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Scattering of Fermions on Monopoles

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Introduction

- Fermion-Monopole scattering processes in an SU(5) GUT

$$u^1 + u^2 + M \to \bar{d}^3 + \bar{e} + M$$

- are unsuppressed by the GUT or Monopole scales
- the relevant scale is the QCD scale ~1GeV — 15 orders of magnitude lower than GUT
- these are anomalous (B+L)-violating processes
- involve light fermions in the J=0 spherical wave which penetrate the 't Hooft-Polyakov monopole core
- Monopole catalysis of proton decay: Rubakov-Callan effect

Rubakov 1981

Callan 1982

Callan 1982

- One can also consider a single fermion, e.g. a massless positron scattering on a GUT monopole

$$\bar{e} + M \to \frac{1}{2}\left(u^1\, u^2\, d^3\, \bar{e}\right) + M$$

- Callan's bosonization formalism for J=0 scattering implies that particles in the final state carry half-integer fermion numbers

- At first sight appears to be highly counter-intuitive:

- we don't have such half-fermion states in perturbation theory and one might expect that asymptotic states (far away from the monopole) should be described by standard QFT perturbative Fock-space states

- However, it is known that magnetically and electrically charged states are always entangled; they carry non-vanishing total orbital momentum J even at infinite separations. Asymptotic states are not described by tensor products of 1-particle ones

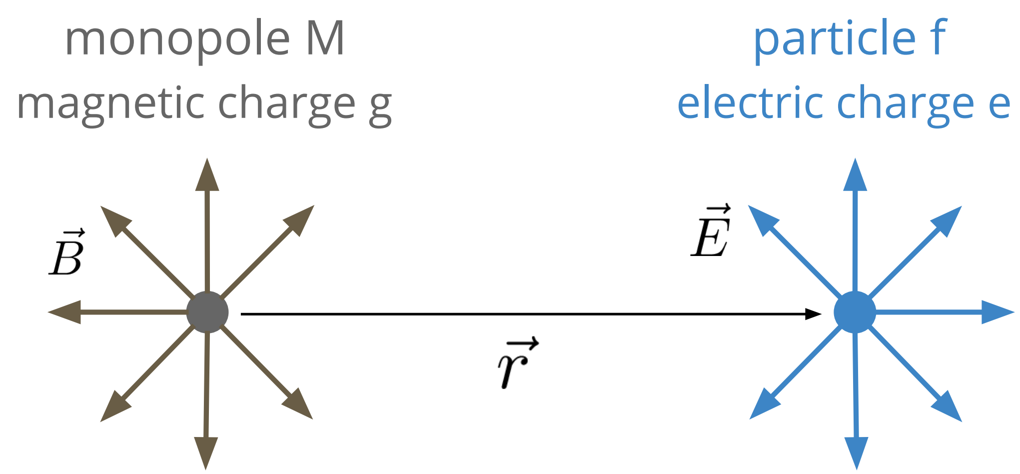
In this talk we'll study such processes and construct their scattering amplitudes

Recall:

from Telem (Planck 2021)

Monopole and Charge: Extra Classical Angular Momentum

Thomson 1904



$$\vec{J}_{\text{field}} = \frac{1}{4\pi}\int \vec{r}\,' \times \left(\vec{E}\times\vec{B}\right)\, d^3r\,' = -\frac{g}{4\pi}\int \left(\vec{\nabla}\,'\cdot\vec{E}\right)\,\hat{r}\,'\, d^3r' = -eg\hat{r}$$

Distance independent!

In the quantum theory $\vec{J}_{\text{field}}$ quantized $\longrightarrow$ $eg = \frac{n}{2}$ Dirac quantization

- Non-vanishing Angular Momentum implies: magnetically and electrically charged states are pairwise entangled. Asymptotic states are not described by tensor products of 1-particle states.

Zwanziger 1972

- For each pair of states with a magnetic and an electric charge there is a new quantum number — a half-integer pairwise helicity — associated with the pair.

Csaki, Hong, Shirman, Telem, Terning, Waterbury 2009.14213

Intriguing, remarkable & unexpected

1. There is no crossing symmetry

One can neither apply crossing to individual particles in the Rubakov-Callan processes

$$u^1 + u^2 + M \to \bar{d}^3 + \bar{e} + M$$

nor would it be allowed by the multi-particle electric--magnetic entanglement.

2. Forward scattering amplitudes often trivially vanish

3. The optical theorem does not apply

For example the complex conjugate amplitude for the process above is the amplitude for

$$\bar{d}^3 + \bar{e} + \overline{M} \to u^1 + u^2 + \overline{M}$$

which involves anti-monopoles rather than monopoles while the fermion states are the same

4. No Lagrangian formulation exists that is both local and Lorentz-invariant.

Intriguing, remarkable & unexpected

5. There is no decoupling of heavy mass-scales from low-energy physics:

   Low-energy lowest partial-wave fermions penetrate the non-Abelian monopole core and result in unsuppressed scattering rates.

   [This effect cannot be described by the low-energy U(1) EFT local Lagrangian formulation of Zwanziger.]

6. There are fermion number violating anomalous, as well as fermion number preserving non-anomalous processes on monopoles that are both unsuppressed;

7. Production of fractional fermion numbers is possible for massless fermions scattered on monopoles thus restructuring the perturbative Fock space.

Fermion-monopole scattering in the SU(2) Model

- an SU(2) gauge theory with the Higgs field in the adjoint representation supports 't Hooft-Polyakov magnetic monopoles

- add Nf flavours of Left-handed Weyl fermion doublets $m_{\rm ferm} \ll M_X/\alpha,$

  $m_{\rm ferm} = 0$ is a good approximation

$$\psi^i_L = \begin{pmatrix} a^i_+ \\ b^i_- \end{pmatrix}_L \,, \quad i = 1, \ldots N_f$$

- corresponding Dirac-conjugate fermions are SU(2) doublets $\overline{(\psi^i_L)} = \bar\psi^i{}_R$

$$\bar\psi^i{}_R = \begin{pmatrix} \bar b^i_+ \\ \bar a^i_- \end{pmatrix}_R$$

- electric fermion charges: $e = \pm 1/2$ in the units of the SU(2) gauge coupling g
  monopole magnetic charge: $g_M = -1$ in units of $4\pi/g$

$$q \equiv e_{a_+/b_-} \cdot g_M = \mp\frac{1}{2}$$

Fermion-monopole scattering in the SU(2) Model

For Left-
handed Weyl fermions in the $J = 0$ wave only their a_+ components exist as incoming waves while their b_- components give the outgoing states in the fermion–monopole scattering.

the Right-handed spinors, $\bar{a}_-$ are incoming and $\bar{b}_+$ are outgoing.

$$\psi^i_L = \begin{pmatrix} a^i_+ \\ b^i_- \end{pmatrix}_L$$

ψ_L	$q = eg_M$	in/out
a_{+L}	$-1/2$	in
b_{-L}	$+1/2$	out

$$\bar{\psi}{}^i{}_R = \begin{pmatrix} \bar{b}^i_+ \\ \bar{a}^i_- \end{pmatrix}_R$$

$(\overline{\psi})_R$	$q = eg_M$	in/out
$\bar{b}_{+R}$	$-1/2$	out
$\bar{a}_{-R}$	$+1/2$	in

Follows from
truncating the theory to $J = 0$ waves for each fermion, and analysing solutions of the I
equation for massless Weyl fermions in the 't Hooft–Polyakov monopole background.

Scattering in the $SU(2)$ theory with $N_f = 2$ flavours

In a scattering process in a gauge theory, electric charge must be conserved.

the energy carried by

fermions is (much) lower than the monopole–dyon mass splitting (~W-boson mass in GUT) =>
charge cannot be deposited on the monopole core by turning it to a dyon.

Starting with a single fermion a^1_{+L} in the initial state, the $J = 0$ state

the in/out selection rules

and the electric charge conservation, allows for the process,

$$a^1_{+L} + M \rightarrow \bar{b}^2_{+R} + M + (\bar{b}b)\,\text{pairs}$$

an anomalous process as the chirality is not conserved.

use a simple selection rule

$$\Delta R^i - \Delta L^i = n\,, \quad \text{for each flavour } i = 1, \ldots, N_f$$

Scattering in the $SU(2)$ theory with $\mathbf{N}_f = 2$ flavours

Combining $$a^1_{+L} + M \to \bar{b}^{\,2}_{+R} + M + (\bar{b}b)\,\text{pairs}$$

with a similar process where flavours1&2 interchanged:

$$a^2_{+L} + M \to \bar{b}^{\,1}_{+R} + M + (\bar{b}b)\,\text{pairs}$$

We now get the scattering process with two incident fermions:

$$\boxed{a^1_{+L} + a^2_{+L} + M \to \bar{b}^{\,1}_{+R} + \bar{b}^{\,2}_{+R} + M + (\bar{b}b)\,\text{pairs}}$$

there are also anomalous processes with $n = -1$ and $n = -2$

$$\bar{a}^1_{-R} + M \to b^{\,2}_{-L} + M + (\bar{b}b)\,\text{pairs},$$
$$\bar{a}^1_{-R} + \bar{a}^2_{-R} + M \to b^{\,1}_{-L} + b^{\,2}_{-L} + M + (\bar{b}b)\,\text{pairs}$$

and non-anomalous processes with $n = 0$

$$\bar{a}^1_{-R} + a^2_{+L} + M \to \bar{b}^{\,1}_{+R} + b^{\,2}_{-L} + M + (\bar{b}b)\,\text{pairs}$$

Scattering in the $\mathbf{N}_f = 4$ model

The similar (allowed by symmetries) scattering process with two incident fermions here is :

$$a^1_{+L} + a^2_{+L} + M \to \bar{b}^{\,3}_{+R} + \bar{b}^{\,4}_{+R} + M + (\bar{b}b)\,\text{pairs}$$

But the more elementary constituent process with a single fermion in the initial state must involve final state particles with half-integer fermion numbers — Callan 1982:

$$a^1_{+L} + M \to \frac{1}{2}\left(b^1_{-L}\,\bar{b}^{\,2}_{+R}\,\bar{b}^{\,3}_{+R}\,\bar{b}^{\,4}_{+R}\right) + M + (\bar{b}b)\,\text{pairs}$$

Final states with half-integer fermion numbers are solitons that appear in the J=0 reduced (effectively 1+1 dimensional (r+t) model) after bosonization — Callan 1982.

Scattering in the $\mathbf{N}_f = 4$ model

Callan 1982:

$$a^1_{+L} + M \to \frac{1}{2}\left(b^1_{-L}\,\bar{b}^{\,2}_{+R}\,\bar{b}^{\,3}_{+R}\,\bar{b}^{\,4}_{+R}\right) + M + (\bar{b}b)\,\text{pairs}$$

Csaki etal (PRL 2022) rejected this process altogether based on the argument that if such massless half-fermion states existed, they would have to be true asymptotic states far from the monopole perturbation theory can be reliable applied. They have proposed instead

Csaki etal 2022:

$$a^1_{+L} + M \to \bar{b}^{\,2}_{+R} + \bar{b}^{\,3}_{+R} + \bar{a}^{\,4}_{-R} + M$$

cannot be in the J=0 state
(it must be incoming rather than outgoing)

It is hard to understand how the outgoing $\bar{a}^{\,4}_{-R}$ fermion could be produced *inside* the monopole core, since it is not in a $J = 0$ single particle state and would experience a very strong Coulomb repulsion from the core.

This must be suppressed

by powers of $E/M_X \ll 1$ where E is the energy

Our 2nd objection against disallowing the Callan-type single-fermion processes is that they can again be combined to correctly reproduce the standard Rubakov-Callan process with two incident fermions (while Csaki et al process cannot).

Indeed, combining:

$$a^1_{+L} + M \to \frac{1}{2}\left(b^1_{-L}\,\bar{b}^{\,2}_{+R}\,\bar{b}^{\,3}_{+R}\,\bar{b}^{\,4}_{+R}\right) + M,$$

$$a^2_{+L} + M \to \frac{1}{2}\left(b^2_{-L}\,\bar{b}^{\,1}_{+R}\,\bar{b}^{\,3}_{+R}\,\bar{b}^{\,4}_{+R}\right) + M,$$

we obtain,

$$\begin{aligned} a^1_{+L} + \left(a^2_{+L} + M\right) &\to a^1_{+L} + M + \frac{1}{2}\left(b^2_{-L}\,\bar{b}^{\,1}_{+R}\,\bar{b}^{\,3}_{+R}\,\bar{b}^{\,4}_{+R}\right) \\ &\to \bar{b}^{\,3}_{+R} + \bar{b}^{\,4}_{+R} + M + \frac{1}{2}\left(\bar{b}^{\,1}_{+R} b^1_{-L}\right) + \frac{1}{2}\left(\bar{b}^{\,2}_{+R} b^2_{-L}\right), \end{aligned}$$

which reproduces correctly

$$a^1_{+L} + a^2_{+L} + M \to \bar{b}^{\,3}_{+R} + \bar{b}^{\,4}_{+R} + M + (\bar{b}b)\,\text{pairs}$$

Scattering amplitudes with pairwise helicities

Lorentz transformation of the out state with electric and magnetic dofs inlcudes a little group phase with pairwise helicities q:

$$U(\Lambda)|p_i, p_M; s_i, s_M; q_{iM}\rangle = \boxed{e^{iq_{iM}\phi_{iM}}} |\Lambda p_i, \Lambda p_M; s'_i, s'_M; q_{iM}\rangle \, \mathcal{D}_{s'_i s_i} \mathcal{D}_{s'_M s_M} \, ,$$

Then the scattering amplitude transforms as: following the formalism in Csaki etal 2020:

$$\tilde{\mathcal{A}}(p_1, \ldots, p_n, p_M | \, k_1, \ldots, k_m, k_M) = \boxed{e^{i\sum_{i=1}^{n} q_{iM}\phi_{iM}} \; e^{i\sum_{l=1}^{m} q_{lM}\phi_{lM}}} \mathcal{A}(\Lambda p_1, \ldots, \Lambda p_n, \Lambda p_M | \, \Lambda k_1, \ldots, \Lambda k_m, \Lambda k_M)$$

Use pairwise helicity spinors for each fermion-monopole pair which transforms as:

$$\Lambda \, |p_{ij}^{\flat\pm}\rangle \; = \; e^{\mp\frac{i}{2}\phi(p_i,p_j,\Lambda)} \, |\Lambda p_{ij}^{\flat\pm}\rangle$$

$$[p_{ij}^{\flat\pm}| \, \tilde{\Lambda} \; = \; e^{\pm\frac{i}{2}\phi(p_i,p_j,\Lambda)} \, [\Lambda p_{ij}^{\flat\pm}|$$

Pairwise spinors

First the momentum pair is Lorentz boosted into the CoM frame

$$k_i = (E_i, 0, 0, p_c)\,, \quad k_j = (E_j, 0, 0, -p_c)$$

Define pairwise momentum variable(s) for the pair:

$$k_{ij}^{\flat\pm} \; = \; p_c\,(1, 0, 0, \pm 1)$$

introduce the pairwise helicity spinors in the CoM frame

$$k_{ij}^{\flat\pm\,\mu}\,\sigma_{\mu\,\alpha,\dot\alpha} \; = \; |k_{ij}^{\flat\pm}\rangle_\alpha\,[k_{ij}^{\flat\pm}|_{\dot\alpha}.$$

Finally, boost to a general Lorentz frame

$$|p_{ij}^{\flat\pm}\rangle_\alpha \; = \; \Lambda_\alpha^{\ \beta}\,|k_{ij}^{\flat\pm}\rangle_\beta\,, \qquad [p_{ij}^{\flat\pm}|_{\dot\alpha} \; = \; [k_{ij}^{\flat\pm}|_{\dot\beta}\,\tilde\Lambda^{\dot\beta}_{\ \dot\alpha}$$

Scattering amplitudes with pairwise helicities

Using the standard all-outgoing conventions for amplitude momenta, the contribution to the amplitude $\tilde{\mathcal{A}}$ from the incoming state $a^1_{+L} + M$ is given by,

$$(a^1_{+L} + M)_{\text{in}} \;\Rightarrow\; \qquad [a^1_{+L}|\, p^{\flat-}_{a^1 M}]$$

uniquely determined by the requirements that: its helicity spinors can involve only the initial states; all (Lorentz) spinor indices must be contracted as this is a $J = 0$ state; Lorentz transformations of the pairwise helicity spinor $|p^{\flat-}_{a^1 M}]$ should give the phase factor $e^{\,iq_{a^1 M}\,\phi}$

The outgoing states contribute to the amplitude:

$$(\frac{1}{2}b^1_{-L} + M)_{\text{out}} \;\Rightarrow\; \qquad \sqrt{\langle b^1_{-L}|\, p^{\flat-}_{b^1 M}\rangle}\,,$$

$$(\frac{1}{2}\bar{b}^{\,i}_{+R} + M)_{\text{out}} \;\Rightarrow\; \qquad \sqrt{[\bar{b}^{\,i}_{+R}|\, p^{\flat-}_{\bar{b}^i M}]}\,, \quad i = 2, 3, 4.$$

It is easy to verify each of these factors transforms with the correct pairwise little group phase $e^{\,iq\,\phi}$, as required

Scattering amplitudes with pairwise helicities

Thus the amplitude for the elementary Callan's process

$$a^1_{+L} + M \to \frac{1}{2}\left(b^1_{-L}\,\bar{b}^{\,2}_{+R}\,\bar{b}^{\,3}_{+R}\,\bar{b}^{\,4}_{+R}\right) + M$$

is given by

$$\tilde{\mathcal{A}} \propto [a^1_{+L}|\,p^{\flat-}_{a^1 M}]\left(\langle b^1_{-L}|\,p^{\flat-}_{b^1 M}\rangle\,[\bar{b}^{\,2}_{+R}|\,p^{\flat-}_{\bar{b}^2 M}]\,[\bar{b}^{\,3}_{+R}|\,p^{\flat-}_{\bar{b}^3 M}]\,[\bar{b}^{\,4}_{+R}|\,p^{\flat-}_{\bar{b}^4 M}]\right)^{1/2}$$

The amplitude for the companion process is obtained by interchanging 1 and 2

$$a^2_{+L} + M \to \frac{1}{2}\left(b^2_{-L}\,\bar{b}^{\,1}_{+R}\,\bar{b}^{\,3}_{+R}\,\bar{b}^{\,4}_{+R}\right) + M$$

$$\tilde{\mathcal{A}} \propto [a^2_{+L}|\,p^{\flat-}_{a^2 M}]\left(\langle b^2_{-L}|\,p^{\flat-}_{b^2 M}\rangle\,[\bar{b}^{\,1}_{+R}|\,p^{\flat-}_{\bar{b}^1 M}]\,[\bar{b}^{\,3}_{+R}|\,p^{\flat-}_{\bar{b}^3 M}]\,[\bar{b}^{\,4}_{+R}|\,p^{\flat-}_{\bar{b}^4 M}]\right)^{1/2}$$

Taking the product, gives the correct result for the process with 2 incident fermions (omitting bbar pairs):

$$a^1_{+L} + a^2_{+L} + M \to \bar{b}^{\,3}_{+R} + \bar{b}^{\,4}_{+R} + M,$$

$$\tilde{\mathcal{A}} \propto [a^1_{+L}|\,p^{\flat-}_{b^1 M}]\,[a^2_{+L}|\,p^{\flat-}_{b^2 M}]\,[\bar{b}^{\,3}_{+R}|\,p^{\flat-}_{\bar{b}^3 M}]\,[\bar{b}^{\,4}_{+R}|\,p^{\flat-}_{\bar{b}^4 M}]$$

Scattering of fermions with SU(5) GUT monopoles

In the minimal GUT theory the 't Hooft–Polyakov monopole lives in the $SU(2)_M$ subgroup of the $SU(5)_{\rm GUT}$. We consider a single generation of massless fermions in this model. Left-handed Weyl fermions transform in the $\bar{5}$ and 10 representations of $SU(5)_{\rm GUT}$ are represented by $N_f = 4$ of $SU(2)_M$ doublets

$$\psi_L^i = \begin{pmatrix} a_+^i \\ b_-^i \end{pmatrix}_L \Rightarrow \begin{pmatrix} \bar{u}_L^1 \\ u_L^2 \end{pmatrix}, \begin{pmatrix} -\bar{u}_L^2 \\ u_L^1 \end{pmatrix}, \begin{pmatrix} d_L^3 \\ \bar{e}_L \end{pmatrix}, \begin{pmatrix} e_L \\ -\bar{d}_L^3 \end{pmatrix},$$

	ψ_L^1	ψ_L^2	ψ_L^3	ψ_L^4	$q = e_M g_M$	in/out
a_{+L} :	$\bar{u}_L^1$	$\bar{u}_L^2$	d_L^3	e_L	$-1/2$	in
b_{-L} :	u_L^2	u_L^1	$\bar{e}_L$	$\bar{d}_L^3$	$+1/2$	out

$$\bar{\psi}^i_R = \begin{pmatrix} \bar{b}_+^i \\ \bar{a}_-^i \end{pmatrix}_R \Rightarrow \begin{pmatrix} \bar{u}_R^2 \\ u_R^1 \end{pmatrix}, \begin{pmatrix} \bar{u}_R^1 \\ -u_R^2 \end{pmatrix}, \begin{pmatrix} e_R \\ \bar{d}_R^3 \end{pmatrix}, \begin{pmatrix} -d_R^3 \\ \bar{e}_R \end{pmatrix}$$

	$(\bar{\psi})_R^1$	$(\bar{\psi})_R^2$	$(\bar{\psi})_R^3$	$(\bar{\psi})_R^4$	$q = e_M g_M$	in/out
$\bar{b}_{+R}$:	$\bar{u}_R^2$	$\bar{u}_R^1$	e_R	d_R^3	$-1/2$	out
$\bar{a}_{-R}$:	u_R^1	u_R^2	$\bar{d}_R^3$	$\bar{e}_R$	$+1/2$	in

Scattering amplitudes with pairwise helicities

Scattering amplitudes for elementary J=0 Callan's processes in SU(5) GUT are as follows

$$\bar{u}\,^1_L + M \to \frac{1}{2}\left(u^2_L\,\bar{u}\,^1_R\,e_R\,d^3_R\right) + M,$$

$$\tilde{\mathcal{A}} \propto [\bar{u}\,^1_L|\,p^{\flat-}_{u^1 M}]\left(\langle u\,^2_L|\,p^{\flat-}_{u^2 M}\rangle\,[\bar{u}^1_R|\,p^{\flat-}_{\bar{u}^1 M}]\,[e_R|\,p^{\flat-}_{eM}]\,[d^3_R|\,p^{\flat-}_{d^3 M}]\right)^{1/2}$$

$$\bar{e}\,_R + M \to \frac{1}{2}\left(d^3_R\,u^2_L\,u^1_L\,\bar{e}_L\right) + M,$$

$$\tilde{\mathcal{A}} \propto \langle \bar{e}_R|\,p^{\flat-}_{\bar{e}M}\rangle\left([d^3_R|\,p^{\flat-}_{d^3 M}]\,\langle u^2_L|\,p^{\flat-}_{u^2 M}\rangle\,\langle u^1_L|\,p^{\flat-}_{u^1 M}\rangle\,\langle \bar{e}_L|\,p^{\flat-}_{\bar{e}M}\rangle\right)^{1/2}$$

Scattering amplitudes with pairwise helicities

And the J=0 scattering amplitudes for Rubakov-Callan 2-fermion processes are

$$\bar{u}^1_L + \bar{u}^2_L + M \to d^3_R + e_R + M,$$
$$\tilde{\mathcal{A}} \propto [\bar{u}^1{}_L|\, p^{\flat -}_{\bar{u}^1 M}]\, [\bar{u}^2{}_L|\, p^{\flat -}_{\bar{u}^2 M}]\, [d^3_R|\, p^{\flat -}_{d^3 M}]\, [e_R|\, p^{\flat -}_{eM}]$$

$$u^1_R + u^2_R + M \to \bar{d}^3_L + \bar{e}_L + M,$$
$$\tilde{\mathcal{A}} \propto \langle u^1_R|\, p^{\flat -}_{u^1 M}\rangle\, \langle u^2_R|\, p^{\flat -}_{u^2 M}\rangle\, \langle \bar{d}^3_L|\, p^{\flat -}_{\bar{d}^3 M}\rangle\, [\bar{e}_L|\, p^{\flat -}_{\bar{e} M}],$$

which describe the monopole catalysis of anti-proton and proton decays

Conclusions

- We re-examined scattering processes involving massless fermions and magnetic monopoles:
- 1) in the minimal SU(2) model that supports 't Hooft-Polyakov monopoles and al
- 2) in the SU(5) GUT theory with a single family of massless fermions
- Derived helicity amplitudes for fermion-monopole scattering in events with a single fermion in the initial state and fractional fermion numbers in the final state
- and provided non-trivial tests on such processes by combining them to reproduce the amplitudes for processes with 2 fermions in the initial- and integer fermion numbers in the final state.
- These processes are unsuppressed, they do not depend on the monopole or the GUT mass scales scale even at low energies; they are instrumental for the monopole catalysis of proton decay and interesting on their own right

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