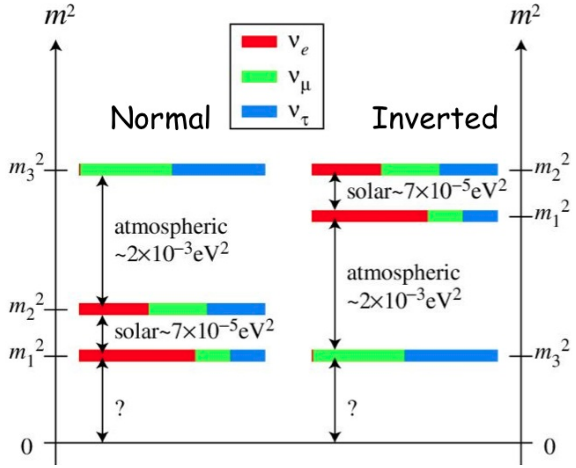
Status of neutrino mass models

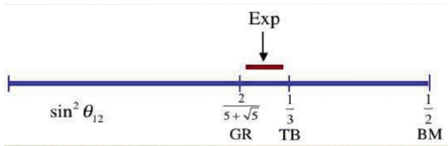
G. Ross, Invisibles 13, Lumley Castle,July 2013

Neutrino mixing

Symmetry or anarchy?

$$U_\Theta = \begin{pmatrix} \cos\Theta & \sin\Theta & 0 \\ \frac{-\sin\Theta}{\sqrt{2}} & \frac{\cos\Theta}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin\Theta}{\sqrt{2}} & \frac{-\cos\Theta}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} P$$



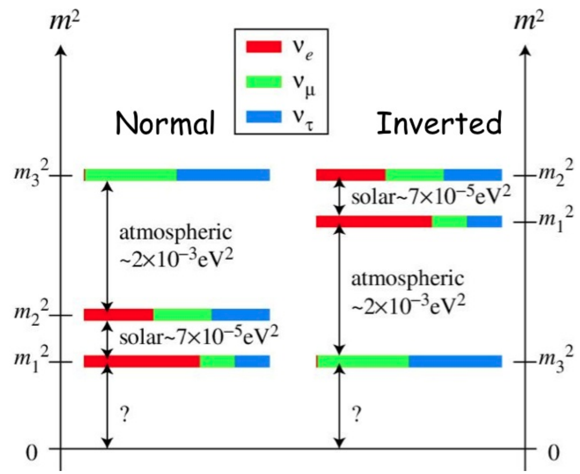


Tri-bi-maximal mixing: $\tan\Theta = 1/\sqrt{2}$ — Harrison, Perkins, Scott

Golden ratio mixing: $\tan\Theta = 2/(1+\sqrt{5}) \equiv 1/\phi$ — Datta et al; Kajyama et al

Bi^2-maximal mixing: $\tan\Theta = 1$ — Barger et al; Fukugita et al; Davidson, King

Neutrino mixing



$$U_\Theta = \begin{pmatrix} \cos\Theta & \sin\Theta & 0 \\ \frac{-\sin\Theta}{\sqrt{2}} & \frac{\cos\Theta}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin\Theta}{\sqrt{2}} & \frac{-\cos\Theta}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

Parameter	Best fit	1σ range	3σ range
$\sin^2\theta_{12}/10^{-1}$	3.0	2.87 – 3.13	2.7 – 3.4
$\sin^2\theta_{13}/10^{-2}$	2.3	2.07 – 2.53	1.6 – 3.0
$\sin^2\theta_{23}/10^{-1}$ (NH)	4.1	4.075 – 4.137	3.4 – 6.7
$\sin^2\theta_{23}/10^{-1}$ (IH)	5.9	5.68 – 6.11	3.35 – 6.63
δ/π	1.67	0.9 – 2.03	0–2

Gonzalez-Garcia, Maltoni, Salvado, Schwetz
see also:
Forero, Tortola, Valle
Fogli, Lisi, Marrone, Montanino, Palazzo, Rotuno

Symmetries of the mass matrices

$$M_l = Diag(m_e, m_\mu, m_\tau)$$

$$M_l = h^T M_l h^* \quad e.g.\, Z_3, h = Diag(1, e^{2i\pi/3}, e^{4i\pi/3})$$

$$M_\nu = U_{PMNS} Diag(m_\perp, m_\odot, m_@) U_{PMNS}^T$$

$Z_2 \times Z_2$ Klein symmetry $M_\nu = S^T M_\nu S$

$$S = U_{PMNS}^* Diag(\pm 1, \pm 1, \pm 1) U_{PMNS}, \quad \det S = 1$$

Symmetries of the mass matrices

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$$S = U_{PMNS}^* \, Diag(\pm 1, \pm 1, \pm 1) U_{PMNS}, \quad \det S = 1$$

Choice of $Z_2 \times Z_2$ symmetry $\Rightarrow$ mass matrix structure

e.g.

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \mu \leftrightarrow \tau \quad \begin{cases} \nu_@ = (\nu_\mu - \nu_\tau)/\sqrt{2} \quad (-,-) \\ \theta_{13} = 0 \end{cases}$$

Bi-maximal

$$S_{TBM} = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \quad \begin{cases} \nu_\odot = (\nu_e + \nu_\mu + \nu_\tau)/\sqrt{3} \quad (+,+) \\ \nu_\perp = \left(2\nu_e - (\nu_\mu + \nu_\tau)/\sqrt{2}\right) \quad (+,-) \end{cases}$$

Tri-maximal

Origin of symmetries

Direct:

$$G_{family} \xrightarrow{\langle \phi_\nu \rangle} Z_2 \times Z_2$$

$$\xrightarrow{\langle \phi_l \rangle} Z_3^l$$

$$e.g. \quad U, S_{TBM}, Z_3^l \subset S_4 \cong (Z_2 \times Z_2) \rtimes S_3 \subset SU(3), \quad U \subset A_4 \; (S_{TBM} \text{ "accidental"})$$

Emergent: $Z_2 \times Z_2 \not\subset G_{family} \quad e.g. \quad G_{family} = \Delta(27) \subset SU(3)$

$$L_{eff}^{\nu} = a\,\psi_i \phi_{123}^i \psi_j \phi_{123}^j + b\,\psi_i \phi_{23}^i \psi_j \phi_{23}^j$$

$$\phi_{123} \propto (1,1,1), \quad \phi_2 \propto (1,0,0), \quad \phi_3 \propto (0,0,1)$$

{ Symmetric under T, S_{TBM}

Vacuum alignment

Symmetries $\Longrightarrow$ Tr-Bi-Maximal, Golden Ratio, ...

$$U_\Theta = \begin{pmatrix} \cos\Theta & \sin\Theta & 0 \\ \frac{-\sin\Theta}{\sqrt{2}} & \frac{\cos\Theta}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin\Theta}{\sqrt{2}} & \frac{-\cos\Theta}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

$\theta_{13} \neq 0$???

$\Longrightarrow$

Anarchy?

Symmetry breaking perturbations ?

New symmetries ?

Symmetry breaking perturbations

$$L_m^{\nu} = m_{@}(\nu_a + \varepsilon_{ab}\nu_b + ..)^2 + m_{\odot}(\nu_b - \varepsilon_{ab}\nu_a + ..)^2 + m_{?}(\nu_c + ..)^2$$

TBM
$$\begin{cases} \nu_a = \frac{1}{\sqrt{2}}\left(\nu_\mu - \nu_\tau\right) \\ \nu_b = \frac{1}{\sqrt{3}}\left(\nu_e + \nu_\mu + \nu_\tau\right) \\ \nu_c = \frac{1}{\sqrt{6}}\left(2\nu_e - \nu_\mu - \nu_\tau\right) \end{cases}$$

GR
$$\begin{cases} \nu_a = \frac{1}{\sqrt{2}}\left(\nu_\mu - \nu_\tau\right) \\ \nu_b = s_\theta \nu_e + c_\theta(\nu_\mu + \nu_\tau)/\sqrt{2} \\ \nu_c = c_\theta \nu_e - s_\theta(\nu_\mu + \nu_\tau)/\sqrt{2} \end{cases}$$

$t_\theta = 1/\phi$

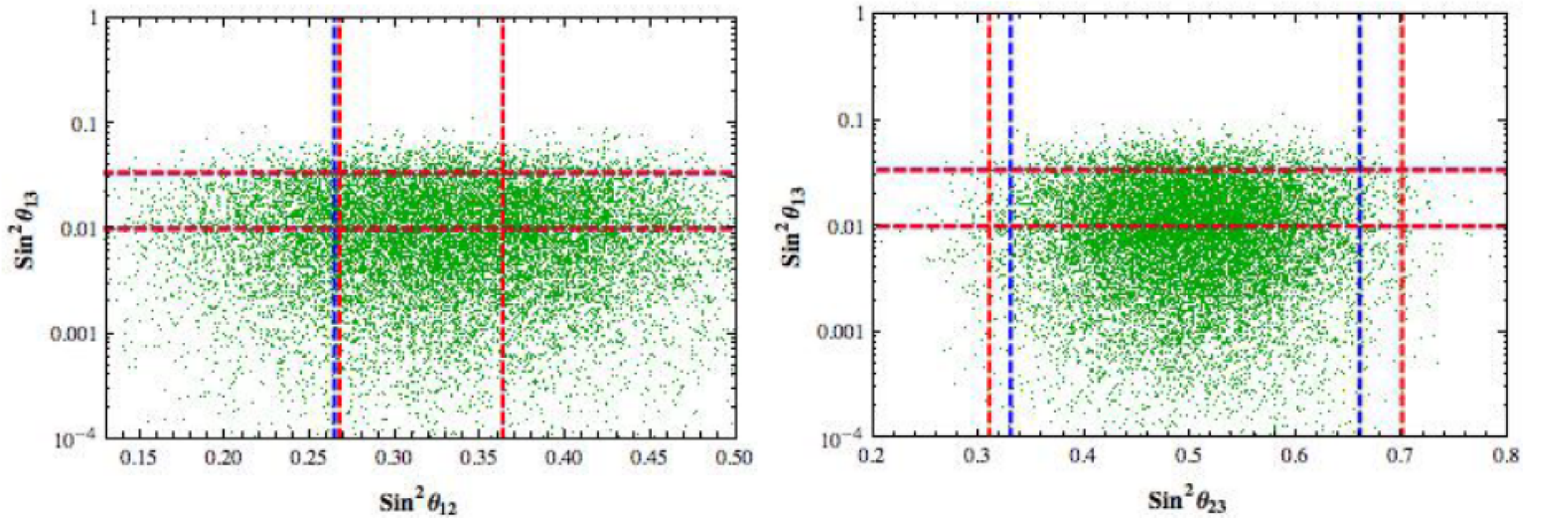
$\theta_{13} \neq 0 \quad \Longrightarrow$ must break $U \quad \Rightarrow \quad \nu_{a,b}$ *and / or* $\nu_{a,c}$ mixing

● General mixing (TBM case):

$$\sin^2\theta_{23} = \frac{1}{2} + \mathcal{R}e(c^e_{23})\,\xi + \frac{1}{\sqrt{3}}\left(\mathcal{R}e(c^\nu_{13}) - \sqrt{2}\,\mathcal{R}e(c^\nu_{23})\right)\,\xi$$

$$\sin^2\theta_{12} = \frac{1}{3} - \frac{2}{3}\mathcal{R}e(c^e_{12} + c^e_{13})\,\xi + \frac{2\sqrt{2}}{3}\,\mathcal{R}e(c^\nu_{12})\,\xi$$

$$\sin\theta_{13} = \frac{1}{6}\left|3\sqrt{2}\,(c^e_{12} - c^e_{13}) + 2\sqrt{3}\left(\sqrt{2}\,c^\nu_{13} + c^\nu_{23}\right)\right|\,\xi\,.$$



c's random

Altarelli, Feruglio, Merlot

- Restricted (bilinear) mixing (TBM case):

$$L_m^{\nu} = m_{@}(\nu_a + \varepsilon_{ab}\nu_b + ..)^2 + m_{\odot}(\nu_b - \varepsilon_{ab}\nu_a + ..)^2 + m_{?}(\nu_c + ..)^2$$

TBM
$$\begin{cases} \nu_a = \frac{1}{\sqrt{2}}\left(\nu_\mu - \nu_\tau\right) \\ \nu_b = \frac{1}{\sqrt{3}}\left(\nu_e + \nu_\mu + \nu_\tau\right) \\ \nu_c = \frac{1}{\sqrt{6}}\left(2\nu_e - \nu_\mu - \nu_\tau\right) \end{cases}$$

$$U = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{c}{\sqrt{3}} & \frac{s}{\sqrt{3}}e^{-i\delta} \\ -\frac{1}{\sqrt{6}} & \frac{c}{\sqrt{3}} - \frac{s}{\sqrt{2}}e^{i\delta} & \frac{c}{\sqrt{2}} + \frac{s}{\sqrt{3}}e^{-i\delta} \\ -\frac{1}{\sqrt{6}} & \frac{c}{\sqrt{3}} + \frac{s}{\sqrt{2}}e^{i\delta} & -\frac{c}{\sqrt{2}} + \frac{s}{\sqrt{3}}e^{-i\delta} \end{pmatrix}$$

$\theta_{13} \neq 0 \implies$ must break $U \Rightarrow \nu_{a,b}$ *or* $\nu_{a,c}$ mixing

$\delta\theta_{12}$ small $\implies$ residual Z_2 symmetry $\equiv$ bilinear mixing

$S_{TBM} \Rightarrow \nu_{a,b}, \quad S_{TBM}T \Rightarrow \nu_{a,c}$

$S_{GR} \Rightarrow \nu_{a,b}, \quad S_{GR}T \Rightarrow \nu_{a,c}$

Hall, GGR
Luhn

Model	ν perturbation	s^l_{12}	δ/π (1σ)	δ/π (3σ)	s^2_{12}	s^2_{23}
TBM	ν_{ab} mixing (NH)	0	$\pm(0.36-0.47)$	$\pm(0.05-1)$	0.33	-
	(IH)	0	$\pm(0.51$–$0.67)$	$\pm(0.08-1)$	0.33	-
	(NH)	$\sqrt{\frac{m_e}{m_\mu}}$	$\pm(0.58-1)$	$0-2$	0.29–0.38	-
	(IH)	$\sqrt{\frac{m_e}{m_\mu}}$	$\pm(0-0.51)$	$0-2$	0.29–0.38	-
TBM						-
						-
						-
						-
TBM						0.45–0.56
GR						-
						-
	(NH)	$\sqrt{\frac{m_e}{m_\mu}}$	$\pm(1.65$–$2.07)$	$0-2$	0.25–0.3	-
	(IH)		$\pm(1$–$1.65)$	$0-2$	0.25–0.3	-
GR	ν_{ac} mixing (NH)					
GR	(IH)					
	(NH)					
	(IH)					
GR	None					.56
BM						.487
	Fit to data (NH) [6]					.414
	(IH) [6]					.67

$$s^{\nu}_{23} = \left| \frac{1}{\sqrt{2}} + e^{-i\delta} \quad s_{13} \right|$$

$$L^{\nu}_{m} = m_{@}\left((\nu_\mu - \nu_\tau)/\sqrt{2} + s_{13}e^{-i\delta}(\nu_e + \nu_\mu + \nu_\tau)\right)^2 + m_{\odot}\left((\nu_e + \nu_\mu + \nu_\tau)/\sqrt{3} - \sqrt{\frac{3}{2}} s_{13} e^{i\delta}(\nu_\mu - \nu_\tau)\right)^2$$

θ_{13}

$$U = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{c}{\sqrt{3}} & \frac{s}{\sqrt{3}} e^{-i\delta} \\ -\frac{1}{\sqrt{6}} & \frac{c}{\sqrt{3}} - \frac{s}{\sqrt{2}} e^{i\delta} & \frac{c}{\sqrt{2}} + \frac{s}{\sqrt{3}} e^{-i\delta} \\ -\frac{1}{\sqrt{6}} & \frac{c}{\sqrt{3}} + \frac{s}{\sqrt{2}} e^{i\delta} & -\frac{c}{\sqrt{2}} + \frac{s}{\sqrt{3}} e^{-i\delta} \end{pmatrix}$$

Model	ν perturbation	s^l_{12}	δ/π (1σ)	δ/π (3σ)	s^2_{12}	s^2_{23}
TBM	ν_{ab} mixing (NH)	0	$\pm(0.36-0.47)$	$\pm(0.05-1)$	0.33	-
	(IH)	0	$\pm(0.51–0.67)$	$\pm(0.08-1)$	0.33	-
	(NH)	$\sqrt{\frac{m_e}{m_\mu}}$	$\pm(0.58-1)$	$0-2$	0.29–0.38	-
	(IH)	$\sqrt{\frac{m_e}{m_\mu}}$	$\pm(0-0.51)$	$0-2$	0.29–0.38	-
TBM	ν_{ac} mixing (NH)	0	$\pm$(0-0.38)	$0-2$	0.33	-
	(IH)	0	$\pm$(0.51-1)	$0-2$	0.33	-
	(NH)	$\sqrt{\frac{m_e}{m_\mu}}$	$\pm$(0-0.4)	$0-2$	0.29–0.38	-
	(IH)	$\sqrt{\frac{m_e}{m_\mu}}$	$\pm$(0.5-1)	$0-2$	0.29–0.38	-
TBM	None	$\sqrt{2}\,s_{13}$	$0.7-1.3$	$0.5-1.5$	-	0.45–0.56
GR	ν_{ab} mixing (NH)	0	$\pm(0.15-0.33)$	$\pm$(-0.03–0.79)	0.276	-
	(IH)	0	$\pm(0.4-0.7)$	$\pm$(0.06–1.22)	0.276	-
	(NH)	$\sqrt{\frac{m_e}{m_\mu}}$	$\pm$(1.65–2.07)	$0-2$	0.25–0.3	-
	(IH)	$\sqrt{\frac{m_e}{m_\mu}}$	$\pm$(1–1.65)	$0-2$	0.25–0.3	-
GR	ν_{ac} mixing (NH)	0	$-0.39-0.39$	$0-2$	0.276	-
GR	(IH)	0	$-0.39-0.39$	$0-2$	0.276	-
	(NH)	$\sqrt{\frac{m_e}{m_\mu}}$	$\pm$(0–0.36)	$0-2$	0.25–0.3	-
	(IH)	$\sqrt{\frac{m_e}{m_\mu}}$	$\pm$(0.52–1)	$0-2$	0.25–0.3	-
GR	None	$\sqrt{2}\,s_{13}$	$\pm(0.35-0.4)$	$\pm(0.32-0.45)$	-	0.45-0.56
BM		-	-	0.75-1.25	-	0.485–0.487
	Fit to data (NH) [6]		0.9–2.03	0–2	0.287–0.313 (1σ)	0.408–0.414
	(IH) [6]		0.9–2.03	0–2	0.287–0.313 (1σ)	0.34–0.67

θ_{13} charged lepton or neutrino origin?

$$s_{23} \approx s_{23}^{\nu} - \theta_{23}^{l} c_{23}^{\nu} e^{i\delta_{23}}$$

$$s_{12} \approx s_{12}^{\nu} - \theta_{12}^{l} c_{23}^{\nu} c_{12}^{\nu} e^{i\delta_{12}}$$

$$\theta_{13} e^{-i\delta_{13}} = \theta_{13}^{\nu} e^{-i\delta_{13}^{\nu}} - \theta_{12}^{l} s_{23}^{\nu} e^{-i(\delta_{23}^{\nu}+\delta_{12}^{e})}$$

$\sqrt{\frac{m_e}{m_\mu}} \simeq \frac{\theta_C}{3}$ (1,1) texture zero †

Cabibbo haze:

$$\theta_{13}^{\nu} = 0, \quad \theta_{13} \approx \theta_{12}^{l}\, s_{23}^{\nu} \approx \frac{\theta_{12}^{l}}{\sqrt{2}}$$

$M^{q,l}$: small mixing ...dominated by θ_C

M^{ν} : tri-bi-maximal

$$L_m^{\nu} = m_{@}\left[\tfrac{1}{\sqrt{2}}(\nu_\mu - \nu_\tau)\right]^2 + m_{\odot}\left[\tfrac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)\right]^2$$

Datta, Everett, Ramond

Marzocca, Petkov, Romanino, Spinrath

Antusch et al

If $\theta_{12}^{l} = \theta_C$ (GUT?), $\theta_{13} = 9^0$!

...but $\theta_{12}^{l} = \theta_C$ inconsistent with other plausible GUT relations †

Symmetry fights back - I

Klein symmetry: $Z_2 \times Z_2' : \quad S_{TBM,GR},\ (U \times CP)_{Diag}$

Harrison, Scott
Feruglio, Hagedorn, Ziegler
Ding, King, Luhn, Stuart
Talbert, GGR

	S,U	$S,(U\times CP)_{Diag}$	$SU,(U\times CP)_{Diag}$
ν_a	$(+,-)$	$(+,\mp)$	$(-,\mp)$
ν_b	$(+,+)$	$(+,\pm)$	$(+,\pm)$
ν_c	$(-,+)$	$(-,\pm)$	$(-,\pm)$
$Mixing$	$-$	$\nu_a, \pm i\nu_b$	$\nu_a, \pm i\nu_c$

$$U = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{c}{\sqrt{3}} & \frac{s}{\sqrt{3}} e^{\pm i\pi/2} \\ -\frac{1}{\sqrt{6}} & \frac{c}{\sqrt{3}} - \frac{s}{\sqrt{2}} e^{\mp i\pi/2} & \frac{c}{\sqrt{2}} + \frac{s}{\sqrt{3}} e^{\pm i\pi/2} \\ -\frac{1}{\sqrt{6}} & \frac{c}{\sqrt{3}} + \frac{s}{\sqrt{2}} e^{\mp i\pi/2} & -\frac{c}{\sqrt{2}} + \frac{s}{\sqrt{3}} e^{\pm i\pi/2} \end{pmatrix}$$

Symmetry fights back - I

Klein symmetry: $Z_2 \times Z_2' : \quad S_{TBM,GR}, \ (U \times CP)_{Diag}$

Feruglio, Hagedorn, Ziegler
Ding, King, Luhn, Stuart

Generalised CP $\quad \varphi(x) \xrightarrow{H_{CP}} X_{\mathbf{r}}\, \varphi^*(x') \qquad (S_4 \rtimes H_{CP})$

I.

$$\sin\alpha_{21} = \sin\alpha_{31} = \sin\delta_{CP} = 0 \ ,$$

$$\sin^2\theta_{13} = \tfrac{2}{3}\sin^2\theta, \quad \sin^2\theta_{12} = \tfrac{1}{2+\cos 2\theta}, \quad \sin^2\theta_{23} = \tfrac{1}{2}\left[1 + \tfrac{\sqrt{3}\sin 2\theta}{2+\cos 2\theta}\right]$$

II.

$$\sin\alpha_{21} = 0, \quad \sin\alpha_{31} = 0, \quad |\sin\delta_{CP}| = 1,$$

$$\sin^2\theta_{13} = \tfrac{2}{3}\sin^2\theta, \quad \sin^2\theta_{12} = \tfrac{1}{2+\cos 2\theta}, \quad \sin^2\theta_{23} = \tfrac{1}{2}$$

III.

$$\sin^2\theta_{13} = \frac{1}{6}\left(2 - \sqrt{3}\cos 2\theta\right), \qquad \sin^2\theta_{12} = \frac{2}{4+\sqrt{3}\ \cos 2\theta} \ ,$$

$$\sin^2\theta_{23}^{1st} = \frac{2}{4+\sqrt{3}\ \cos 2\theta}, \quad \text{or} \quad \sin^2\theta_{23}^{2nd} = 1 - \frac{2}{4+\sqrt{3}\ \cos 2\theta} \ .$$

$$|\sin\alpha_{21}| = \left|\frac{\sqrt{3} + 2\cos 2\theta}{2+\sqrt{3}\cos 2\theta}\right|, \qquad |\sin\alpha_{31}'| = \left|\frac{4\sqrt{3}\ \sin 2\theta}{5 - 3\cos 4\theta}\right|,$$

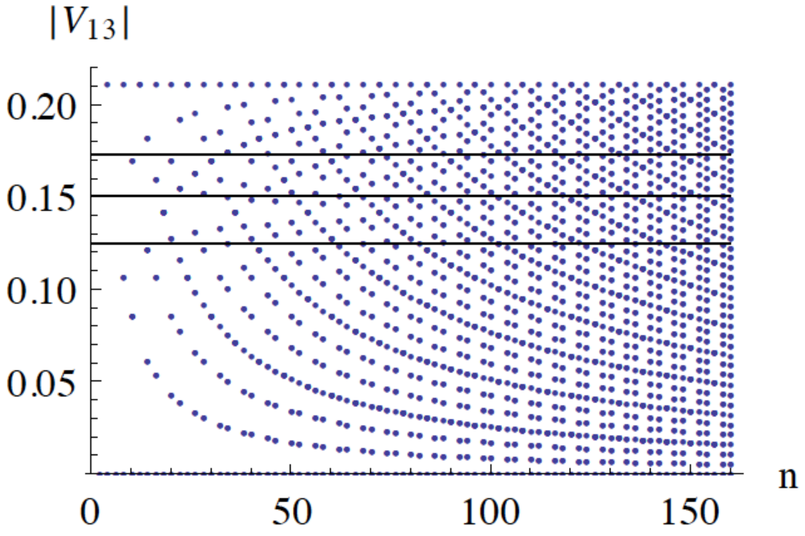
$$|\sin\delta_{CP}| = \left|\frac{\sqrt{4 - 2\sqrt{3}\ \cos 2\theta}\,(4+\sqrt{3}\cos 2\theta)\sin 2\theta}{5 - 3\cos 4\theta}\right| \ .$$

Symmetry fights back - II

Direct: $Z_2 \times Z_2' \times Z_3 \subset G_{family}$

$G_{family} = \Delta(600):\quad \sin^2\theta_{13} = 0.028, \quad \sin^2\theta_{23} = 0.38$ Lam

$G_{family} = \Delta(6n^2) \cong (Z_n \times Z_n) \rtimes S_3, \quad n$ even



Trimaximal mixing, $\theta_{23} = 45° \mp \theta_{13}/\sqrt{2}, \quad \delta = 0, \pi$ King, Neder, Stuart

Symmetry fights back - III

Emergent:

$$\langle \phi_{\rm atm} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} v_{\rm atm}, \qquad \langle \phi_{\rm sol} \rangle = \frac{1}{\sqrt{21}} \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} v_{\rm sol}$$

King

$$\mathcal{W}^{\nu}_{\rm Yuk} \sim \frac{1}{\Lambda} H_2 (\phi_{\rm atm} \cdot L) N_{\rm atm} + \frac{1}{\Lambda} H_2 (\phi_{\rm sol} \cdot L) N_{\rm sol}$$

$$\mathcal{W}_R \sim \xi_{\rm atm} N^2_{\rm atm} + \xi_{\rm sol} N^2_{\rm sol}$$

$2\,\nu_R$

(1,1) Texture Zero

$$m^{\nu} \sim \frac{v_2^2}{\Lambda^2} \left(\frac{\langle \phi_{\rm atm} \rangle \langle \phi_{\rm atm} \rangle^T}{\langle \xi_{\rm atm} \rangle} + \frac{\langle \phi_{\rm sol} \rangle \langle \phi_{\rm sol} \rangle^T}{\langle \xi_{\rm sol} \rangle} \right)$$

1 parameter fit (m_2 / m_3)

$$\theta_{12} \approx 34^\circ, \theta_{23} \approx 41^\circ, \theta_{13} \approx 9.5^\circ, \delta \approx 106^\circ, \eta = \frac{2\pi}{5}$$

Vacuum alignment (A_4 model)

$$\langle \phi_{\rm atm} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} v_{\rm atm}, \qquad \langle \phi_{\rm sol} \rangle = \frac{1}{\sqrt{21}} \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} v_{\rm sol} \qquad , \eta = \frac{2\pi}{5}$$

King

$$W_{A_4}^{\text{flavon},R} = gP\left(\frac{\xi_{\rm atm}^5}{\Lambda^3} - M^2\right) + g'P'\left(\frac{\xi_{\rm sol}^5}{\Lambda'^3} - M'^2\right)$$

$$W_{A_4}^{\text{flavon},\ell} \sim A_e \varphi_e \varphi_e + A_\mu \varphi_\mu \varphi_\mu + A_\tau \varphi_\tau \varphi_\tau + O_{e\mu} \varphi_e \varphi_\mu + O_{e\tau} \varphi_e \varphi_\tau + O_{\mu\tau} \varphi_\mu \varphi_\tau$$

$$\begin{aligned} W_{A_4}^{\text{flavon},\nu} = \; & A_{\nu_2}(g_1 \varphi_{\nu_2}\varphi_{\nu_2} + g_2 \varphi_{\nu_2}\xi_{\nu_2}) + A_{\nu_2'}(g_1' \varphi_{\nu_2'}\varphi_{\nu_2'} + g_2' \varphi_{\nu_2'}\xi_{\nu_2'}) \\ & + O_{e\nu_3} g_3 \varphi_e \varphi_{\nu_3} + O_{\nu_2\nu_3} g_4 \varphi_{\nu_2}\varphi_{\nu_3} + O_{\nu_1\nu_2} g_5 \varphi_{\nu_1}\varphi_{\nu_2} + O_{\nu_1\nu_3} g_6 \varphi_{\nu_1}\varphi_{\nu_3} \\ & + O_{e\nu_3'} g_3' \varphi_e \varphi_{\nu_3'} + O_{\nu_2'\nu_3'} g_4' \varphi_{\nu_2'}\varphi_{\nu_3'} + O_{\nu_1'\nu_2'} g_5' \varphi_{\nu_1'}\varphi_{\nu_2'} + O_{\nu_1'\nu_3'} g_6' \varphi_{\nu_1'}\varphi_{\nu_3'} \\ & + O_{\mu\nu_5} g_7 \varphi_\mu \varphi_{\nu_5} + O_{\nu_1'\nu_5} g_8 \varphi_{\nu_1'}\varphi_{\nu_5} + O_{\mu\nu_6} g_9 \varphi_\mu \varphi_{\nu_6} + O_{\nu_5\nu_6} g_{10} \varphi_{\nu_5}\varphi_{\nu_6} \end{aligned}$$

$$\Delta W_{A_4}^{\text{flavon},\ell} \sim \sum_{l=e,\mu,\tau} \frac{P}{\Lambda}\left((\varphi_l \cdot \varphi_l)\rho_l - M^3\right) + P\left(\frac{\rho_l^3}{\Lambda} - M^2\right),$$

$$\Delta W_{A_4}^{\text{flavon},\nu} \sim \sum^{6} \frac{P}{\Lambda}\left(((\varphi_{\nu_i}) \cdot \varphi_{\nu_i})\rho_{\nu_i} - M^3\right) + P\left(\frac{\rho_{\nu_i}^5}{\Lambda^3} - M^2\right)$$

Symmetry fights back - IV

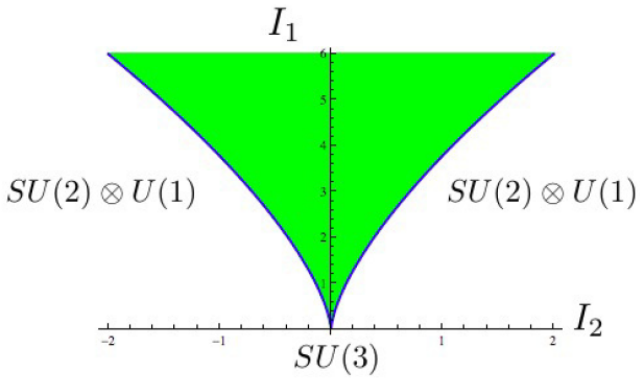
Mixing and masses from an extremum principle

$$-\mathcal{L}_Y = \bar{q}_L Y_D H D_R + \bar{q}_L Y_U \tilde{H} U_R + \bar{\ell}_L Y_E H E_R + \bar{\ell}_L Y_\nu \tilde{H} N + \text{h.c.} + \frac{M}{2} N \gamma_0 N$$

$$G^{local}_{family} = [SU(3)]^5 \otimes O(3) \xrightarrow{\langle Y_i \rangle} ?$$

Alonso, Gavela, Isidori, Maiani
Alonso, Gavela, Hernandez, Merlo, Rigolin
Grinstein, Redi, Villadoro

Y_i dynamical variables- "Natural extrema"



e.g. x SU(3) adjoint:

$$V(I_1, I_2): \quad I_1 = Tr(x^2), I_2 = Det(x)$$

"Natural extrema"

Quarks:

$$SU(3)_q \otimes SU(3)_U \otimes SU(3)_D \longrightarrow SU(2)_q \otimes SU(2)_U \otimes SU(2)_D \otimes U(1)$$

$$m_q = \text{diag}(0,0,c) \qquad U_{\text{CKM}} = 1$$

Leptons: $SU(3)_l \otimes O(3) \rightarrow SU(2)_l \otimes U(1)$

$$\hat{m}_\nu = \frac{v^2}{M}\,\text{diag}(y_1^2, y_2y_3, y_2y_3)\ ,$$

$$U_{\text{PMNS}}^{(0)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

Difference due to Majorana masses restricting redefinition of neutrinos

Add perturbations:

$$m_\nu = \frac{v^2 y}{M}\begin{pmatrix} 1+\delta+\sigma & \epsilon+\eta & \epsilon-\eta \\ \epsilon+\eta & \delta+\kappa & 1 \\ \epsilon-\eta & 1 & \delta-\kappa \end{pmatrix}$$

Quasi degenerate neutrinos

Normal or inverted hierarchy

2 large mixing angles

Θ_{13} generically small

Epilogue

- Masses -Froggatt-Nielsen

  -Xtra-dimension/Composite

Epilogue

- Masses -Froggatt-Nielsen $\left(\langle\theta\rangle/\Lambda\right)^n$

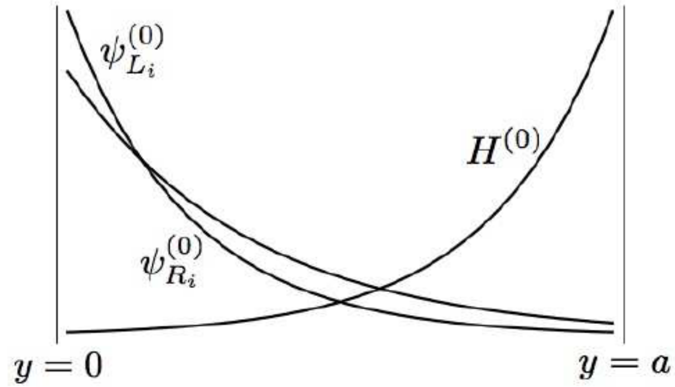
  -Xtra-dimension/Composite

5D mass parameters

$$Y_{4D,ij} \sim \int_0^a dy\, Y_{5D,ij}(y)\, e^{-(M_{L_i}+M_{R_j})y+M_H(y-a)}$$

$(M_{L_i}+M_{R_j}>M_H)$ $\swarrow$ $\searrow$ $(M_{L_i}+M_{R_j}<M_H)$

$$\sim \tilde{Y}_{0,ij}\, e^{-M_H a} \ll \sim \tilde{Y}_{a,ij}\, e^{-(M_{L_i}+M_{R_j})a}$$



Flavour blind
..Dirac neutrinos
θ_{13} large

Flavour hierarchy
.. q, l

Agashe, Okui, Sundrum

4D strongly coupled AdS/CFT analogue

CFT – Walking Technicolour

Leptons elementary - couple to Higgs via fermionic operators in strong sector

Exponential suppression factors come from RG running with large scaling dimensions

Epilogue

- Masses

  -Froggatt-Nielsen

  -Xtra-dimension/Composite

  } Majorana or Dirac

  Normal/Inverted/
  Quasi-degenerate (O(3))

Epilogue

- Masses

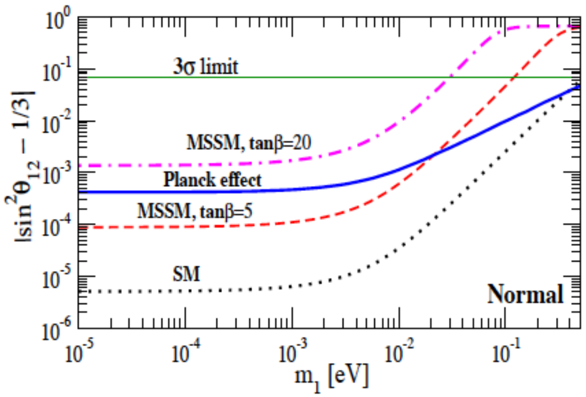
-Froggatt-Nielsen $\left(\langle\theta\rangle/\Lambda\right)^n$

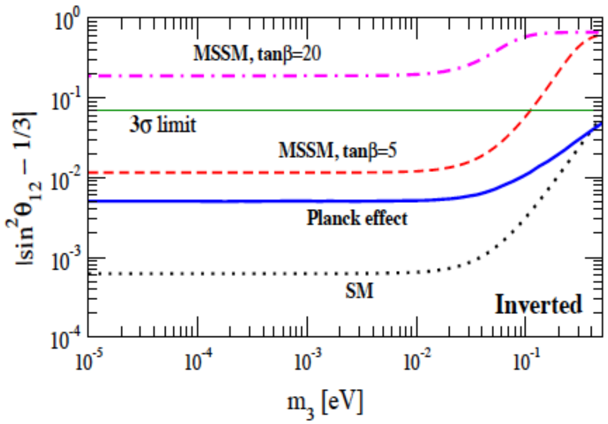
-Xtra-dimension/Composite

} Majorana or Dirac

Normal/Inverted/
Quasi-degenerate (O(3)) †

† Radiative generation of θ_{13}





Dighe, Goswami, Rodejohann
Ellis, Lola
...

Epilogue

- Masses

  -Froggatt-Nielsen

  -Xtra-dimension/Composite

  -Texture (zeros)

e.g. $\sin\theta_{13} = \sqrt{\frac{2}{3}\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}}$ $T\chi M$

Krishnan, Harrison, Scott

Epilogue

- Masses
  - -Froggatt-Nielsen
  - -Xtra-dimension/Composite
  - -Texture (zeros)
- Quark-lepton unification?
  - -Spontaneous breaking (natural extrema)
  - -Hierarchical see-saw

Hierarchical see-saw (Sequential Dominance)

Quarks, charged leptons, neutrinos can have similar Dirac mass

$$L_{Dirac}^{q,l,\nu} = \alpha\, \psi_i \overline{\phi}_3^i \psi_j^c \overline{\phi}_3^j + \beta \left(\psi_i \overline{\phi}_{123}^i \psi_j^c \overline{\phi}_{23}^j + \psi_i \overline{\phi}_{23}^i \psi_j^c \overline{\phi}_{123}^j \right) + \gamma\, \psi_i \overline{\phi}_{23}^i \psi_j^c \overline{\phi}_{23}^j \Sigma_{45} \qquad \alpha > \beta$$

$$\frac{M^{Dirac}}{m_3} = \begin{pmatrix} <\varepsilon^4 & \varepsilon^3+\varepsilon^4 & -\varepsilon^3+\varepsilon^4 \\ \varepsilon^3+\varepsilon^4 & a\varepsilon^2+\varepsilon^3 & -a\varepsilon^2+\varepsilon^3 \\ -\varepsilon^3+\varepsilon^4 & -a\varepsilon^2+\varepsilon^3 & 1 \end{pmatrix}$$

$\varepsilon^d = 0.15,\ \ a^d = 1$

$\varepsilon^l = 0.15,\ \ a^e = -3$

$\varepsilon^u = 0.05,\ \ a^u = 1$

$\varepsilon^\nu = 0.05,\ \ a^\nu = 0$

$m_b \simeq 3m_\tau$

$m_s \simeq m_\mu$ ((1,1) T.Z.)

$m_d \simeq 9m_e$

Hierarchical see-saw (Sequential Dominance)

Quarks, charged leptons, neutrinos can have similar Dirac mass

$$L^{q,l,\nu}_{Dirac} = \alpha\, \psi_i \overline{\phi}^i_3 \psi^c_j \overline{\phi}^j_3 + \beta \left(\psi_i \overline{\phi}^i_{123} \psi^c_j \overline{\phi}^j_{23} + \psi_i \overline{\phi}^i_{23} \psi^c_j \overline{\phi}^j_{123} \right) + \gamma\, \psi_i \overline{\phi}^i_{23} \psi^c_j \overline{\phi}^j_{23} \Sigma_{45} \qquad \alpha > \beta$$

$$\frac{M^{Dirac}}{m_3} = \begin{pmatrix} <\varepsilon^4 & \varepsilon^3 + \varepsilon^4 & -\varepsilon^3 + \varepsilon^4 \\ \varepsilon^3 + \varepsilon^4 & a\varepsilon^2 + \varepsilon^3 & -a\varepsilon^2 + \varepsilon^3 \\ -\varepsilon^3 + \varepsilon^4 & -a\varepsilon^2 + \varepsilon^3 & 1 \end{pmatrix}$$

$\varepsilon^d = 0.15,\ a^d = 1$

$\varepsilon^l = 0.15,\ a^e = -3$

$\varepsilon^u = 0.05,\ a^u = 1$

$\varepsilon^\nu = 0.05,\ a^\nu = 0$

$m_b \simeq 3m_\tau$

$m_s \simeq m_\mu$

$m_d \simeq 9m_e$

Majorana mass structure

$M_1 < M_2 << M_3$

$$L_M = \theta \psi^c_i \psi^c_j \left(a\overline{\phi}^i_3 \overline{\phi}^i_3 + b\varepsilon^3 \overline{\phi}^i_{23} \overline{\phi}^i_{23} + c\varepsilon^2 \overline{\phi}^i_{123} \overline{\phi}^i_{123} \right)$$

small

$<\phi_{23}> = \varepsilon(0,1,-1),$

$<\phi_{123}> = \varepsilon^2(1,1,1)$

$$L^\nu_{eff} / H^2 = \frac{\beta^2}{M_1} \psi_i \phi^i_{123} \psi_j \phi^j_{123} + \frac{\beta^2}{M_2} \psi_i \phi^i_{23} \psi_j \phi^j_{23} + \frac{(\alpha+\beta)^2}{M_3} \psi_i \phi^i_3 \psi_j \phi^j_3$$

$$\Rightarrow \frac{m_\odot}{m_@} = O(\varepsilon)$$

Epilogue

- Masses

  -Froggatt-Nielsen $\left(\langle\theta\rangle / \Lambda\right)^n$

  -Xtra-dimension/Composite

  -Texture (zeros)

- Quark-lepton unification?

  -Spontaneous breaking (natural extrema)

  -Hierarchical see-saw

- Symmetry/Anarchy?

Epilogue

- Masses

  -Froggatt-Nielsen $\left({\langle\theta\rangle}/{\Lambda} \right)^n$

  -Xtra-dimension/Composite

  -Texture (zeros)

- Quark-lepton unification?

  -Spontaneous breaking (natural extrema)

  -Hierarchical see-saw

- Symmetry/Anarchy?

  -Sparse data; departure from e.g. pure tri-bi-maximal mixing…
  will need precision measurement and prediction to decide

Mass relations:

$$\theta_C = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta}\sqrt{\frac{m_u}{m_c}} \right|$$

$$m_\tau(M_{GUT}) = m_b(M_{GUT})$$

$$m_\mu(M_{GUT}) = 3m_s(M_{GUT})$$

$$m_e(M_{GUT}) = \frac{1}{3}m_s(M_{GUT})$$

$$M^d = m_b \begin{pmatrix} <\varepsilon^4 & \varepsilon^3 & . \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ . & . & 1 \end{pmatrix}$$

Gatto et al, Weinberg, Fritzsch

$$M^l = m_b \begin{pmatrix} <\varepsilon^4 & 1\varepsilon^3 & . \\ 1\varepsilon^3 & 3\varepsilon^2 & \varepsilon^2 \\ . & . & 1 \end{pmatrix}$$

Georgi-Jarlskog

Parameters	Input SUSY Parameters					
$\tan\beta$	1.3	10	38	50	38	38
γ_b	0	0	0	0	-0.22	$+0.22$
γ_d	0	0	0	0	-0.21	$+0.21$
γ_t	0	0	0	0	0	-0.44
Parameters	**Comparison with GUT Mass Ratios**					
$(m_b/m_\tau)(M_X)$	$1.00^{+0.04}_{-0.4}$	0.73(3)	0.73(3)	0.73(4)	1.00(4)	1.00(4)
$(3m_s/m_\mu)(M_X)$	$0.70^{+0.8}_{-0.05}$	0.69(8)	0.69(8)	0.69(8)	0.9(1)	0.6(1)
$(m_d/3\,m_e)(M_X)$	0.82(7)	0.83(7)	0.83(7)	0.83(7)	1.05(8)	0.68(6)
$(\frac{\det Y^d}{\det Y^e})(M_X)$	$0.57^{+0.08}_{-0.26}$	0.42(7)	0.42(7)	0.42(7)	0.92(14)	0.39(7)

GGR, Serna

...but $\theta_{12}^{l} = \theta_C$ inconsistent with other plausible GUT relations

$$\theta_C = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta} \sqrt{\frac{m_u}{m_c}} \right|$$

$$m_\tau(M_{GUT}) = m_b(M_{GUT})$$

$$m_\mu(M_{GUT}) = 3m_s(M_{GUT})$$

$$m_e(M_{GUT}) = \frac{1}{3} m_s(M_{GUT})$$

$$M^d = m_b \begin{pmatrix} <\varepsilon^4 & \varepsilon^3 & <\varepsilon^3 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ . & . & 1 \end{pmatrix} \rightarrow \left| \frac{V_{td}}{V_{ts}} \right| = \sqrt{\frac{m_d}{m_s}}$$

Hall, Rasin

$$M^l = m_b \begin{pmatrix} <\varepsilon^4 & 1\varepsilon^3 & . \\ 1\varepsilon^3 & 3\varepsilon^2 & \varepsilon^2 \\ . & . & 1 \end{pmatrix}$$

$$\theta_{12}^{l} = \sqrt{\frac{m_e}{m_\mu}} = \frac{\theta_C}{3}$$

$$\boxed{\theta_{13} = \frac{\theta_{12}^{l}}{\sqrt{2}} = 3^0}$$

...needs additional neutrino contribution

How do we distinguish between these possibilities?

...correlations between mixing angles and phase

Vacuum alignment

e.g. $Z_3 \ltimes Z_n$

ϕ_i	$Z_3\phi_i$	$Z_n^{'}\phi_i$
ϕ_1	$\rightarrow \phi_2$	$\rightarrow \alpha\,\phi_1$
ϕ_2	$\rightarrow \phi_3$	$\rightarrow \alpha^2\phi_2$
ϕ_3	$\rightarrow \phi_1$	$\rightarrow \alpha^{-3}\phi_3$

$\alpha^n = 1$

Choice of discrete symmetry

Vacuum structure : $Z_3 \ltimes Z_n \rightarrow \begin{cases} Z_3, & \langle\phi\rangle = (1,1,1) \quad \lambda > 0 \\ Z_n, & \langle\phi\rangle = (0,0,1) \quad \lambda < 0 \end{cases}$

$$V(\phi) = -m^2\,\phi^{\dagger i}\phi_i + \ldots + \lambda\, m^2 \phi^{\dagger i}\phi_i\phi^{\dagger i}\phi_i$$

Vacuum alignment

$$P \supset \langle P \rangle \phi_{23}\phi_{123}^2 \rightarrow m_{3/2}\phi_{23}\phi_{123}^2$$

$$V_{tree} = m_{3/2}^2 |\phi_{123}|^4 + m_{3/2}^2 |\phi_{123}\phi_{23}|^2$$

$$V_{rad} = \alpha m_{3/2}^2 |\phi_2|^4 + \beta m_{3/2}^2 |\phi_3|^4 + \gamma m_{3/2}^2 |\phi_2\phi_3|^4 + \delta m_{3/2}^2 |\phi_2\phi_{23}|^2 + \ldots$$

$$s_{12}^{\nu 2} = s_{23}^{\nu 2} = 0.5$$

$$\phi_{123} \propto (1,1,1), \quad \phi_2 \propto (1,0,0), \quad \phi_3 \propto (0,0,1), \quad \phi_{23} \propto (0,1,-1)$$

$$\alpha, \beta < 0, \quad \gamma, \delta > 0$$

Bi-maximal mixing "perturbation"

$$s_{12}^{\nu 2} = s_{23}^{\nu 2} = 0.5 \qquad V_{BM}^{\nu} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$V_{BM}^{l} = \begin{pmatrix} \cos\alpha & -e^{-i\delta^l}\sin\alpha & 0 \\ e^{-i\delta^l}\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned} s_{13} &\approx \alpha/\sqrt{2} \\ s_{12}^2 &\approx 1/2 + \alpha\cos\delta/\sqrt{2} \\ s_{23}^2 &\approx 1/2 - \alpha^2/4 \\ \delta &= \delta^l. \end{aligned}$$