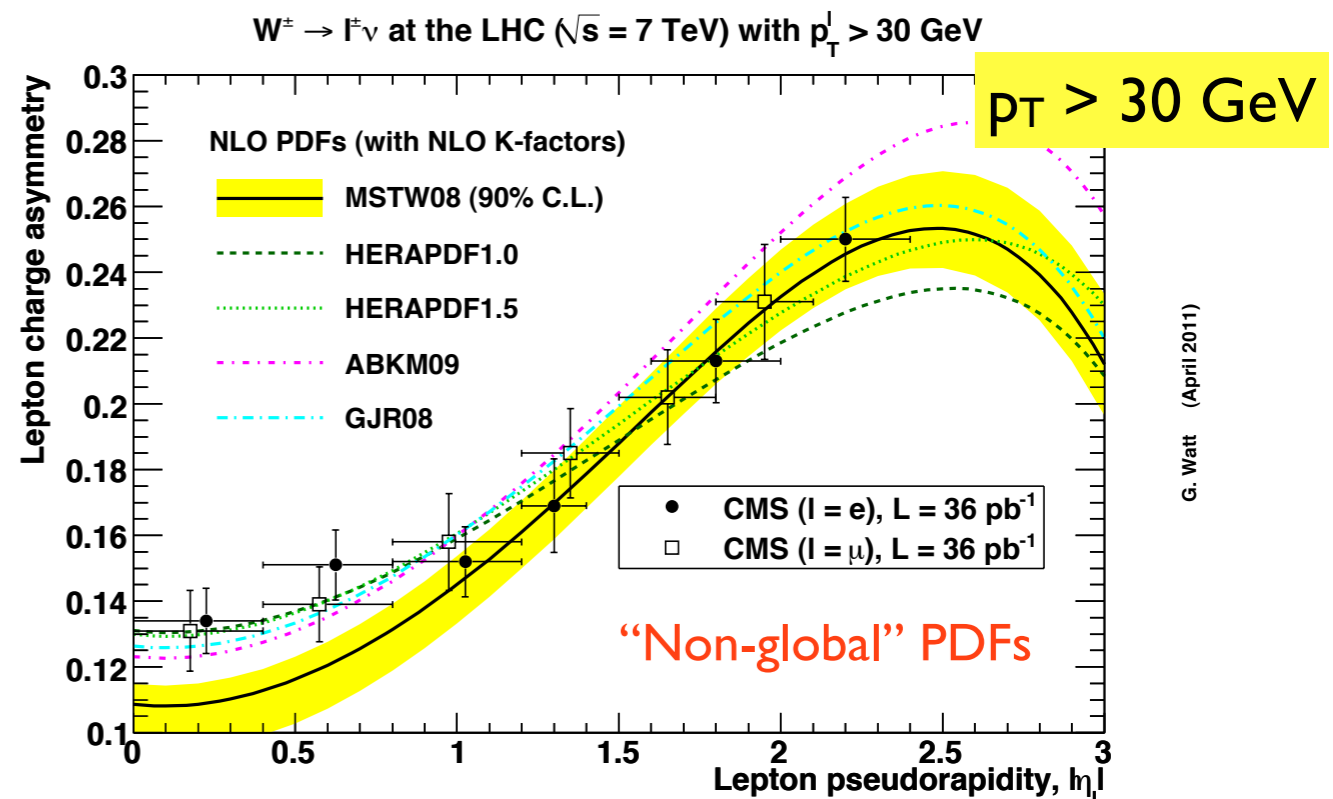
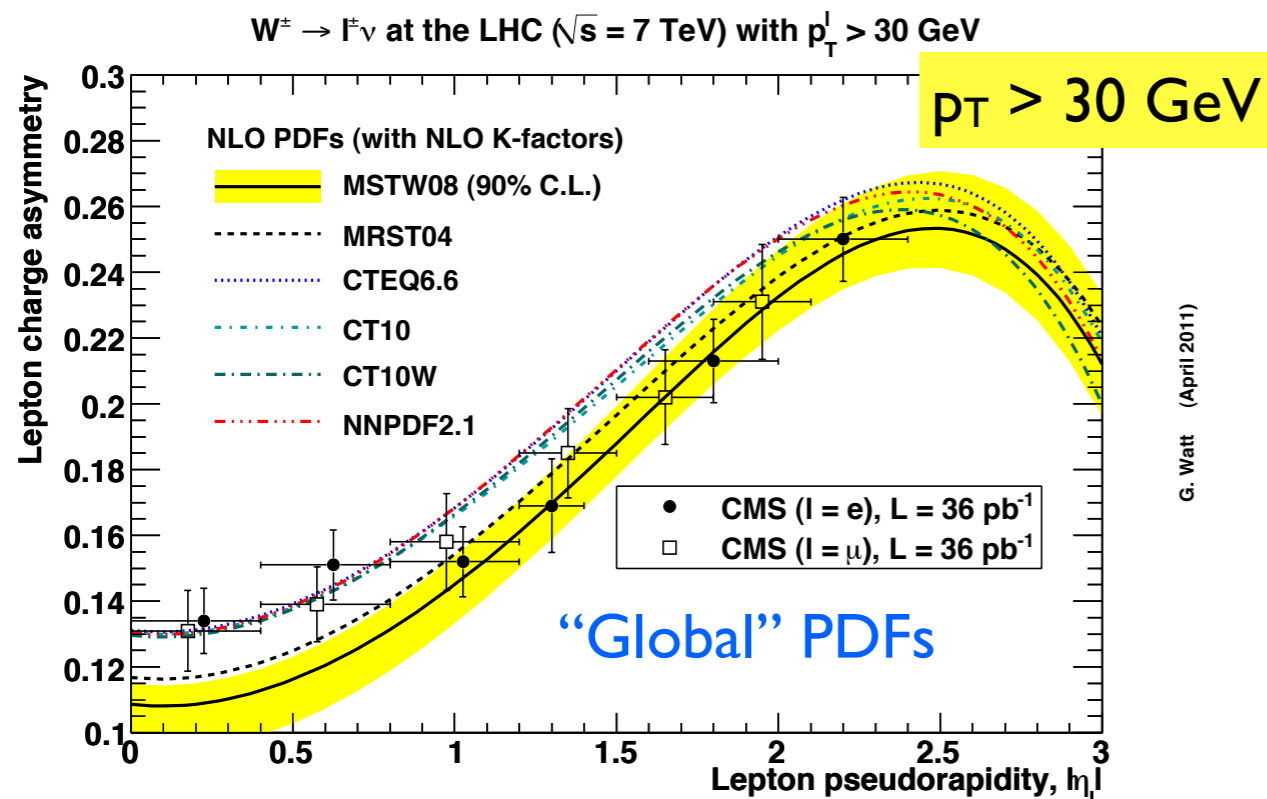
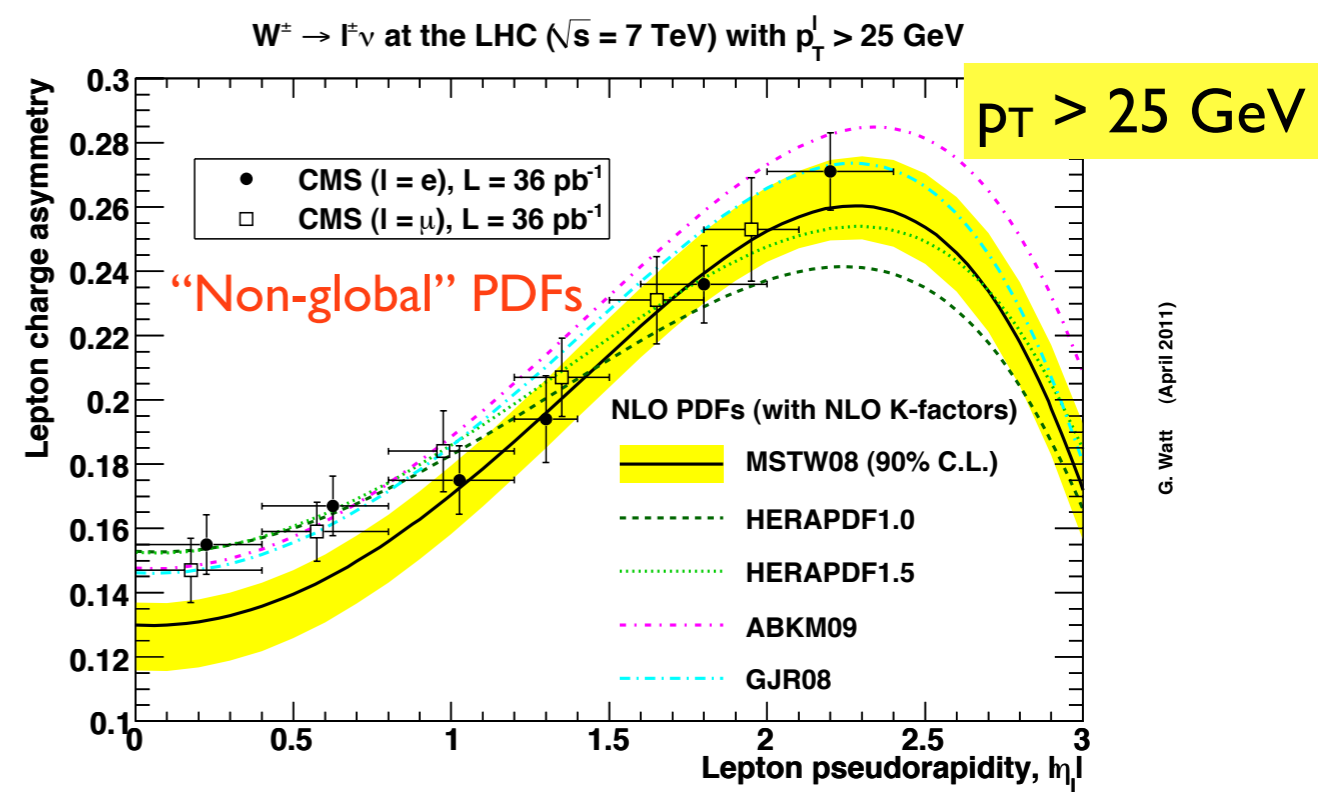
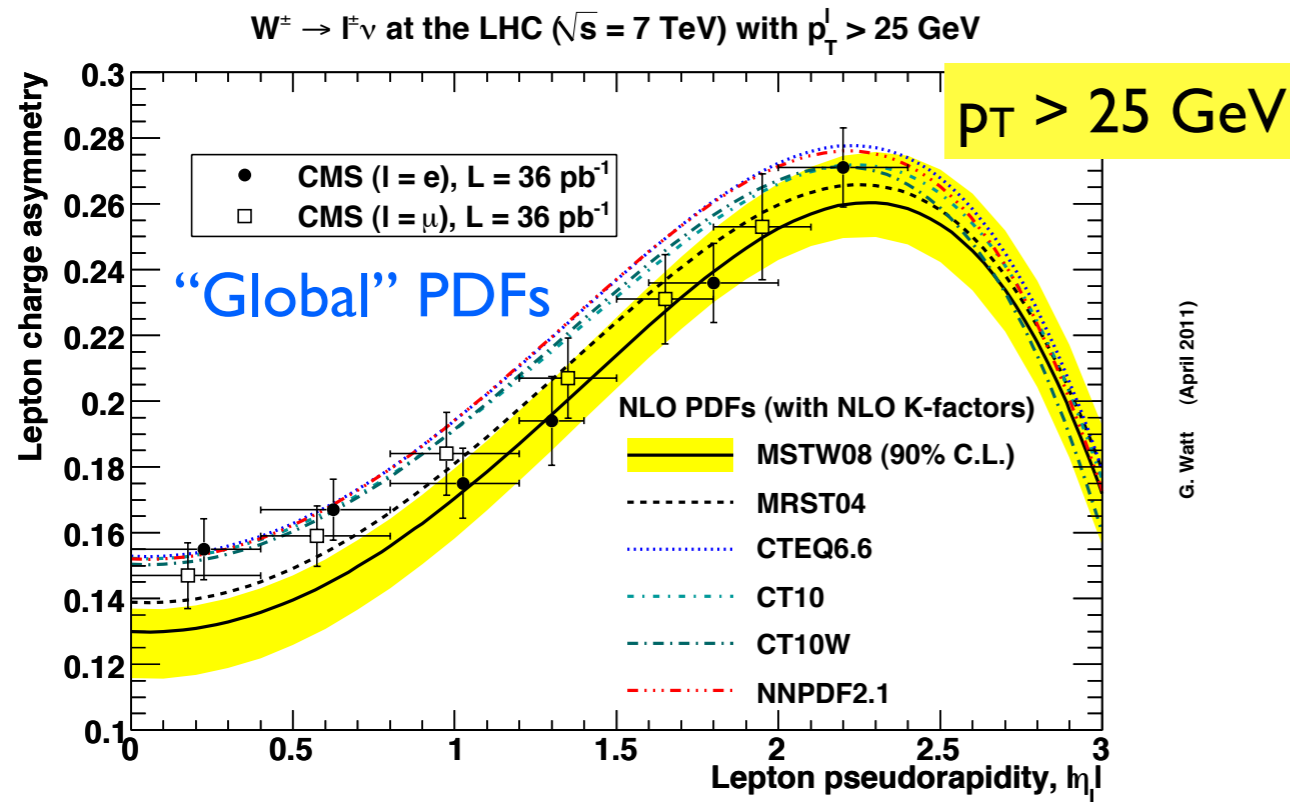


$W^\pm \rightarrow l^\pm \nu$ at the LHC ($\sqrt{s} = 7$ TeV) with $p_T^l > 20$ GeV, $E_T^\nu > 25$ GeV, $M_T > 40$ GeV

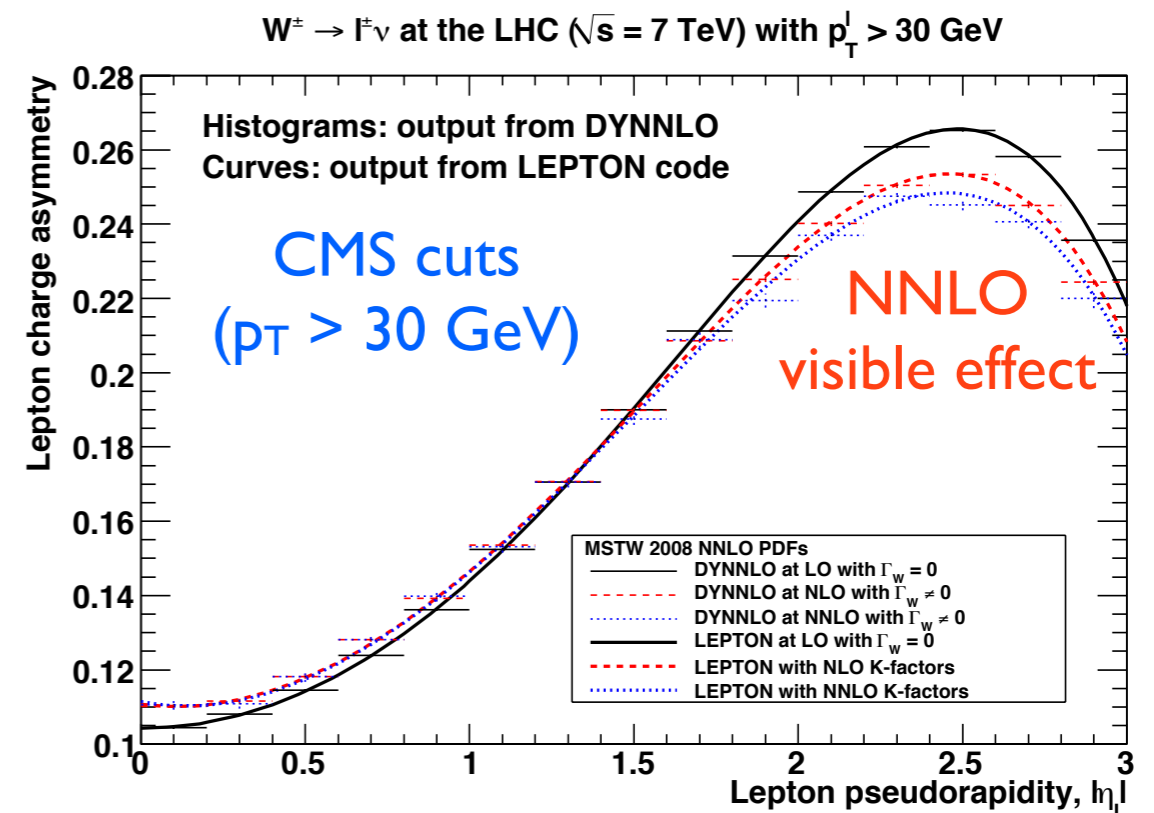
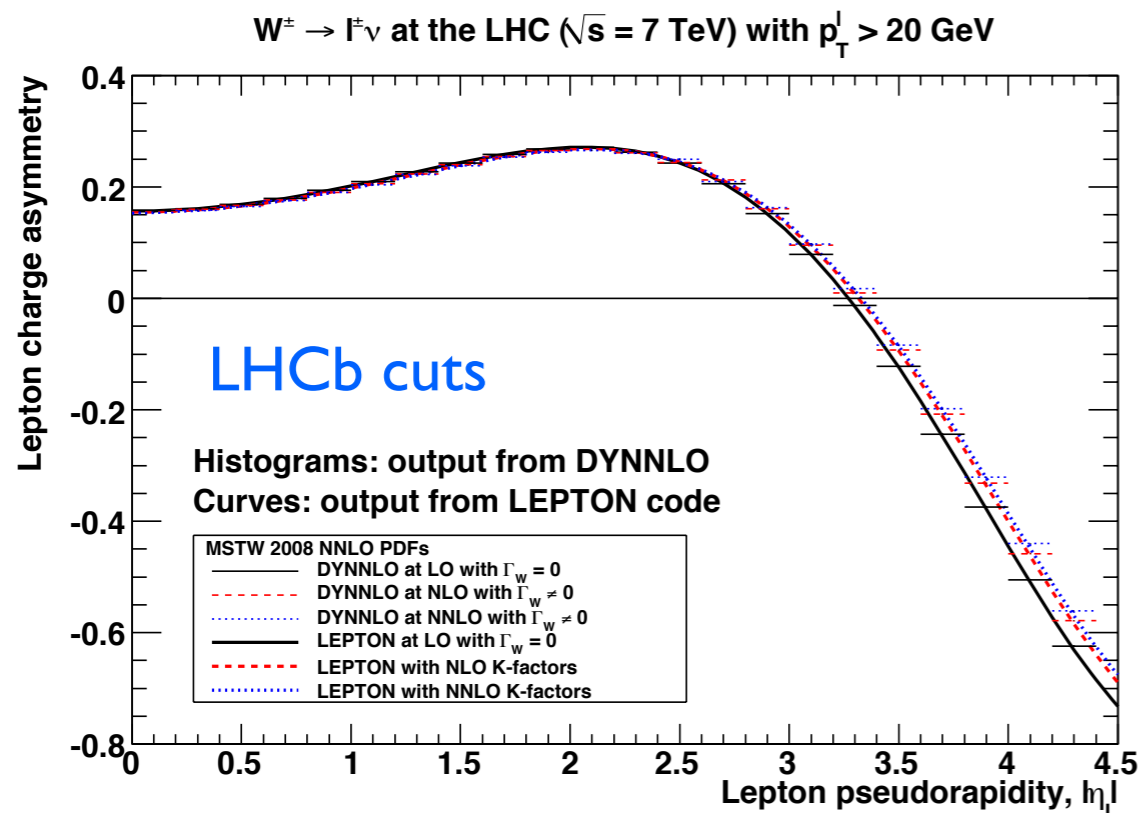
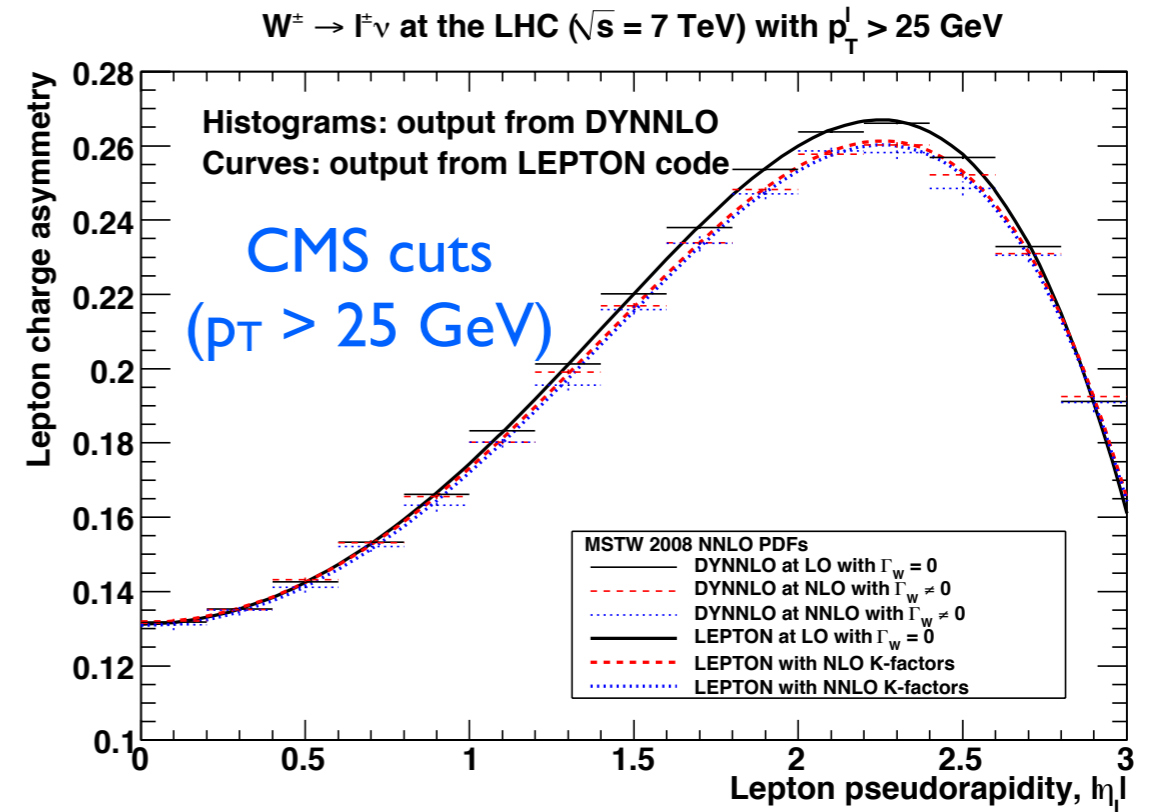
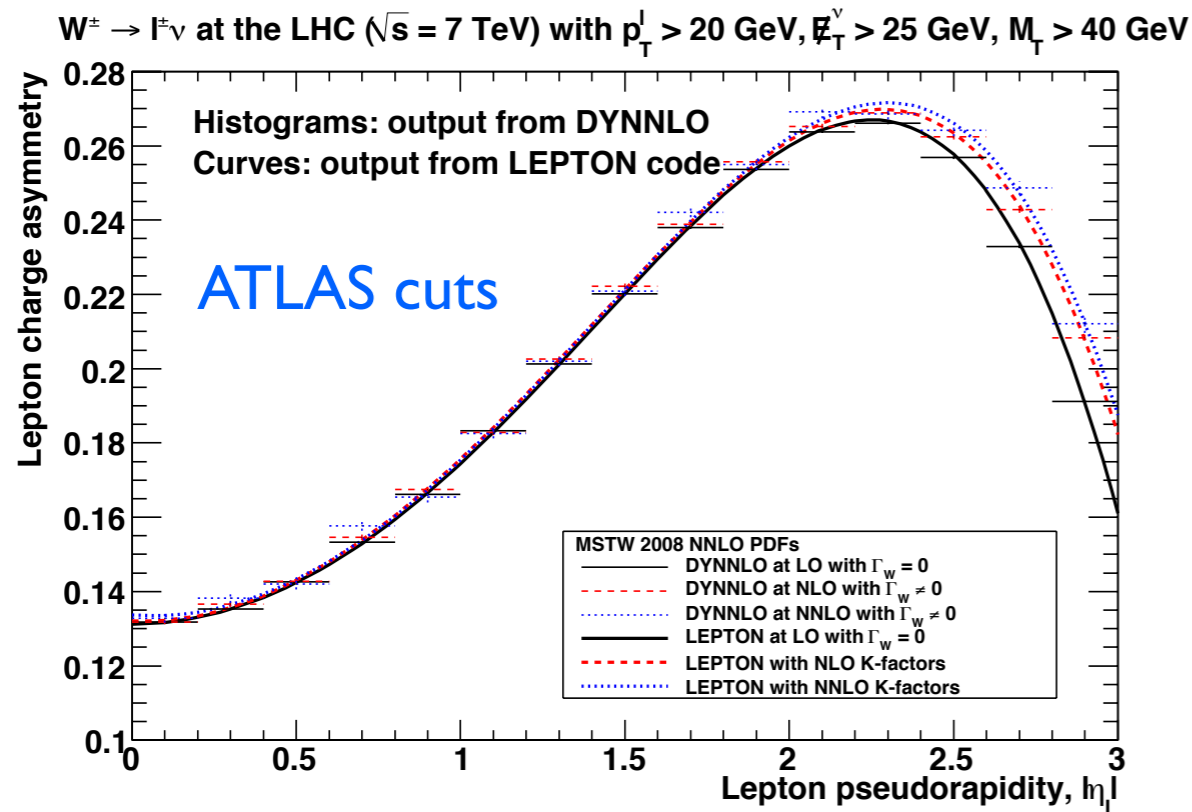


- Wide spread in predictions using different PDF sets \Rightarrow input to future global fits.

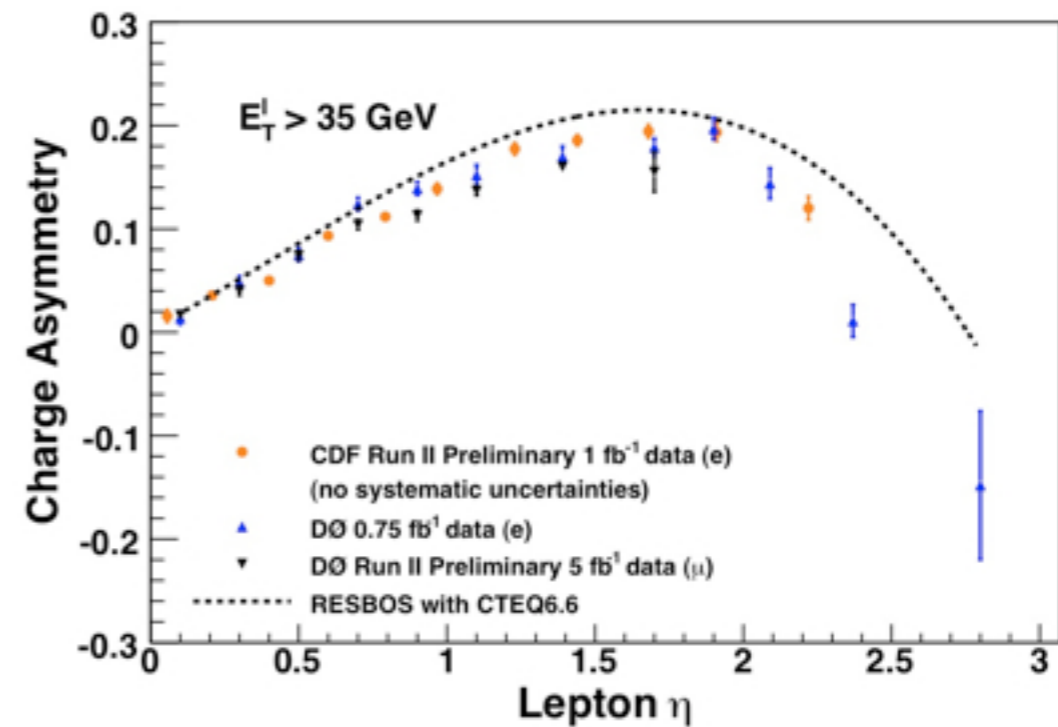
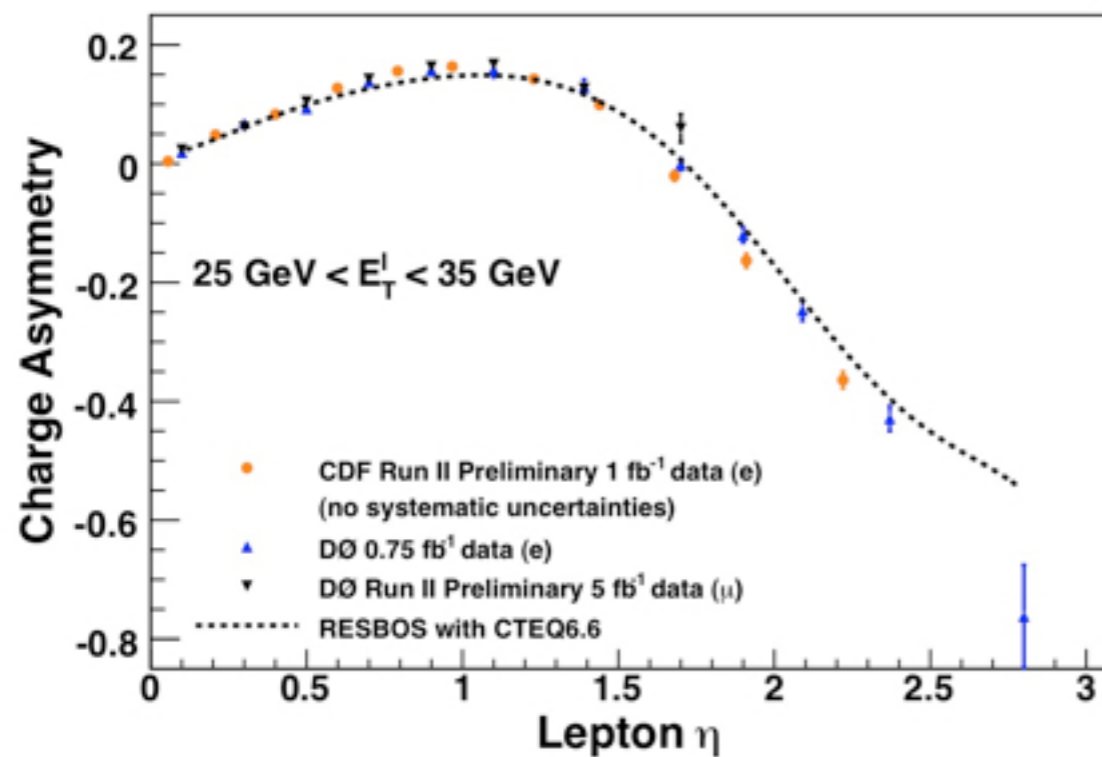
Description of CMS data



Size of NLO/NNLO corrections



Tevatron $W^\pm \rightarrow l^\pm \nu$ asymmetry

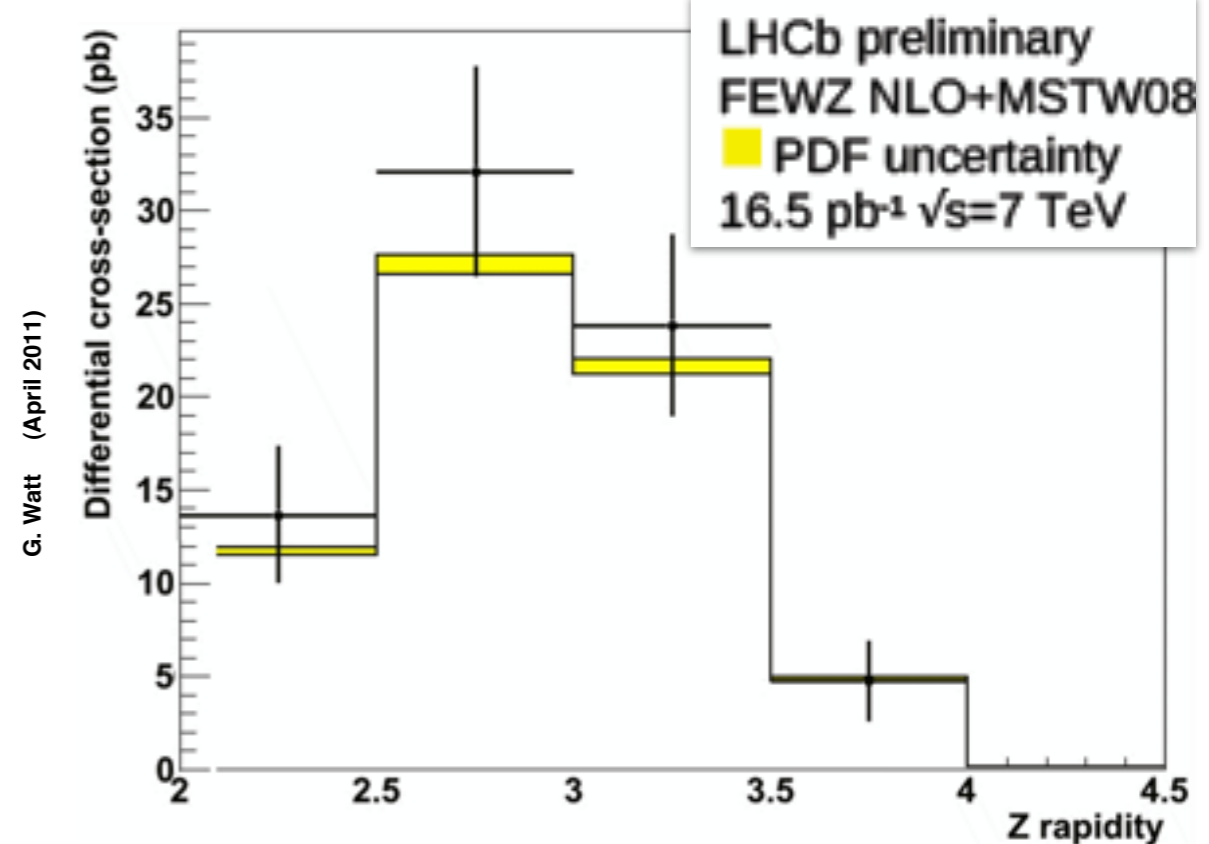
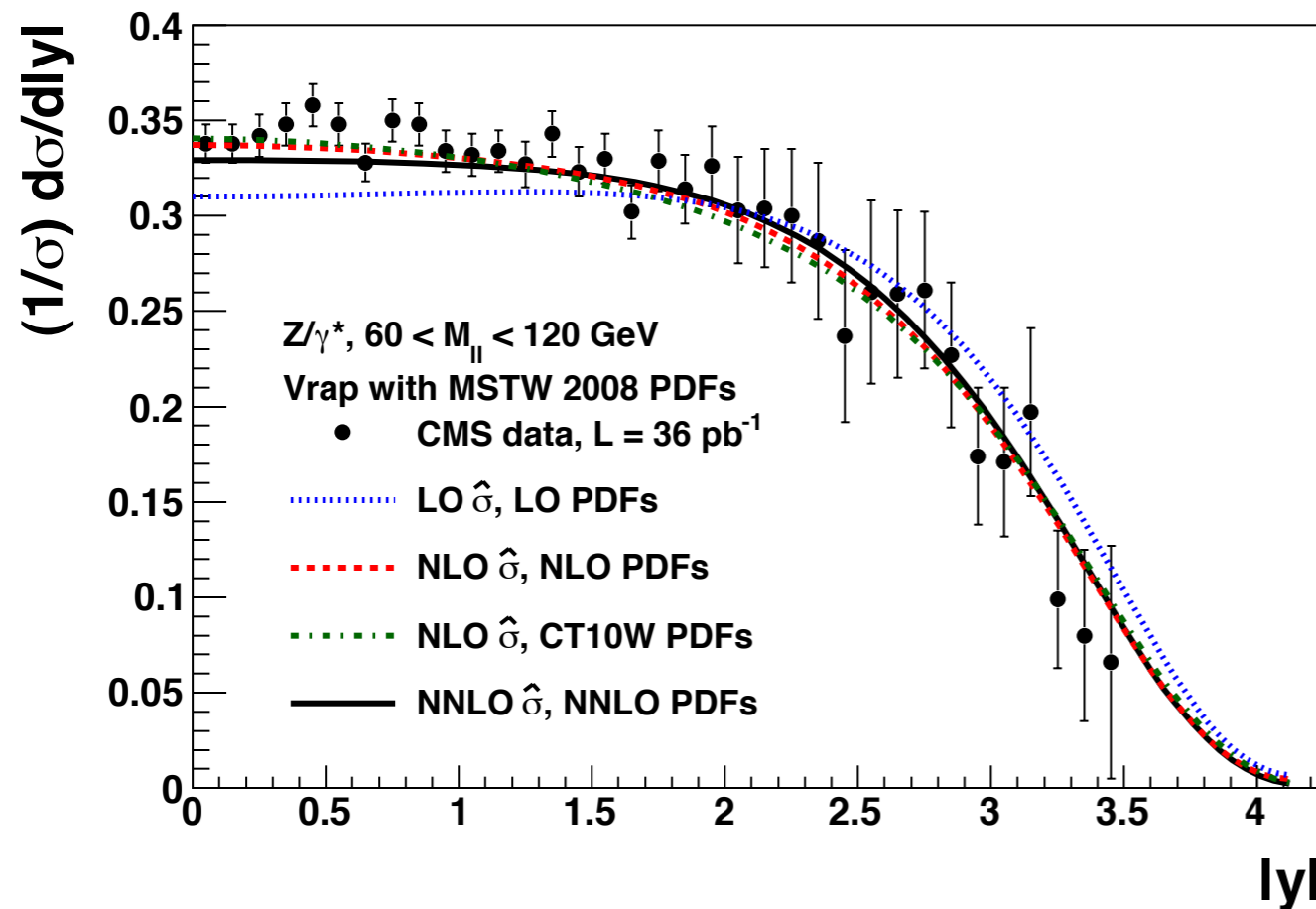


- Unresolved issues with Tevatron data split into E_T bins, especially for $E_T > 35 \text{ GeV}$.
- See MSTW write-up [[arXiv:1006.2753](https://arxiv.org/abs/1006.2753)].
- Precise LHC measurements in lepton p_T bins might help, otherwise more inclusive generally provides a safer PDF constraint.

Z/ γ^* rapidity from CMS/LHCb

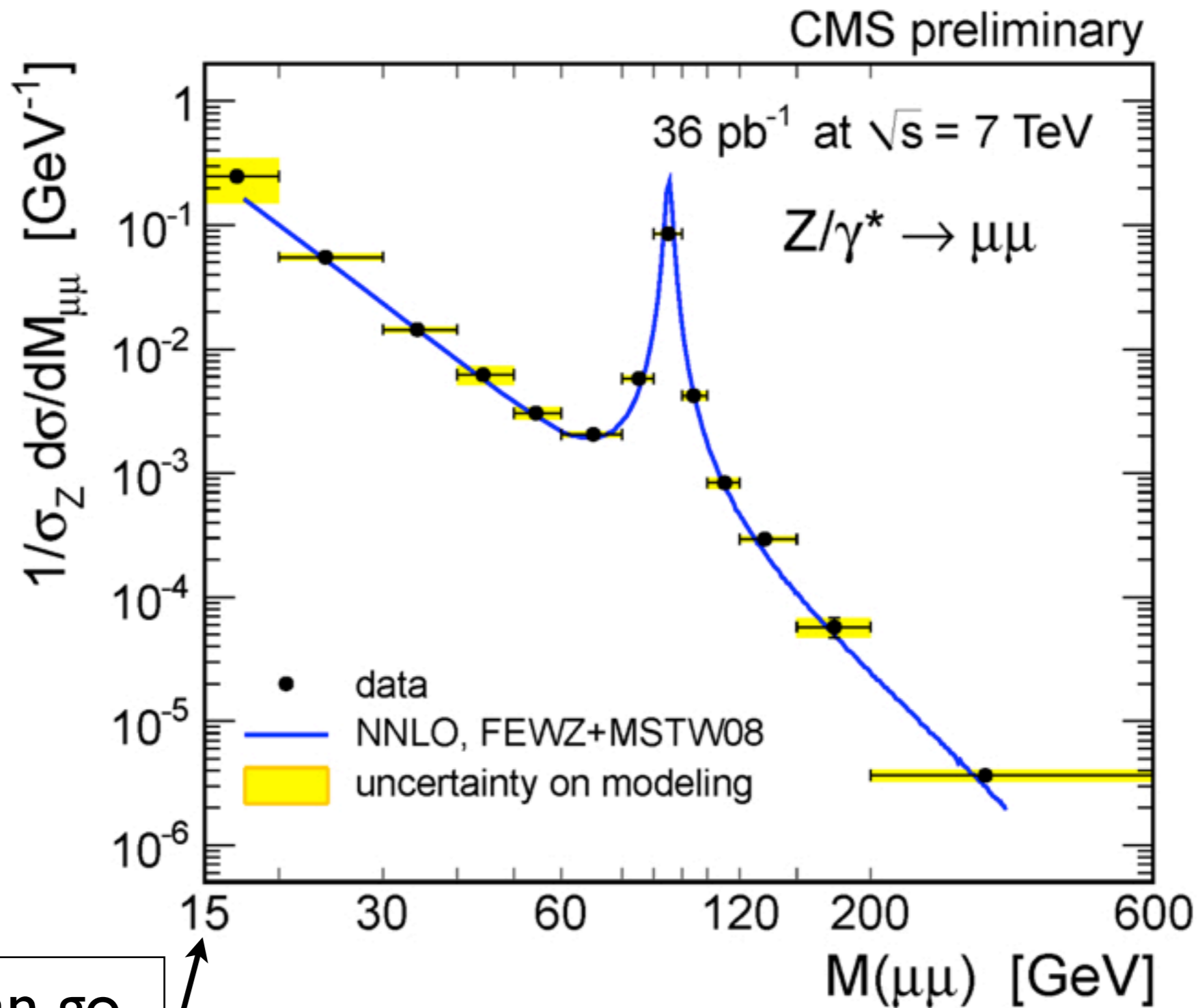
$$\frac{d\sigma_Z}{dy} \sim 0.29 u(x_1)\bar{u}(x_2) + 0.37 d(x_1)\bar{d}(x_2), \quad x_{1,2} = \frac{M_Z}{\sqrt{s}} \exp(\pm y)$$

Z/ γ^* rapidity shape distribution at the LHC ($\sqrt{s} = 7$ TeV)



- Preliminary CMS data [[CMS PAS EWK-10-010](#)].
- CMS: NLO/NNLO PDFs favoured over LO.
- LHCb: both muons with $2 < \eta_{\mu} < 4.5$.

DY away from the Z peak

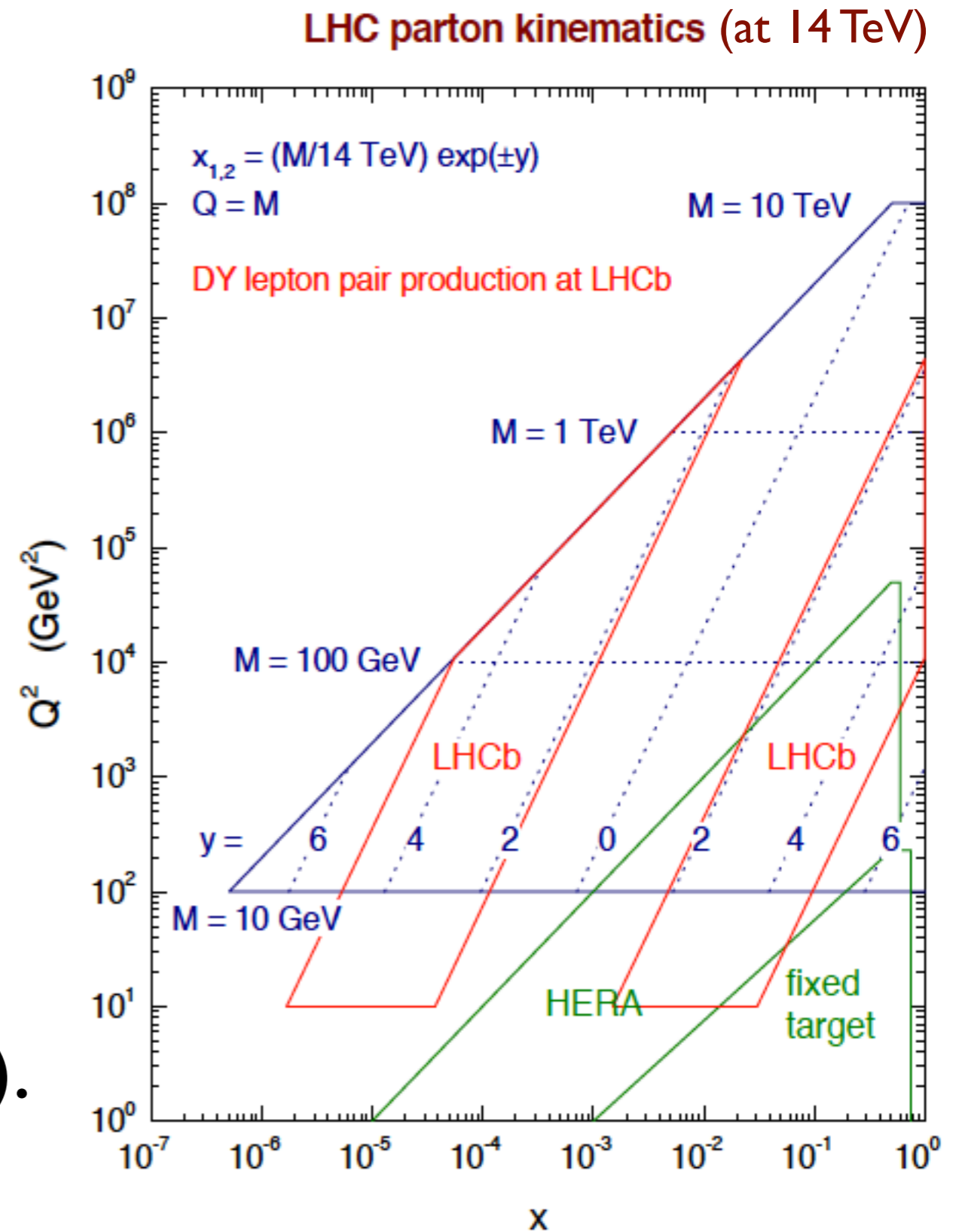


LHCb can go to lower $M_{\mu\mu}$!

[CMS PAS EWK-10-007]

Low-mass Drell-Yan at LHCb

- Measure low-mass ($M_{\mu\mu} > 2.5 \text{ GeV}$) Drell-Yan at forward rapidities to access **small- x PDFs** beyond the reach of HERA.
- Expect need to resum $\alpha_s \log(1/x)$ terms (**BFKL**) in addition to usual $\alpha_s \log(Q^2)$ terms (**DGLAP**).

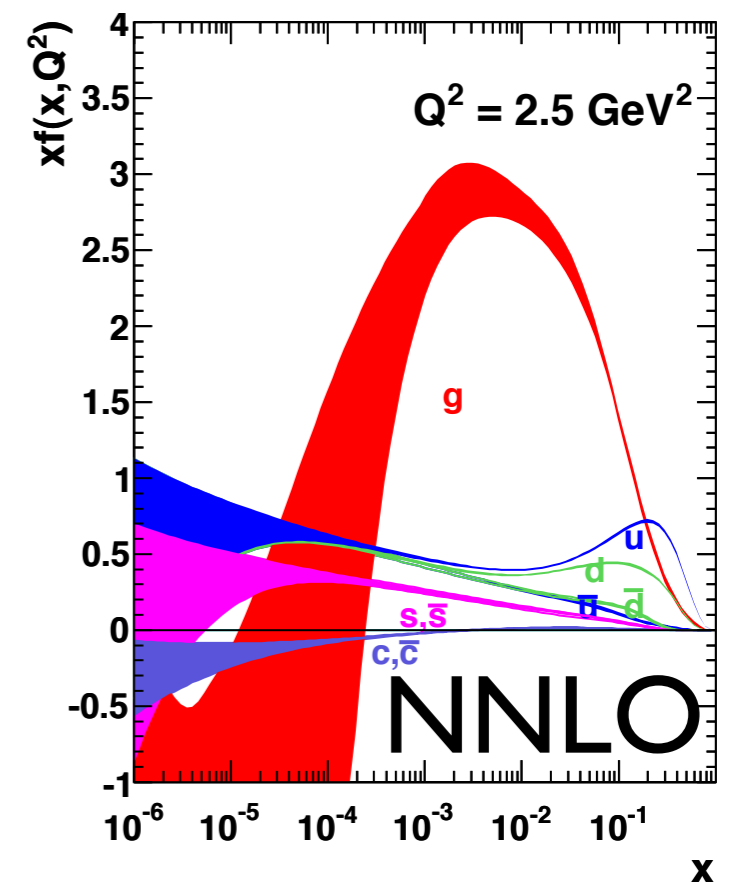
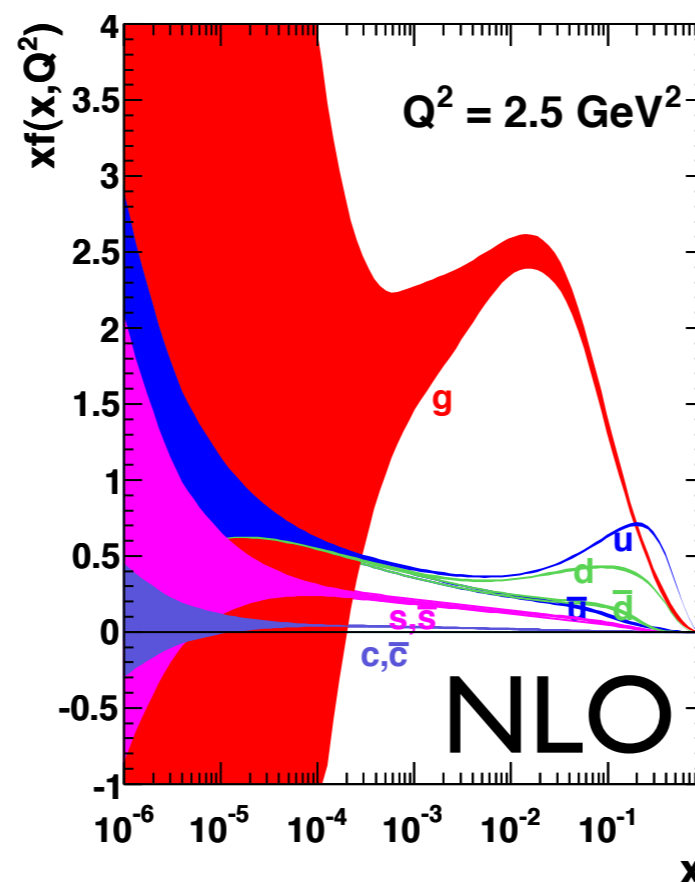
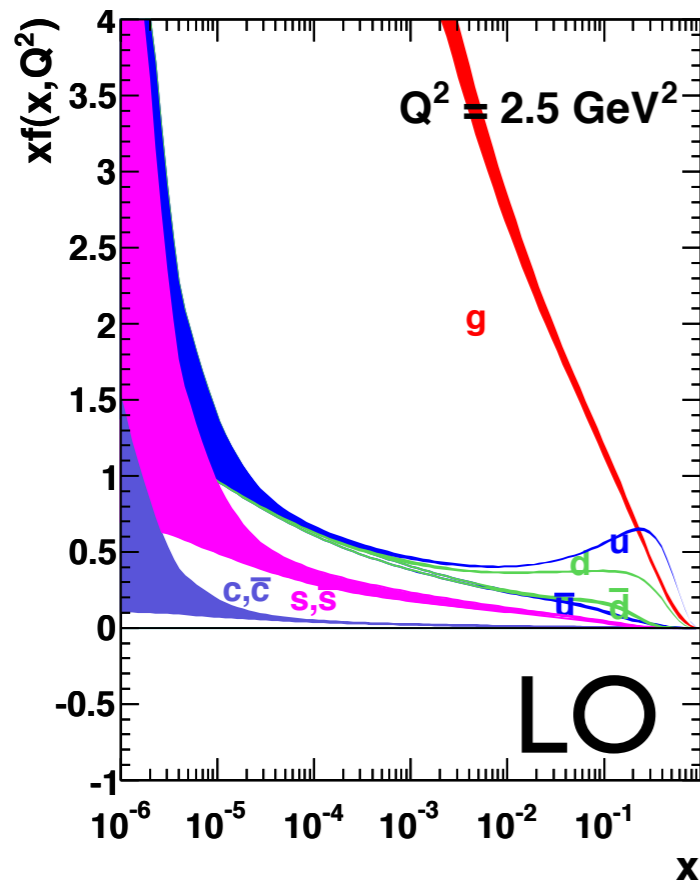


PDFs at low x and Q^2

MSTW 2008 LO PDFs (68% C.L.)

MSTW 2008 NLO PDFs (68% C.L.)

MSTW 2008 NNLO PDFs (68% C.L.)

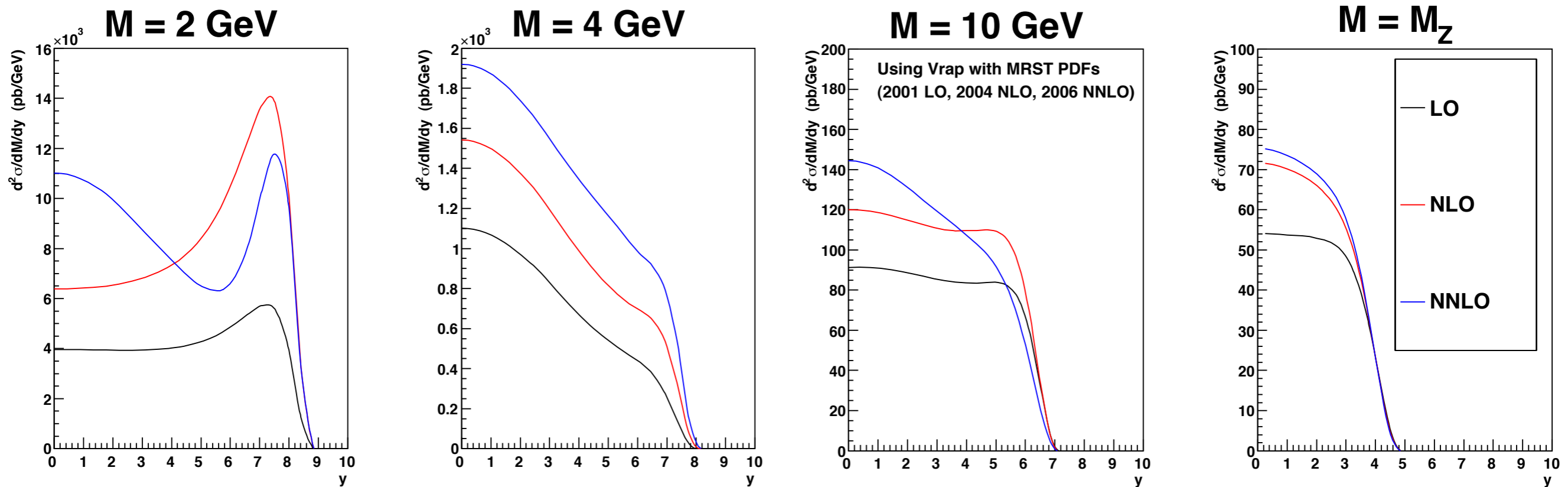


- Little or no data constraint for $x < 10^{-4}$.
- Large differences in small- x PDFs at different perturbative orders, particularly for the gluon.

Drell-Yan rapidity distributions

- Look at LO \rightarrow NLO \rightarrow NNLO behaviour.
- Match order of PDF to order of calculation.

γ^*/Z rapidity at LHC ($\sqrt{s} = 14$ TeV)

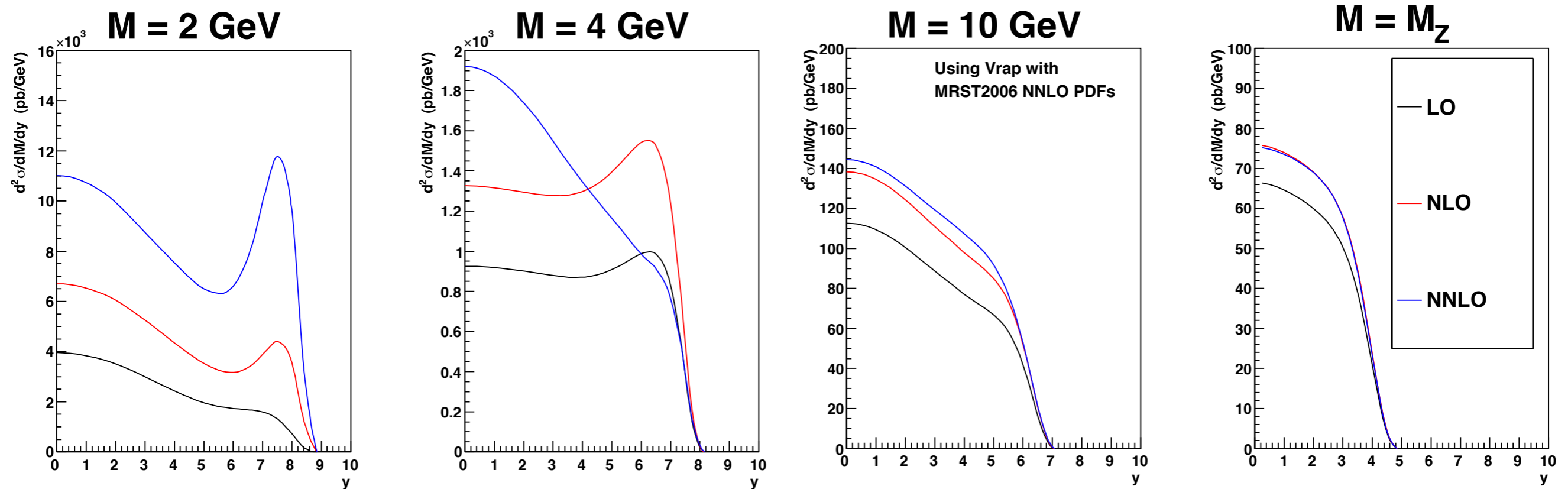


- Rapidity distributions unstable for $M^2 \ll s$.

Drell-Yan rapidity distributions

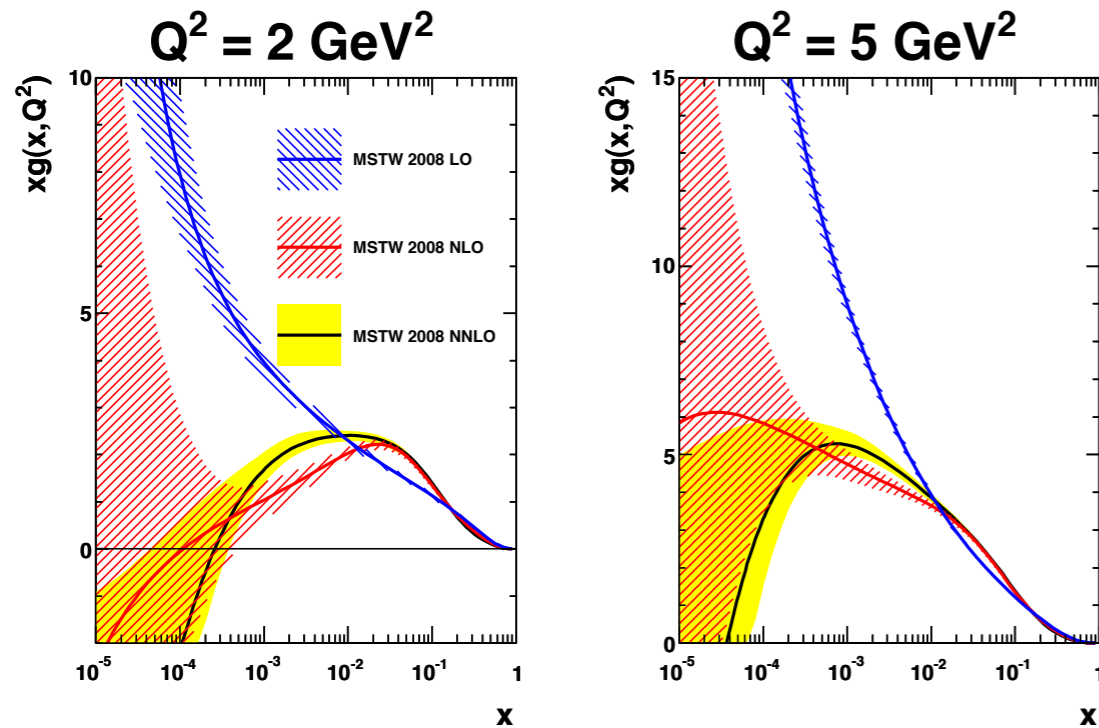
- Same NNLO PDF at each perturbative order.
⇒ Isolate differences from partonic cross section.

γ^*/Z rapidity at LHC ($\sqrt{s} = 14$ TeV)

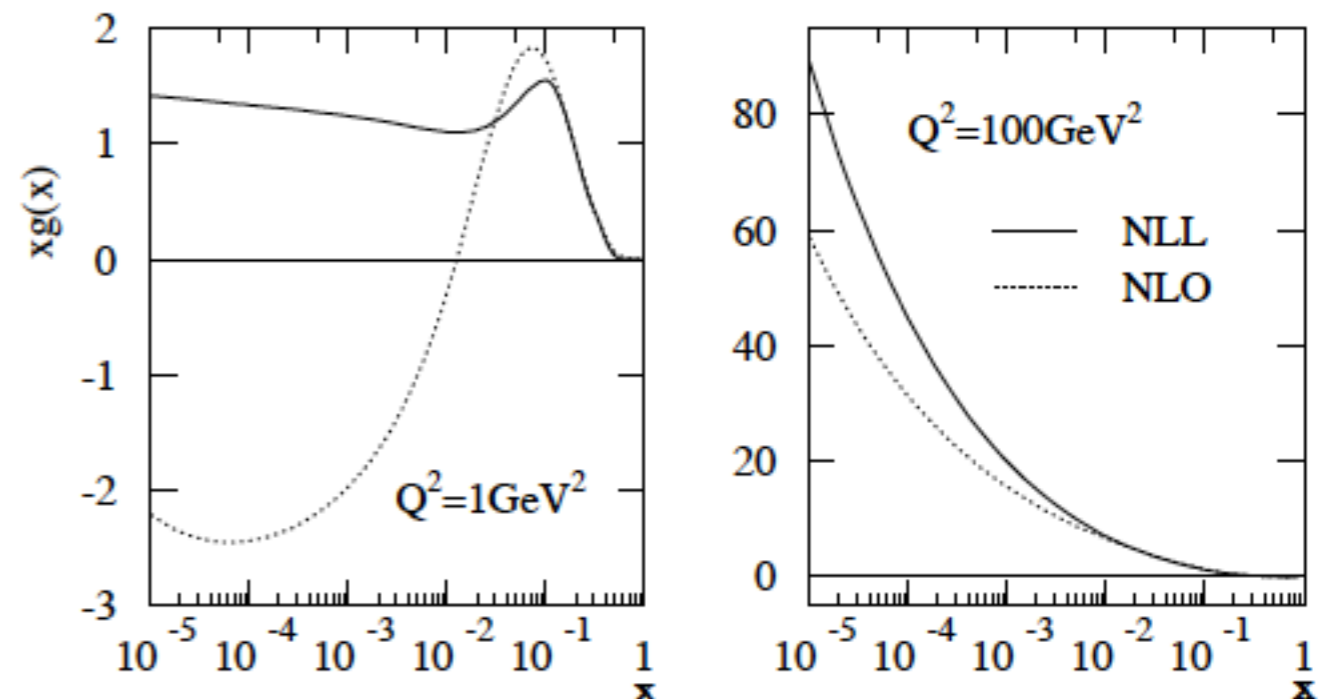
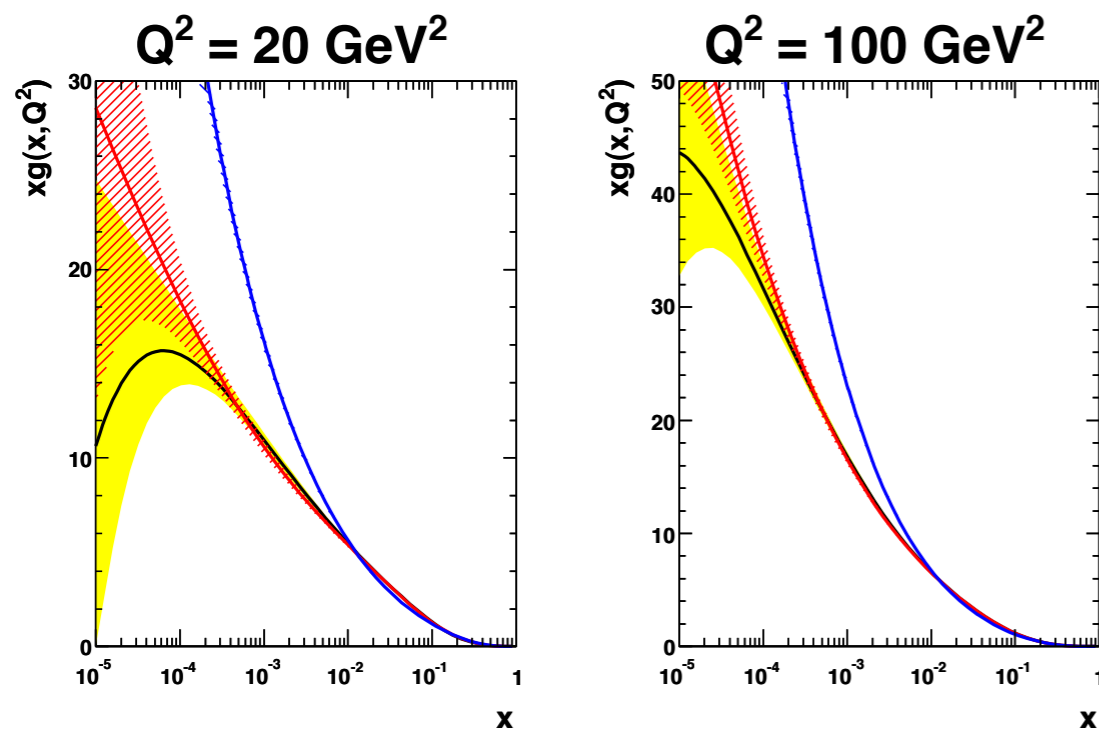


- DY coefficient functions unstable for $M^2 \ll s$.

Small-x gluon distribution



- Gluon perturbatively unstable as the scale Q^2 is lowered.
- Behaviour improved by small-x resummation: C. D. White and R. S. Thorne [[hep-ph/0611204](#)].

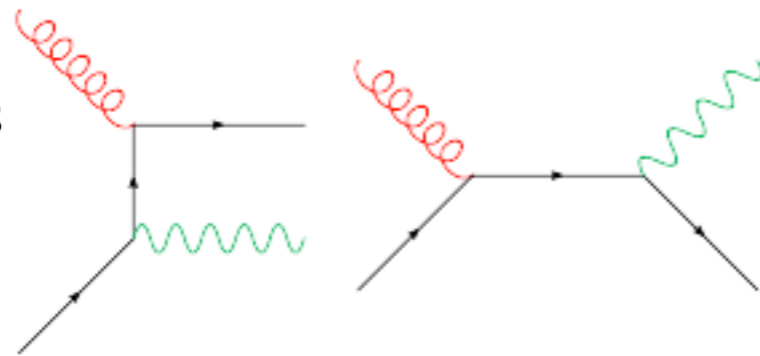


Small- x resummation for Drell-Yan at LHC

S. Marzani and R. D. Ball [[arXiv:0812.3602](#), [arXiv:0906.4729](#), [arXiv:1006.2314](#)]

Resummed coefficient functions

$$g^* q \rightarrow \gamma^* q$$

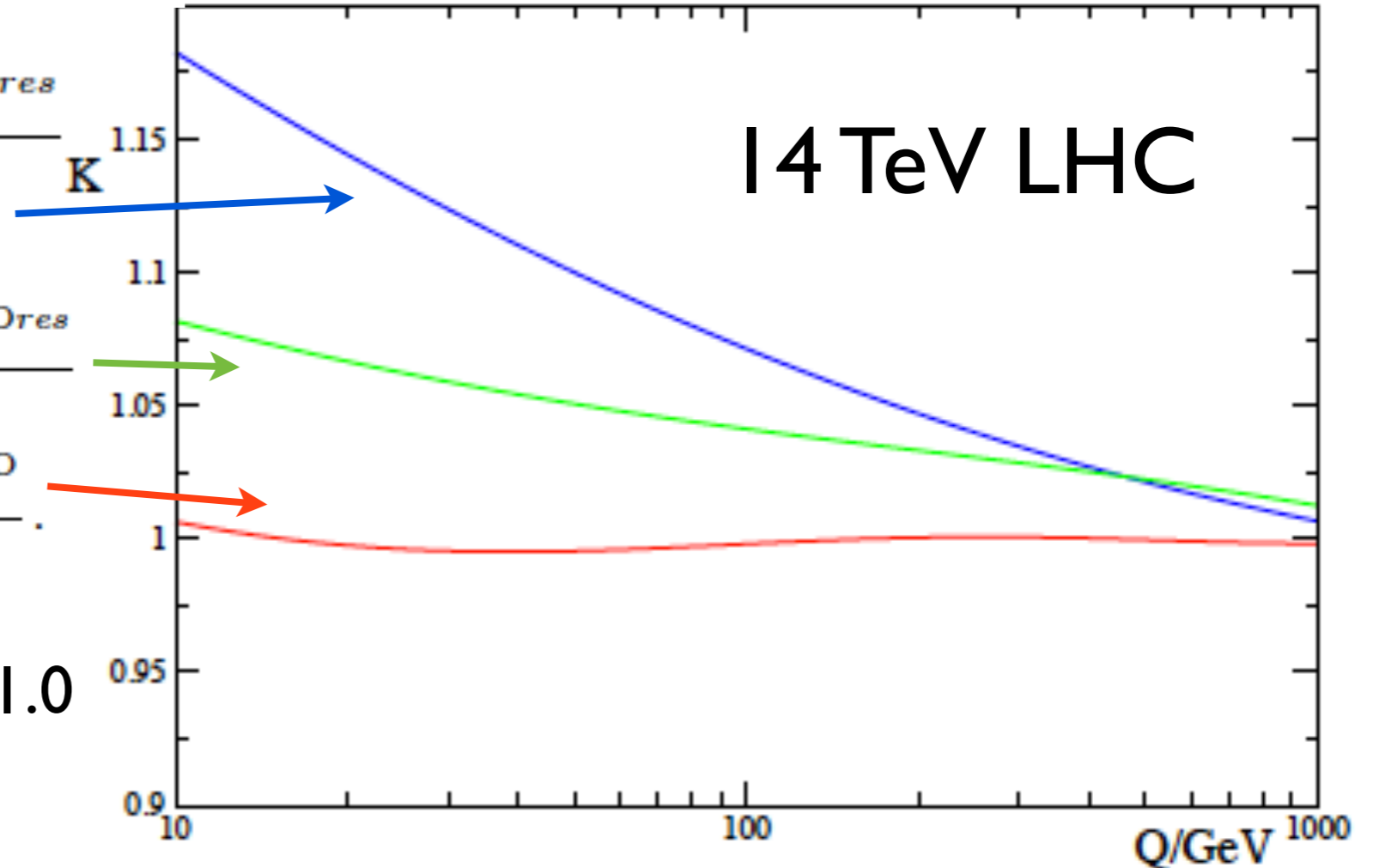


Resummed evolution kernels:
Altarelli, Ball, Forte [[arXiv:0802.0032](#)]

$$K^{NLO} = \frac{D_{ij}^{NLOres} \otimes f_i^{NLOres} \otimes f_j^{NLOres}}{D_{ij}^{NLO} \otimes f_i^{NLO} \otimes f_j^{NLO}}$$

$$K_1^{NNLO} = \frac{D_{ij}^{NNLOres} \otimes f_i^{NLOres} \otimes f_j^{NLOres}}{D_{ij}^{NNLO} \otimes f_i^{NLO} \otimes f_j^{NLO}}$$

$$K_2^{NNLO} = \frac{D_{ij}^{NNLOres} \otimes f_i^{NNLO} \otimes f_j^{NNLO}}{D_{ij}^{NNLO} \otimes f_i^{NNLO} \otimes f_j^{NNLO}}$$

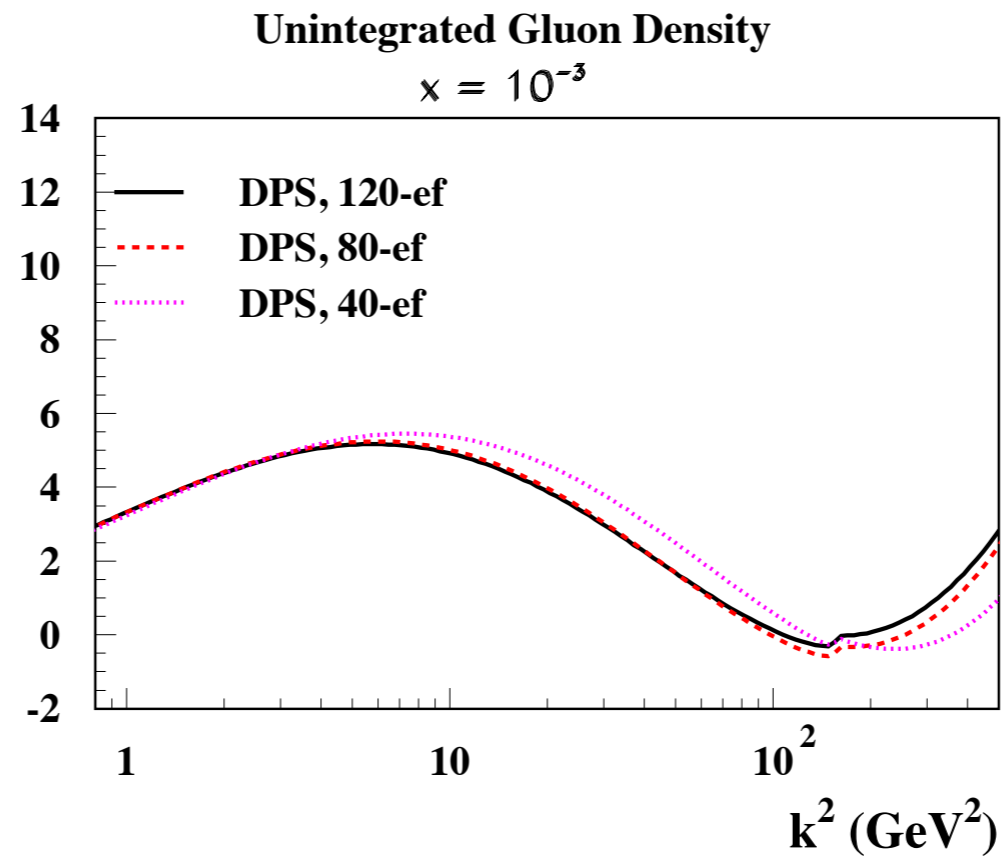
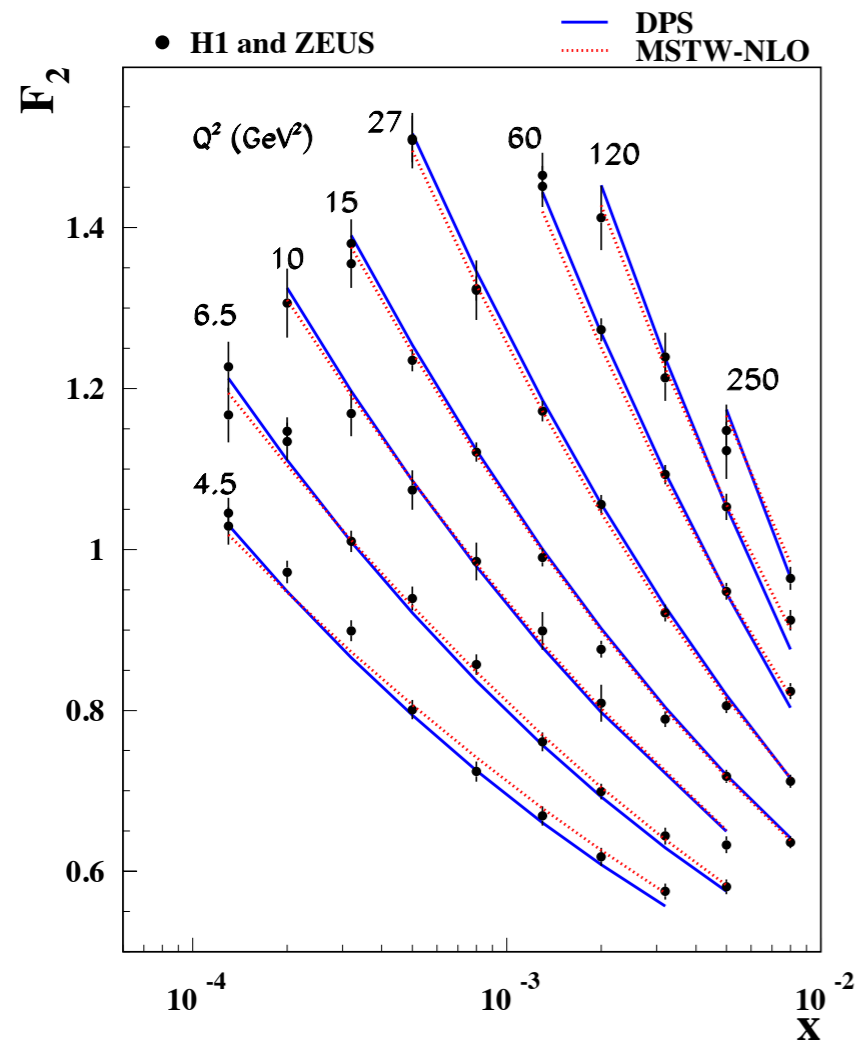


- Input PDFs from NNPDF1.0 at $Q_0 = 3$ GeV (no refit).

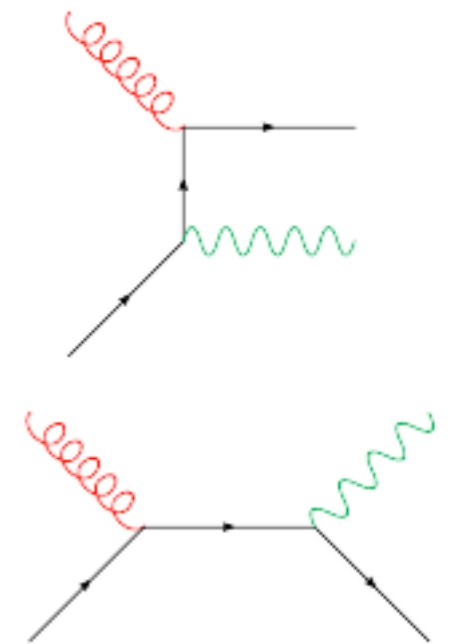
- Larger effects expected for DY rapidity distributions (in progress).
Method applied to $gg \rightarrow H$ by Caola, Forte, Marzani [[arXiv:1010.2743](#)].

Discrete Pomeron Solution to BFKL equation

H. Kowalski, L. N. Lipatov, D.A. Ross, G.Watt [[arXiv:1005.0355](https://arxiv.org/abs/1005.0355)]

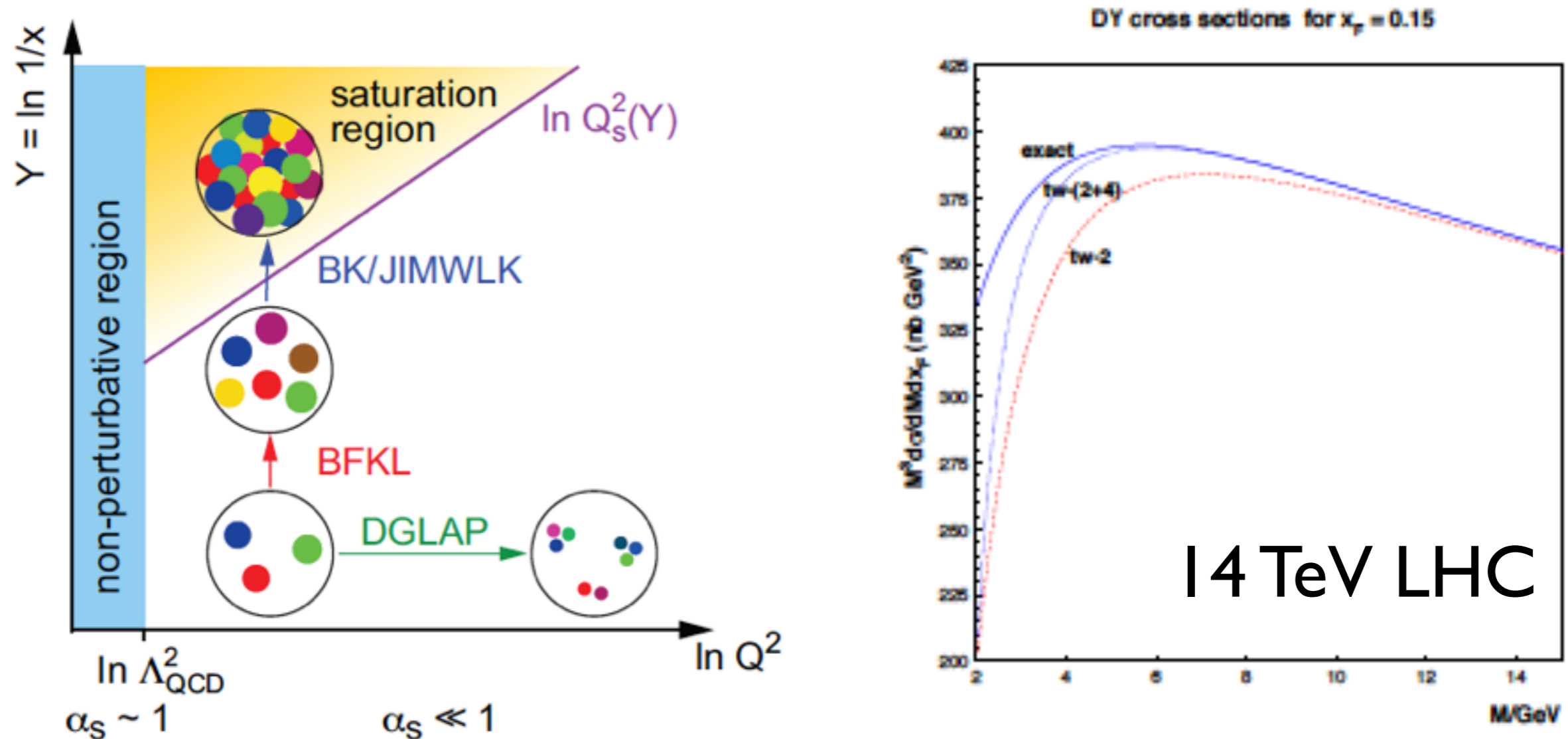


$$g^* q \rightarrow \gamma^* q$$



- Successful BFKL description of HERA data.
- Could use “unintegrated” gluon density with $g^* q \rightarrow \gamma^* q$ subprocess to calculate forward Drell-Yan at LHC.

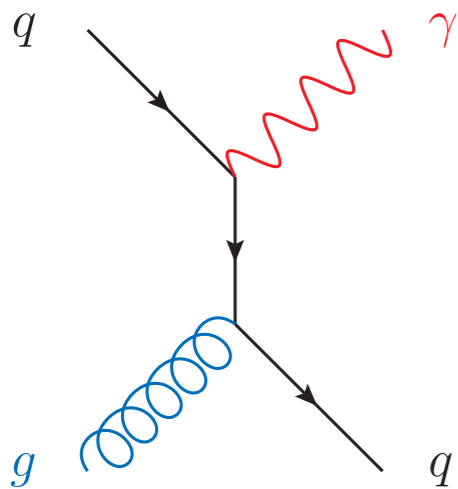
Parton saturation effects



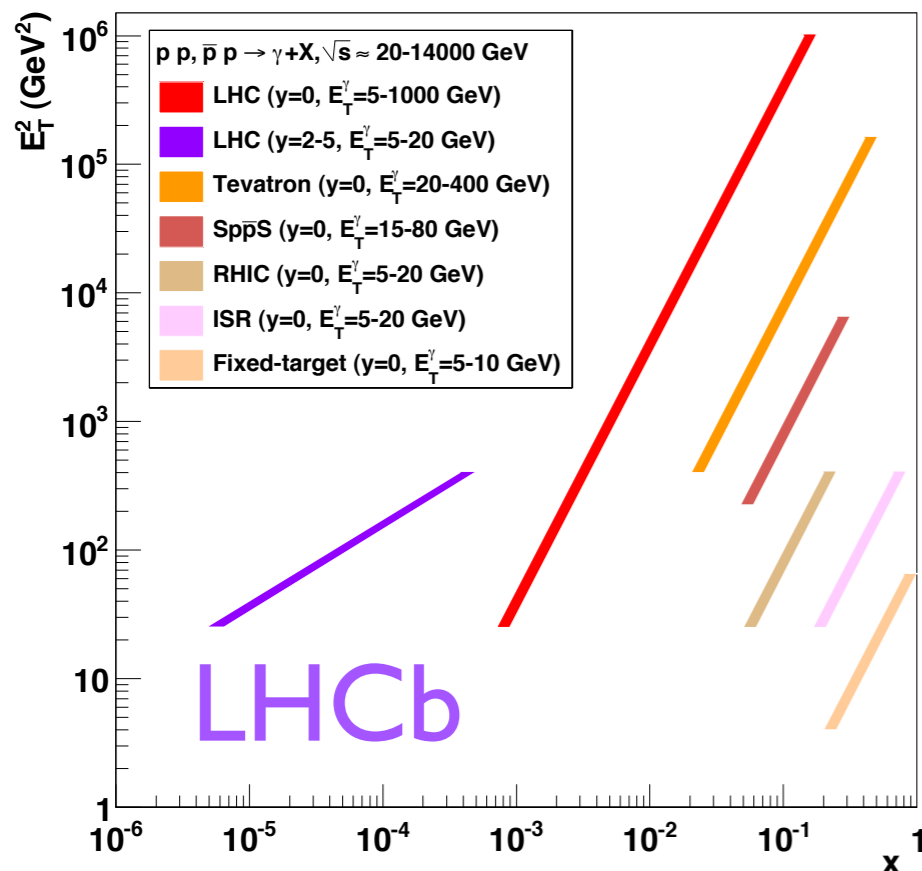
Golec-Biernat, Lewandowska, Staśto [[arXiv:1008.2652](https://arxiv.org/abs/1008.2652)]

- *Twist* expansion in powers of (Q_s^2 / M^2) .
- Leading-twist sufficient for M larger than ~ 6 GeV.
Twist expansion divergent for $M < 4$ GeV.

Isolated photon production



- Early PDF fits used fixed-target data on **prompt photon production** to provide a gluon constraint, but later omitted due to problems with both data and theory.
- Situation at LHC looks more promising.

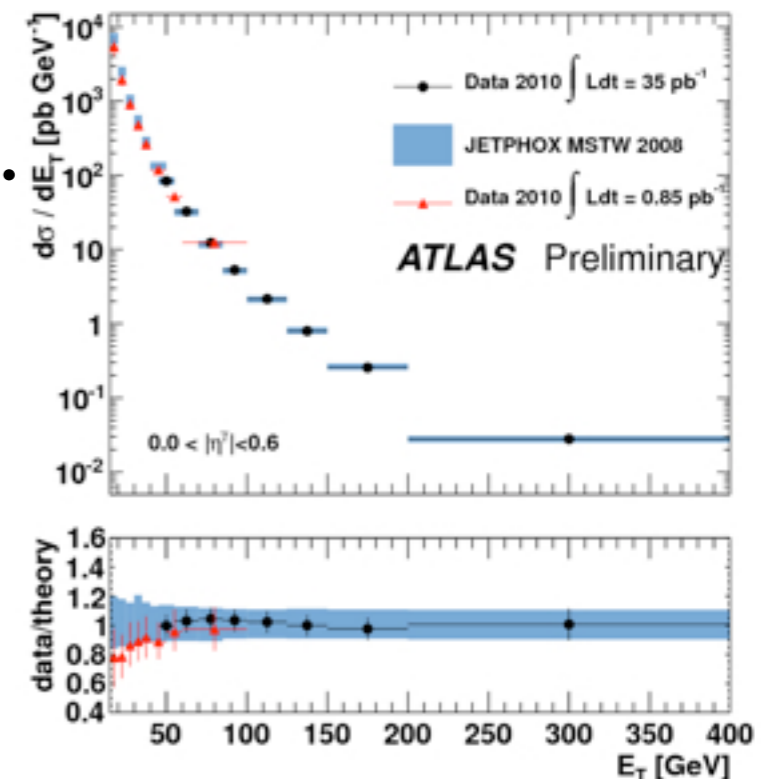


- LHCb measurement of isolated photons could constrain gluon.

$$x \sim x_T = \frac{2E_T^\gamma}{\sqrt{s}}$$

- Variations: $gs \rightarrow Wc$,
 $gc \rightarrow \gamma c$, $gb \rightarrow \gamma b$,
 $gc \rightarrow Zc$, $gb \rightarrow Zb$.

[[ATLAS-CONF-2011-058](#)]



Summary

- **Parton Distribution Functions (PDFs)** are a **non-negotiable** input to almost all theory predictions at the LHC.
- LHCb electroweak physics programme can provide significant PDF constraints complementary to ATLAS and CMS:
 - W^+, W^-, Z cross sections (and their ratios).
 - $W^\pm \rightarrow l^\pm \nu$ charge asymmetry in forward region.
 - Z/γ^* rapidity distribution in Z mass window.
- LHCb can uniquely measure forward low-mass Drell-Yan to probe small- x PDFs where resummation/saturation needed.
- Other PDF constraints: isolated photons, jets, heavy flavours.