Helsinki Higgs Forum, 14-16 Dec. 2016

Higgs effects in Cosmology and at FCC

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Motivation

After the Higgs discovery the SM still leaves some fundamental questions unanswered:

- It accommodates $v = 246\ \mathrm{GeV}$ and $m_h \simeq 125\ \mathrm{GeV}$ essentially as input parameters, but the SM does not explain the origin and smallness of the EWSB scale $\{v\,,\, m_h\} \ll M_{\mathrm{Pl}}$
- There is no Dark Matter in the SM
- The Generation of the matter-anti-matter asymmetry of the Universe (BAU) is impossible within the SM
- Need to include Neutrino masses and oscillations
- Robust particle physics implementation of Cosmological Inflation is missing
- The minimal SM Higgs potential is unstable at high scale
- Strong CP problem, axions, etc.

1. Minimal BSM-ing: Higgs Portals to New Physics

There is just a single occurrence of a non-dynamical scale in the Standard Model: the $\mu^2_{\rm SM}$ parameter.

$$V_{\rm cl}^{\rm SM}(H) = -\frac{1}{2}\,\mu^2_{\rm SM}\,H^\dagger H + \lambda_h (H^\dagger H)^2$$

Replace $\mu^2_{\rm SM}$ by a Higgs portal interaction with a new scalar Φ:

$$V_{\rm cl}(H,\Phi) = \lambda_h (H^\dagger H)^2 + \lambda_\phi (\Phi^\dagger \Phi)^2 - \lambda_{\rm P} (H^\dagger H)(\Phi^\dagger \Phi)$$

$V_{\rm cl}$ is now scale-invariant.

If the VEV of Φ , i.e. $\frac{1}{\sqrt{2}}\langle\phi\rangle$, can be generated quantum mechanically, it will trigger the electro-weak symmetry breaking (EWSB):

$$\mu^2_{\rm SM} = \lambda_{\rm P} |\langle\phi\rangle|^2 \quad = m_h^2 = 2\,\lambda_h\, v^2$$

1. Minimal BSM-ing: Higgs Portals to New Physics

Coleman-Weinberg mechanism (1973) – 1st example of the dimensional transmutation:

a massless scalar field Φ coupled to a gauge field dynamically generates a non-trivial $\langle\phi\rangle$ via a dimensional transmutation of the log-running couplings. $\langle\phi\rangle$ is generated before the self-coupling λ_ϕ becomes negative in the IR [tracing $\lambda_\phi(\mu)$ with a positive beta function from the UV to the IR].

- Classical scale invariance is not an exact symmetry. It is broken anomalously by running couplings in a controlled way.
- The symmetry-breaking order parameter is the dynamical scale $\langle\phi\rangle \ll M_{UV}$ which then feeds into the EWSB and other features.
- Generic UV regularisation would introduce *large* effects $\sim \alpha\, M_{UV}^2$. To maintain the anomalously broken scale invariance, one should choose a scale-invariance-preserving regularisation scheme – dimensional regularisation – Bardeen 1995.

1. Minimal BSM-ing: Higgs Portals to New Physics

A powerful principle for the BSM model building. No vastly different scales can co-exist in such a theory:

- Scale invariance requires that all scales associated with new physics are generated dynamically. No large input scales are allowed; i.e. no thermal Leptogenesis with $\sim 10^9$ GeV Majorana masses; not GUT scale, etc.

The BSM theory is a minimal extension of the SM which should address all the sub-Planckian shortcomings of the SM without introducing scales higher than $\langle\phi\rangle$ which itself is not much higher the electroweak scale.

1. Link between CW scale and the Higgs scale (EWSB)
2. Link with the Leptogenesis scale (BAU)
3. Link with the Dark Matter scale (DM)

DM $\leftrightarrow$ BAU $\leftrightarrow$ EWSB

Classically Scale Invariant Extended Standard Model

- Study different examples of G_{CW}: $\mathrm{U}(1)_{CW}$, $\mathrm{U}(1)_{B-L}$, $\mathrm{SU}(2)_{CW}$;
  also can add more singlets in the Higgs portal;
  (can include strongly-coupled hidden sectors, not only weakly-coupled CW)

- Minimal CSI SM$\times$G$_{CW}$ models have only two free parameters, the portal coupling, $\lambda_{\rm P}$ and the hidden gauge coupling g_{CW}.

- H and Φ scalars mix, giving two higgs mass-eigenstates $m_{h_1} \simeq 125$ GeV and m_{h_2} (which can be $>$ or $< m_{h_1}$).

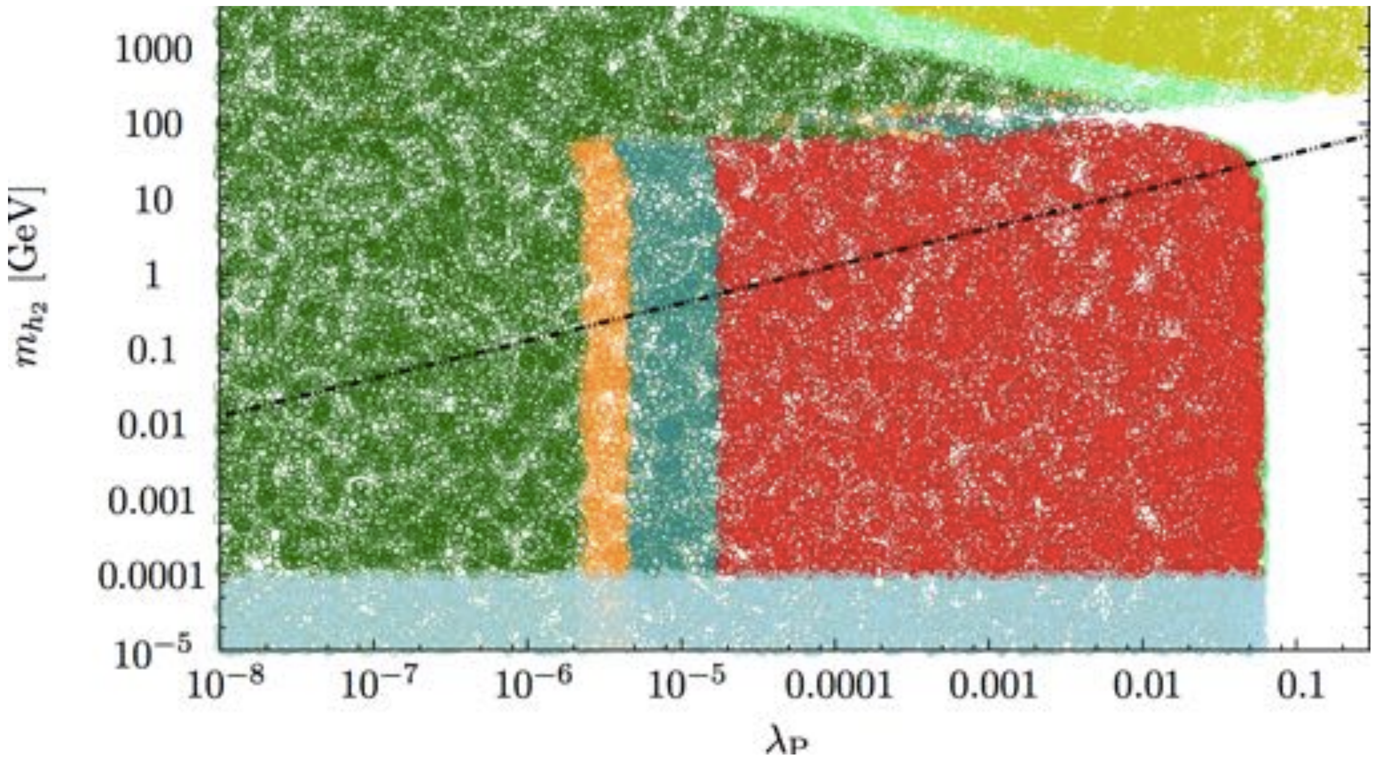
- There is always Z' with $M_{Z'} > m_{h_2}$. Both, m_{h_2} and $M_{Z'}$ can be determined in terms of $\lambda_{\rm P}$ and g_{CW}.
- If $m_{h_1} > 2m_{h_2}$ the SM Higgs can decay into two hidden Higgses which constrains $\lambda_{\rm P} \lesssim 10^{-5}$.

- For $m_{h_2} > m_{h_1}/2$ the coupling $\lambda_{\rm P}$ is much less constrained.

- Collider production of Z' possible if SM quarks couple to the hidden G_{CW} - as in the $\mathrm{U}(1)_{B-L}$ example - but not otherwise.

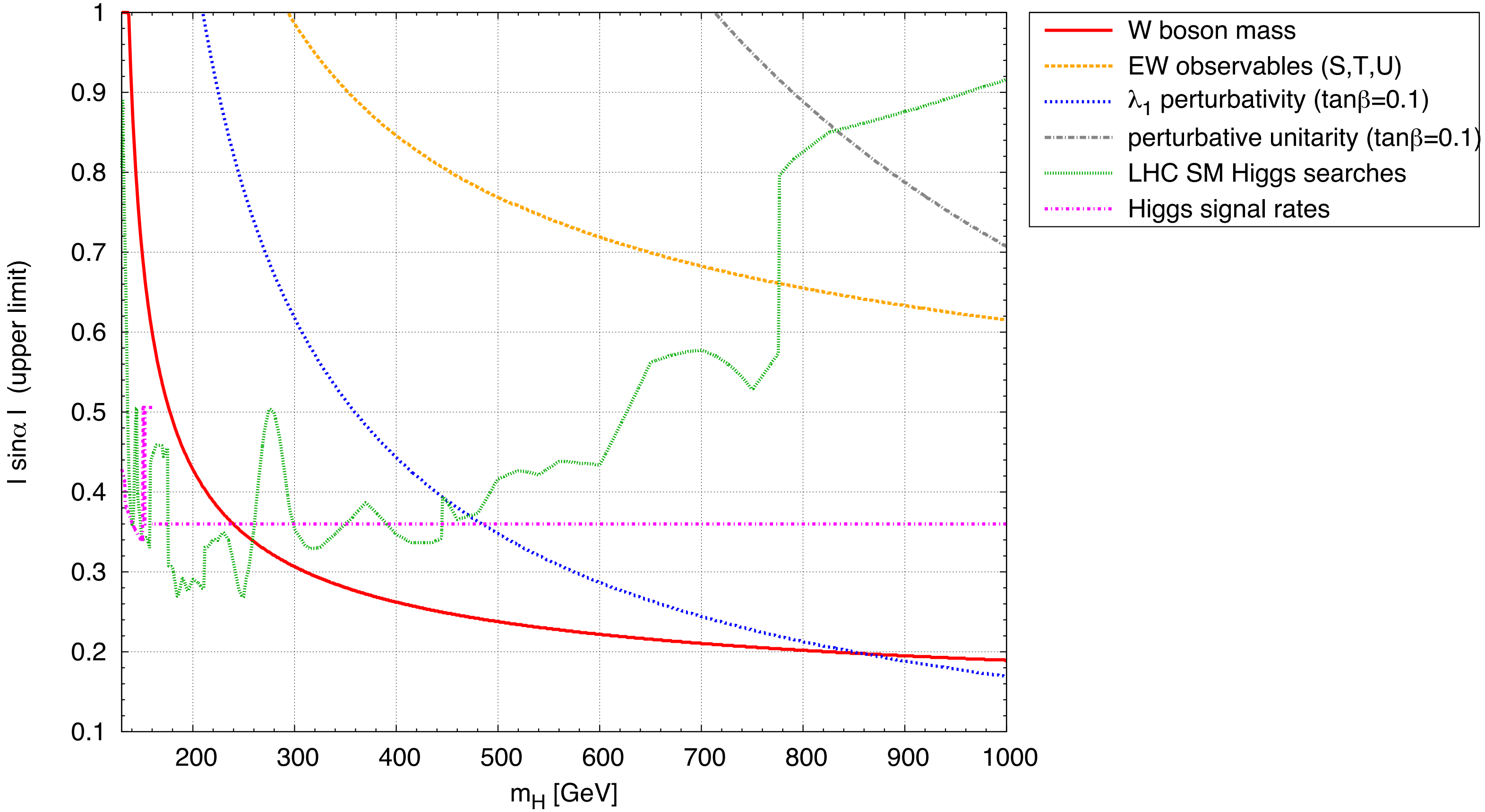
Light $m_{h_2} < (1/2)m_{h_1}$ states are constrained by $\Gamma_{h_1 \to h_2 h_2}$

C. Englert, J. Jaeckel, V. V. Khoze and M. Spannowsky



Red region is excluded by LHC Run 1. Cyan will be probed by HL LHC. Orange region is a projection for a combination of a HL LHC with an LC. Green region is allowed.

Upper bounds on $|sin\alpha|$ for heavier $h_2 \in [130, 1000]$GeV



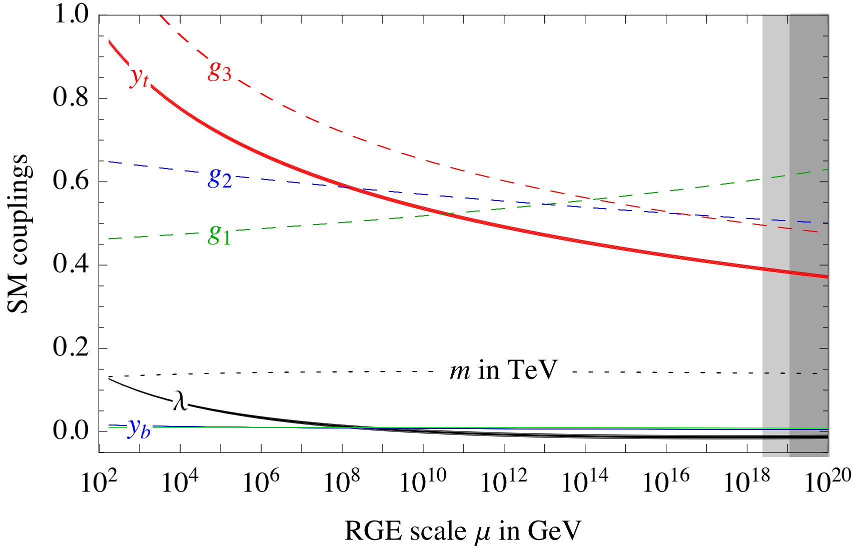
T. Robens and T. Stefaniak, 1601.07880;
D. Lopez-Val and T. Robens, 1406.1043

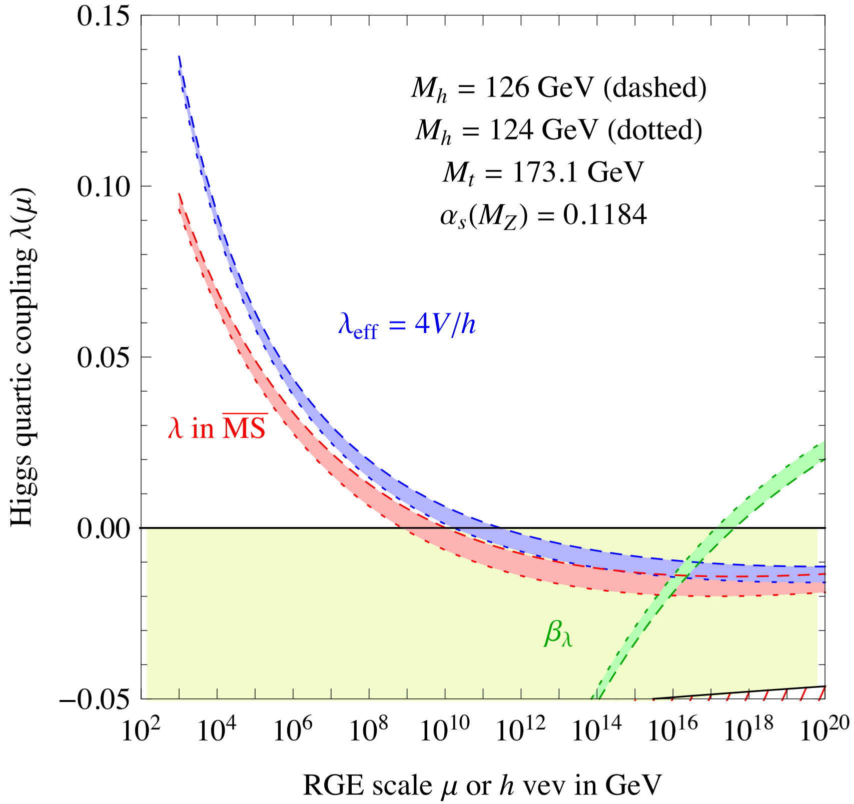
Stabilisation of the Higgs potential

The SM Higgs potential is unstable as the Higgs self-coupling λ turns < 0.

(probably)

… see also Bernd Kniehl's talk





G. Degrassi, S. Di Vita, J. Elias-Miro, J.R. Espinosa, G.F. Giudice. G. Isidori, A. Strumia, 1205.6497
D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio and A. Strumia, 1307.3536

Stabilisation of the Higgs potential

A minimal and robust way to repair the EW vacuum stability is provided by the Higgs portal extension of the SM – just what we have in our theory.

Two effects to stabilise the vacuum:

1. The portal coupling gives a positive contribution to the beta function of the Higgs quartic coupling, $\Delta\beta_\lambda \sim +\lambda_{\rm P}^2$
2. The vev of the second scalar, $\langle\phi\rangle > v$, leads to mixing between ϕ and the Higgs resulting in a threshold correction lifting the SM Higgs λ_H

O. Lebedev, 1203.0156

J. Elias-Miro, J. R. Espinosa, G. F. Giudice, H. M. Lee and A. Strumia, 1203.0237; T. Hambye and A. Strumia, 1306.2329

We will also consider extending the model by adding a real singlet:

$$\text{CSI} \quad \text{SM} \times G_{CW} \oplus \text{singlet } s(x)$$

The singlet gives the inflaton and the Dark Matter candidate plus helps with the Higgs vacuum stabilisation. Values of $\lambda_{Hs} \gtrsim 0.35$ are sufficient to stabilise the Higgs by this effect alone.

Dark Matter

Adding a scalar singlet $s(x)$ to the SM x $SU(2)_{CW}$ model:

$$V_{\rm cl}(H,\phi,s) = \frac{\lambda_{Hs}}{2}|H|^2 s^2 + \frac{\lambda_{\phi s}}{2}|\Phi|^2 s^2 + \frac{\lambda_s}{4}s^4 + V_{\rm cl}(H,\Phi)$$

There are two immediate DM candidates (and one can add more):

1. The $SU(2)_{CW}$ gauge bosons give vector DM. They are stable due to an SO(3) symmetry and no kinetic mixing

   T. Hambye 2008, T. Hambye and A. Strumia 1306.2329

2. The singlet scalar $s(x)$, if present, is stable due to a Z_2 symmetry which is automatic due to CSI and gauge invariance

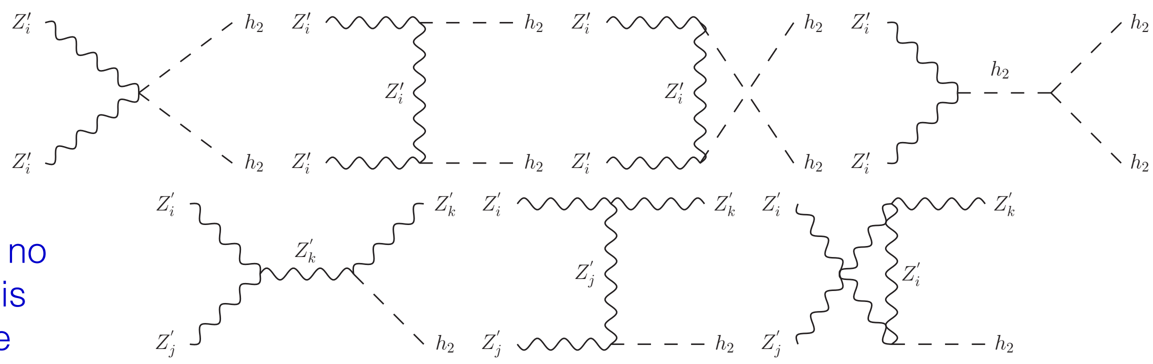
The origin of the dark matter scale is the same as the origin of the EW scale as $m_{DM} \sim \langle\Phi\rangle$. Relic abundance produced by standard freeze out mechanism.

VVK, C. McCabe and G. Ro, arXiv:1403.4953

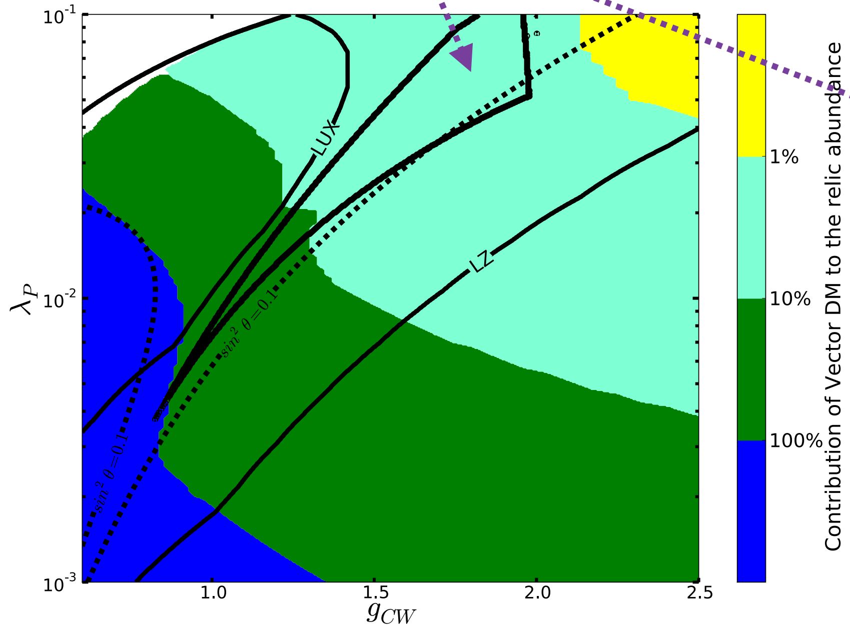
More references: DM in the Higgs portal:

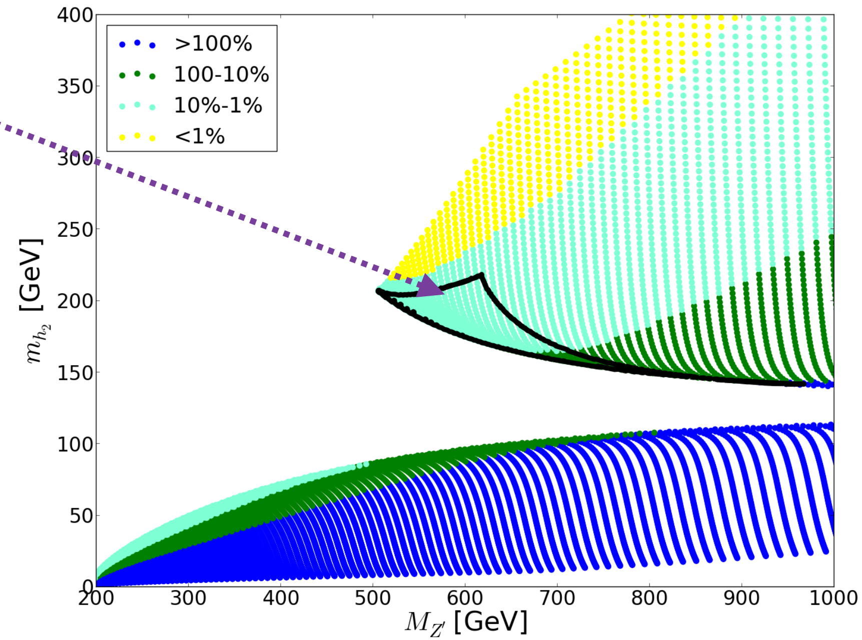
- A. Djouadi, O. Lebedev, Y. Mambrini and J. Quevillon, 1112.3299 – *Higgs Portal DM*
- A. Djouadi, A. Falkowski, Y. Mambrini and J. Quevillon, 1205.3169
- T. Hambye 2008, T. Hambye and A. Strumia 1306.2329 – *Vector DM*
- VVK, C. McCabe and G. Ro, 1403.4953 – *SU(2) Vector & Scalar DM*
- G. Arcadi, C. Gross, O. Lebedev, Y. Mambrini, S. Pokorski and T. Toma, 1611.00365 – *Multi-component incl SU(3) DM*
- G. Arcadi, C. Gross, O. Lebedev, S. Pokorski and T. Toma, 1611.09675 – *SU(N) Vector DM*
- A. Karam and K. Tamvakis, 1508.03031 and 1607.01001.
- VVK and A.. Plascencia, 1605.06834 – *Leptogenesis + Vector DM*

- $SU(2)_{CW}$ Vector Dark Matter annihilation and semi-annihilation:



The SM Higgs potential (with no additional singlets present) is stabilised inside the wedge



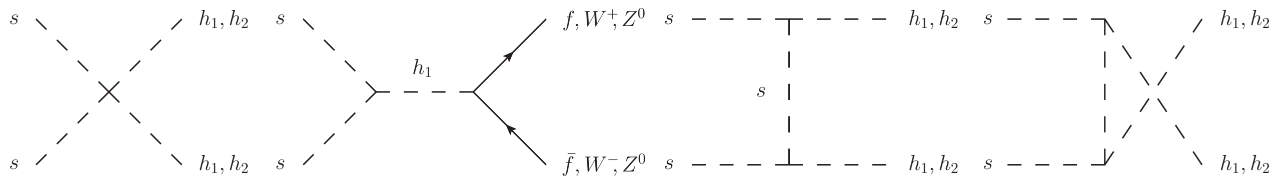


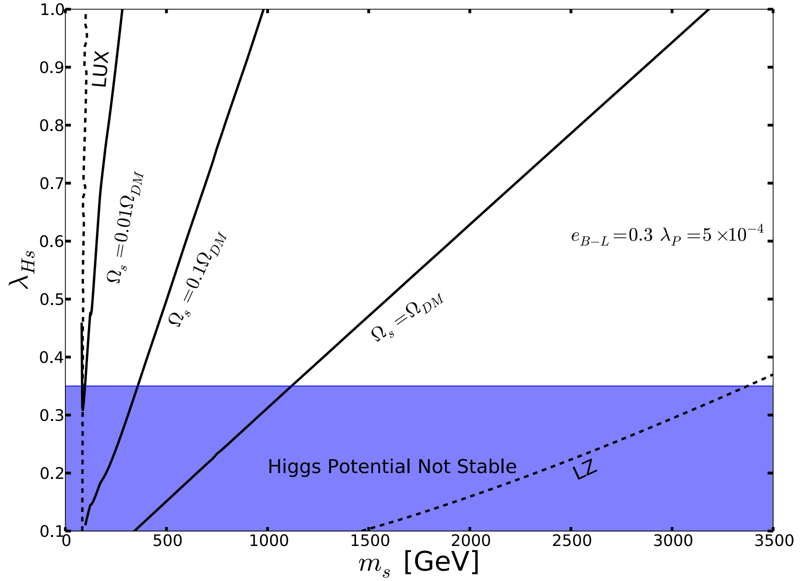
$SU(2)_{CW} \times$ SM: λ_P, g_{CW} plane

[For $SU(2)_{CW} \times$ SM $\oplus$ singlet $s(x)$ – don't need to be inside the wedge]

Scalar singlet Dark Matter

- Scalar Dark Matter annihilation diagrams include:





$U(1)_{B-L} \times$ SM $\oplus$ singlet model

VVK, C. McCabe and G. Ro, arXiv:1403.4953

Leptogenesis via neutrino oscillations

- An attractive scenario for generating BAU is Leptogenesis:
- Standard approach: Lepton asymmetry is generated by out-of-equilibrium decays of heavy sterile Majorana neutrinos into SM leptons at T much above the electroweak scale. The lepton asymmetry is then reprocessed into the baryon asymmetry by electroweak sphalerons.
- Requires extremely heavy masses for sterile neutrinos, $M_N \gtrsim 10^9$ GeV. Inconsistent with the classical scale-invariance.
- We adopt an *alternative* ARS approach to leptogenesis: the lepton flavour asymmetry is produced during oscillations of Majorana neutrinos with masses $200\,\mathrm{MeV} \lesssim M_N \lesssim 500\,\mathrm{GeV}$.

  E. K. Akhmedov, V. A. Rubakov and A. Y. Smirnov, 9803255
  T. Asaka and M. Shaposhnikov, 0505013
  M. Drewes and B. Garbrecht, 1206.5537

- Fits perfectly with classical scale-invariance settings.

  V. V. Khoze and G. Ro, 1307.3764 – Neutrinos $\ni$ CW sector
  V. V. Khoze and A. Plascencia, 1605.06834 – Neutrinos not gauged

Leptogenesis: Neutrinos $\oplus$ CW sector

- Couple sterile Majorana neutrinos N to a singlet scalar σ

$$\mathcal{L}_N = -\frac{1}{2}\left(Y^M_{ij}\sigma\overline{N_i^c}N_j + Y^{M\dagger}_{ij}\sigma\overline{N_i}N_j^c\right) - Y^D_{ia}\overline{N_i}(\varepsilon H)l_{La} - Y^{D\dagger}_{ai}\overline{l_{La}}(\varepsilon H)^\dagger N_i$$

the first two terms give rise to the Majorana masses, $M_{ij} = Y^{\mathrm{M}}_{ij}\langle\sigma\rangle$.
The Yukawa matrices are responsible for CP-violating oscillations of N_i.

- Then add the Coleman-Weinberg gauge sector with the scalar Φ as a separate sector with the portal couplings:

$$V_{\mathrm{cl}} = \lambda_\phi|\Phi|^4 + \lambda_h|H|^4 + \frac{\lambda_\sigma}{4}\sigma^4 - \lambda_{h\phi}|H|^2|\Phi|^2 - \frac{\lambda_{\phi\sigma}}{2}|\Phi|^2\sigma^2 + \frac{\lambda_{h\sigma}}{2}|H|^2\sigma^2$$

Here need to use the Gildener-Weinberg formalism for generating the vevs of *multiple* scalars.

E. Gildener and S. Weinberg, Phys. Rev. **D13** (1976) 3333
A. Karam and K. Tamvakis, 1508.03031
V. V. Khoze and A. Plascencia, 1605.06834

Leptogenesis via neutrino oscillations

- The right-handed neutrinos are produced thermally in the early Universe.
- After being produced, they begin to oscillate, $N_i \leftrightarrow N_j$, between the three different flavour states $i, j = 1, 2, 3$ in the expanding Universe.
- The lepton number of individual flavours is not conserved: complex non-diagonal Majorana matrices induce CP-violating flavour oscillations followed by out-of-equilibrium – due to small Yukawas – decays

$$N_i \leftrightarrow N_j \rightarrow \nu_{Lj}\, h$$

- Require that by the time the temperature cools down to T_{EW}, where electroweak sphaleron processes freeze out, only two out of three neutrino flavours equilibrate with their Standard Model counterparts

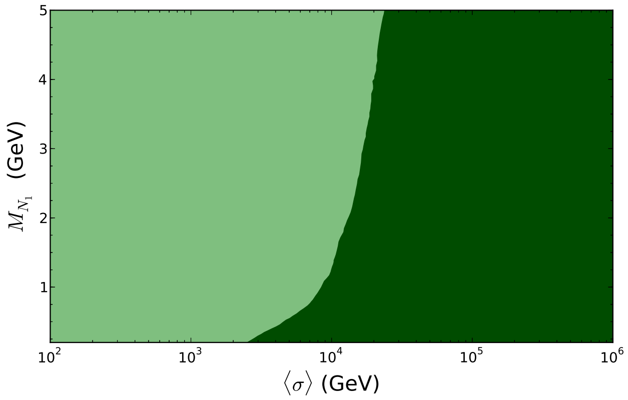
$$\Gamma_2(T_{EW}) > H(T_{EW})\,, \quad \Gamma_3(T_{EW}) > H(T_{EW})\,, \quad \Gamma_1(T_{EW}) < H(T_{EW})$$

where H is the Hubble constant, $H(T) = \frac{T^2}{M_{\mathrm{P}}}$ and $M_{\mathrm{P}} \simeq 10^{18}\,\mathrm{GeV}$. Therefore:

$$\Gamma_1(T_{\mathrm{EW}}) = \frac{1}{2}\sum_i Y^{\mathrm{D}\,\dagger}_{ei} Y^{\mathrm{D}}_{ie}\, \gamma_{av}\, T_{\mathrm{EW}} < H(T_{\mathrm{EW}})\,, \quad \gamma_{av} \approx 3 \times 10^{-3}$$

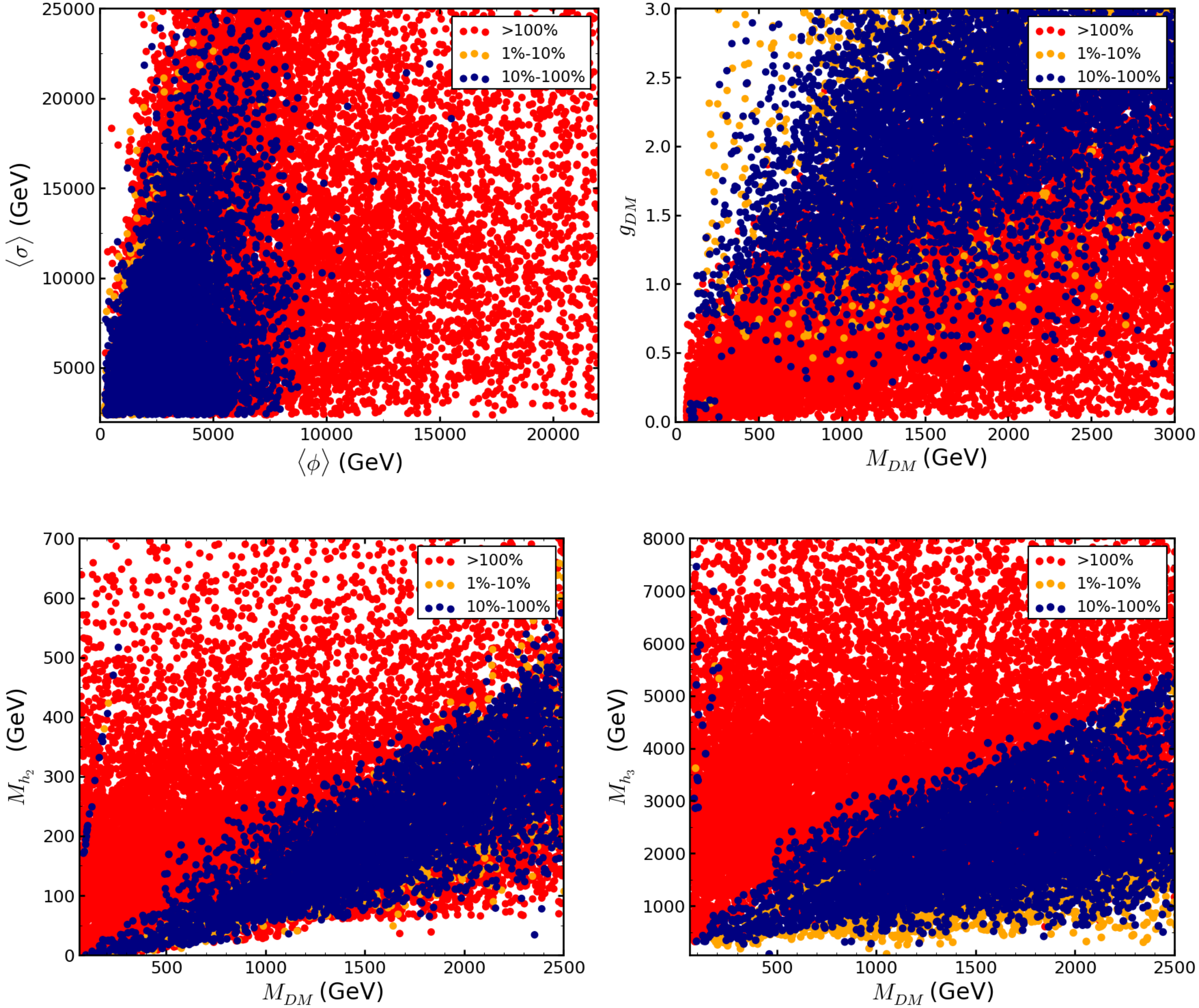
Leptogenenis vs Dark Matter

After having performed a scan over all free parameters in the model we find:
(1) $\langle\phi\rangle < 17$ TeV in order for dark matter not to overclose the universe, and
(2) $\langle\sigma\rangle > 2.5$ TeV in order for leptogenesis to explain the BAU.

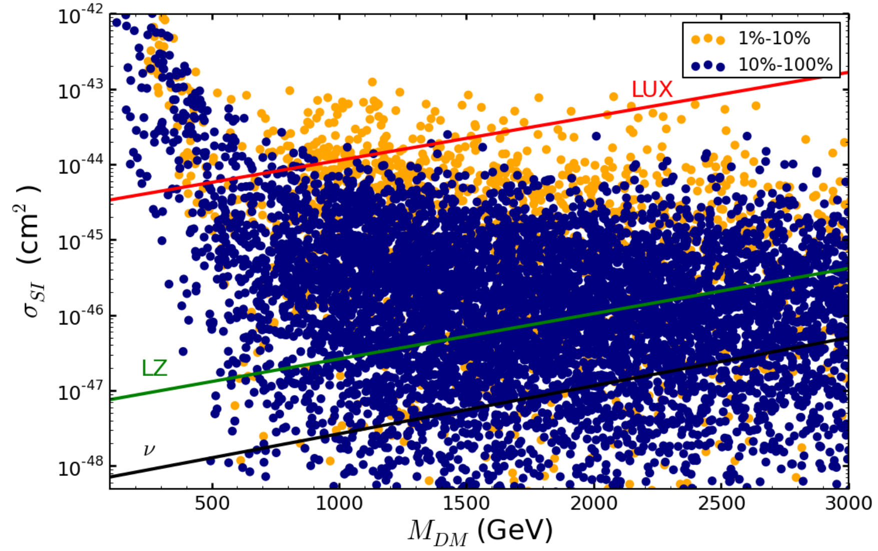


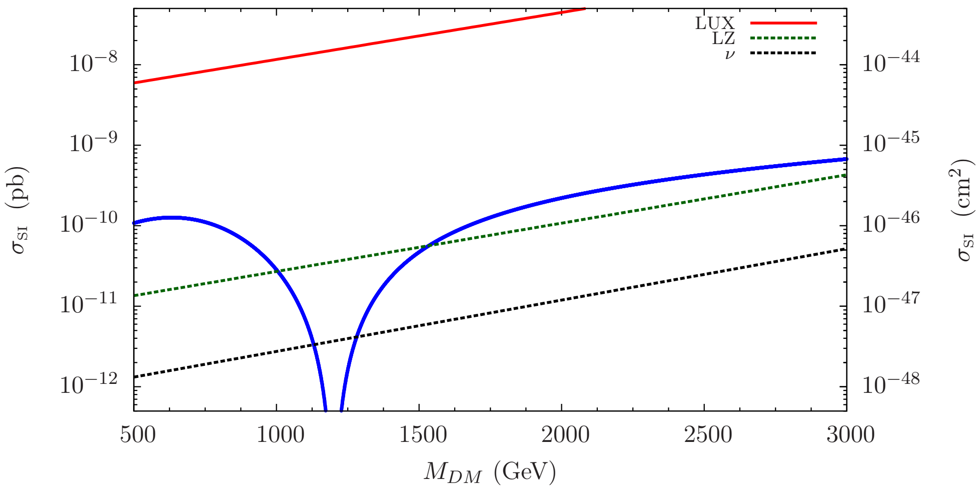
1. $\langle\sigma\rangle \approx \langle\phi\rangle \sim$ TeV. Here there is a strong mixing between the scalar states ϕ and σ. Some fine-tuning of ΔM_{N_i} is required for leptogenesis to work.
2. $\langle\sigma\rangle \gg \langle\phi\rangle \sim$ TeV. In this region σ overlaps maximally with h_2 and is the CW scalar. The radiative symmetry breaking is induced by $\lambda_\sigma \ll 1$ and we get $M_{h_2} \ll M_{h_3}$. Most points have $M_{\rm DM} > M_{h_2}$. Large values of $\langle\sigma\rangle$ require almost no fine-tuning in ΔM_{N_i} for leptogenesis to work.

Leptogenenis vs Dark Matter



Vector Dark Matter





Inflation in the Higgs portal

- Cosmological Inflation was proposed in the 80s to solve the flatness, isotropy, homogeneity, horizon and relic problems in cosmology.
- Confirmed by observations, including the recent Planck data, which favour a simple inflationary scenario with one slow rolling scalar field.
- Relevant energy scales are far higher than can be probed at colliders, the underlying particle physics implementation of inflation is still unknown.
- We will focus on the approach based on renormalisable QFT Lagrangians
- Include a non-minimal coupling of a scalar field to gravity, in addition to the usual Einstein-Hilbert term
- By taking the non-minimal coupling ξ to be (moderately) large $\sim 10^4$, a slow-roll potential for the scalar is generated and inflation takes place

Original approach based on non-minimal scalar-to-gravity coupling:

D. S. Salopek, J. R. Bond and J. M. Bardeen, Phys. Rev. D **40** (1989)
F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B **659** (2008) 703

Inflation in the Higgs portal

- Start with the scalar potential $V(H,\phi)$ of the SM-CW model
- Add a real scalar singlet $s(x)$ coupled in the portal to the Higgs and ϕ
- Couple the theory to gravity with the non-minimal coupling $(\xi_s/2)\, s^2 R$:

$$\mathcal{L}_J = \sqrt{-g_J}\left(-\frac{M^2 R}{2} - \xi_s \frac{s^2 R}{2} + \frac{1}{2} g_J^{\mu\nu}\partial_\mu s\, \partial_\nu s + g_J^{\mu\nu}(D_\mu H)^\dagger D_\nu H + \frac{1}{2} g_J^{\mu\nu}(D_\mu \phi)^\dagger D_\nu \phi \right.$$
$$\left. - \frac{\lambda_s}{4} s^4 - \frac{\lambda_{hs}}{2}|H|^2 s^2 - \frac{\lambda_{\phi s}}{4}|\phi|^2 s^2 - V(H,\phi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \text{Fermions} + \text{Yukawas}\right)$$

- $M \simeq 10^{18}$ GeV denotes the reduced Planck mass; it appears only in the Einstein-Hilbert term and does not couple directly to non-gravitational d.o.f's.

- $(\xi_s/2)\, s^2 R$ is the non-minimal coupling of the singlet $s(x)$ to gravity, R is the scalar curvature. For successful inflation ξ_s should be relatively large, $\xi_s \sim 10^4$.
- Hence we will treat ξ_s and $\sqrt{\xi_s}$ as large parameters $\gg 1$. In this sense, $s(x)$ is distinguished from the two other scalars, H and ϕ, which in our case have either vanishing or small loop-induced non-minimal gravitational couplings.

- Remove the non-minimal scalar-gravity interaction with a transformation to the Einstein frame: $g_{\mu\nu} \to \Omega^{-2} g_{\mu\nu}$, where $\Omega^2 := 1 + \frac{\xi_s s^2}{M^2}$.
- Now the kinetic term for $s(x)$ is no longer normalised canonically
- Perform a field redefinition $s(x) \to \sigma(x)$ so that it gives back the canonically normalised kinetic term:

$$\left(\frac{1}{\Omega^2} + \frac{6\xi_s s^2}{M^2\Omega^4}\right) \frac{g_E^{\mu\nu}\, \partial_\mu s\, \partial_\nu s}{2} = \frac{1}{2}\, g_E^{\mu\nu}\, \partial_\mu \sigma\, \partial_\nu \sigma\,.$$

- At low field values, $s \lesssim 10^{14}$ GeV, the redefinition is $s(x) \approx \sigma(x)$
- At higher values of s, the solution for s in terms of σ is exponential,

$$s(x) = \frac{M}{\sqrt{\xi_s}} \times \exp\left(\frac{\sigma(x)}{\sqrt{6}M}\right) , \quad \text{for } s \gg \frac{M}{\sqrt{\xi_s}}$$

- The Einstein frame potential for the canonically normalised singlet $\sigma(x)$ is now exponentially flat and well-suited for the slow-roll inflation:

$$V_E(s[\sigma]) = \frac{\lambda_s}{4}\frac{s^4(x)}{\Omega^4} = \frac{\lambda_s M^4}{4\,\xi_s^2}\left(1 - \exp\left[-\frac{2\sigma(x)}{\sqrt{6}M}\right]\right)^2 , \quad \text{for } s \gg \frac{M}{\xi_s}$$

- Everything follows from this $V(\sigma)$. The slow-roll inflation parameter is

$$\epsilon := \frac{M^2}{2}\left(\frac{V(\sigma)/d\sigma}{V(\sigma)}\right)^2 = \frac{4\,M^4}{3\,\xi_s^2\,s^2}$$

- Inflation starts at $s_0 \simeq 9.14\,M/\sqrt{\xi_s}$. The CMB normalisation condition at s_0 determines the value of the non-minimal singlet coupling to gravity

$$\xi_s \simeq 4.7 \times 10^4\,\sqrt{\lambda_s}$$

  Inflation ends when $\epsilon = 1$ which corresponds to $s_{\rm end} = (4/3)^{1/4} M/\sqrt{\xi_s}$.

- The spectral index and the tensor-to-scalar perturbation ratios in this model are the same as computed in the Bezrukov-Shaposhnikov Higgs-inflation. They are in agreement with the Planck measurements.

This is a one-field slow-roll inflation model. The singletσ is the inflaton; other scalars decouple during inflation, they are much heavier than the Hubble const.

- This realisation of inflation does not require inclusion of new physics d.o.f's at the the 'low' M/ξ_s and 'intermediate' scale $M/\sqrt{\xi_s}$.
- H and ϕ, are already canonically normalised and there are no non-renormalisable interactions involving sub-Planckian scales.

VVK 1308.6338

Conclusions: Higgs Portals to New Physics

Classical scale invariance: a powerful principle for BSM model building. No vastly different scales can co-exist in such a theory:

The BSM theory is a minimal extension of the SM which should address a multitude of sub-Planckian shortcomings of the SM without introducing scales higher than $\langle \phi \rangle$ which itself is not much higher the electroweak scale.

1. Link between CW scale and the Higgs scale (EWSB)
2. Link with the Dark Matter scale (DM)
3. Link with the Leptogenesis scale (BAU)

$$\text{DM} \leftrightarrow \text{BAU} \leftrightarrow \text{EWSB}$$

4. Higgs stability
5. Consistent implementation of the singlet scalar field slow-roll inflation.

2. SM multi-Higgs production at very high (FCC) energies

$$\mathcal{L}(h) \;=\; \frac{1}{2}\,(\partial h)^2 - \frac{\lambda}{4}\,\left(h^2 - v^2\right)^2 \;,$$

The classical equation for the spatially uniform field $h(t)$,

$$d_t^2 h \;=\; -\lambda\, h^3 + \lambda v^2\, h\,,$$

has a closed-form solution with correct initial conditions $h_{\rm cl} = v + z + \ldots$

$$h_{\rm cl}(t) \;=\; v\, \frac{1+\frac{z(t)}{2v}}{1-\frac{z(t)}{2v}}\,, \quad \text{where} \quad z(t) \;=\; z_0\, e^{iM_h t} \;=\; z_0\, e^{i\sqrt{2\lambda}\, v\, t}$$

$$h_{\rm cl}(t) \;=\; 2v \sum_{n=0}^{\infty} \left(\frac{z(t)}{2v}\right)^n d_n \;=\; v \,+\, 2v \sum_{n=1}^{\infty} \left(\frac{z(t)}{2v}\right)^n \,,$$

i.e. with $d_0 = 1/2$ and all $d_{n\geq 1} = 1$.

$$\mathcal{A}_{1\to n} \;=\; \left(\frac{\partial}{\partial z}\right)^n h_{\rm cl}\Big|_{z=0} \;=\; n!\,(2v)^{1-n}$$ **Factorial growth!!**

L. Brown 9209203

Factorial growth of large-n scalar amplitudes on mass threshold

Above the n-particle thresholds:
solution of the recursion relations

$$\varepsilon = \frac{1}{n\,M_h}\,E_n^{\rm kin} = \frac{1}{n}\,\frac{1}{2M_h^2}\,\sum_{i=1}^{n}\vec{p}_i^{\,2}$$

$$\mathcal{A}_n(p_1\dots p_n) \;=\; n!\,(2v)^{1-n}\,\left(1-\frac{7}{6}\,n\,\varepsilon\;-\;\frac{1}{6}\,\frac{n}{n-1}\,\varepsilon+\mathcal{O}(\varepsilon^2)\right).$$

An important observation is that by exponentiating the order-$n\varepsilon$ contribution, one obtains the expression for the amplitude which solves the original recursion relation to all orders in $(n\varepsilon)^m$ in the large-n non-relativistic limit,

$$\mathcal{A}_n(p_1\dots p_n) \;=\; n!\,(2v)^{1-n}\,\exp\left[-\frac{7}{6}\,n\,\varepsilon\right]\,,\quad n\to\infty\,,\quad \varepsilon\to 0\,,\quad n\varepsilon = \text{fixed}\,.$$

Simple corrections of order ε, with coefficients that are not-enhanced by n are expected, but the expression is correct to all orders $n\varepsilon$ in the double scaling large-n limit. The exponential factor can be absorbed into the z variable so that

$$\varphi(z) = \sum_{n=1}^{\infty} d_n\,\left(z\,e^{-\frac{7}{6}\,\varepsilon}\right)^n\,,$$

remains a solution to the classical equation and the original recursion relations.

Can now integrate over the phase-space

- VVK 1411.2925

Gluon fusion process

$$\mathcal{A}_{gg\to n\times h} = \sum_{\text{polygons}} \mathcal{A}^{\text{polygons}}_{gg\to k\times h^*} \sum_{n_1+\ldots+n_k=n} \prod_{i=1}^{k} \mathcal{A}_{h_i^*\to n_i\times h}$$

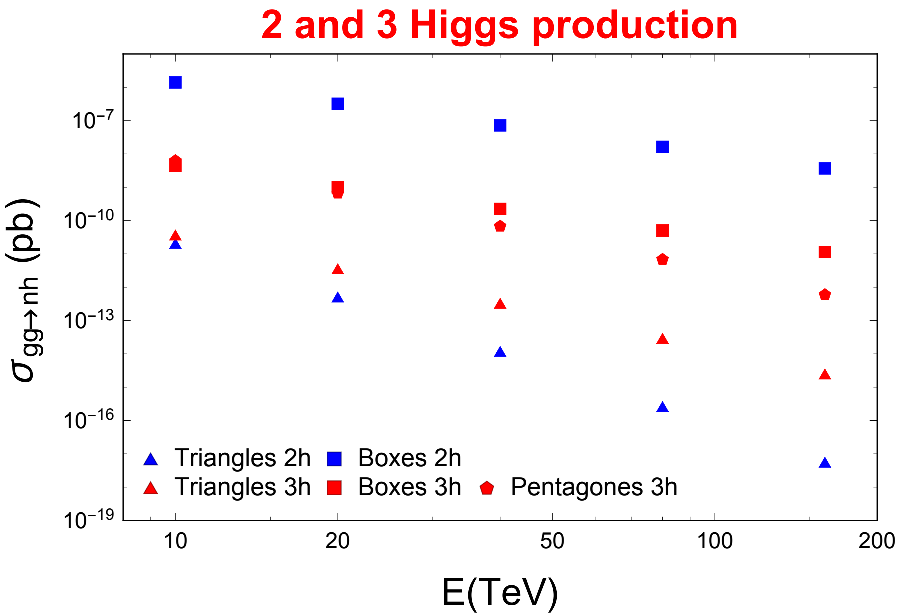
1-loop polygons:
triangles, boxes,
pentagons, hexagons, etc
Compute numerically
in the high-energy limit
MadGraph5_aMC@NLO

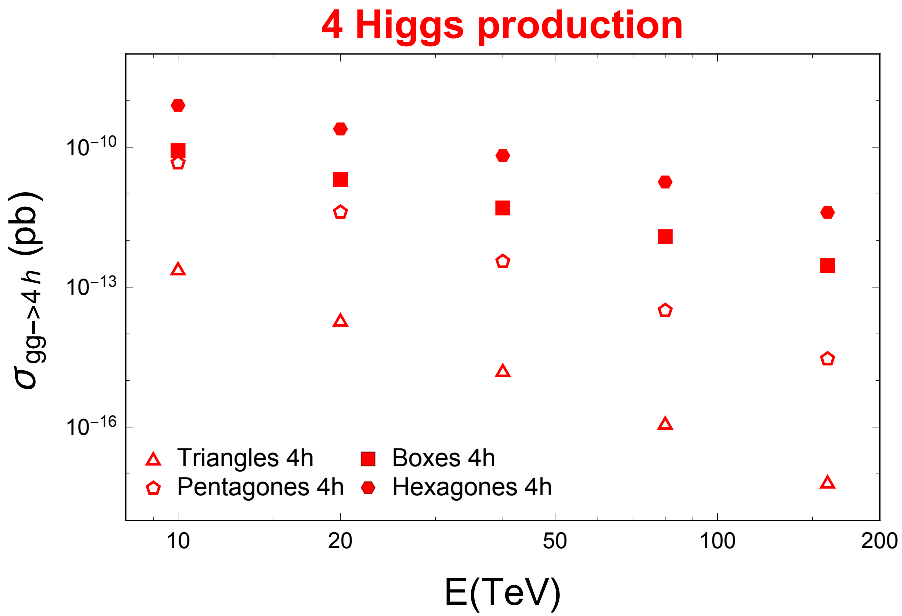
Tree-level 1->n multi-Higgs
processes. Compute at fixed
multiplicities n=5,6,7 at all energies
(i.e. arbitrary epsilon)
and scale to large n
using known n-dependence
of the holy grail function.

- VVK 1504.05023

- Degrande-VVK-Mattelaer 1605.06372

Polygon contributions:

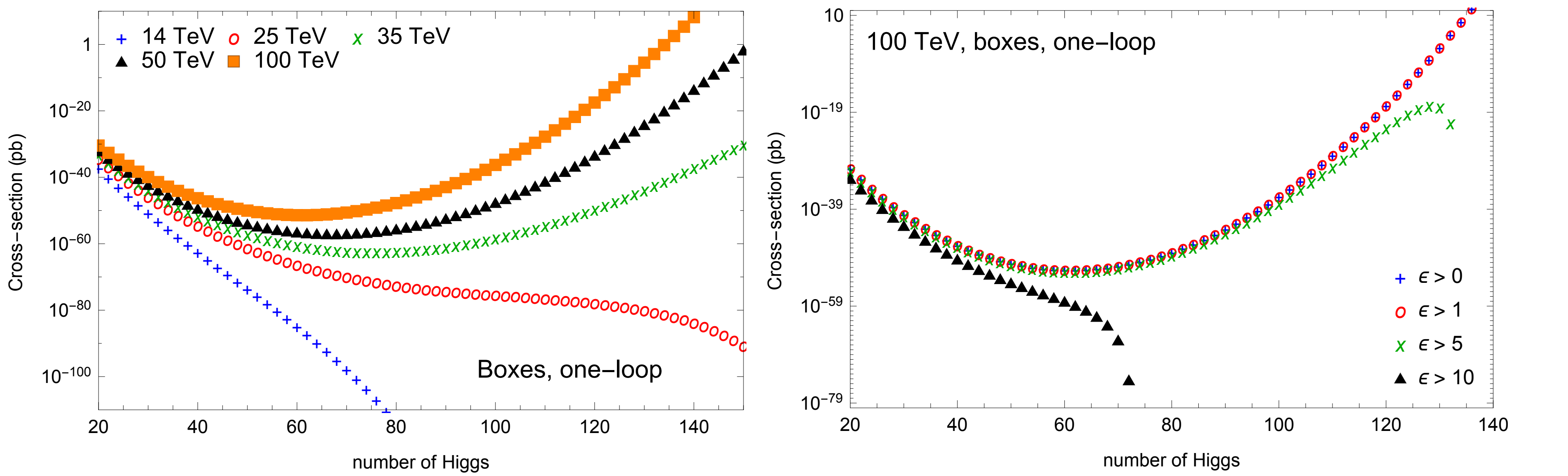




$s \gg m_t\,,\, M_h$ limit

	$\sigma_{gg\to hh}$	$\sigma_{gg\to hhh}$	$\sigma_{gg\to hhhh}$
Triangles	$y_t^2 \frac{m_t^2 M_h^2}{s^3} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \frac{M_h^2}{v^2}$	$y_t^2 \frac{m_t^2}{s^2} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \frac{M_h^4}{v^4}$	$y_t^2 \frac{m_t^2}{s^2} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \frac{M_h^6}{v^6}$
Boxes	$y_t^4 \frac{1}{s}$	$y_t^4 \frac{1}{s} \frac{M_h^2}{v^2}$	$y_t^4 \frac{1}{s} \frac{M_h^4}{v^4}$
Pentagons	–	$y_t^6 \frac{m_t^2}{s^2} \log^4\left(\frac{m_t}{\sqrt{s}}\right)$	$y_t^6 \frac{m_t^2}{s^2} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \frac{M_h^2}{v^2}$
Hexagons	–	–	$y_t^8 \frac{1}{s}$

Combine with tree-level multi-H branchings & convolute with Parton Distribution Functions



Left panel: Cross-sections for multi-Higgs production at proton colliders including the PDFs for different energies of the proton-proton collisions plotted as the function of the Higgs multiplicity. Only the contributions from the boxes are included. The right panel illustrates the dependence on average energy variable ε by applying a sequence of cuts on ε at 100 TeV.

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Summary: multi-Higgs production

- At (not too high) high energies perturbative Standard Model exhibits a formal breakdown. Perturbative unitarity is broken. OPTIONS:

- At high energies (multiplicities) the Standard Model is fundamentally non-perturbative (?)

- The theory classicalizes: the ultra-high multiplicity processes will completely dominate everything else (?)

- New physics beyond the Standard Model has to set in before the cross-sections become large, i.e. as early as at ~50 TeV (?)

- New theoretical approaches & computational techniques have to be developed to determine the relevant energy scale