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[extended version]

# High-energy Higgspllosion

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IPPP Durham

- VVK & Spannowsky 1704.03447, 1707.01531, 1809.11141
- VVK 1806.05648 & 1705.04365
- VVK & Reiness: Semiclassical Review 1810.01722



## In the first part of this talk: I'll outline the main idea of Higgspllosion

- Consider  $n \sim 150$  Higgs bosons produced in a final state at  $n \lambda \gg 1$ . Kinematically possible for scattering at  $E \sim 100$  TeV
- **HIGGSPLOSION**:  $n$ -particle rates computed in a weakly-coupled theory become unsuppressed above certain critical values of  $n$  and  $E = \sqrt{s} = \sqrt{p^2}$

$$m \Gamma_n(p^2) = \text{Im}_n \Sigma(p^2) > p^2, m^2$$

## In the second part of this talk:

- we'll consider an intrinsically Non-perturbative — **semiclassical set-up**  
 $n \propto \sqrt{s}/m \propto 1/\lambda \gg 1$
- it incorporates correctly the tree-level results and
- the leading-order quantum effects = leading loops
- **compute quantum effects in the large  $\lambda$   $n$  limit**



already known

new

Conclusions & summary

**Higgspllosion:** *few* particles  $\longrightarrow$  *many* particles processes

$$\sqrt{s} : \quad \underset{\text{few}}{X} \longrightarrow \underset{\text{many}}{n \times \phi},$$

Scattering process :  $|X(\sqrt{s})\rangle = |2\rangle \rightarrow |n\rangle \Rightarrow$  cross section  $\sigma_n(\sqrt{s})$ ,

Resonance decay :  $|X(\sqrt{s})\rangle = |1^*\rangle \rightarrow |n\rangle \Rightarrow$  partial width  $\Gamma_n(s)$ .

A non-perturbative semiclassical approach can be used to compute such processes. The semiclassical approach assumes that the initial state  $X$  can be approximated by a point-like injection of energy: via a local operator  $O(x)$

$$|X\rangle = \mathcal{O}(0) |0\rangle$$

Ideal for  $1^* \longrightarrow n$  Higgspllosion is when:  $\frac{m}{s} \Gamma_n(s) \gtrsim 1$

For for  $2 \longrightarrow n$  OK for s-channel but not for t-channel where there is an impact parameter

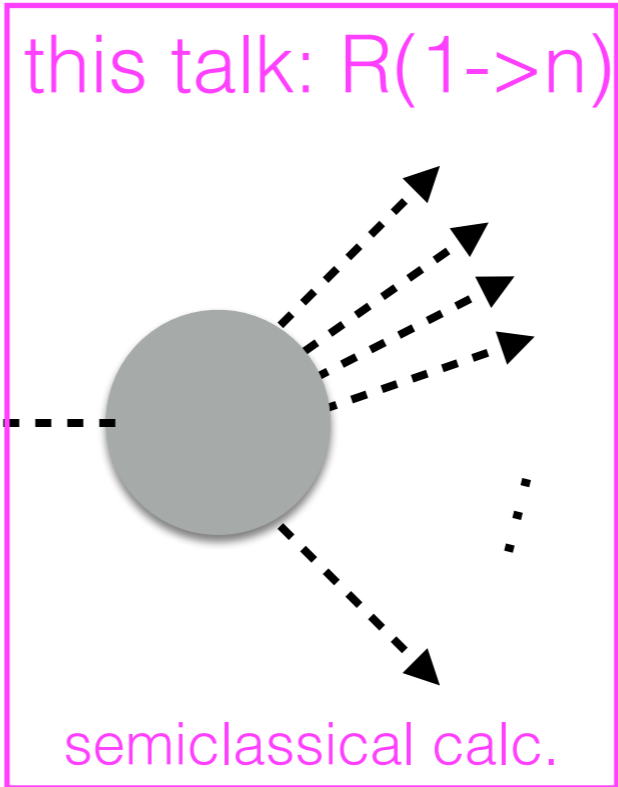
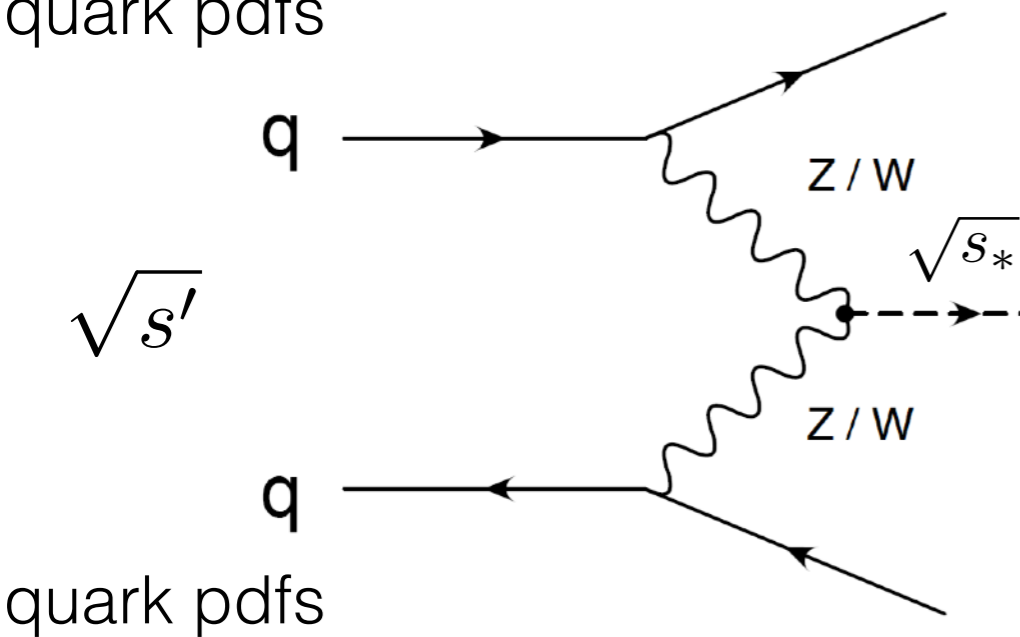
1)

# 1->n processes of interest

for Higgspllosion

e.g.: Vector boson fusion in high-energy pp collisions at ~100 TeV

quark pdfs



n non-relativistic Higgses  
Higgspllosion at  $\sqrt{s_*}$

$$\text{Im } \Sigma_n(s_*) \sim \Gamma_n(s_*)$$

$$\frac{i}{s_* - m_h^2 - \text{Re}\tilde{\Sigma}(s_*) + im_h\Gamma(s_*)} \quad \text{Propagator with Higgsdispersion at } \sqrt{s_*}$$

- VVK & Spannowsky 1704.03447, 1707.01531

2)

## Schwinger-propagator and optical theorem

The optical theorem relates the  $1^* \rightarrow n h$  amplitudes to the imaginary part of the self-energy (valid to all orders)

$$- \text{Im} \Sigma_R(p^2) = m \Gamma(p^2) \quad \longleftrightarrow \quad - \text{Im} \left( \text{---} \text{---} \text{---} \right) = m \text{---} \text{---} \text{---}$$

where  $\Gamma(s) = \sum_{n=2}^{\infty} \Gamma_n(s)$  and  $\Gamma_n(s) = \frac{1}{2m} \int \frac{d\Phi_n}{n!} |\mathcal{M}(1 \rightarrow n)|^2$

and thus 
$$\Delta_R(p) = \frac{i}{p^2 - m^2 - \text{Re} \Sigma_R(p^2) + im\Gamma(p^2) + i\epsilon}$$

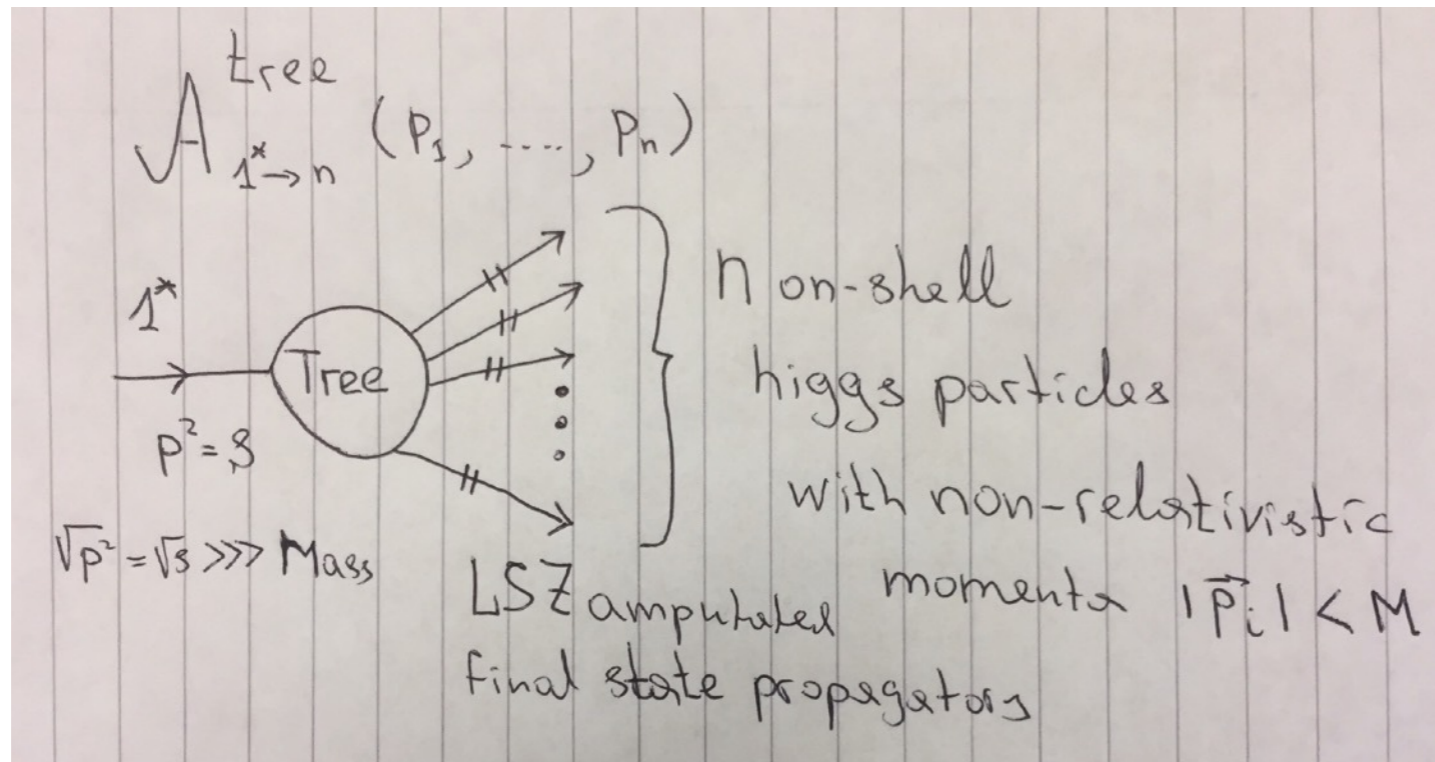
No non-pert. information about the Real part of Sigma but it cannot cancel Imaginary part

Higgsplodes when

$$m\Gamma_n(p^2) \gtrsim p^2$$



# Warm-up: Compute 1 → n amplitudes@LO with non-relativistic final momenta:



see classic 1992-1994 papers:  
Brown; Voloshin;  
Argyres, Kleiss, Papadopoulos  
Libanov, Rubakov, Son, Troitski

more recently: VVK 1411.2925

$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 - \frac{\lambda}{4}(h^2 - v^2)^2$$

prototype of the SM Higgs  
in the unitary gauge

Tree-level  $1^* \rightarrow n$  amplitudes in the limit  $\varepsilon \rightarrow 0$  for any  $n$  are given by

$$\mathcal{A}_n(p_1, \dots, p_n) = n! \left( \frac{\lambda}{2M_h^2} \right)^{\frac{n-1}{2}} \left( 1 - \frac{7}{6}n\varepsilon - \frac{1}{6} \frac{n}{n-1} \varepsilon + \mathcal{O}(\varepsilon^2) \right)$$

factorial growth

amplitude on the n-particle threshold

$$\varepsilon = \frac{1}{n M_h} E_n^{\text{kin}} = \frac{1}{n} \frac{1}{2M_h^2} \sum_{i=1}^n \vec{p}_i^2$$

kinetic energy per particle per mass

In the large- $n$ -non-relativistic limit the result is

$$\mathcal{A}_n(p_1, \dots, p_n) = n! \left( \frac{\lambda}{2M_h^2} \right)^{\frac{n-1}{2}} \exp \left[ -\frac{7}{6}n\varepsilon \right], \quad n \rightarrow \infty, \quad \varepsilon \rightarrow 0, \quad n\varepsilon = \text{fixed}$$

## Square the amplitude & integrate over the n-particle phase-space:

The cross-section and/or the  $n$ -particle partial decay  $\Gamma_n$

$$\Gamma_n(s) = \int d\Phi_n \frac{1}{n!} |\mathcal{A}_{h^* \rightarrow n \times h}|^2$$

The  $n$ -particle Lorentz-invariant phase space volume element

$$\int d\Phi_n = (2\pi)^4 \delta^{(4)}(P_{\text{in}} - \sum_{j=1}^n p_j) \prod_{j=1}^n \int \frac{d^3 p_j}{(2\pi)^3 2p_j^0},$$

in the large- $n$  non-relativistic limit with  $n\varepsilon_h$  fixed becomes,

$$\Phi_n \simeq \frac{1}{\sqrt{n}} \left( \frac{M_h^2}{2} \right)^n \exp \left[ \frac{3n}{2} \left( \log \frac{\varepsilon_h}{3\pi} + 1 \right) + \frac{n\varepsilon_h}{4} + \mathcal{O}(n\varepsilon_h^2) \right]$$

We find:

$$\Gamma_n^{\text{tree}}(s) \sim \exp \left[ n \left( \log \frac{\lambda n}{4} - 1 \right) + \frac{3n}{2} \left( \log \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} n\varepsilon + \mathcal{O}(n\varepsilon^2) \right]$$

Son 1994;

Libanov, Rubakov, Troitskii 1997; more recently: VVK 1411.2925

Can also include *loop corrections* to amplitudes on thresholds:

The 1-loop corrected threshold amplitude for the pure  $n$  Higgs production:

$$\phi^4 \text{ with SSB : } \mathcal{A}_{1 \rightarrow n}^{\text{tree}+1\text{loop}} = n! (2v)^{1-n} \left( 1 + n(n-1) \frac{\sqrt{3}\lambda}{8\pi} \right)$$

There are strong indications, based on the analysis of leading singularities of the multi-loop expansion around singular generating functions in scalar field theory, that the 1-loop correction exponentiates,

*Libanov, Rubakov, Son, Troitsky 1994*

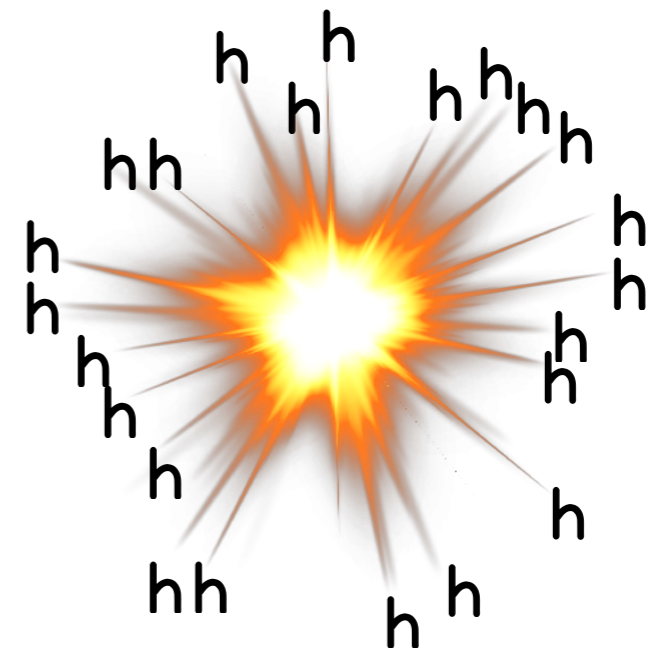
$$\mathcal{A}_{1 \rightarrow n} = \mathcal{A}_{1 \rightarrow n}^{\text{tree}} \times \exp [B \lambda n^2 + \mathcal{O}(\lambda n)]$$

in the limit  $\lambda \rightarrow 0, n \rightarrow \infty$  with  $\lambda n$  fixed. Here  $B$  is determined from the 1-loop calculation (as above) – *Smith; Voloshin 1992*):

$$B = + \frac{\sqrt{3}}{4\pi}$$

Really need to switch to the regime of  $\lambda n \gg 1$

For this we need a non-perturbative — semiclassical approach — next slide...

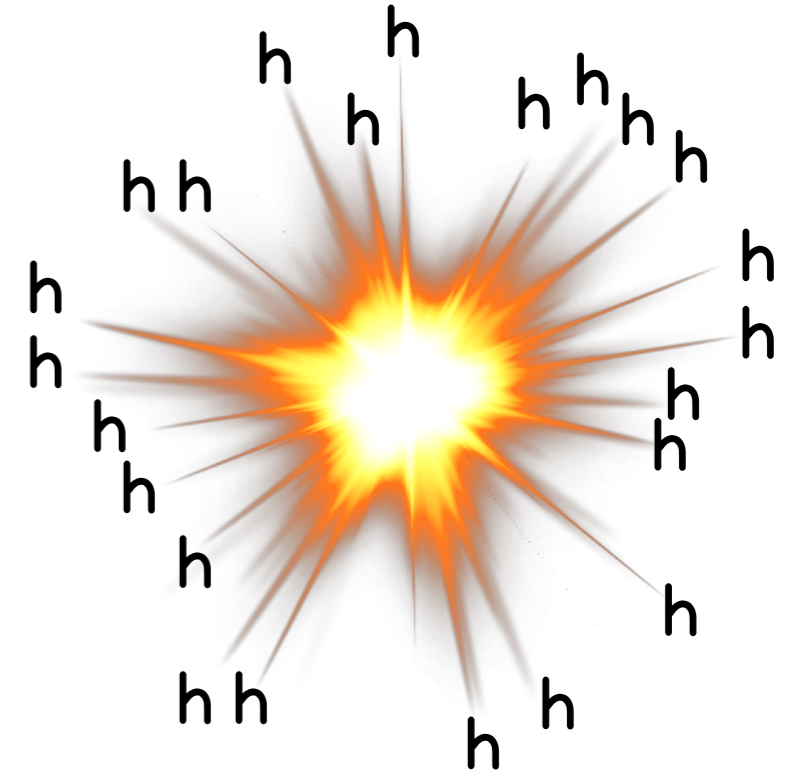




# Explosive growth of 1->n perturbative process

$$\mathcal{R}_n(s) := \frac{1}{2M_h^2} \int d\Pi_n |\mathcal{M}(1 \rightarrow n)|^2$$

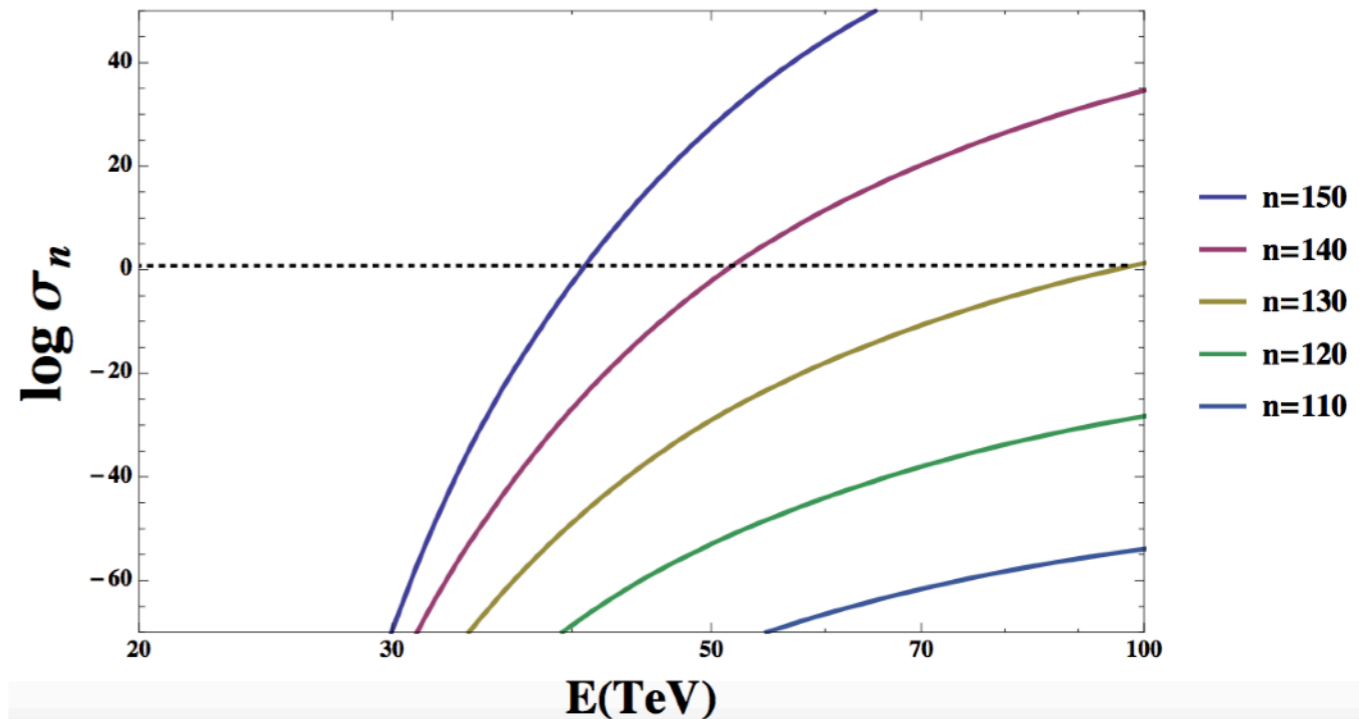
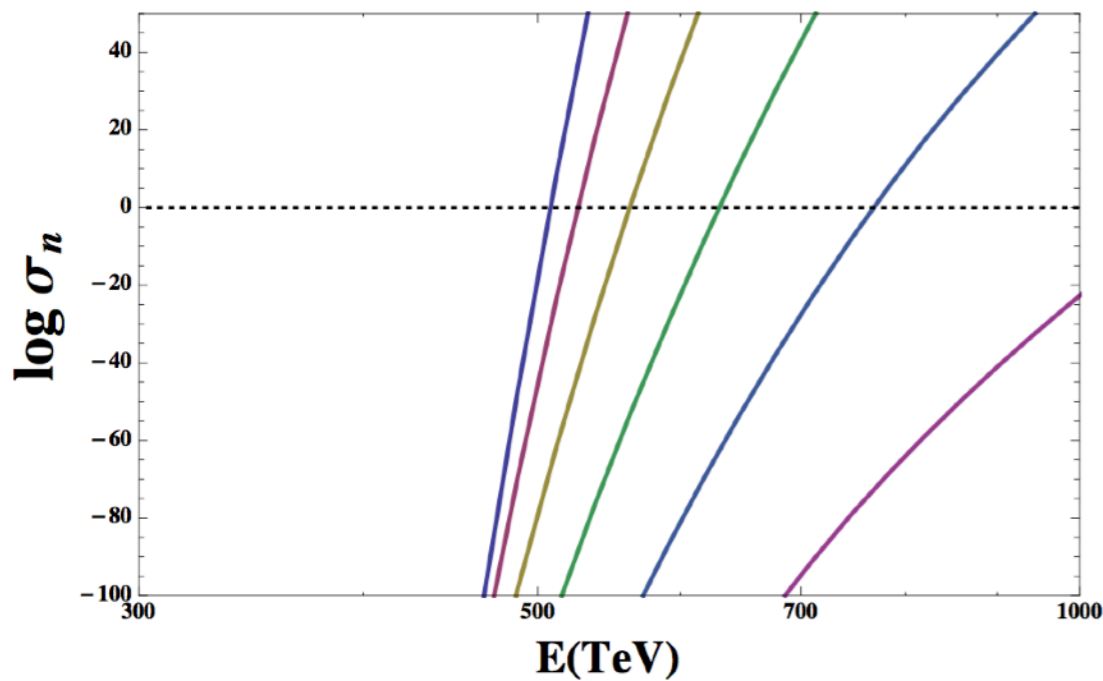
$$\mathcal{R}(\lambda; n, \varepsilon) = \exp \left[ n \left( \log \frac{\lambda n}{4} - 1 \right) + \frac{3n}{2} \left( \log \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} n \varepsilon \right]$$



**Extreme energy dependence** for  $1^* \rightarrow n$  cross section  
including 1-loop result reduces 'ignition' scale

$\log \sigma_n^{\text{tree}}$

$\log \sigma_n^{\text{loop}}$

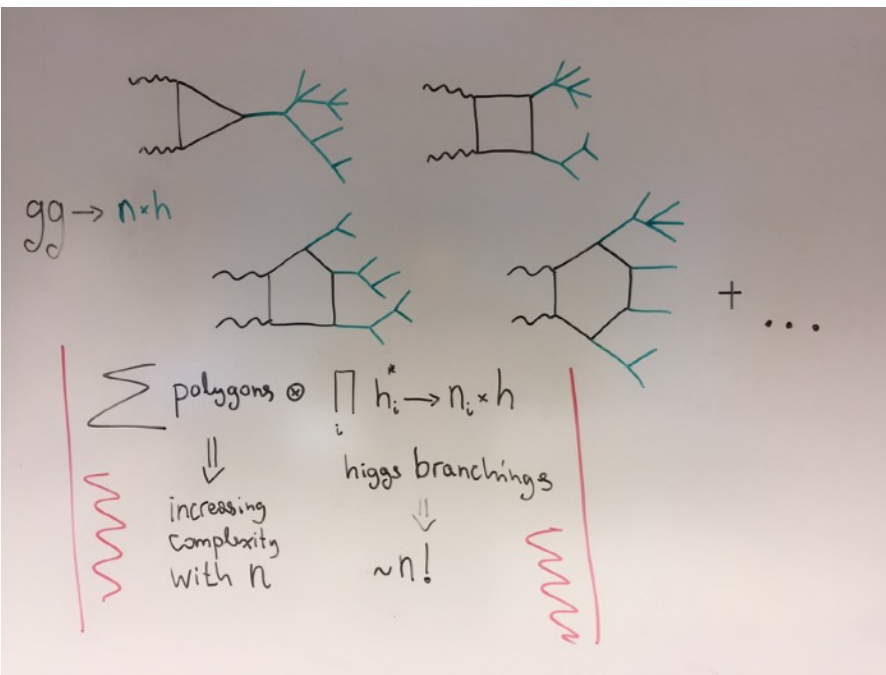


*Higgspersion of the propagator due to Im Sigma not yet included here!*

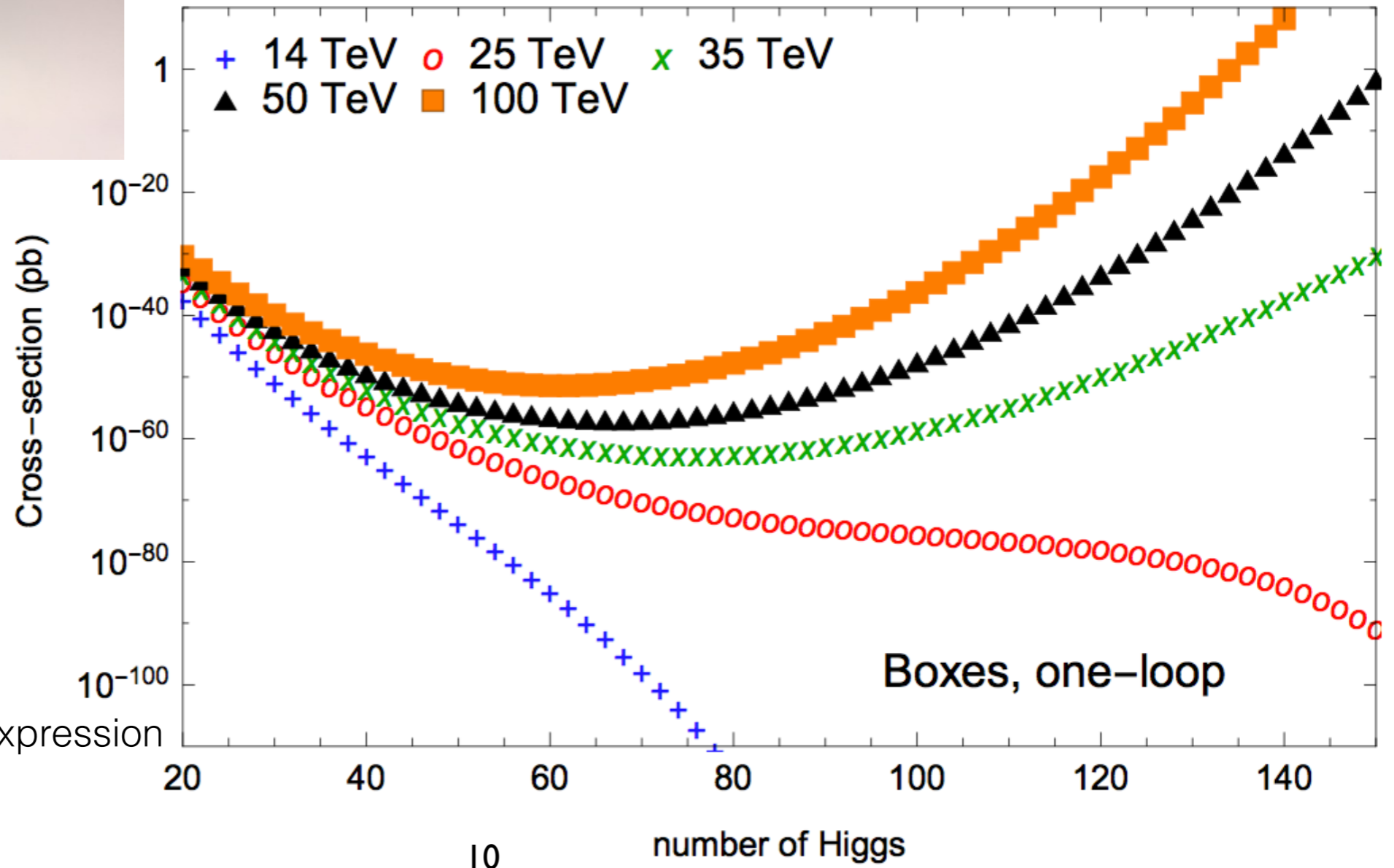
**We'll need a non-perturbative semiclassical approach: Part 2 of the talk**

# Application to gluon fusion:

It was argued that these results can be used to assess what collider energy needed to test where perturbation theory becomes strong [in gluon fusion]



Degrande, VVK, Mattelaer '16



using the 1-loop improved expression

# Factorial growth of tree-level amplitudes at threshold is captured by classical solutions

$$\mathcal{L}(h) = \frac{1}{2} (\partial h)^2 - \frac{\lambda}{4} (h^2 - v^2)^2,$$

The classical equation for the spatially uniform field  $h(t)$ ,

$$d_t^2 h = -\lambda h^3 + \lambda v^2 h,$$

has a closed-form solution with correct initial conditions  $h_{\text{cl}} = v + z + \dots$

$$h_0(z_0; t) = v \left( \frac{1 + z_0 e^{imt} / (2v)}{1 - z_0 e^{imt} / (2v)} \right), \quad m = \sqrt{2\lambda}v$$

$$h_0(z) = v + 2v \sum_{n=1}^{\infty} \left( \frac{z}{2v} \right)^n, \quad z = z(t) = z_0 e^{imt}$$

$$\mathcal{A}_{1 \rightarrow n} = \left( \frac{\partial}{\partial z} \right)^n h_{\text{cl}} \Big|_{z=0} = n! (2v)^{1-n}$$

**Factorial growth**  
L. Brown 9209203

# Classical Solutions & singularities in complex time:

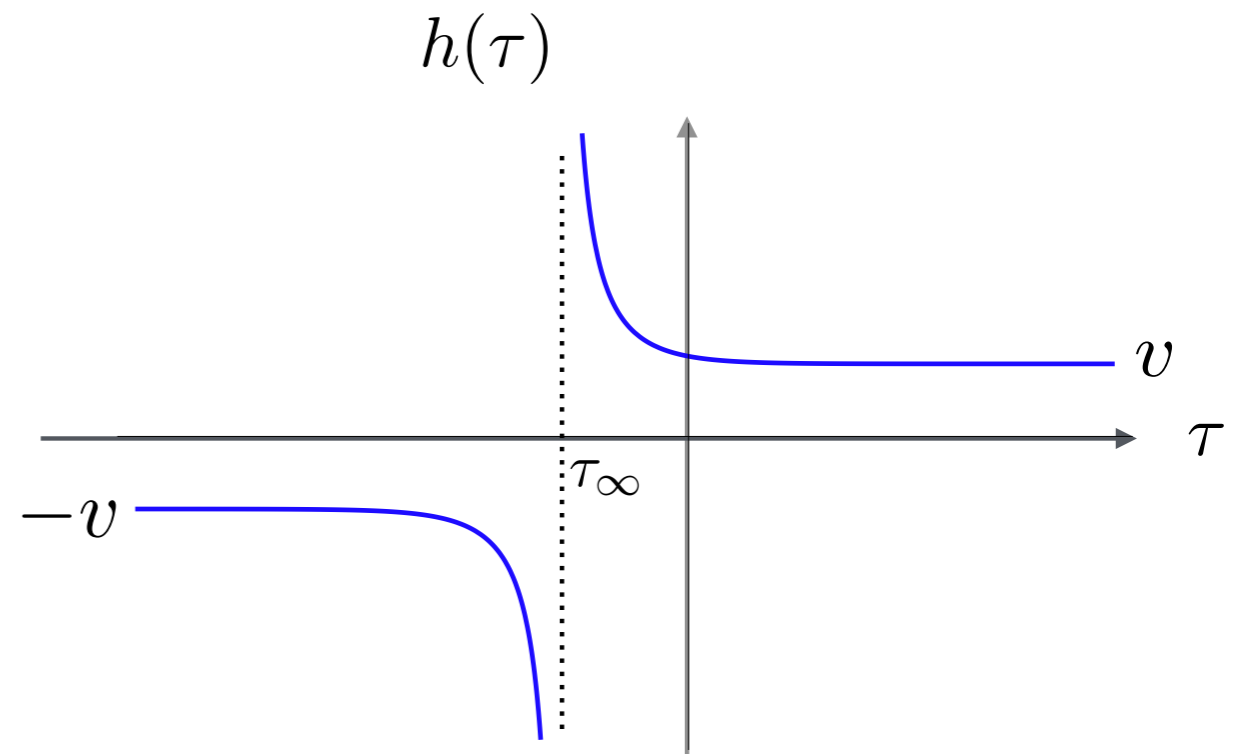
$$t \longrightarrow t_{\mathbb{C}} = t + i\tau$$

$$h_0(t_{\mathbb{C}}) = v \left( \frac{1 + e^{im(t_{\mathbb{C}} - i\tau_{\infty})}}{1 - e^{im(t_{\mathbb{C}} - i\tau_{\infty})}} \right),$$

$$\tau_{\infty} := \frac{1}{m} \log \left( \frac{z_0}{2v} \right)$$

Our simple example of a classical solution  
(corresponding to the tree-level Amplitudes)

$$h_0(\tau) = v \left( \frac{1 + e^{-m(\tau - \tau_{\infty})}}{1 - e^{-m(\tau - \tau_{\infty})}} \right)$$



Such singular complex-valued solutions will emerge in the semiclassical approach

# Part II

## Main idea of the semiclassical approach

$\mathcal{R}_n(E)$  is the probability rate for a local operator  $\mathcal{O}(0)$  to create  $n$  particles of total energy  $E$  from the vacuum,

$$\mathcal{R}_n(E) = \int \frac{1}{n!} d\Phi_n \langle 0 | \mathcal{O}^\dagger S^\dagger P_E | n \rangle \langle n | P_E S \mathcal{O} | 0 \rangle$$

$P_E$  is the projection operator on states with fixed energy  $E$ .

$$\mathcal{O} = e^{j h(0)},$$

and the limit  $j \rightarrow 0$  is taken in the computation of the probability rates,

$$\mathcal{R}_n(E) = \lim_{j \rightarrow 0} \int \frac{1}{n!} d\Phi_n \langle 0 | e^{j h(0)\dagger} S^\dagger P_E | n \rangle \langle n | P_E S e^{j h(0)} | 0 \rangle.$$

Note: non-dynamical (non-propagating) initial state  $\mathcal{O}|0\rangle$ .

The semi-classical (steepest descent) limit:

$$\varepsilon = \frac{E - nm}{nm}$$

$$\lambda \rightarrow 0, \quad n \rightarrow \infty, \quad \text{with } \lambda n = \text{fixed}, \quad \varepsilon = \text{fixed}.$$

Evaluate the path integral in this double-scaling limit.  
 $n$  enters via the coherent state formalism.

# Main idea of the semiclassical approach

Note (1):

The initial state is not semiclassical, it contains few rather than many particles.

Rubakov et al & Son argued that it can be approximated in the semiclassical method by a certain local operator acting on the vacuum:

$$|X\rangle = \mathcal{O}(0) |0\rangle$$

$$\mathcal{O}(x) = j^{-1} e^{j\phi(x)},$$

$j$  is a constant  $j = c/\lambda$ . Finally one takes the limit  $c \rightarrow 0$  (or equivalently  $j \rightarrow 0$ )

A refinement:

operator localized in the vicinity of a point  $x$

$$\mathcal{O}_g(x) = \int d^4x' g(x' - x) \mathcal{O}(x'),$$

smear  $\mathcal{O}(x)$  with a wave packet / test function

$$|X\rangle = \mathcal{O}_g(0) |0\rangle = \int d^4x' g(x') \mathcal{O}(x') |0\rangle$$

# The Semiclassical formalism of Son: results in four steps

1. Solve the classical equation without the source-term:

$$\frac{\delta S}{\delta h(x)} = 0$$

a complex-valued solution  $h(x)$  with a point-like singularity at  $x^\mu = 0$ .  
The singularity is due to  $\mathcal{O}(x = 0)$ .

2. Impose the initial and final-time boundary conditions:

$$\lim_{t \rightarrow -\infty} h(x) = v + \int \frac{d^3 k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} a_{\mathbf{k}}^\dagger e^{ik_\mu x^\mu}$$

$$\lim_{t \rightarrow +\infty} h(x) = v + \int \frac{d^3 k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} \left( b_{\mathbf{k}} e^{\omega_{\mathbf{k}} T - \theta} e^{-ik_\mu x^\mu} + b_{\mathbf{k}}^\dagger e^{ik_\mu x^\mu} \right)$$

- [Son hep-ph/055338](#)

# The Semiclassical formalism of Son: results in four steps

3. Compute  $E$  and  $n$  of the final state using the  $t \rightarrow +\infty$  asymptotics

$$E = \int d^3k \omega_{\mathbf{k}} b_{\mathbf{k}}^\dagger b_{\mathbf{k}} e^{\omega_{\mathbf{k}}T - \theta}, \quad n = \int d^3k b_{\mathbf{k}}^\dagger b_{\mathbf{k}} e^{\omega_{\mathbf{k}}T - \theta}$$

At  $t \rightarrow -\infty$  the energy and the particle number are vanishing.

The energy changes discontinuously from 0 to  $E$  at the singularity at  $t = 0$ .

4. Eliminate the  $T$  and  $\theta$  parameters in favour of  $E$  and  $n$ .  
Finally, compute the function  $W(E, n)$

$$W(E, n) = ET - n\theta - 2\text{Im}S[h]$$

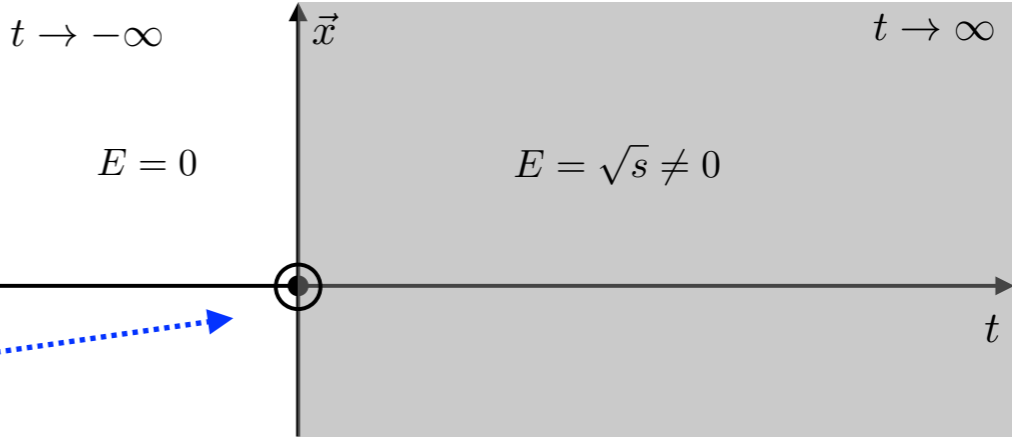
on the set  $\{h(x), T, \theta\}$  and find the semiclassical rate  $\mathcal{R}_n(E) = \exp[W(E, n)]$



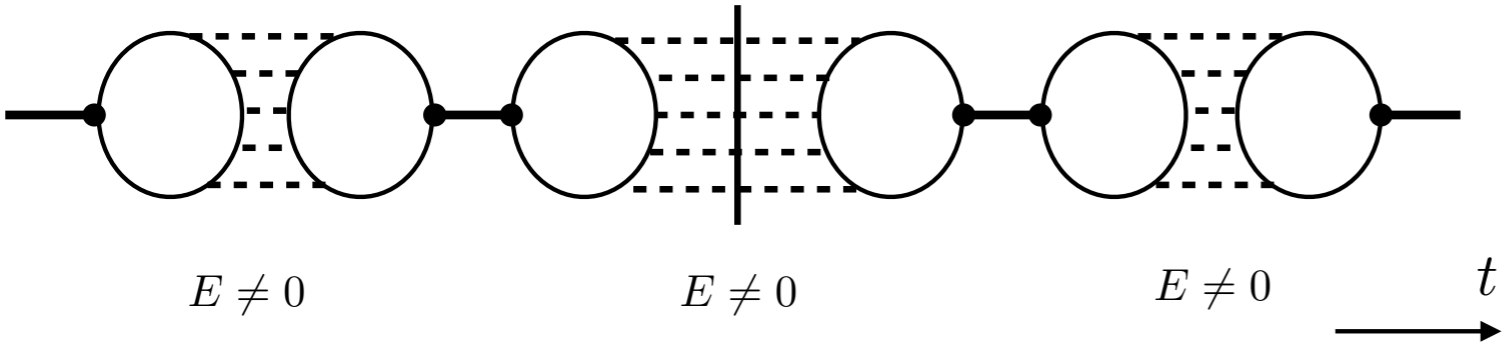
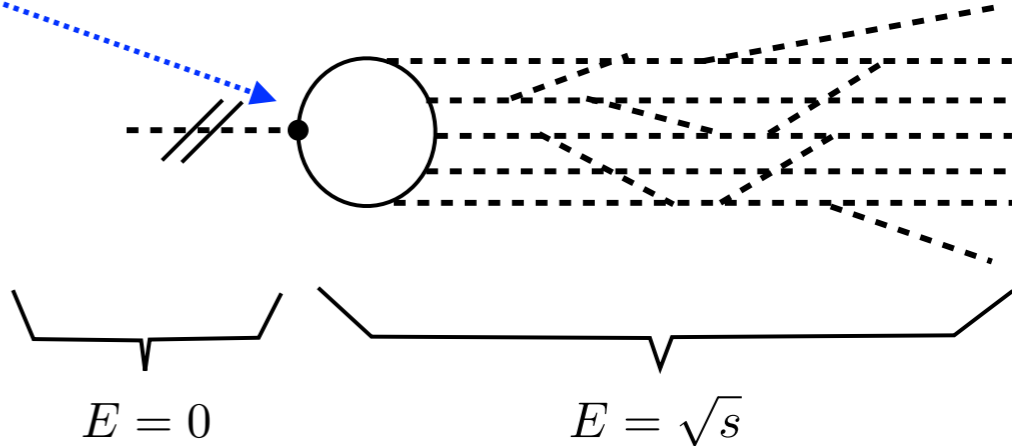
# Main idea of the semiclassical approach

Note (2):  $\text{Im } \Sigma(s)$

The classical solutions that we use have a single point-like singularity in Minkowski space at the point  $x=0$  where the operator  $O(0)$  is located.



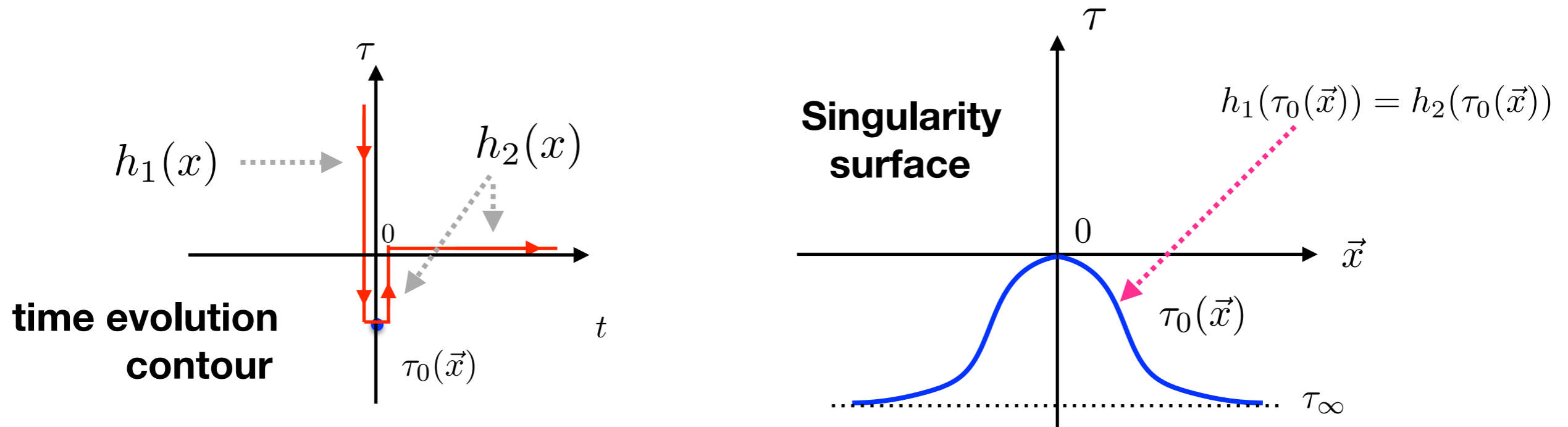
Such configurations contribute to **1PI matrix elements** i.e. precisely to  $\text{Im } \Sigma(s)$



1-particle-reducible contributions to would require multiple singularities, i.e. multiple energy jumps.

# Refining the method in complex time

- In Euclidean space-time  $(\tau, \vec{x})$  the solution is singular on a 3-dimensional surface  $\tau = \tau_0(\vec{x})$ .

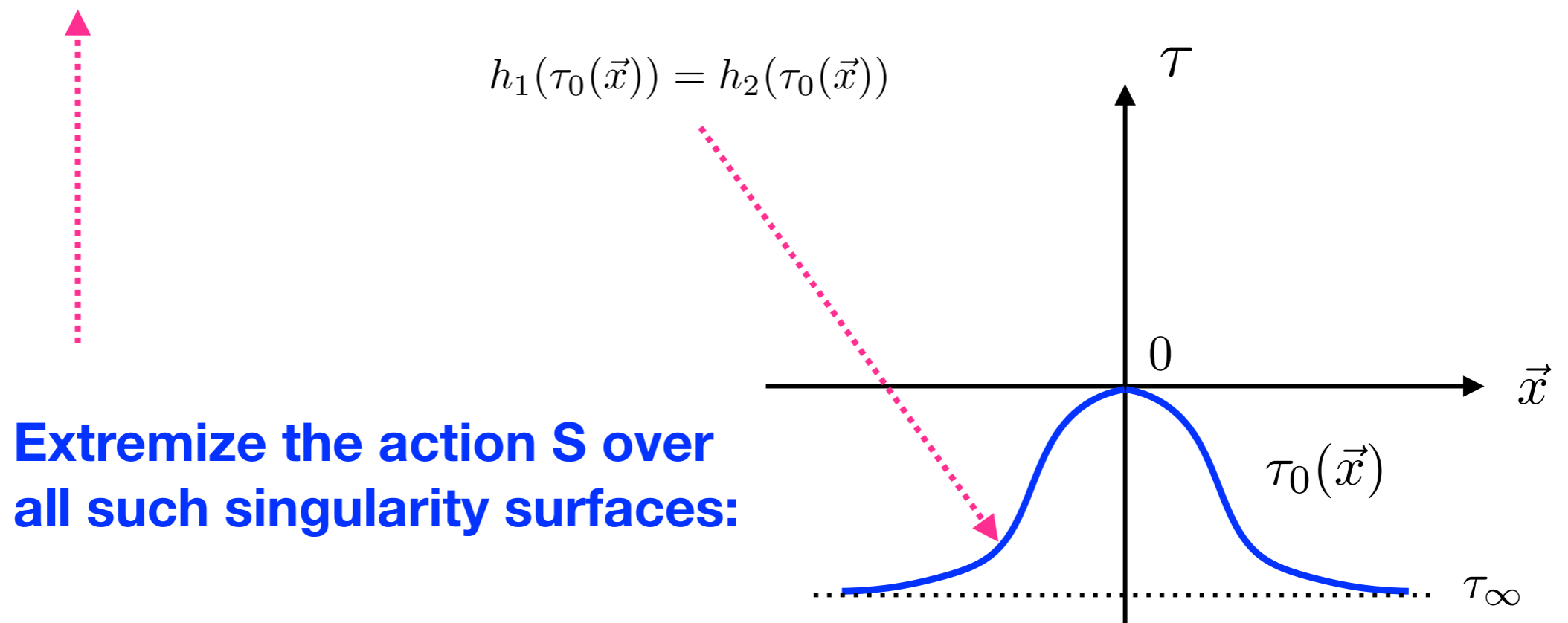


- Find a classical trajectory  $h_1(\tau, \vec{x})$  satisfying initial time boundary cond-s.
- Find another classical trajectory  $h_2(\tau, \vec{x})$  satisfying final time conditions.
- $h_1$  and  $h_2$  are singular on  $\tau_0(\vec{x})$  and  $h_1(\tau_0(\vec{x}), \vec{x}) = h_2(\tau_0(\vec{x}), \vec{x})$
- Extremize the action  $S$  over all singularity surfaces  $\tau_0(\vec{x})$ .

- For the combined configuration  $h(x)$  to solve classical equations everywhere, including the  $\tau_0$  surface:

need to extremize the action integral over all singularity surfaces  $\tau = \tau_0(\vec{x})$  containing the point  $t = 0 = \vec{x}$ .

$$iS[h] = \int d^3x \left( \left| \int_{+\infty}^{\tau_0(\vec{x})} d\tau \mathcal{L}_{\text{Eucl}}(h_1) \right| - \left| \int_{\tau_0(\vec{x})}^0 d\tau \mathcal{L}_{\text{Eucl}}(h_2) \right| + i \int_0^{\infty} dt \mathcal{L}(h_2) \right)$$



# Computing the semiclassical rate

Classical solution singular on generic  $\tau_0(x)$  surfaces:

$$h(t_{\mathbb{C}}, \vec{x}) = v \left( \frac{1 + e^{im(t_{\mathbb{C}} - i\tau_{\infty})}}{1 - e^{im(t_{\mathbb{C}} - i\tau_{\infty})}} \right) + \tilde{\phi}(t_{\mathbb{C}}, \vec{x})$$

Find that:

$$\begin{aligned} W(E, n) &= ET - n\theta - 2\text{Re}S_{\text{Eucl}}[h] \\ &= n \log \frac{\lambda n}{4} + \frac{3n}{2} \left( \log \frac{3\pi}{\varepsilon} + 1 \right) - 2nm\tau_{\infty} - 2\text{Re}S_{\text{Eucl}}[h] \end{aligned}$$



$$W(E, n)^{\text{tree}}$$

agrees with the known result  
of tree-level contributions



$$\Delta W^{\text{quant}}$$

need to compute by extremizing  
w.r.t  $\tau_0(x)$

# Computing the semiclassical rate

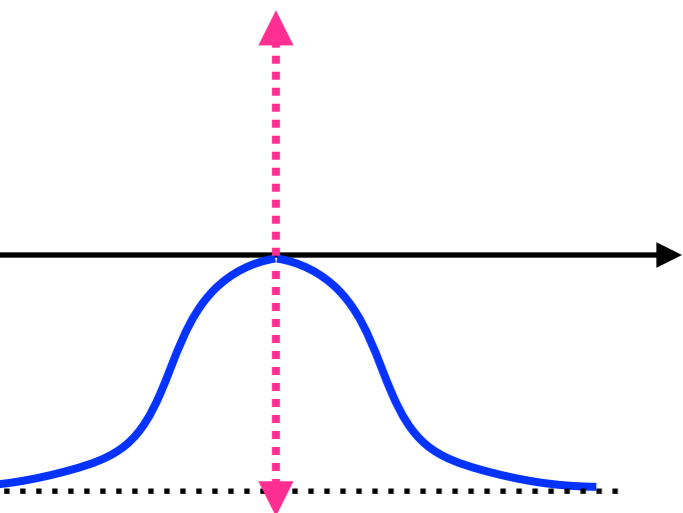
$$\begin{aligned} \Delta W^{\text{quant}} &= -2nm\tau_\infty - 2\text{Re} S_{\text{Eucl}}^{(1,2)} \\ &= 2nm|\tau_\infty| + 2 \int d^3x \left[ \int_{\tau_0(\vec{x})}^{+\infty} d\tau \mathcal{L}_{\text{Eucl}}(h_1) - \int_{\tau_0(\vec{x})}^0 d\tau \mathcal{L}_{\text{Eucl}}(h_2) \right] \end{aligned}$$

Use *thin wall* approximation:

Force x height

E=0 configuration

E=mn configuration



Surface-energy

$$\frac{1}{2} \Delta W^{\text{quant}} = nm|\tau_\infty| - \underbrace{\int_{A+i\epsilon}^{0+i\epsilon} d\tau L_{\text{Eucl}}(h_2; \tau_0(\vec{x}))}_{\equiv S_{\text{Eucl}}[\tau_0(\vec{x})]} + \frac{4\pi}{3} \mu R^3$$

Force x height

Surface-energy

Mechanical analogy: surface at equilibrium of forces.

• [Gorsky & Voloshin hep-ph/9305219](#)

• [VVK 1806.05648](#)

# Computing the semiclassical rate for $\lambda n \gg 1$

Use *thin wall* approximation:

$$S_{\text{Eucl}}[\tau_0(r)] = \int_{\tau_\infty}^0 d\tau 4\pi\mu r^2 \sqrt{1 + \dot{r}^2} \equiv \int_{\tau_\infty}^0 d\tau L(r, \dot{r})$$

Surface tension  $\mu = \int_{-\infty-i\epsilon}^{+\infty-i\epsilon} d\tau \left( \frac{1}{2} \left( \frac{dh}{d\tau} \right)^2 + \frac{\lambda}{4} (h^2 - v^2)^2 \right) = \frac{m^3}{3\lambda}$

Conjugate momentum

Hamiltonian => Energy

$$p = \frac{\partial L(r, \dot{r})}{\partial \dot{r}} = 4\pi\mu \frac{r^2 \dot{r}}{\sqrt{1 + \dot{r}^2}} \quad H(p, r) = L(r, \dot{r}) - p \dot{r}$$

$$\frac{1}{2} \Delta W^{\text{quant}} = (E - nm)\tau_\infty - \int_R^0 p(E) dr + \frac{4\pi}{3} \mu R^3$$

Quantum rate on the stationary trajectory:

$$\frac{1}{2} \Delta W^{\text{quant}}_{\text{stationary}} = - \int_R^0 p(E) dr + \frac{4\pi}{3} \mu R^3, \quad E = nm$$

- [Gorsky & Voloshin hep-ph/9305219](#)
- [VVK 1806.05648](#)

# Computing the semiclassical rate

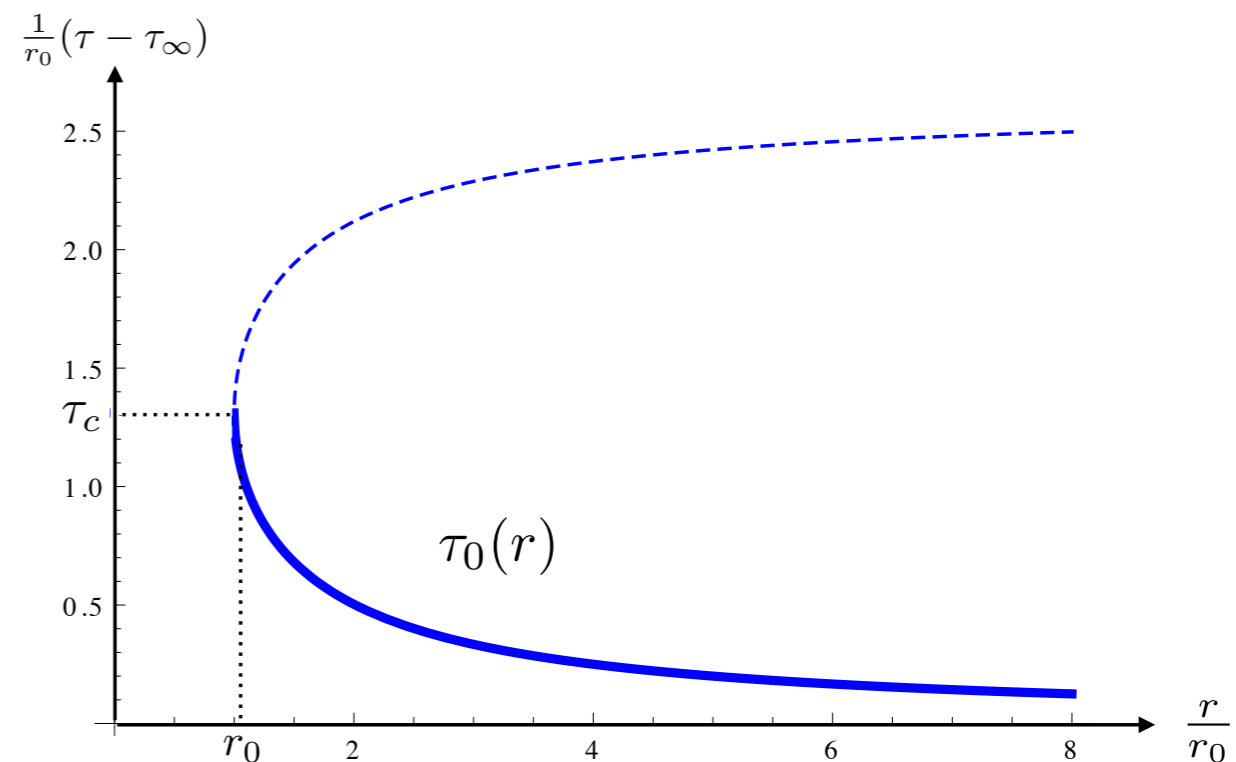
Use *thin wall* approximation:

$$\frac{1}{2} \Delta W_{\text{stationary}}^{\text{quant}} = - \int_R^0 p(E) dr + \frac{4\pi}{3} \mu R^3, \quad E = nm$$

**final result**

$$\Delta W^{\text{quant}} = \frac{E^{3/2}}{\sqrt{\mu}} \frac{2}{3} \frac{\Gamma(5/4)}{\Gamma(3/4)} = \frac{1}{\lambda} (\lambda n)^{3/2} \frac{2}{\sqrt{3}} \frac{\Gamma(5/4)}{\Gamma(3/4)} \simeq 0.854 n \sqrt{\lambda n}$$

Classical trajectory  $\tau(r)$ :



Justifies the thin wall approximation:

$$rm \geq r_0 m = m \left( \frac{E}{4\pi\mu} \right)^{1/2} \propto \left( \frac{\lambda E}{m} \right)^{1/2} = \sqrt{\lambda n} \gg 1,$$

$$r_0^2 = \frac{E}{4\pi\mu}$$

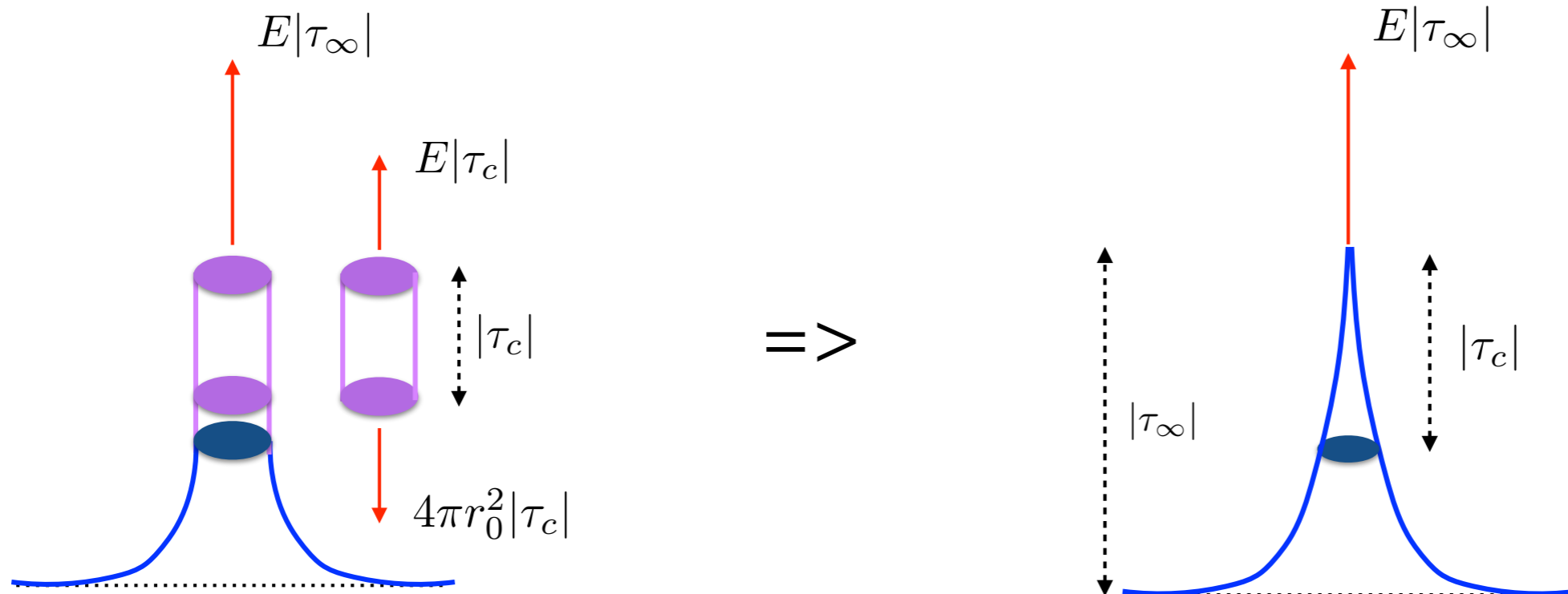
# Computing the semiclassical rate

Use *thin wall* approximation:

$$\frac{1}{2} \Delta W_{\text{stationary}}^{\text{quant}} = - \int_R^0 p(E) dr + \frac{4\pi}{3} \mu R^3, \quad E = nm$$

**final result**

$$\Delta W^{\text{quant}} = \frac{E^{3/2}}{\sqrt{\mu}} \frac{2}{3} \frac{\Gamma(5/4)}{\Gamma(3/4)} = \frac{1}{\lambda} (\lambda n)^{3/2} \frac{2}{\sqrt{3}} \frac{\Gamma(5/4)}{\Gamma(3/4)} \simeq 0.854 n \sqrt{\lambda n}$$



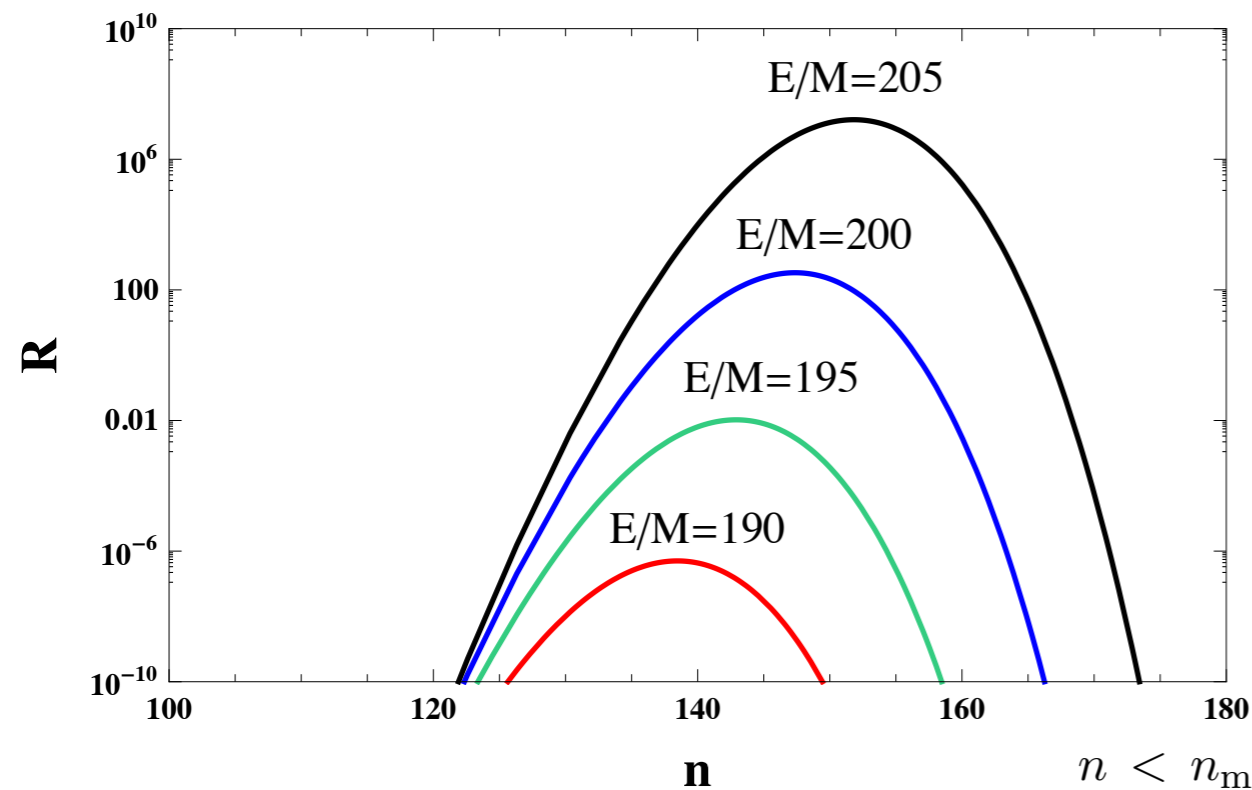


# Summary of the semiclassical result

VVK 1705.04365  
1806.05648

$\lambda \rightarrow 0, \quad n \rightarrow \infty, \quad \text{with } \lambda n = \text{fixed} \gg 1, \quad \varepsilon = \text{fixed} \ll 1$

$$\mathcal{R}_n(E) = e^{W(E,n)} = \exp \left[ \frac{\lambda n}{\lambda} \left( \log \frac{\lambda n}{4} + 0.85 \sqrt{\lambda n} - 1 + \frac{3}{2} \left( \log \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} \varepsilon \right) \right]$$



$$E/m = (1 + \varepsilon) n$$

positive  
(quantum effects)

negative  
(phase space)

Can always make this term win =>  
**unsuppressed R at high Energies**

VVK & Spannowsky 1704.0344

Higher order corrections are suppressed by extra powers of  $\lambda \rightarrow 0$  and  $1/n \rightarrow 0$  and by  $\mathcal{O}(1/\sqrt{\lambda n})$  as well as by  $\mathcal{O}(\varepsilon)$ .

Thus we have computed the rate  $R$  in the large  $\lambda n$  limit:

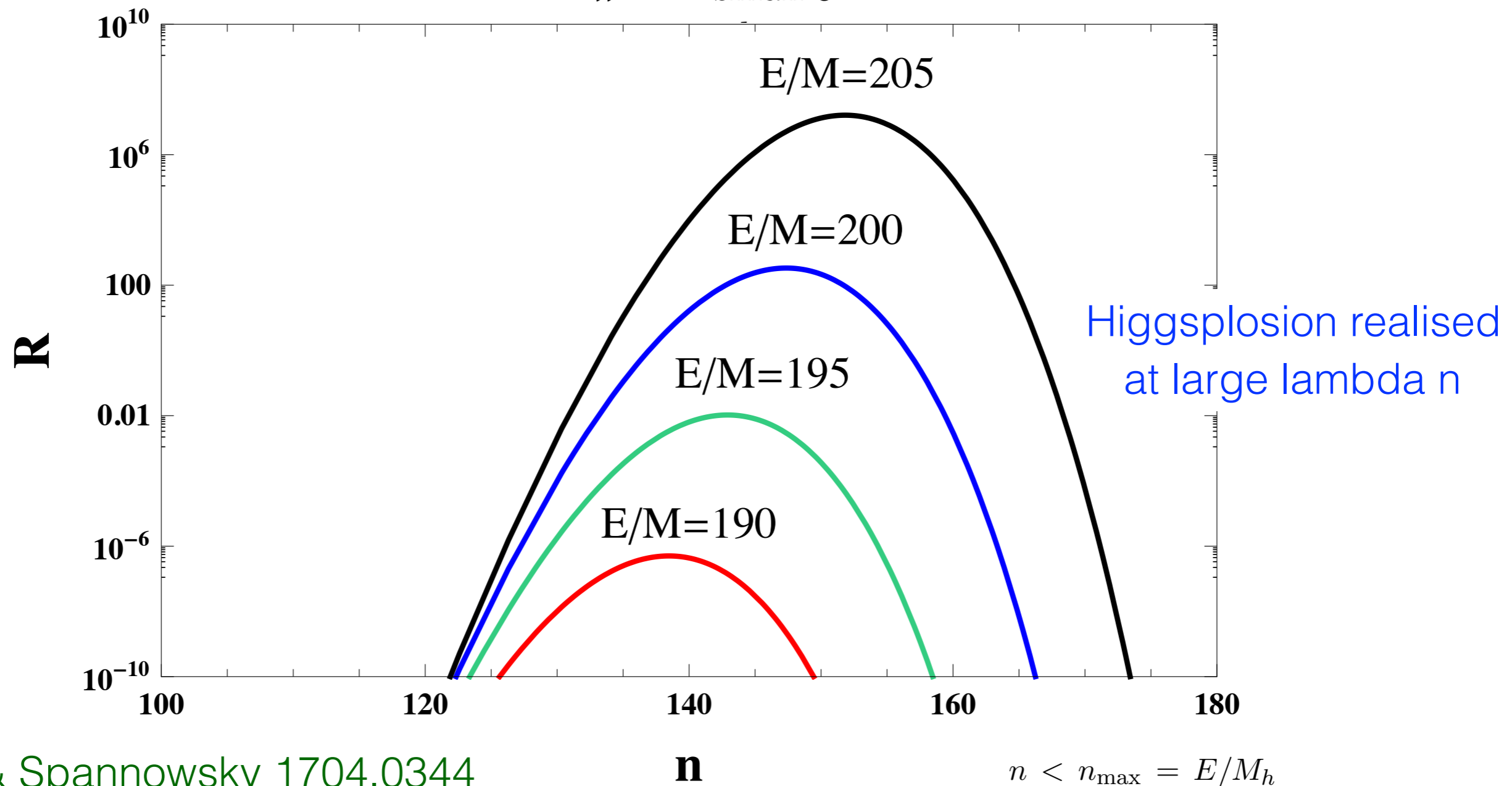
using the semi-classical approach and the thin-wall approximation

$$\mathcal{R} = \exp \left[ \frac{\lambda n}{\lambda} \left( \log \frac{\lambda n}{4} + 3.02 \sqrt{\frac{\lambda n}{4\pi}} - 1 + \frac{3}{2} \left( \log \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} \varepsilon \right) \right]$$

$\lambda n \gg 1$     small  $\varepsilon$

VVK 1705.04365

VVK 1806.05648

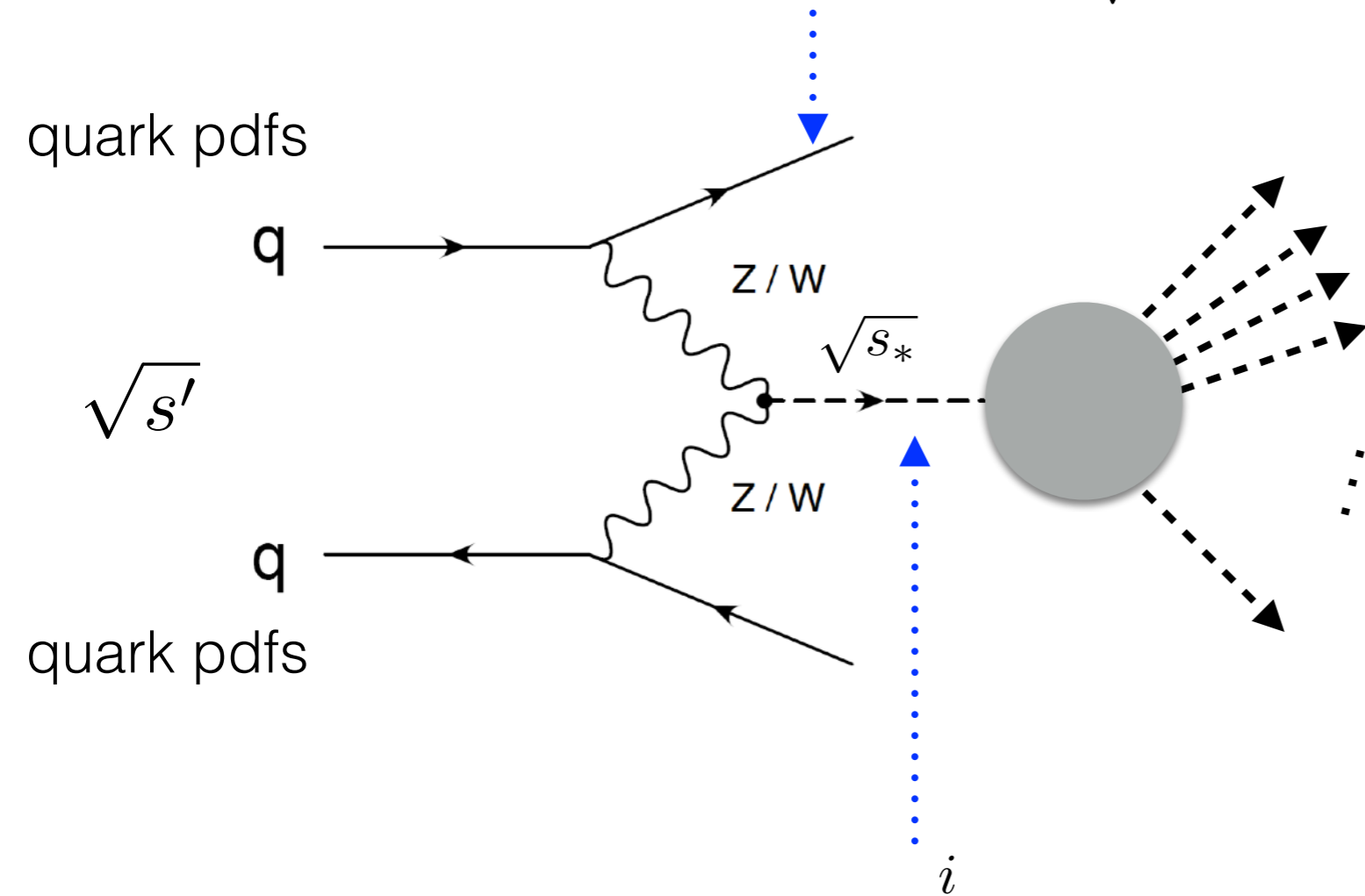


VVK & Spannowsky 1704.0344

# Applications:

## Vector boson fusion at high-energy pp colliders (FCC)

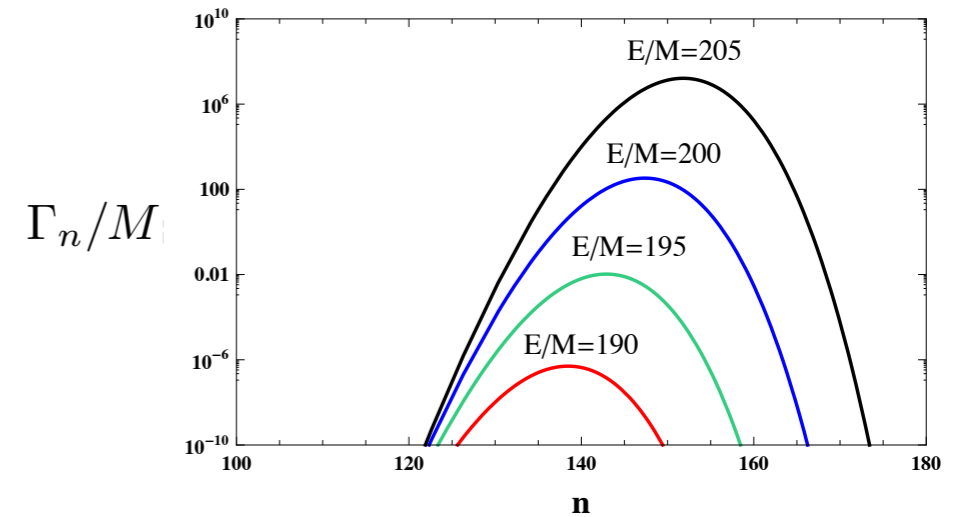
energy excess over  $\sqrt{s_*}$  carried away by jets



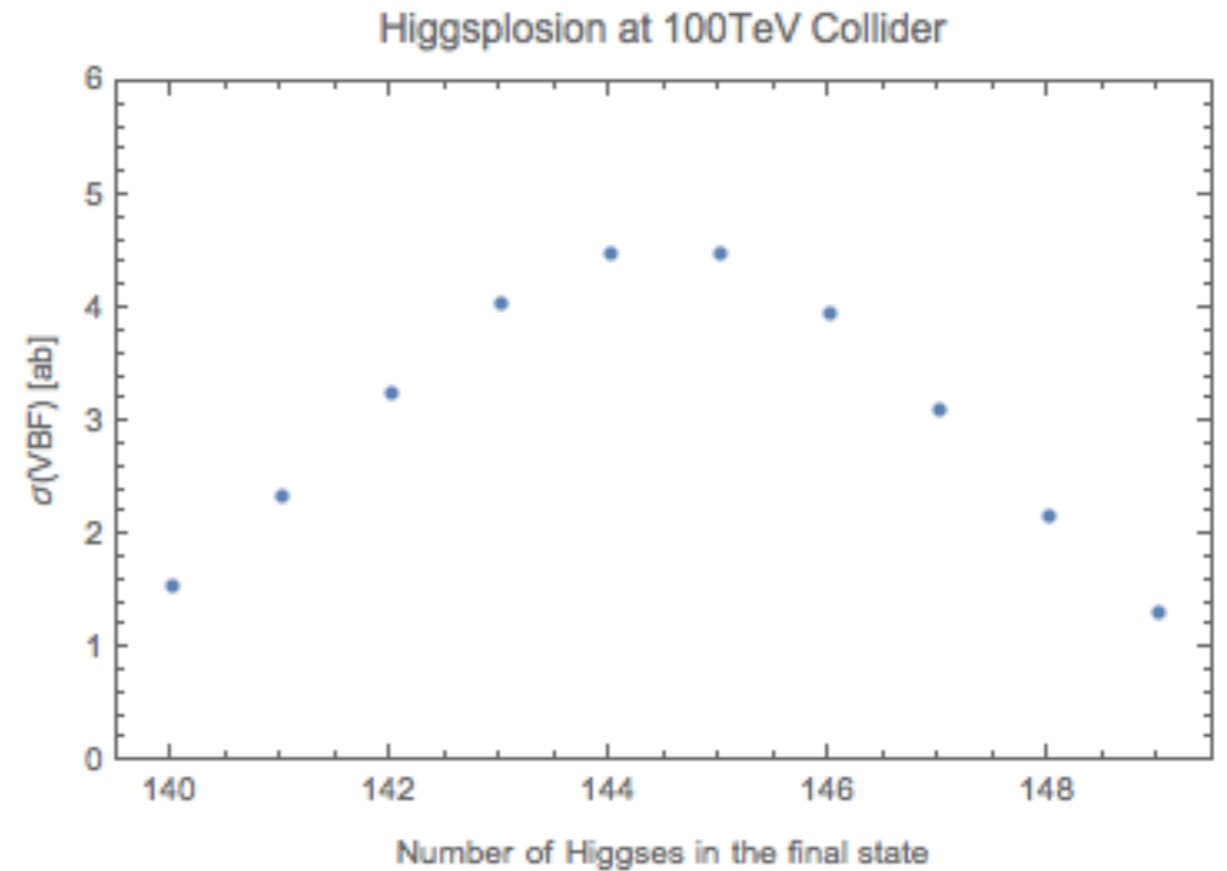
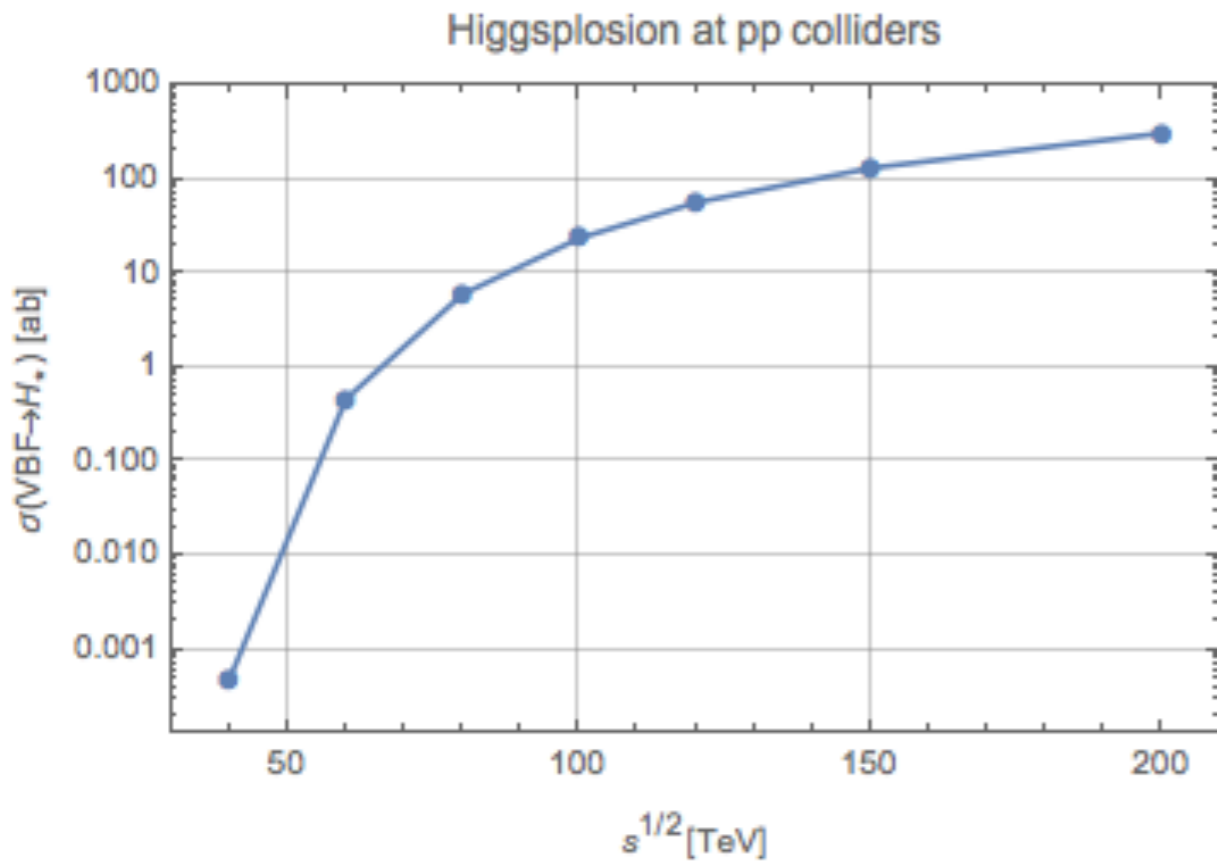
n non-relativistic Higgses  
Higgs plosion at  $\sqrt{s_*}$

$$s_* - m_h^2 - \text{Re}\tilde{\Sigma}(s_*) + im_h\Gamma(s_*)$$

Propagator with Higgspersion at  $\sqrt{s_*}$



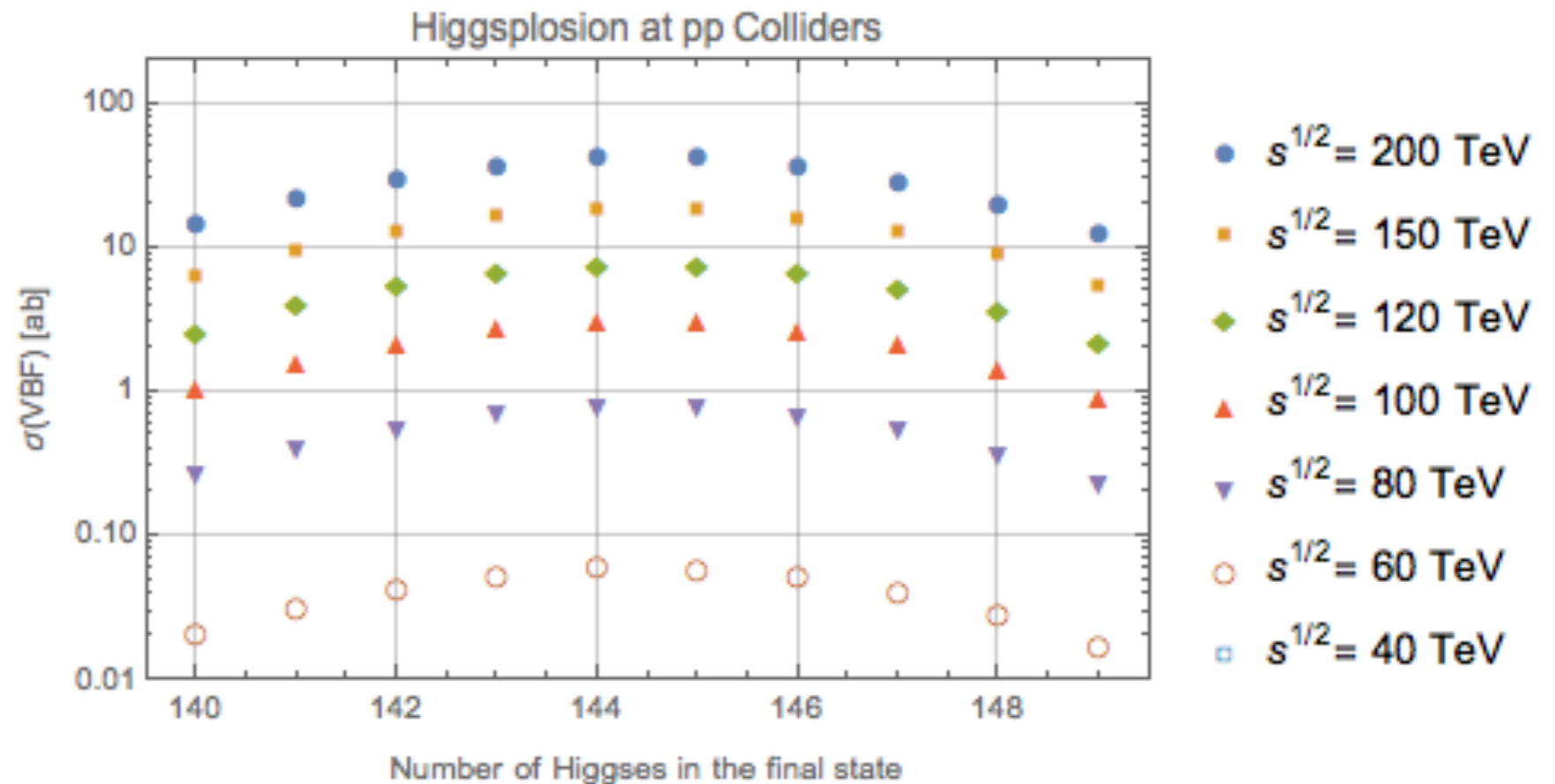
# Vector boson fusion at high-energy pp colliders (FCC)



using  $p_{t \text{ jet}} > 40$  GeV

VVK, J Scholtz, M Spannowsky

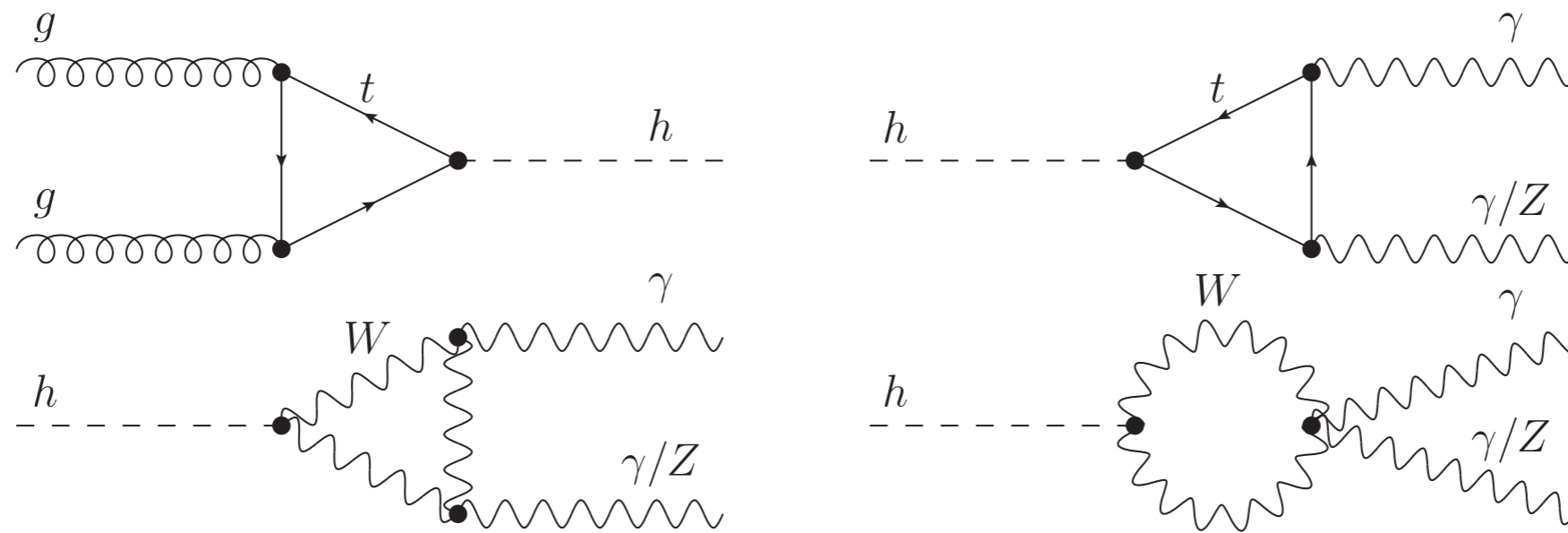
preliminary: no Higgs decays into SM d.o.f included;  
& no vector bosons in final states



# Effects of Higgspllosion on Precision Observables

- VVK, J Reiness, M Spannowsky, P Waite 1709.08655

Here focus on a class of observables which have no tree-level contributions

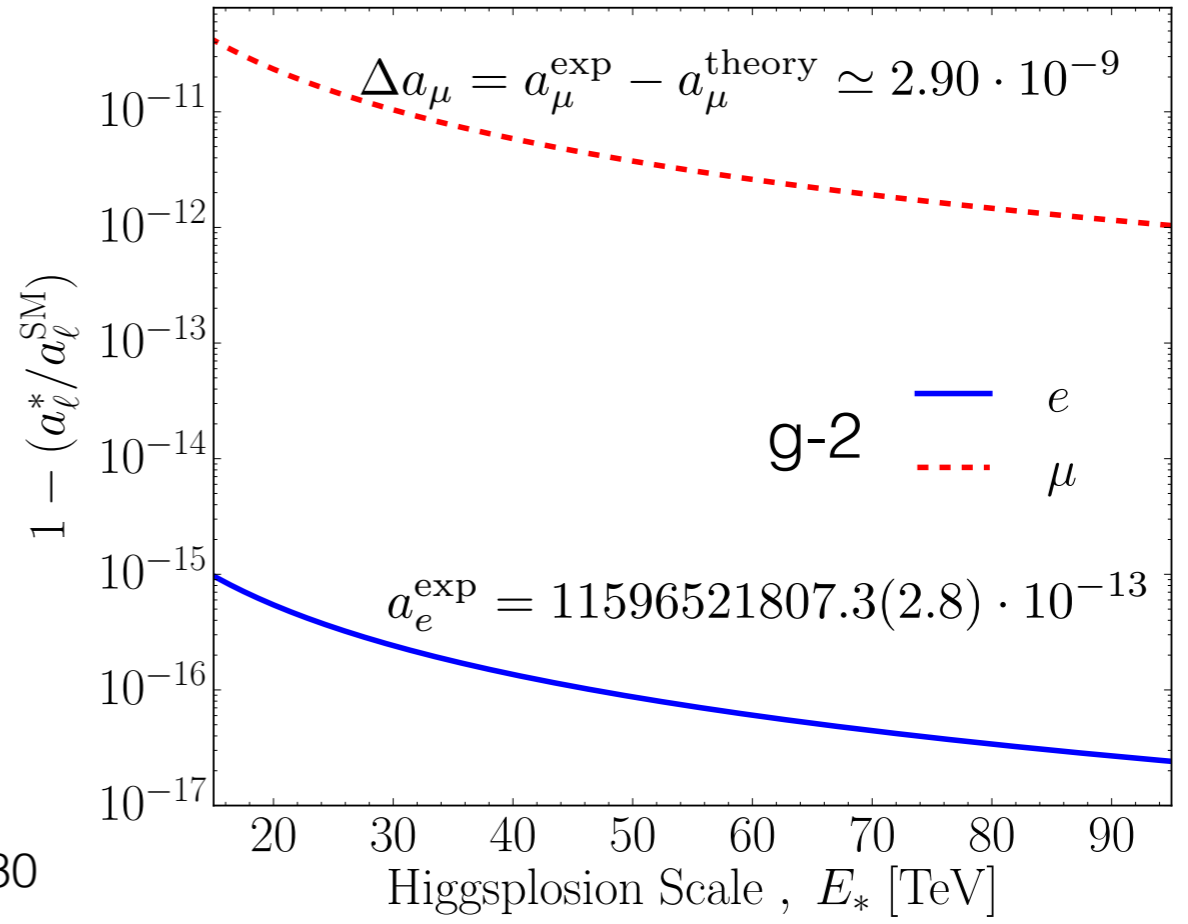
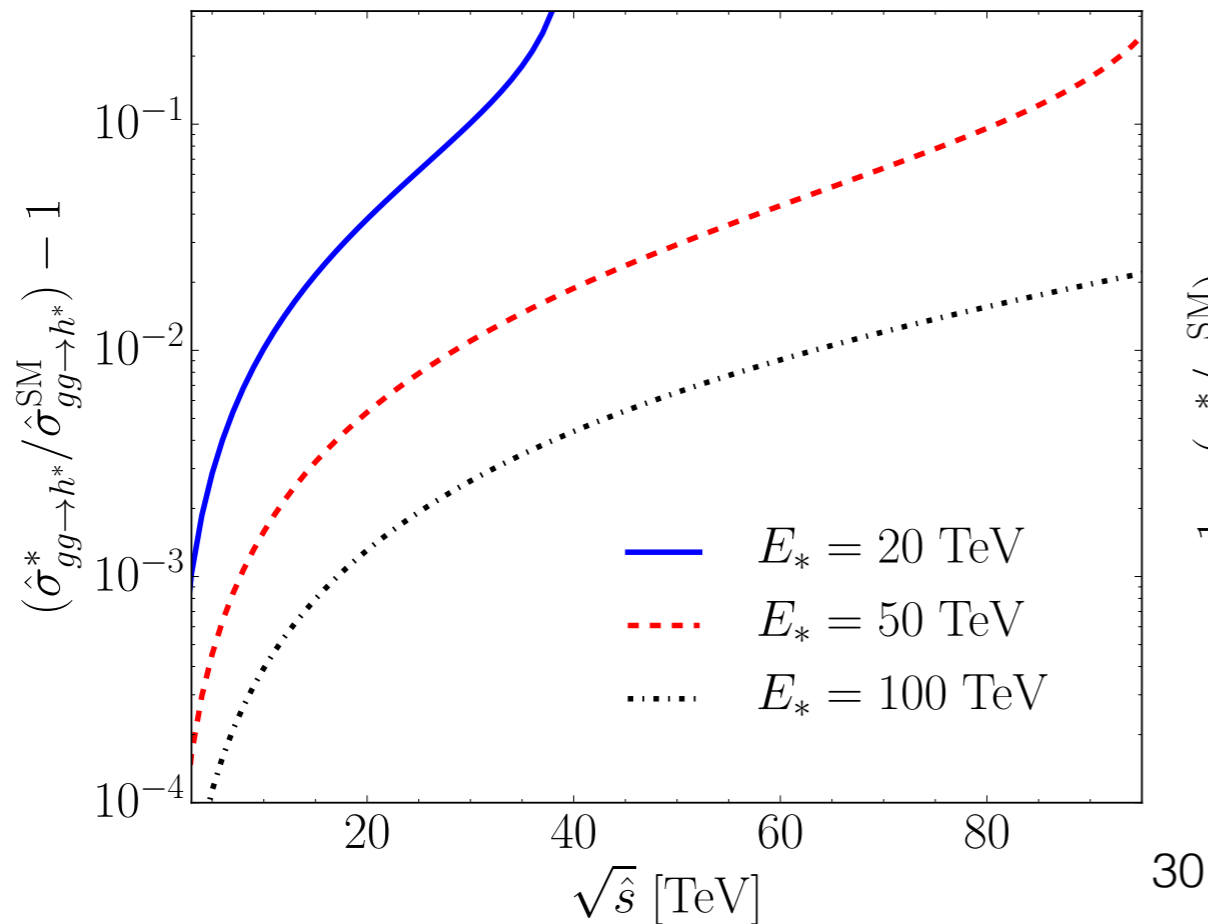
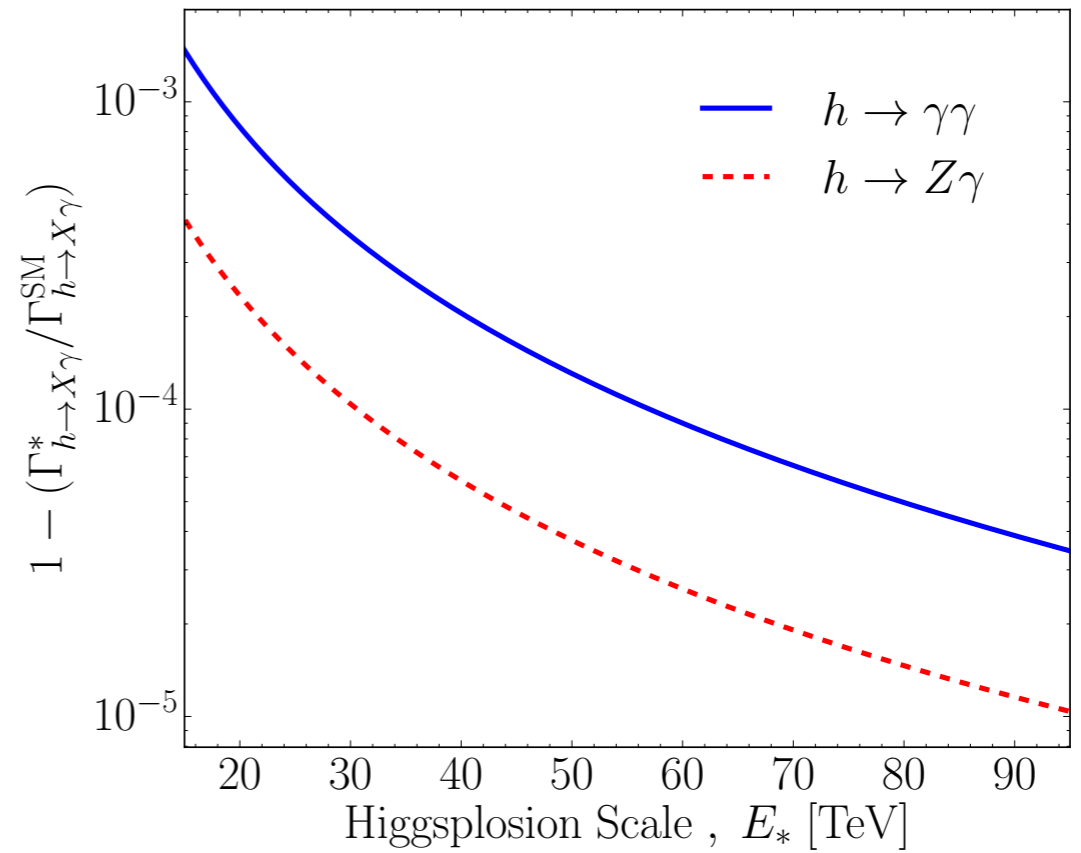
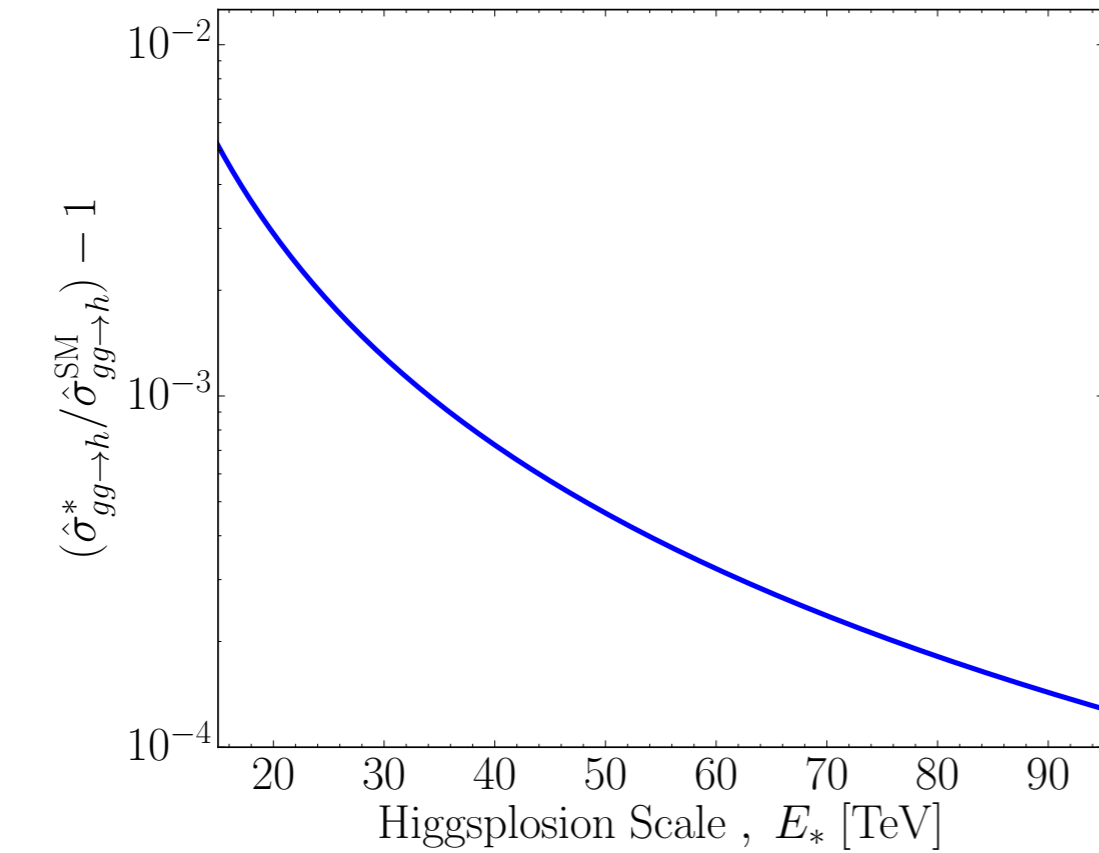


At LHC energies effects of Higgspllosion are small (next slide).

However  $O(1)$  effects can be achieved for these loop-induced processes if the interactions are probed close to  $\sim 2E^*$ .

# Effects of Higgspllosion on Precision Observables

• VVK, J Reiness, M Spannowsky, P Waite 1709.086655



## Conclusions:

$$\Delta_R(p) = \frac{i}{p^2 - m^2 - \text{Re} \Sigma_R(p^2) + im\Gamma(p^2) + i\epsilon}$$



Loop integrals are effectively cut off at  $E_*$  by the exploding width  $\Gamma(p^2)$  of the propagating state into the high-multiplicity final states.

The incoming highly energetic state decays rapidly into the multi-particle state made out of soft quanta with momenta  $k_i^2 \sim m^2 \lll E_*^2$ .

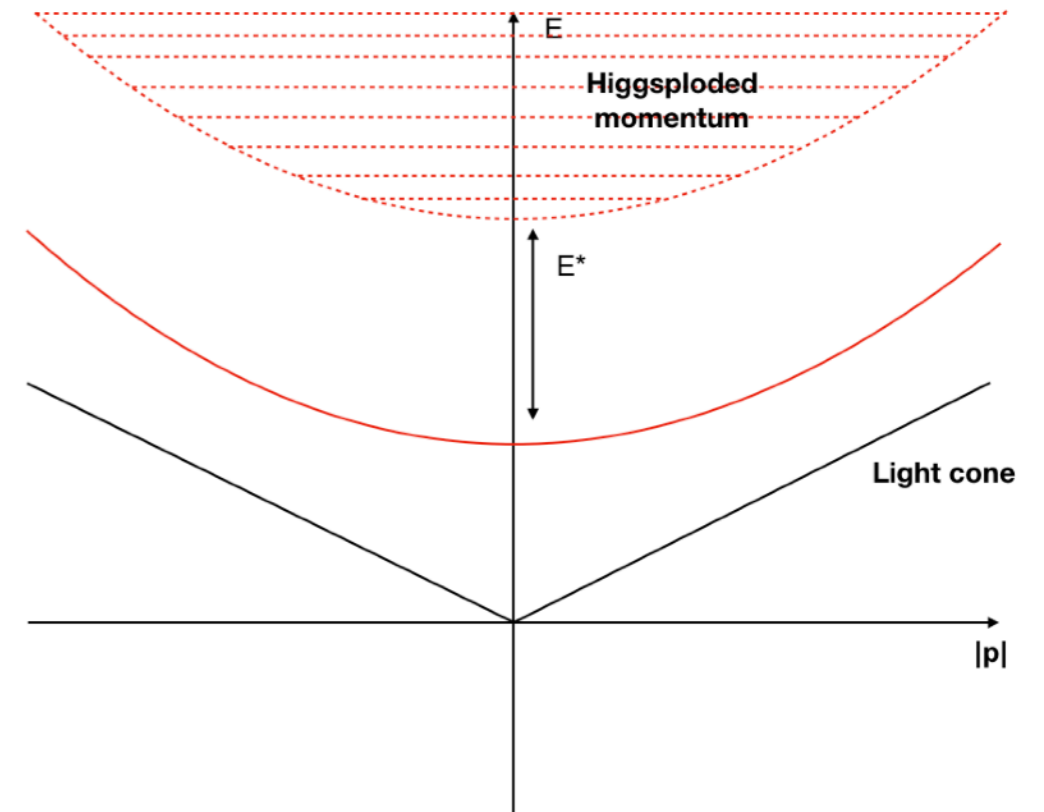
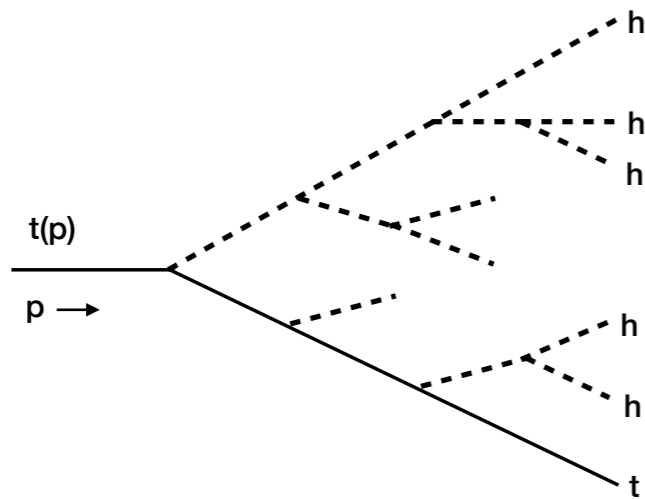
The width of the propagating degree of freedom becomes much greater than its mass: it is no longer a simple particle state.

In this sense, it has become a composite state made out of the  $n$  soft particle quanta of the same field  $\phi$ .

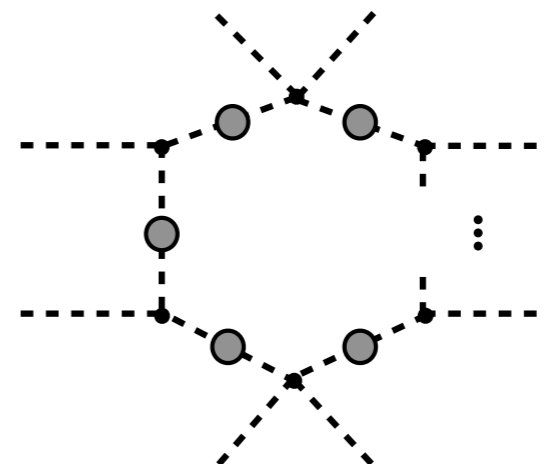
- [VVK & Spannowsky 1704.03447, 1707.01531](#)

# Consequences of Higgspllosion

- All particles Higgsplode if **virtual** enough  
e.g. top quark, Z boson and even graviton  
Higgsplodes



- As all virtual particles Higgsplode, virtual corrections are regulated by higgspersing propagators





# Consequences of Higgspllosion

- As all loop-diagrams are regulated, i.e. quantum fluctuations are exponentially suppressed, the Standard Model develops an asymptotic fixed point.

→ Classical/Deterministic theory

→ Any highly virtual or a very heavy particle rapidly decays into a large number of relatively soft Higgs bosons. A composite state.

→ Above higgspllosion scale, quantum fluctuations are damped

- SM is embedded into asymptotically safe theory

→ coupling constants stop running above the higgspllosion scale

# Consequences of Higgspllosion

- SM has new physical scale

(close analogy to Sphaleron)

$$E_* = C \frac{m_h}{\lambda} \quad \text{with } C = \text{const.}$$

$$M_{\text{sph}} = \text{const} \frac{m_W}{\alpha_w}$$

Scaling behaviour of propagator:

$$\Delta(x) := \langle 0 | T(\phi(x) \phi(0)) | 0 \rangle \sim \begin{cases} m^2 e^{-m|x|} & : \text{ for } |x| \gg 1/m \\ 1/|x|^2 & : \text{ for } 1/E_* \ll |x| \ll 1/m \text{ ,} \\ E_*^2 & : \text{ for } |x| \lesssim 1/E_* \end{cases}$$

for  $|x| \lesssim 1/E_*$  one enters the Higgspllosion regime



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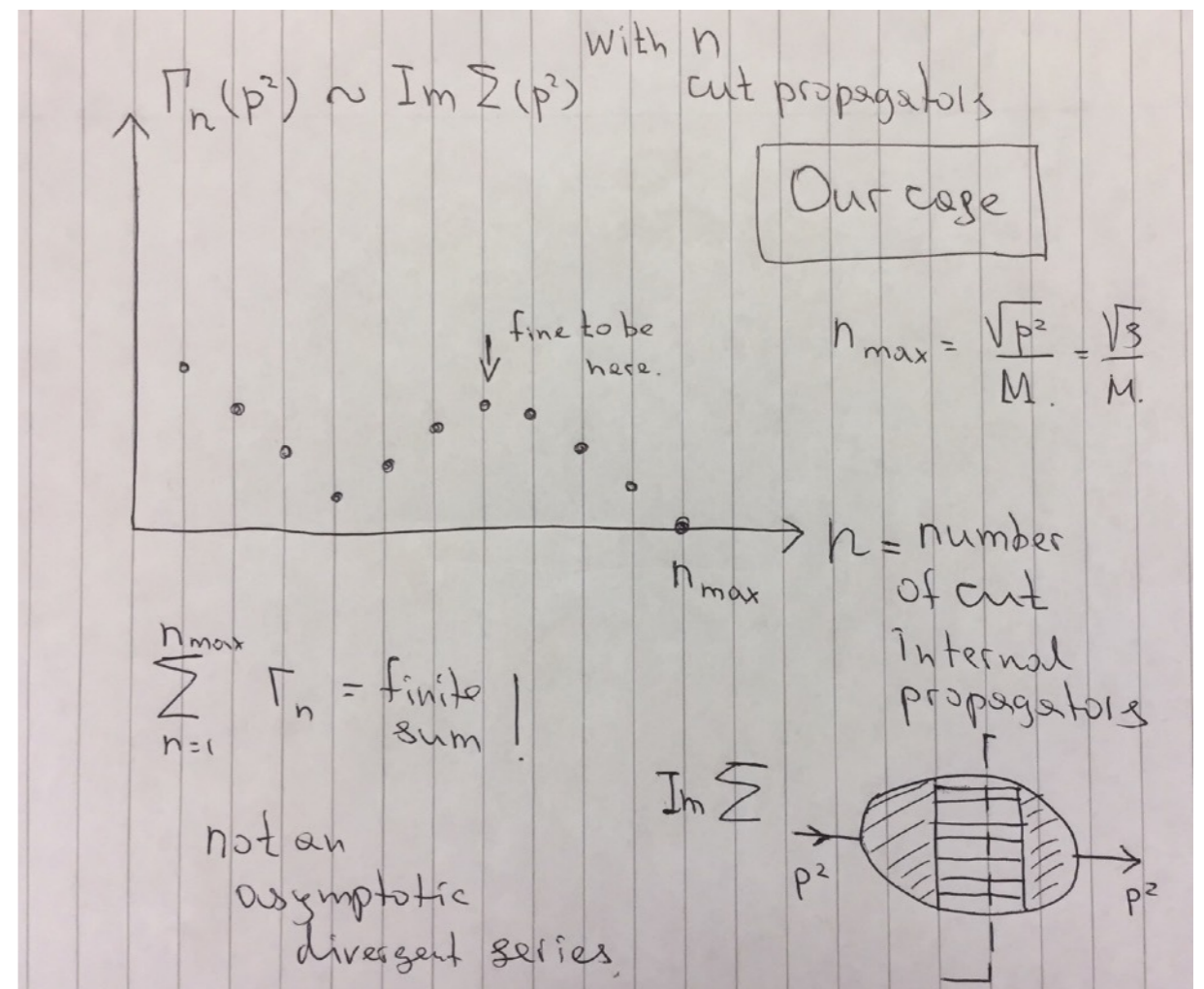
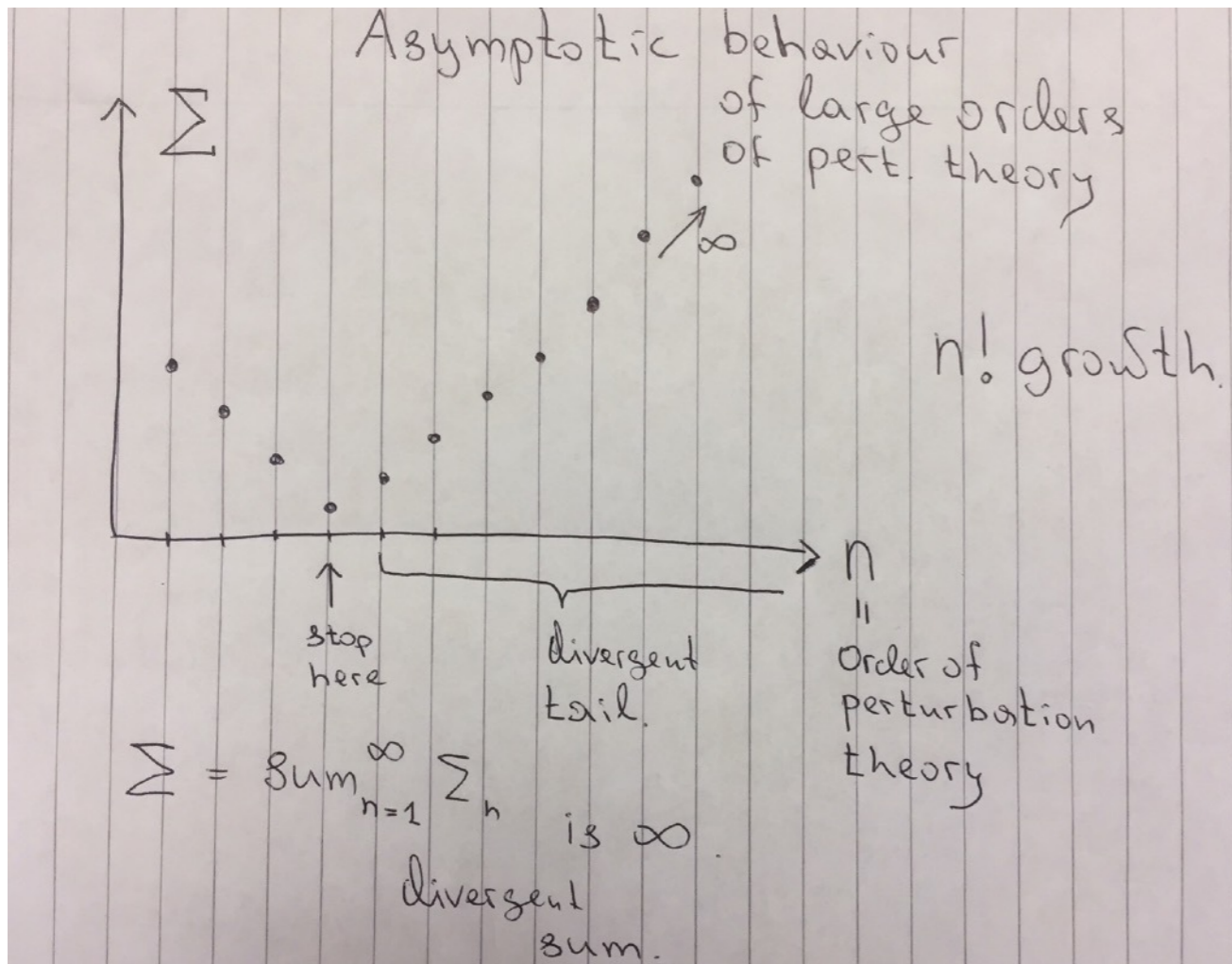

$$s_* - m_h^2 - \text{Re} \tilde{\Sigma}(s_*) + im_h \Gamma(s_*)$$

Propagator with Higgspllosion

# Extra slides

- The  $n!$  growth of *perturbative* amplitudes is not entirely surprising: the *number of contributing Feynman diagrams is known to grow factorially with  $n$* . [In scalar QFT there are no partial cancellations between individual diagrams (unlike QCD).]
- Important to distinguish between the two types of large- $n$  corrections:
  - (a) present case where the *leading-order* tree-level contribution to the  $1^* \rightarrow n$  Amplitude grows factorially with the particle multiplicity  $n$  of the final state.
- (b) *higher-order* perturbative corrections to some leading-order quantities

# Contrast asymptotic growth of higher-order corrections in perturbation theory with the $\sim n!$ contributions to $\Gamma_n(s)$



Not the same types of beasts

It is the decay width  $\Gamma_n(s)$  which is the central object of interest and the driving force of Higgspllosion.