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Large effects from small QCD instantons:

soft bombs @ hadron colliders

Valya Khoze

IPPP Durham

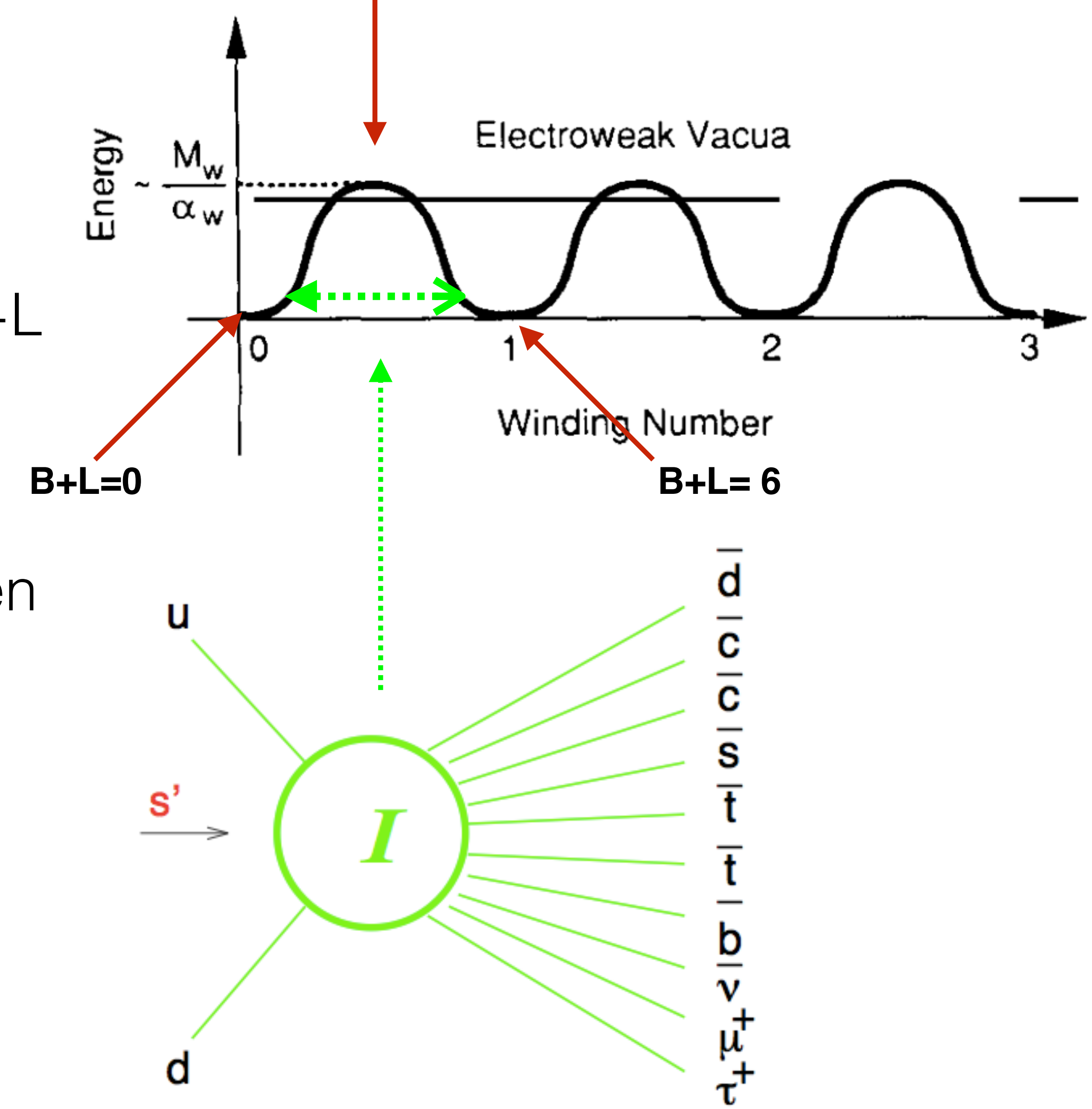
with Frank Krauss (IPPP) & Matthias Schott (Mainz)

[1st part of the forthcoming paper]

Warm-up: Electro-Weak Instantons

$$E_{\rm sph} = c_{\rm sph}\, \frac{m_W}{\alpha_W} \approx 10\,\mathrm{TeV}$$

- Yang-Mills vacuum has a nontrivial structure
- The saddle-point at the top of the barrier is the *sphaleron.* New EW scale ~ 10 TeV
- Transitions between the vacua change B+L (result of the ABJ anomaly):
  Delta (B+L)= 3 x (1+1) ; Delta (B-L)=0
- *Instantons* are tunnelling solutions between the vacua. They mediate B+L violation
- 3 x (1 lepton + 3 quarks) = 12 fermions
  12 left-handed fermion doublets are involved
- There are EW processes which are not described by perturbation theory!



$$\boxed{q + q \to 7\bar{q} + 3\bar{l} + n_W W + n_Z Z + n_h H}$$

Warm-up: Electro-Weak Instantons

- All instanton contributions come with an exponential suppression due to the instanton action:

$$\mathcal{A}^{\rm inst} \propto e^{-S^{\rm inst}} = e^{-2\pi/\alpha_w - \pi^2\rho^2 v^2}\,, \quad \sigma^{\rm inst} \propto e^{-4\pi/\alpha_w} \simeq 5 \times 10^{-162}$$

- This is precisely the expected semiclassical price to pay for a quantum mechanical tunnelling process.

- At leading order, the instanton acts as a point-like vertex with a large number n of external legs => n! factors in the amplitude.

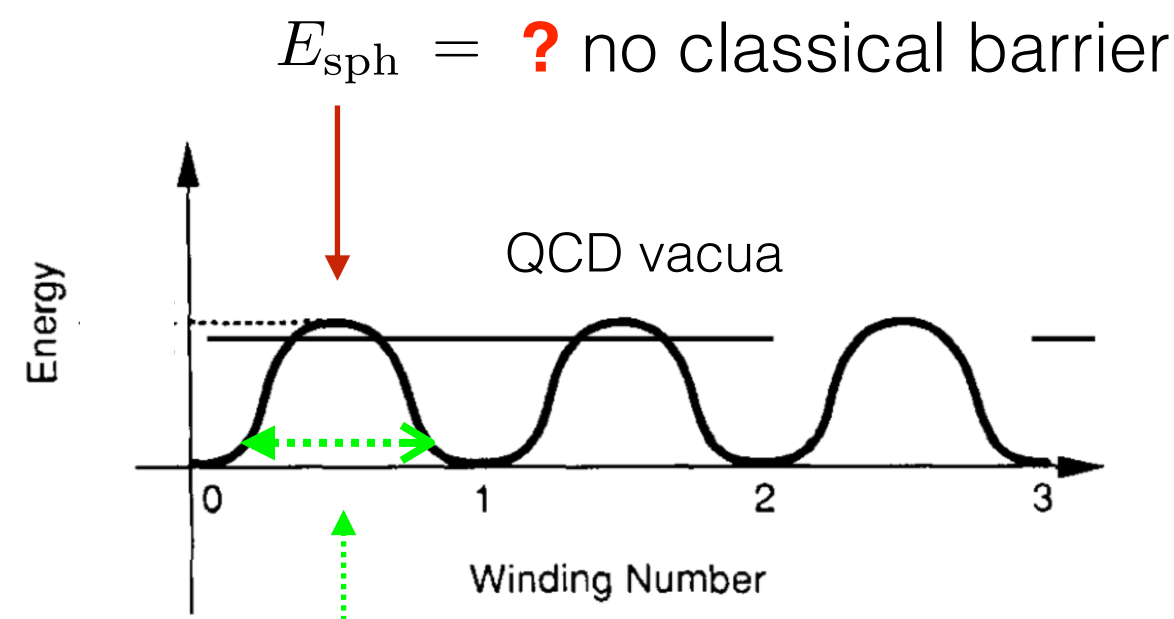
$$q + q \to 7\bar{q} + 3\bar{l} + n_W W + n_Z Z + n_h H$$

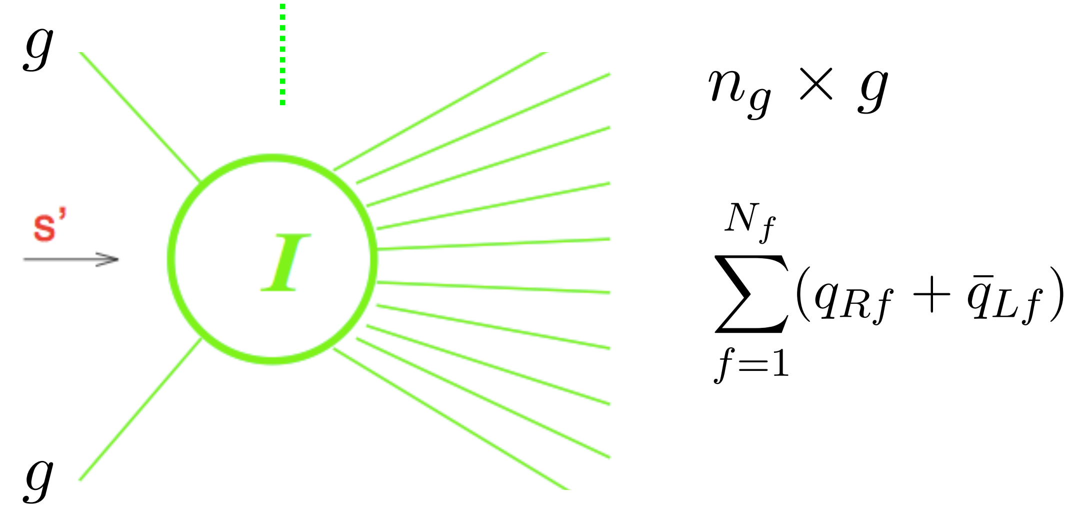
- As the number of W's, Z's and H's produced in the final state at sphaleron-like energies is allowed to be large, ~ 1/alpha, **the instanton cross-section receives exponential enhancement with energy**

Ringwald 1990; McLerran, Vainshtein, Voloshin 1990; ….

QCD Instantons

- Yang-Mills vacuum has a nontrivial structure
- At the classical level there is no barrier in QCD. The *sphaleron is a quantum effect*
- Transitions between the vacua change chirality (result of the ABJ anomaly).
- All light quark-anti-quark pairs must participate in the reaction
- *Instantons* are tunnelling solutions between the vacua.
- Not described by perturbation theory.

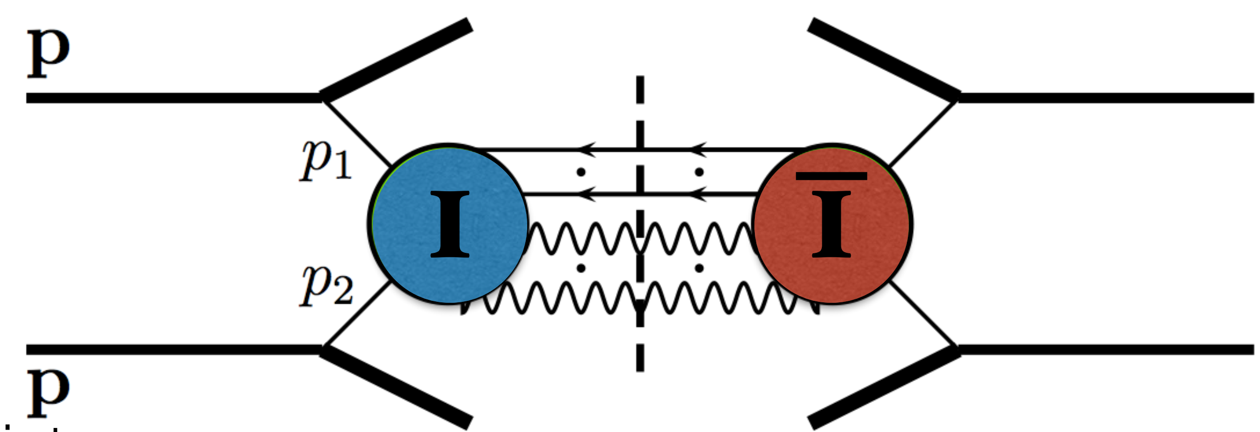


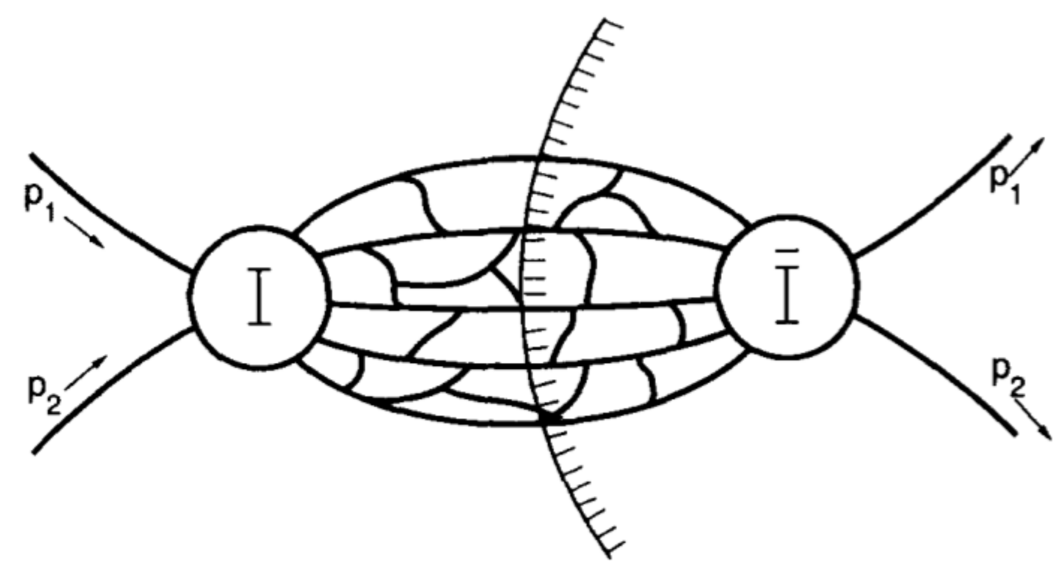


$$\boxed{g + g \to n_g \times g + \sum_{f=1}^{N_f} (q_{Rf} + \bar{q}_{Lf})}$$

The Optical Theorem approach: to include final state interactions

- Crossection is obtained by |squaring| the instanton amplitude.
- Final states have been instrumental in combatting the exp. suppression.
- Now also the interactions between the final states (and the improvement on the point-like I-vertex) are taken into account.
- Use the Optical Theorem to compute *Im* part of the 2->2 amplitude in around the Instanton-Antiinstanton configuration.
- Higher and higher energies correspond to shorter and shorter I-Ibar separations R. At R=0 they annihilate to perturbative vacuum.
- The suppression of the EW instanton cross-section is gradually reduced with energy.





VVK & Ringwald 1991

The Optical Theorem approach: to include final state interactions

- Instanton — anti-instanton valley configuration has Q=0; it interpolates between infinitely separated instanton—anti-instanton and the perturbative vacuum at z=0

$$G_{4\,\mathrm{Eucl}} \sim \int d^4R \, d\rho_I d\rho_{\bar{I}} \ldots \exp\left[i(p_1+p_2)\cdot R - S_{I\bar{I}}(z) - \pi^2 v^2(\rho_I^2+\rho_{\bar{I}}^2)\right]$$

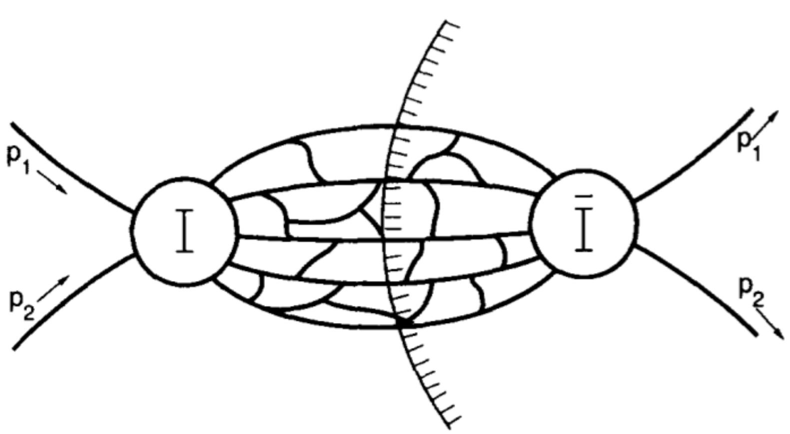
instanton separation

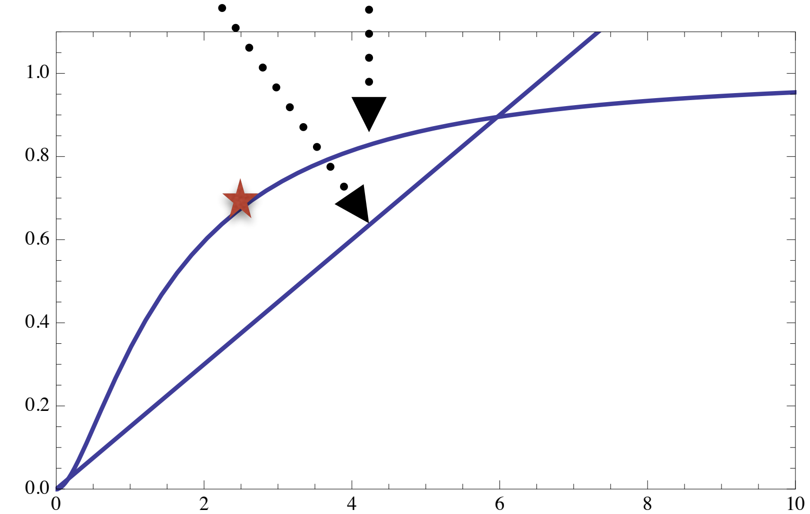
instanton sizes

$$z \sim \frac{R^2+\rho_I^2+\rho_{\bar{I}}^2}{\rho_I \rho_{\bar{I}}}$$

Higgs vev:
EW theory — **not in QCD!**

$$\sigma_{B+L} \sim \mathrm{Im} \int d^4R \, d\rho_I d\rho_{\bar{I}} \ldots \exp\left[ER - S_{I\bar{I}}(R) - \pi^2 v^2(\rho_I^2+\rho_{\bar{I}}^2)\right]$$



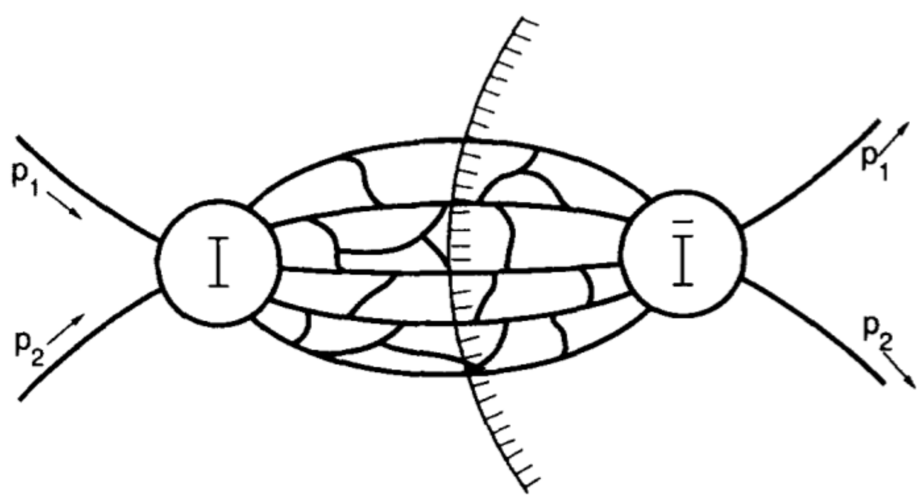


Higgs vev cuts-off large instantons

- Exponential suppression is gradually reduced with energy
- no radiative corrections from hard initial states included in this approximation

Now: in QCD

$$\sigma_{\rm tot}^{\rm (cl)\,inst} = \frac{1}{s}\,{\rm Im}\,\mathcal{A}_4^{I\bar{I}}(p_1,p_2,-p_1,-p_2)$$
$$\simeq \frac{1}{s}\,{\rm Im}\int_0^\infty d\rho \int_0^\infty d\bar{\rho}\,\int d^4R\,\int d\Omega\; D(\rho)D(\bar{\rho})\; e^{-S_{I\bar{I}}}\;\mathcal{K}_{\rm ferm}\times$$
$$A_{LSZ}^{\rm inst}(p_1)\,A_{LSZ}^{\rm inst}(p_2)\,A_{LSZ}^{\overline{\rm inst}}(-p_1)\,A_{LSZ}^{\overline{\rm inst}}(-p_2)\,,$$



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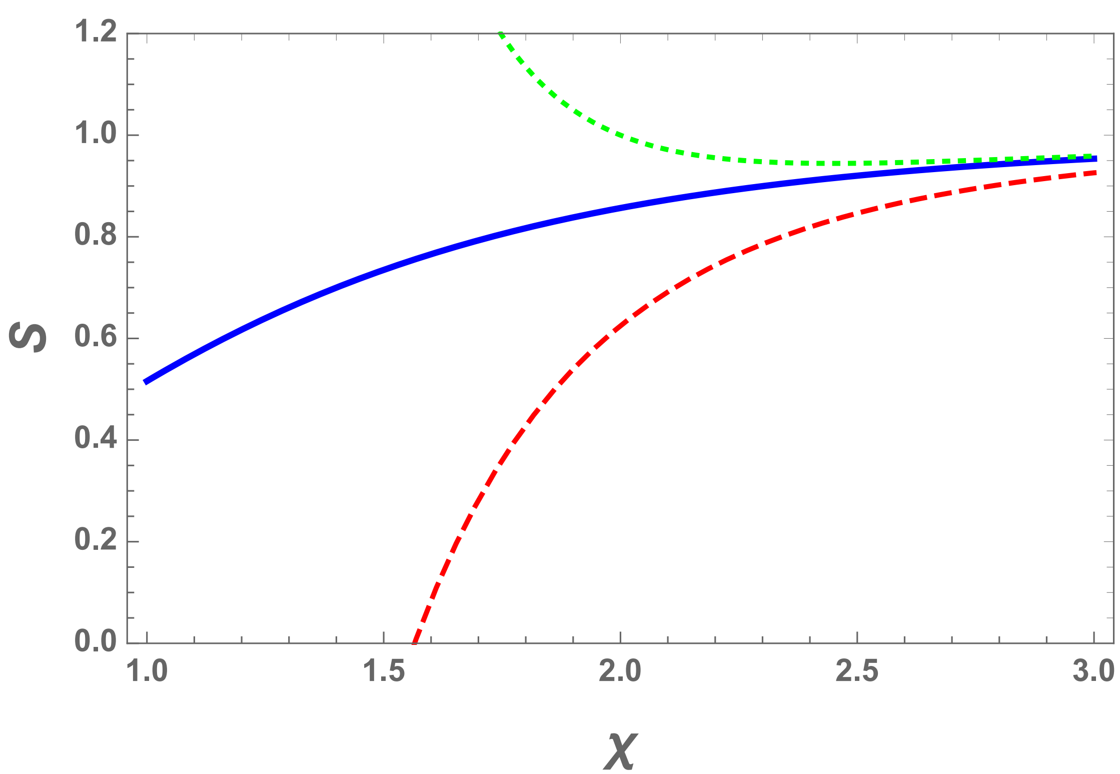
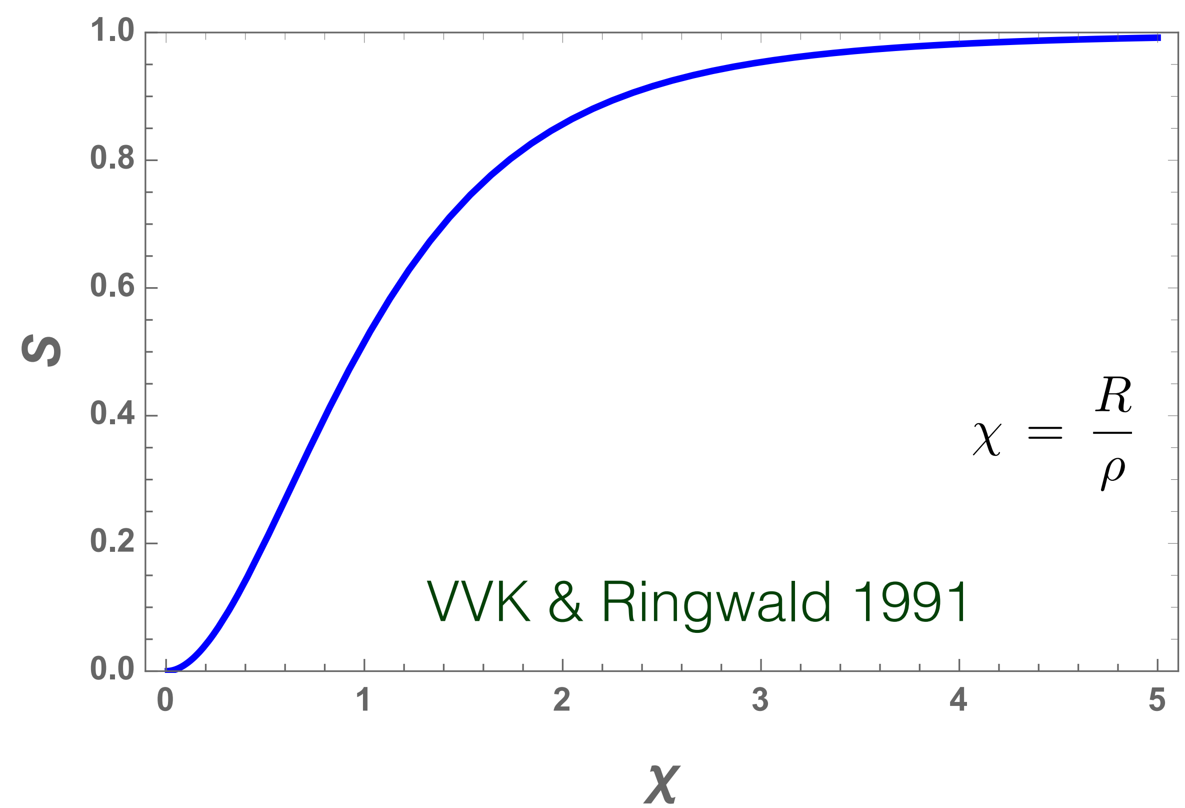
$$A_{LSZ}^{\rm inst}(p_1)\,A_{LSZ}^{\rm inst}(p_2)\,A_{LSZ}^{\overline{\rm inst}}(-p_1)\,A_{LSZ}^{\overline{\rm inst}}(-p_2)\,,$$

$$S_{I\bar I}(\rho,\bar\rho,R) = \frac{4\pi}{\alpha_s(\mu_r)}\,\hat{\mathcal{S}}$$

$$\mathcal{S}(\chi) \simeq 1 - 6/\chi^4 + 24/\chi^6 + \ldots$$





Now: in QCD

$$D(\rho, \mu_r) \;=\; \kappa\, \frac{1}{\rho^5} \left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^6 \, (\rho\mu_r)^{b_0}$$

$$\begin{aligned}
\sigma_{\rm tot}^{\rm (cl)\,inst} \;&=\; \frac{1}{s}\, {\rm Im}\, \mathcal{A}_4^{I\bar{I}}(p_1, p_2, -p_1, -p_2) \\
&\simeq\; \frac{1}{s}\, {\rm Im} \int_0^\infty d\rho \int_0^\infty d\bar{\rho} \int d^4R \int d\Omega \;\, D(\rho) D(\bar{\rho}) \; e^{-S_{I\bar{I}}} \;\, \mathcal{K}_{\rm ferm} \times \\
&\quad A_{LSZ}^{\rm inst}(p_1)\, A_{LSZ}^{\rm inst}(p_2)\, A_{LSZ}^{\overline{\rm inst}}(-p_1)\, A_{LSZ}^{\overline{\rm inst}}(-p_2)\,,
\end{aligned}$$

fermion prefactor
from Nf qq-bar pairs

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\end{aligned}$$

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$$A_{LSZ}^{\rm inst}(p_1)\, A_{LSZ}^{\rm inst}(p_2)\, A_{LSZ}^{\overline{\rm inst}}(-p_1)\, A_{LSZ}^{\overline{\rm inst}}(-p_2) \;=\; \frac{1}{36} \left(\frac{2\pi^2}{g} \rho^2 \sqrt{s'} \right)^4 \; e^{iR\cdot(p_1+p_2)}$$

$$\exp\left(R_0 \sqrt{s} \;-\; \frac{4\pi}{\alpha_s(\mu_r)}\, \hat{\mathcal{S}}(z) \right)$$

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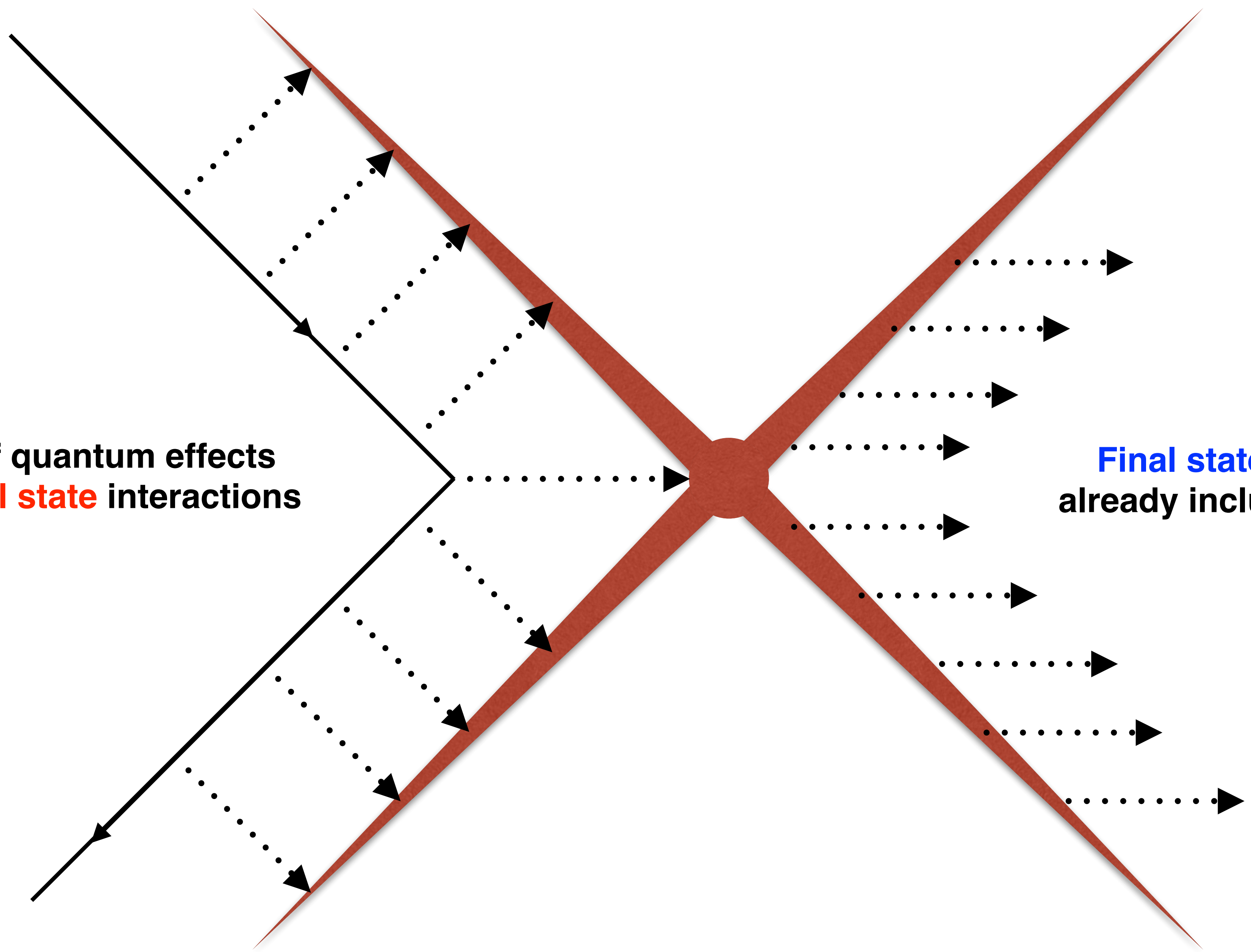
$$\exp\left(R_0\sqrt{s}\,-\,\frac{4\pi}{\alpha_s(\mu_r)}\,\hat{\mathcal{S}}(z)\right)$$

But the instanton size has not been stabilised.
In QCD - **rho** is a ***classically flat direction —***
*need to i**nclude and re-sum quantum corrections**!*

Initial state interactions in the instanton approach

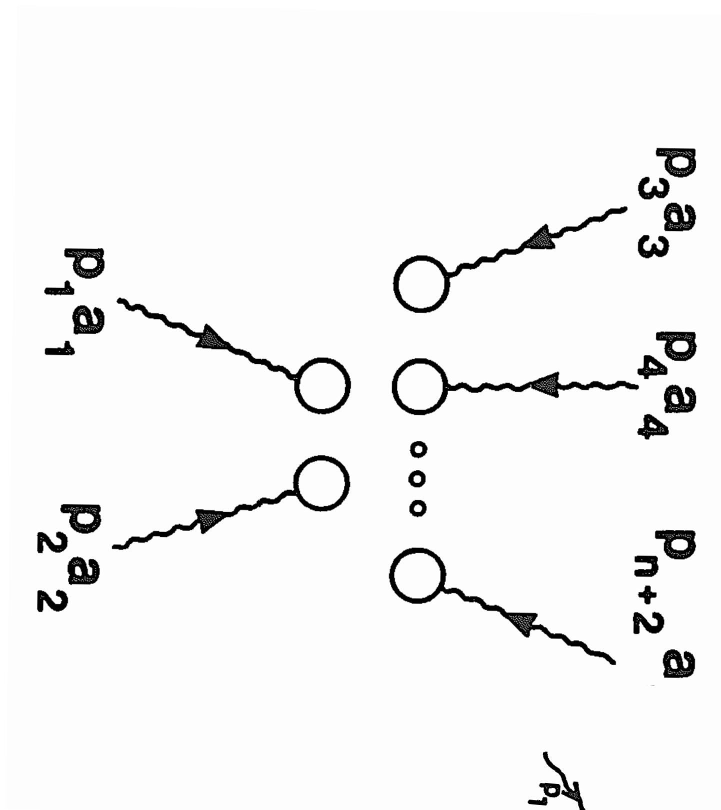


Cartoon of quantum effects due to Initial state interactions

Final states and their re-scattering already included in the optical theorem

Gluon propagator in the instanton background.

Re-sum all in-in quantum corrections



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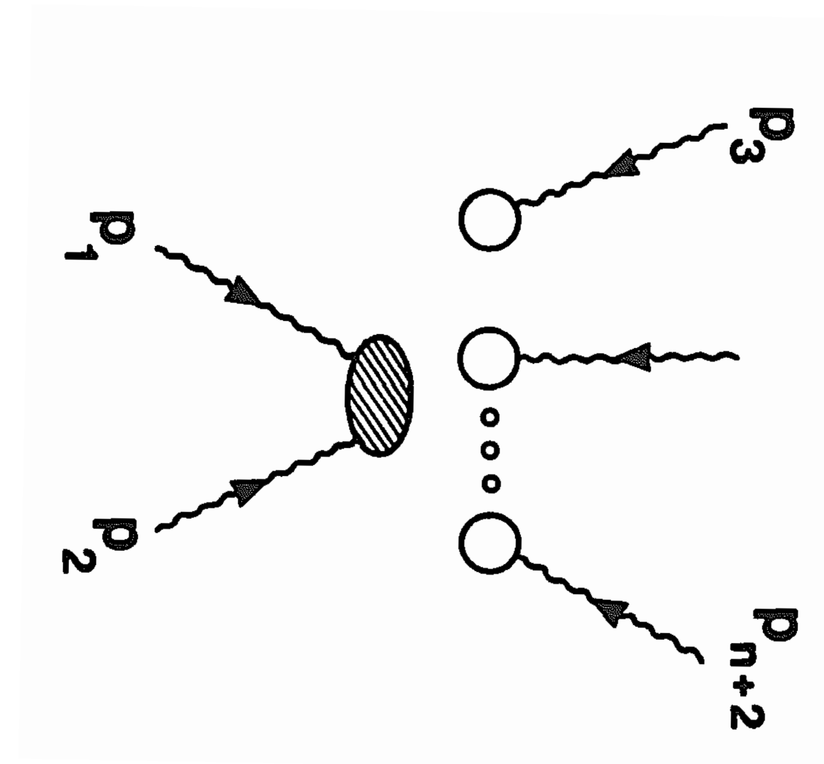

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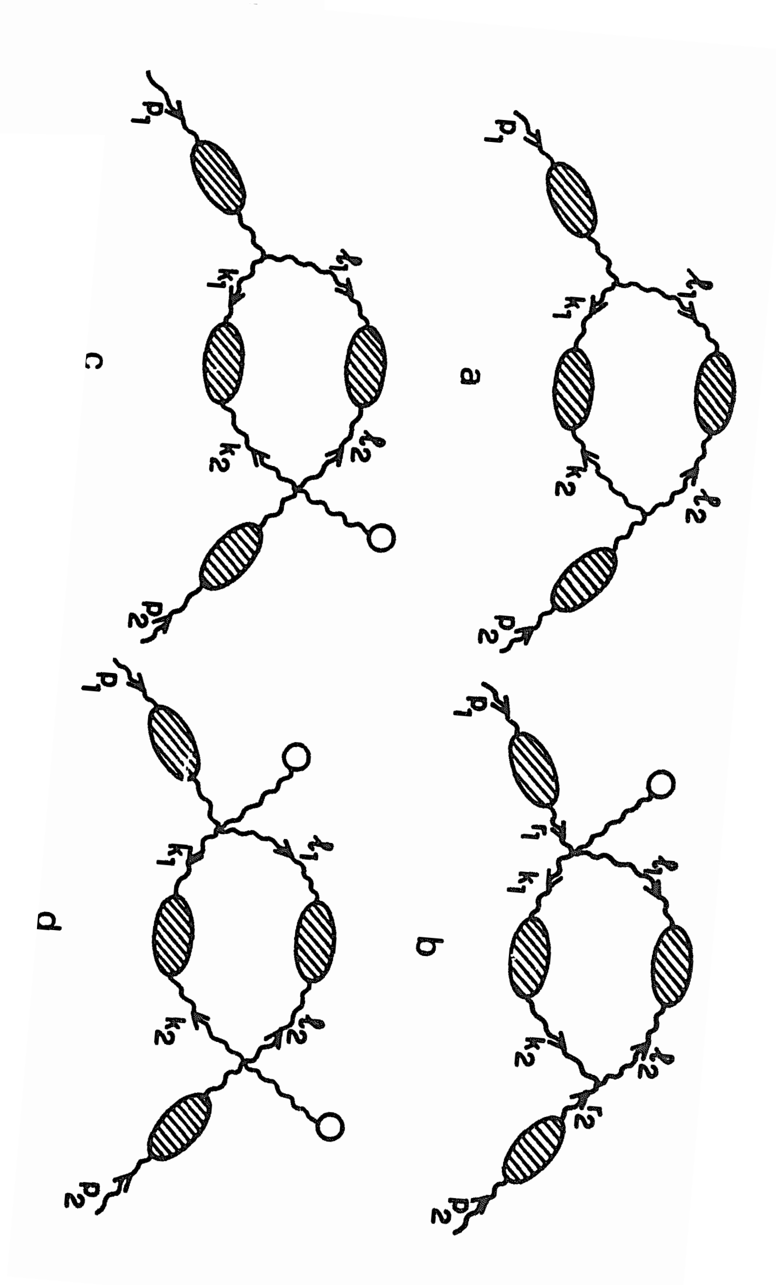

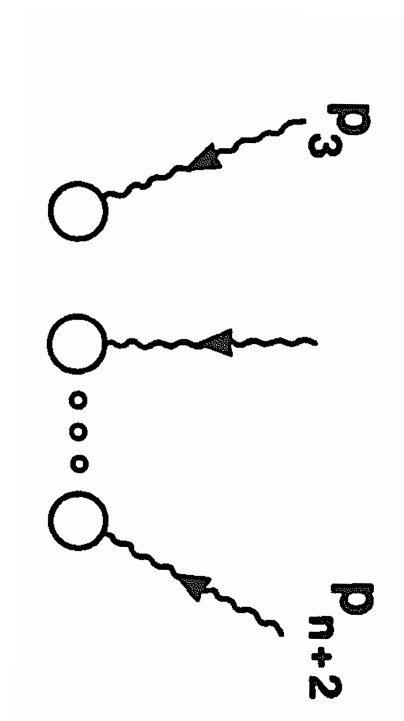

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Mueller 1991

Combined effect of initial and final states interactions in QCD

$$\hat{\sigma}_{\rm tot}^{\rm inst} \simeq \frac{1}{s'}\,{\rm Im}\,\frac{\kappa^2\pi^4}{36\cdot 4}\int\frac{d\rho}{\rho^5}\int\frac{d\bar\rho}{\bar\rho^5}\int d^4R\int d\Omega\left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^{14}(\rho^2\sqrt{s'})^2(\bar\rho^2\sqrt{s'})^2\,\mathcal{K}_{\rm ferm}$$
$$(\rho\mu_r)^{b_0}(\bar\rho\mu_r)^{b_0}\;\exp\left(R_0\sqrt{s'}\;-\;\frac{4\pi}{\alpha_s(\mu_r)}\,\hat{\mathcal{S}}(z)\;-\;\frac{\alpha_s(\mu_r)}{16\pi}(\rho^2+\bar\rho^2)\,s'\,\log\left(\frac{s'}{\mu_r^2}\right)\right)$$

Instanton size is cut-off by $\sim\sqrt{s}$
this is what sets the
effective QCD sphlarenon scale

Mueller's result for
quantum corrections
due to in-in states
interactions

Basically, in QCD one can never reach the effective
sphaleron barrier — it's hight grows with the energy.

=> Among other things, no problems with unitarity.

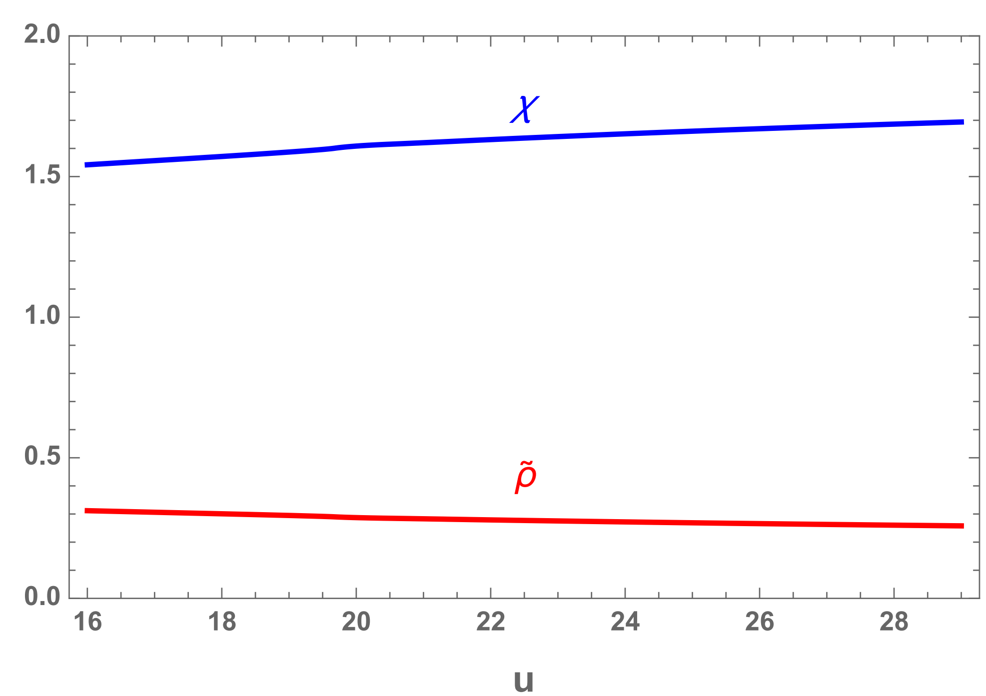
Combined effect of initial and final states interactions in QCD

$$\hat{\sigma}_{\rm tot}^{\rm inst} \simeq \frac{1}{s'}\,{\rm Im}\,\frac{\kappa^2\pi^4}{36\cdot 4}\int\frac{d\rho}{\rho^5}\int\frac{d\bar\rho}{\bar\rho^5}\int d^4R\int d\Omega\left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^{14}(\rho^2\sqrt{s'})^2(\bar\rho^2\sqrt{s'})^2\,\mathcal{K}_{\rm ferm}$$
$$(\rho\mu_r)^{b_0}(\bar\rho\mu_r)^{b_0}\,\exp\left(R_0\sqrt{s'}-\frac{4\pi}{\alpha_s(\mu_r)}\hat{\mathcal{S}}(z)-\frac{\alpha_s(\mu_r)}{16\pi}(\rho^2+\bar\rho^2)\,s'\,\log\left(\frac{s'}{\mu_r^2}\right)\right)$$

1. Extermise the holy-grail function in the exponent by finding a saddle-point in variables:

$$\mathcal{F}=\rho\chi\sqrt{s}-\frac{4\pi}{\alpha_s(\rho)}\mathcal{S}(\chi)-\frac{\alpha_s(\rho)}{4\pi}\rho^2 s\,\log(\sqrt{s}\rho)$$

$$\tilde\rho=\frac{\alpha_s(\rho)}{4\pi}\sqrt{s}\rho\,,\qquad \chi=\frac{R}{\rho}$$



$u=\sqrt{s}/\mu=\sqrt{s}\rho$

Combined effect of initial and final states interactions in QCD

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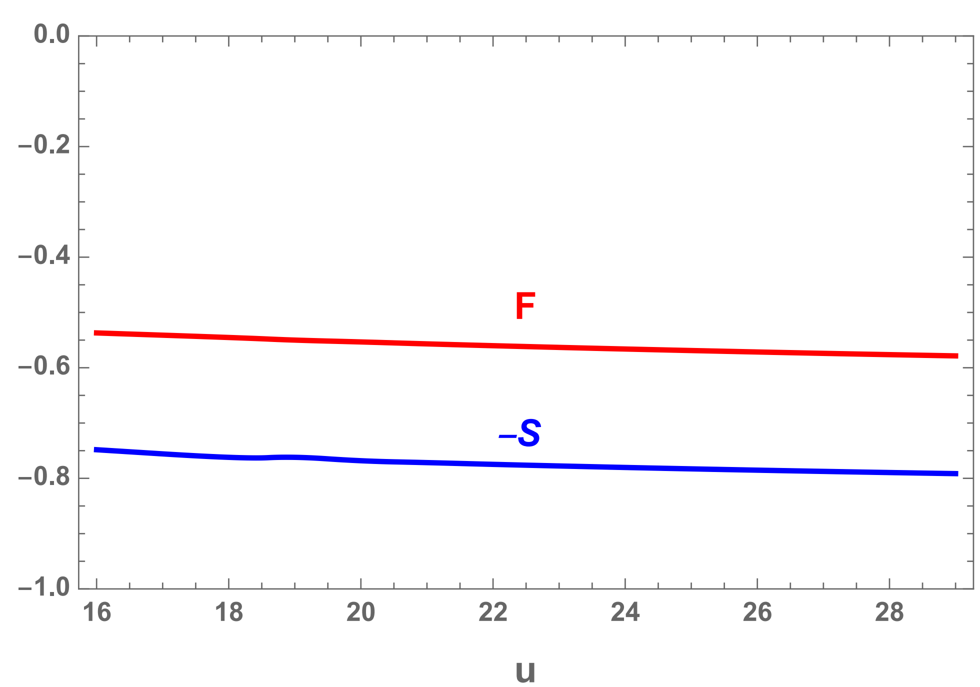
- Il action



The holy-grail

in units of $\frac{4\pi}{\alpha_s}$

$u = \sqrt{s}/\mu = \sqrt{s}\rho$

Combined effect of initial and final states interactions in QCD

$$\hat{\sigma}_{\rm tot}^{\rm inst} \simeq \frac{1}{s'}\,{\rm Im}\,\frac{\kappa^2\pi^4}{36\cdot 4}\int\frac{d\rho}{\rho^5}\int\frac{d\bar\rho}{\bar\rho^5}\int d^4R\int d\Omega\left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^{14}(\rho^2\sqrt{s'})^2(\bar\rho^2\sqrt{s'})^2\,\mathcal{K}_{\rm ferm}$$

$$(\rho\mu_r)^{b_0}(\bar\rho\mu_r)^{b_0}\;\exp\left(R_0\sqrt{s'}-\frac{4\pi}{\alpha_s(\mu_r)}\hat{\mathcal{S}}(z)-\frac{\alpha_s(\mu_r)}{16\pi}(\rho^2+\bar\rho^2)\,s'\,\log\left(\frac{s'}{\mu_r^2}\right)\right)$$

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$$\tilde\rho\;=\;\frac{\alpha_s(\rho)}{4\pi}\,\sqrt{s}\rho\,,\qquad \chi\;=\;\frac{R}{\rho}$$

2. Carry out all integrations using the steepest descent method evaluating the determinants of quadratic fluctuations around the saddle-point solution

3. Pre-factors are very large — they compete with the semiclassical exponent which is very small!

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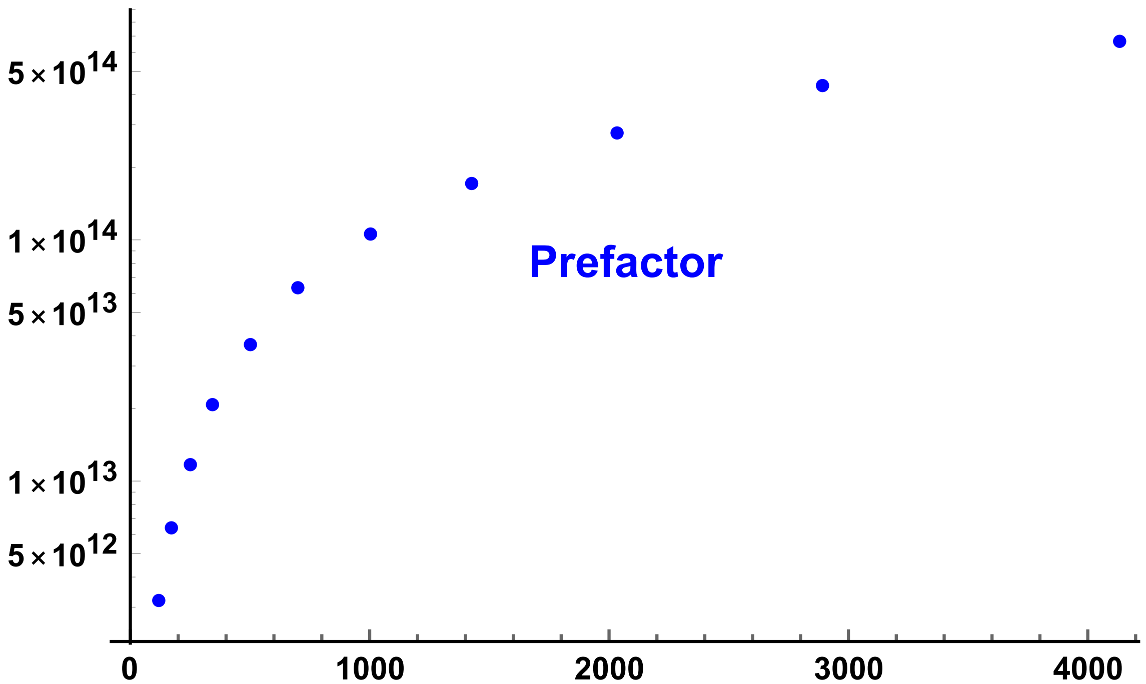
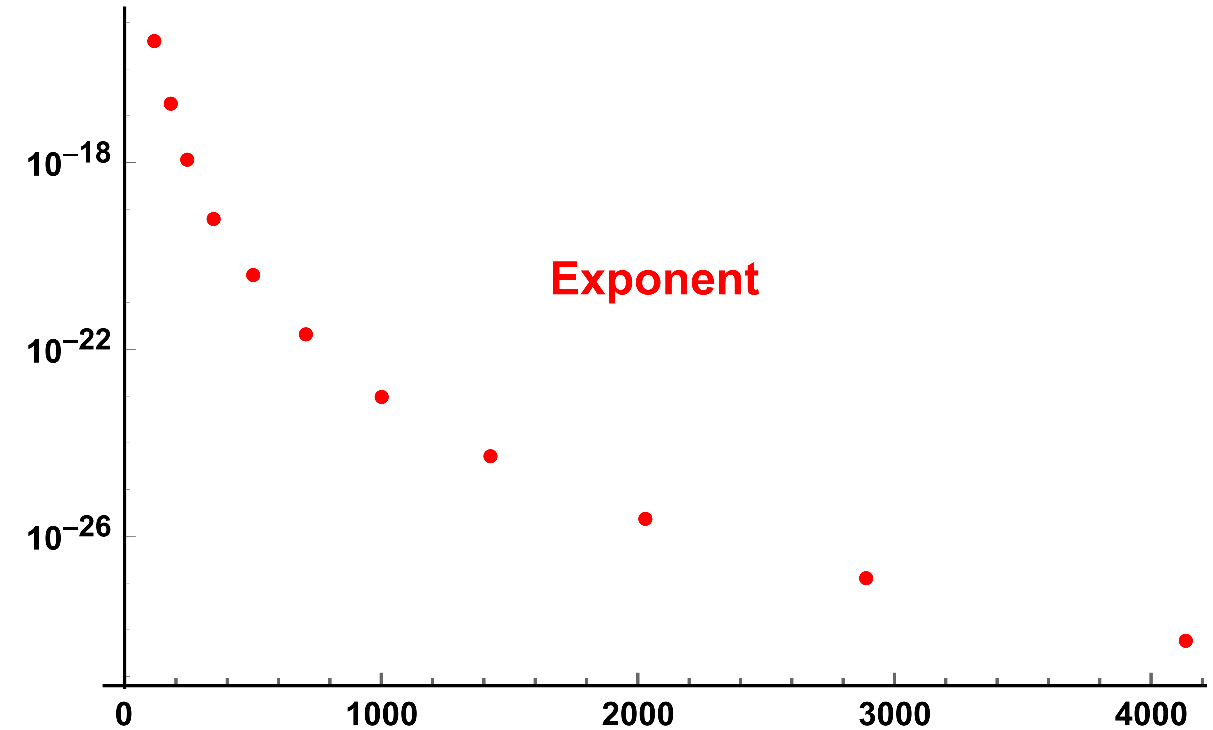
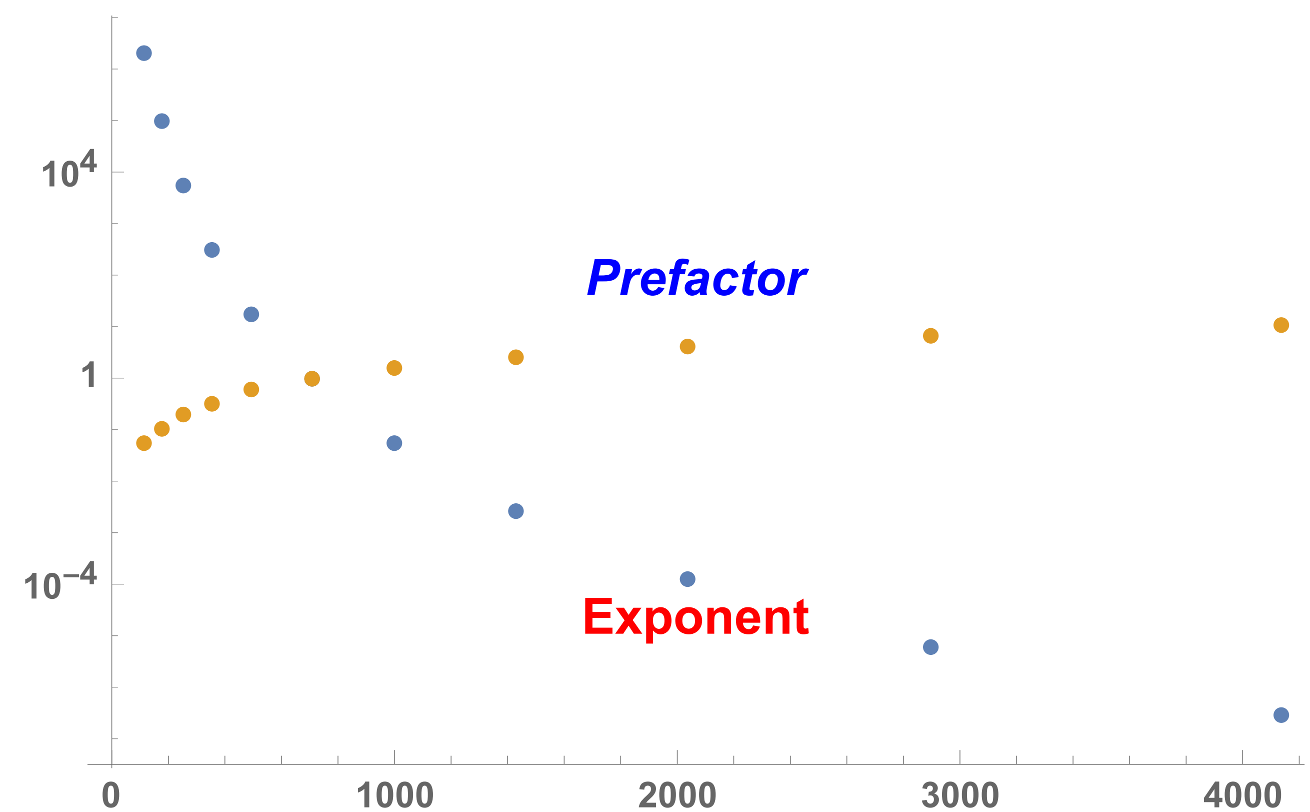
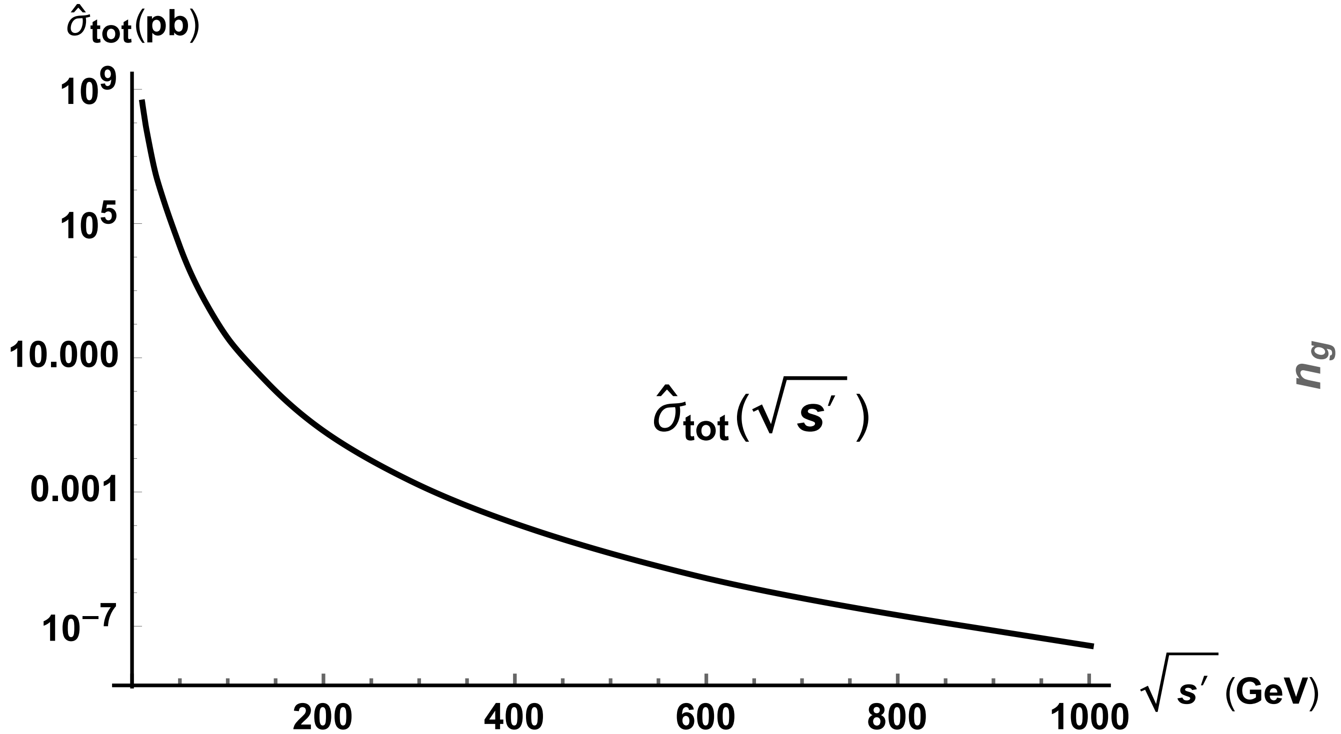
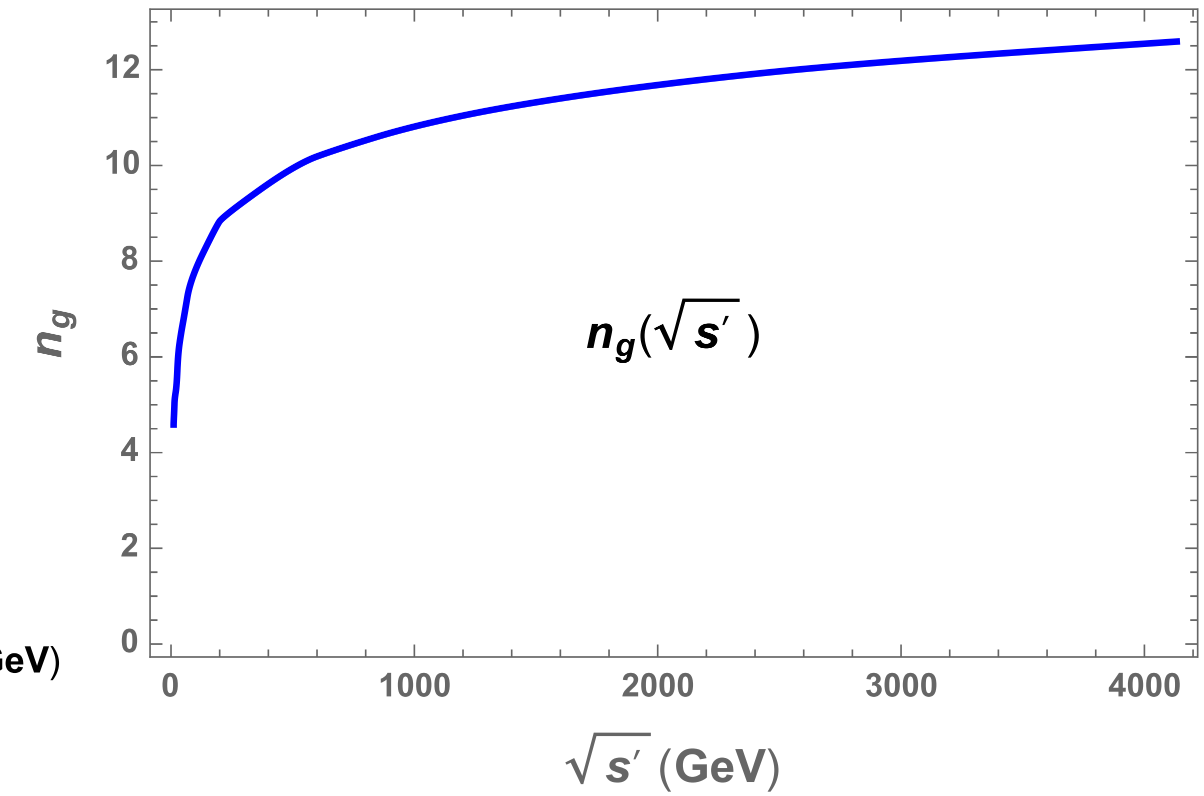


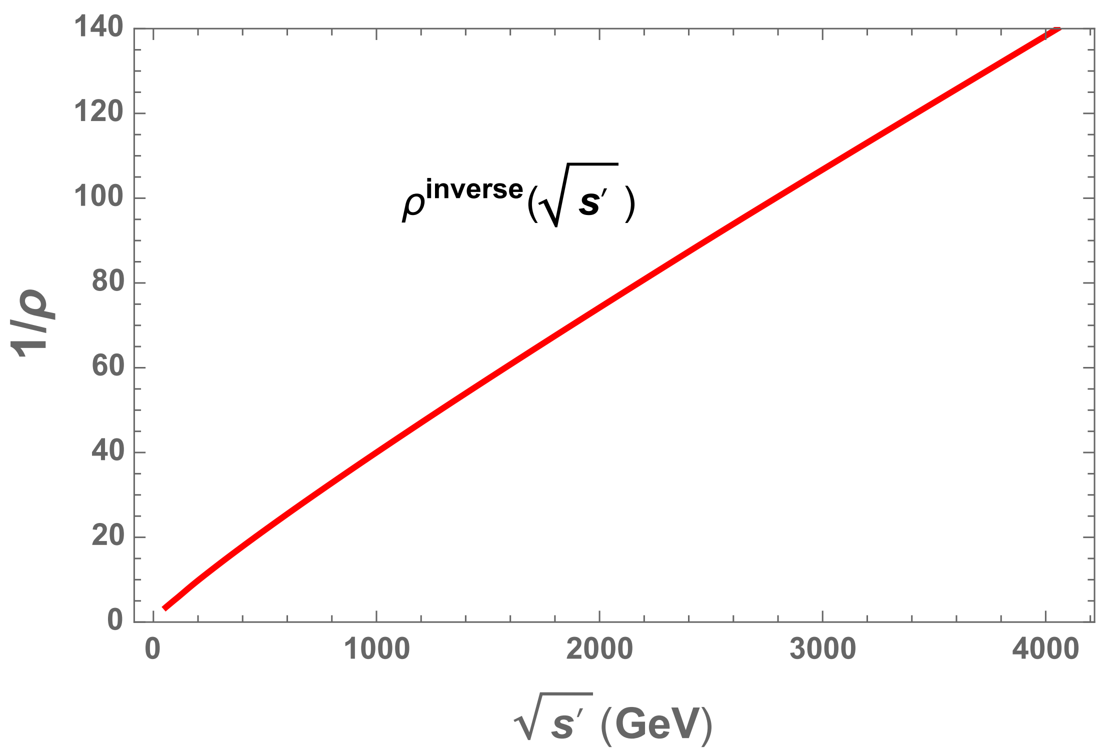


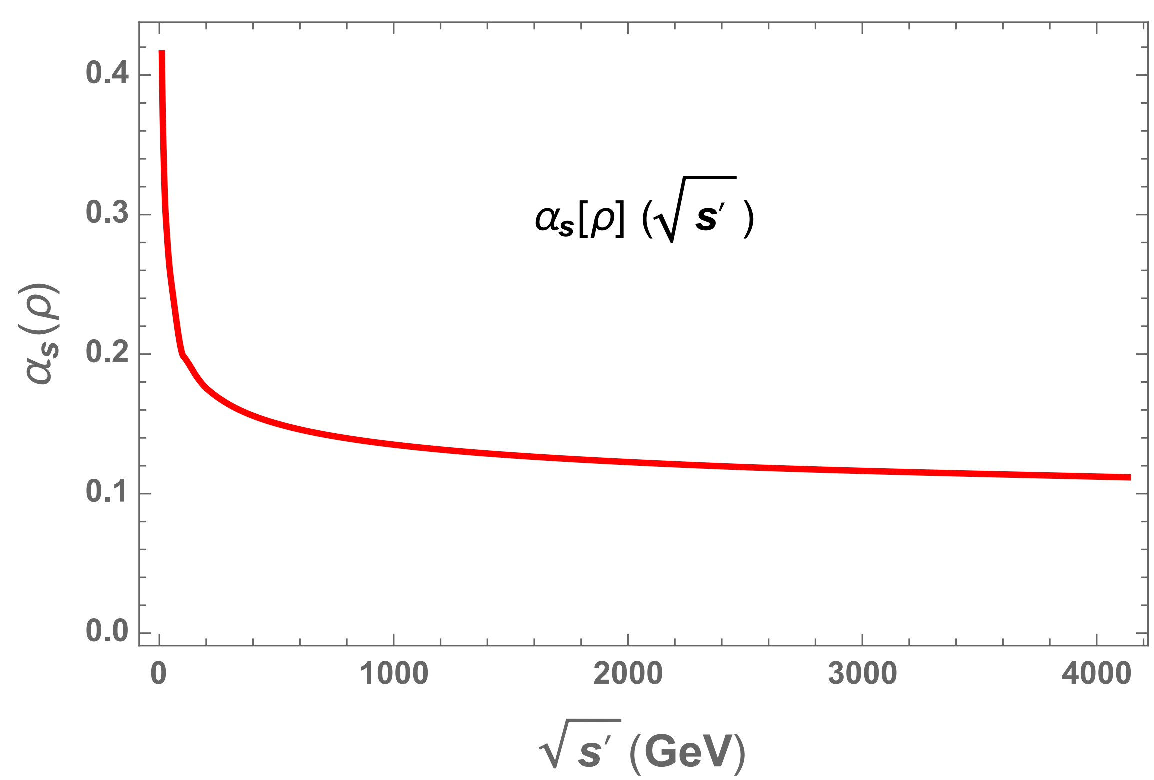


Results


$\hat{\sigma}_{tot}$(pb)
10^9
10^5
10.000
0.001
10^{-7}
200
400
600
800
1000
$\sqrt{s'}$ (GeV)
$\hat{\sigma}_{tot}(\sqrt{s'})$


n_g
0
2
4
6
8
10
12
$n_g(\sqrt{s'})$
0
1000
2000
3000
4000
$\sqrt{s'}$ (GeV)


$1/\rho$
0
20
40
60
80
100
120
140
$\rho^{inverse}(\sqrt{s'})$
0
1000
2000
3000
4000
$\sqrt{s'}$ (GeV)


$\alpha_s(\rho)$
0.0
0.1
0.2
0.3
0.4
$\alpha_s[\rho]$ $(\sqrt{s'})$
0
1000
2000
3000
4000
$\sqrt{s'}$ (GeV)

Criticism of EW sphaleron production in high-E collisions

The sphaleron is a semiclassical configuration with

$$\boxed{\mathrm{Size_{sph}} \sim m_W^{-1}\,, \quad E_{\rm sph} = \mathrm{few} \times m_W/\alpha_W \simeq 10\,\mathrm{TeV}.}$$

It is 'made out' of $\sim 1/\alpha_W$ particles (i.e. it decays into $\sim 1/\alpha_W$ W's, Z's, H's).

$$\boxed{2_{\text{initial hard partons}} \;\to\; \text{Sphaleron} \;\to\; (\sim 1/\alpha_W)_{\text{soft final quanta}}}$$

The sphaleron production out of 2 hard partons is unlikely.

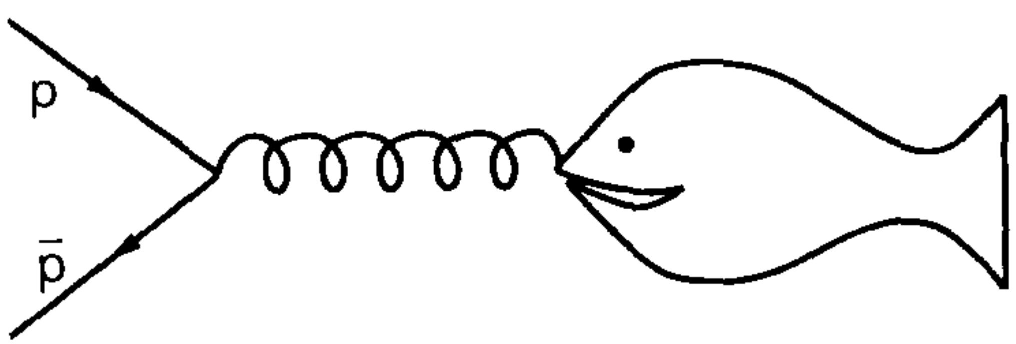


Fig. 3. "You can't make a fish in a p$\bar{\rm p}$ collider."

from Mattis PRpts 1991

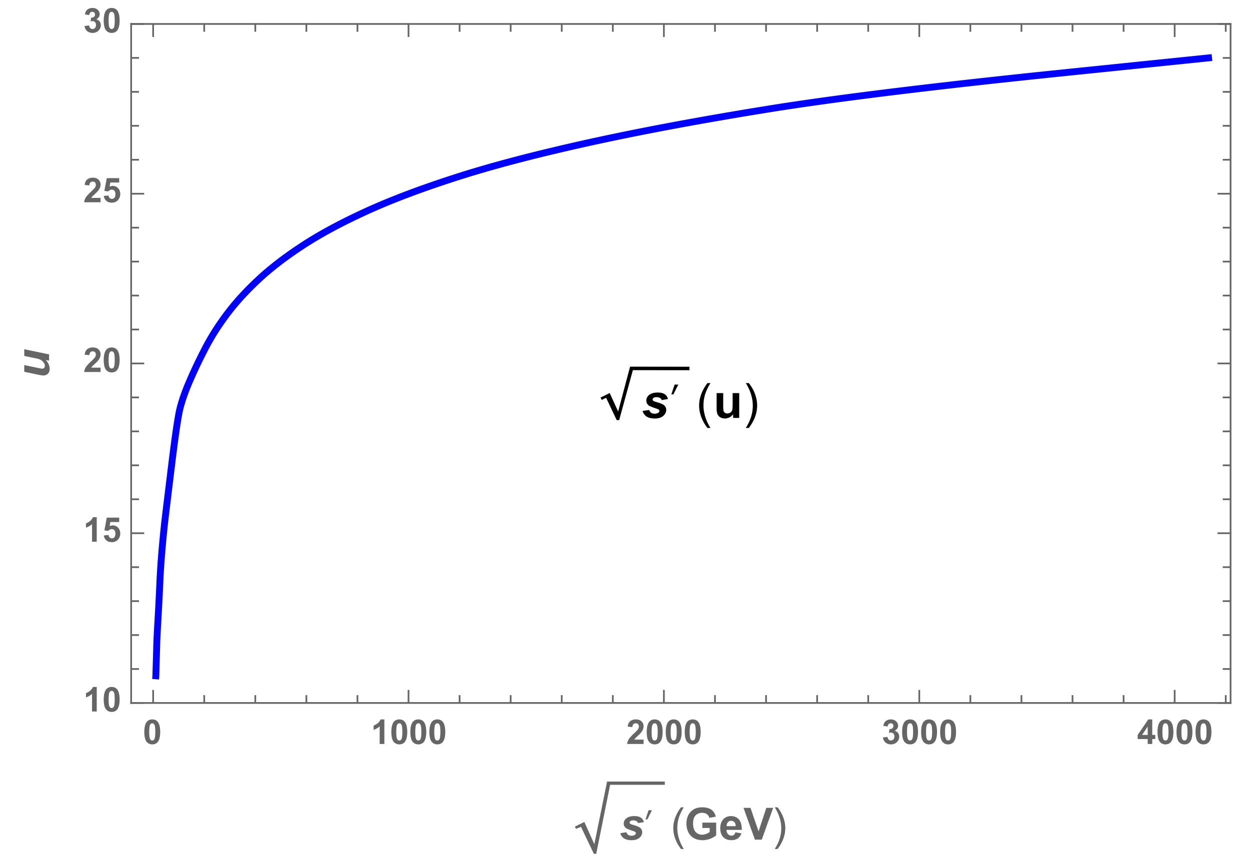
But in QCD instantons are small

[A `small fish' compared to the EW case]

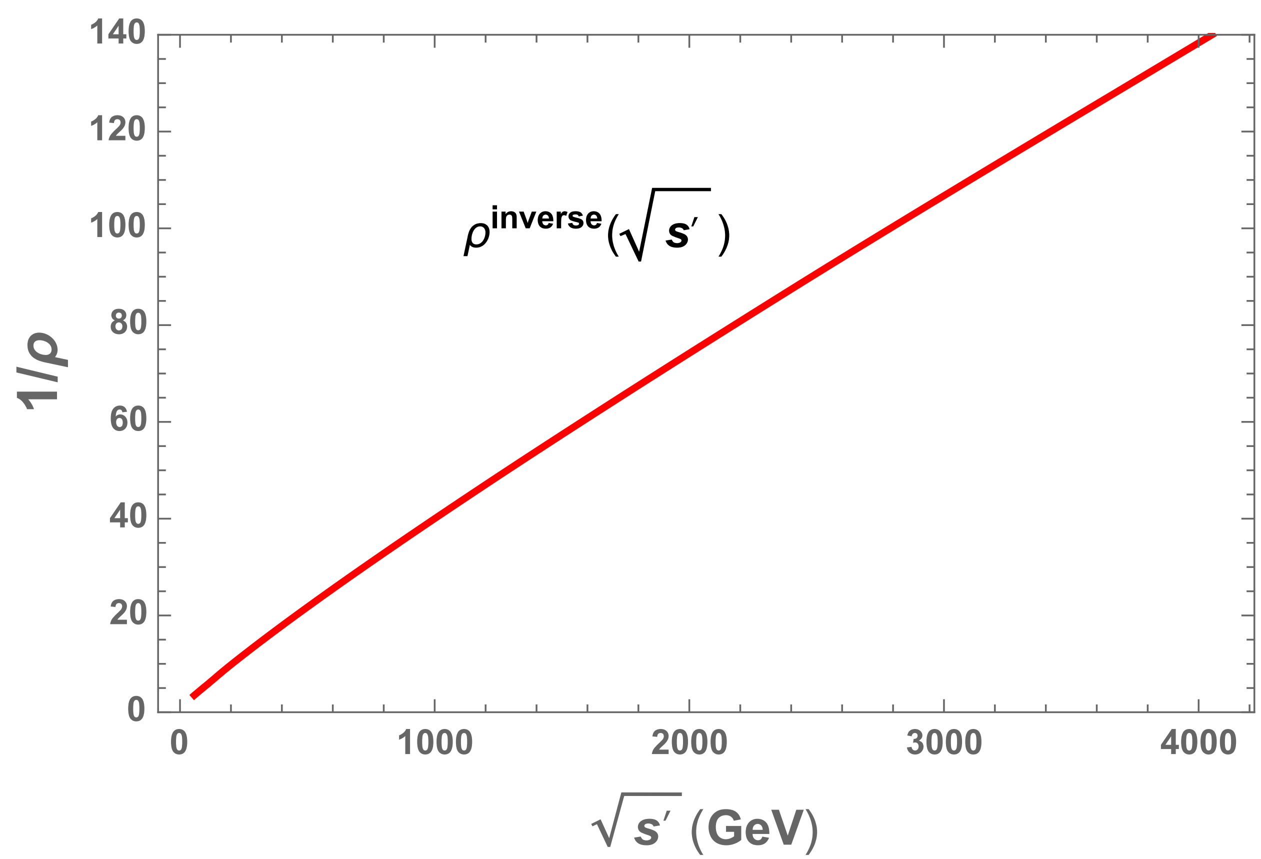
This criticism does not apply to our QCD calculation

Results a) Instanton size

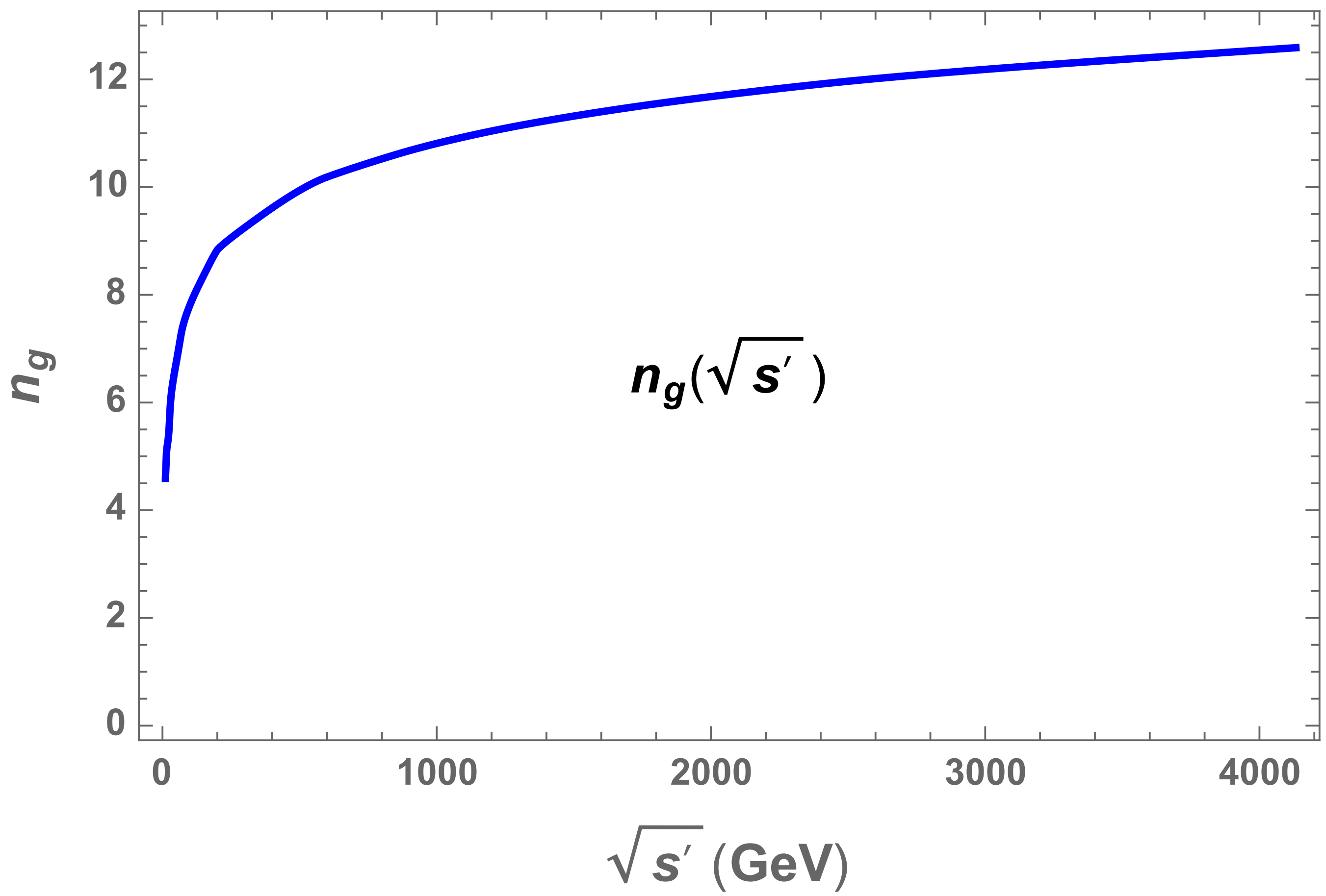
$$u = \sqrt{s}\rho$$

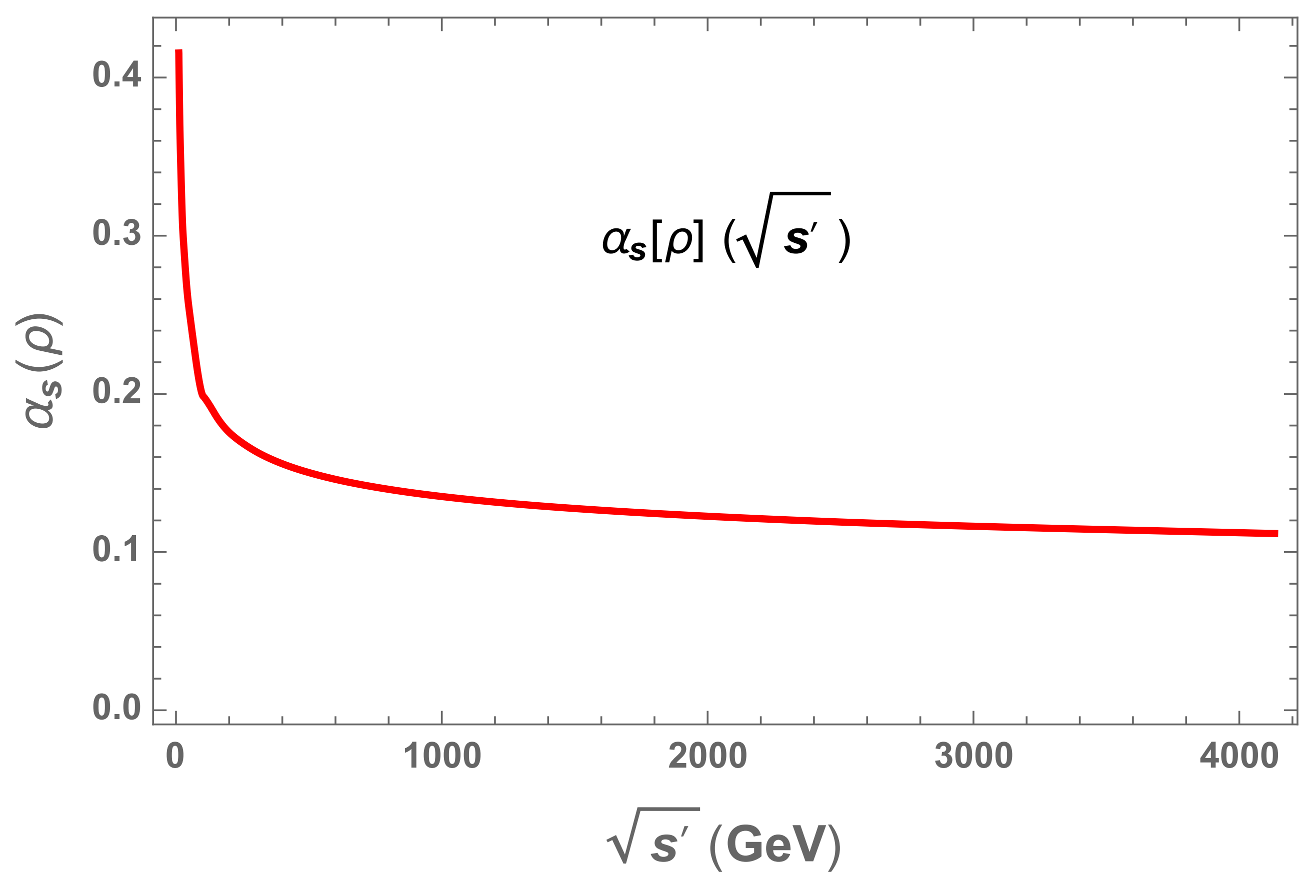


$$\frac{1}{\rho}\,(\mathrm{GeV})$$

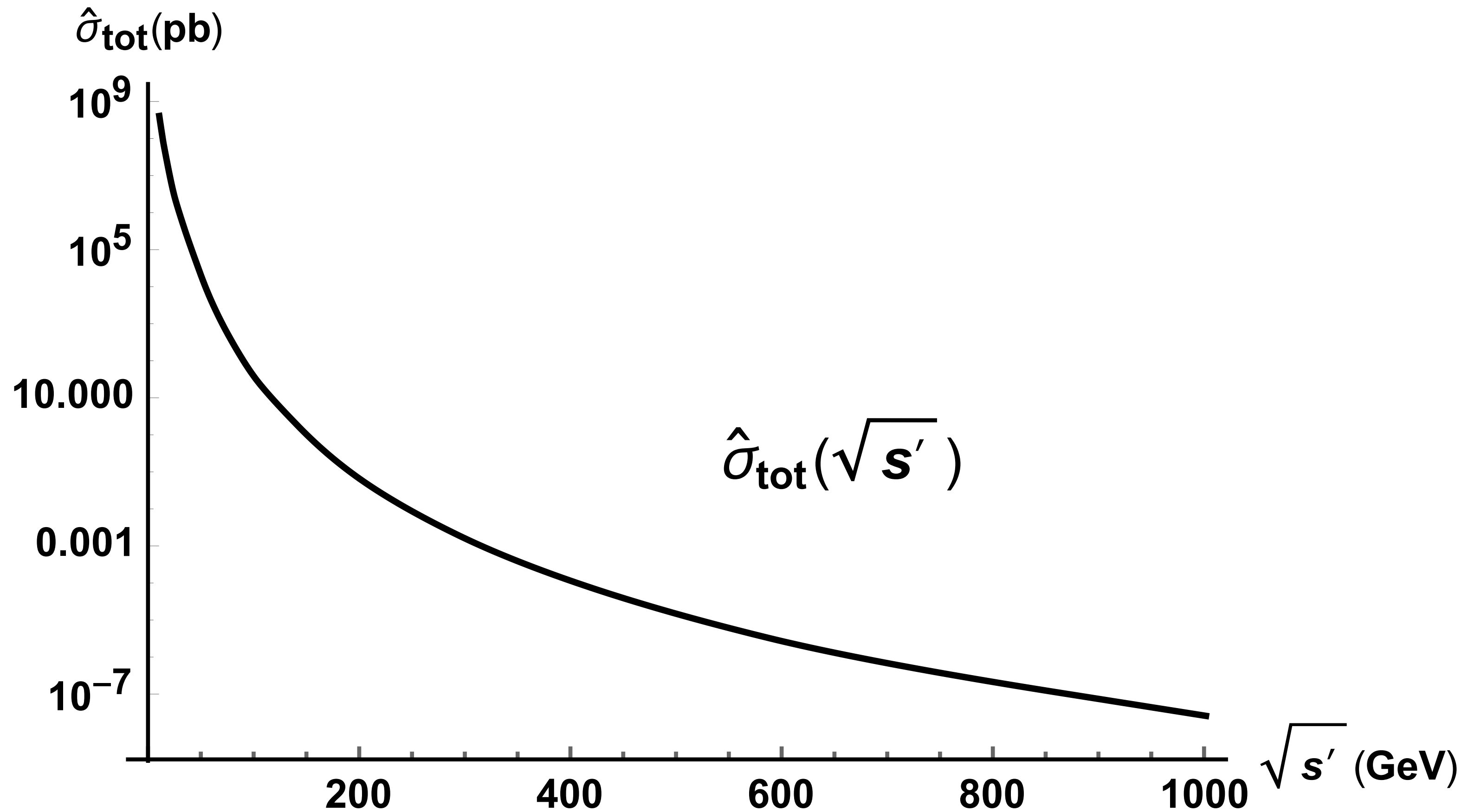


Results b) <number of gluons>





Results c) partonic cross-section



$\sqrt{s'}$ (GeV)	ρ^{inverse}(GeV)	$\alpha_s[\rho]$	n_g	$\hat{\sigma}_{tot}$(pb)
40.77	2.718	0.2669	6.471	1.110×10^5
56.07	3.504	0.2449	6.915	1.105×10^4
61.84	3.638	0.2235	7.280	3145.
89.63	4.979	0.2058	7.670	107.7
118.0	6.212	0.1950	8.248	9.275
174.4	8.720	0.1804	8.604	0.2413
246.9	11.76	0.1693	9.045	0.009685
349.9	15.90	0.1594	9.486	0.0003907
496.3	21.58	0.1504	9.928	0.00001588
704.8	29.37	0.1424	10.37	6.440×10^{-7}
1002.	40.07	0.1351	10.81	2.500×10^{-8}