WHAT

HAVE WE LEARNED IN THE PAST 20 YEARS OF PARTICLE PHYSICS?

$$N_\nu \cong 3$$

Σ LEP

2.989 ± 0.012

$(< 3.001 \ 1\sigma)$

***

PRIMORDIAL

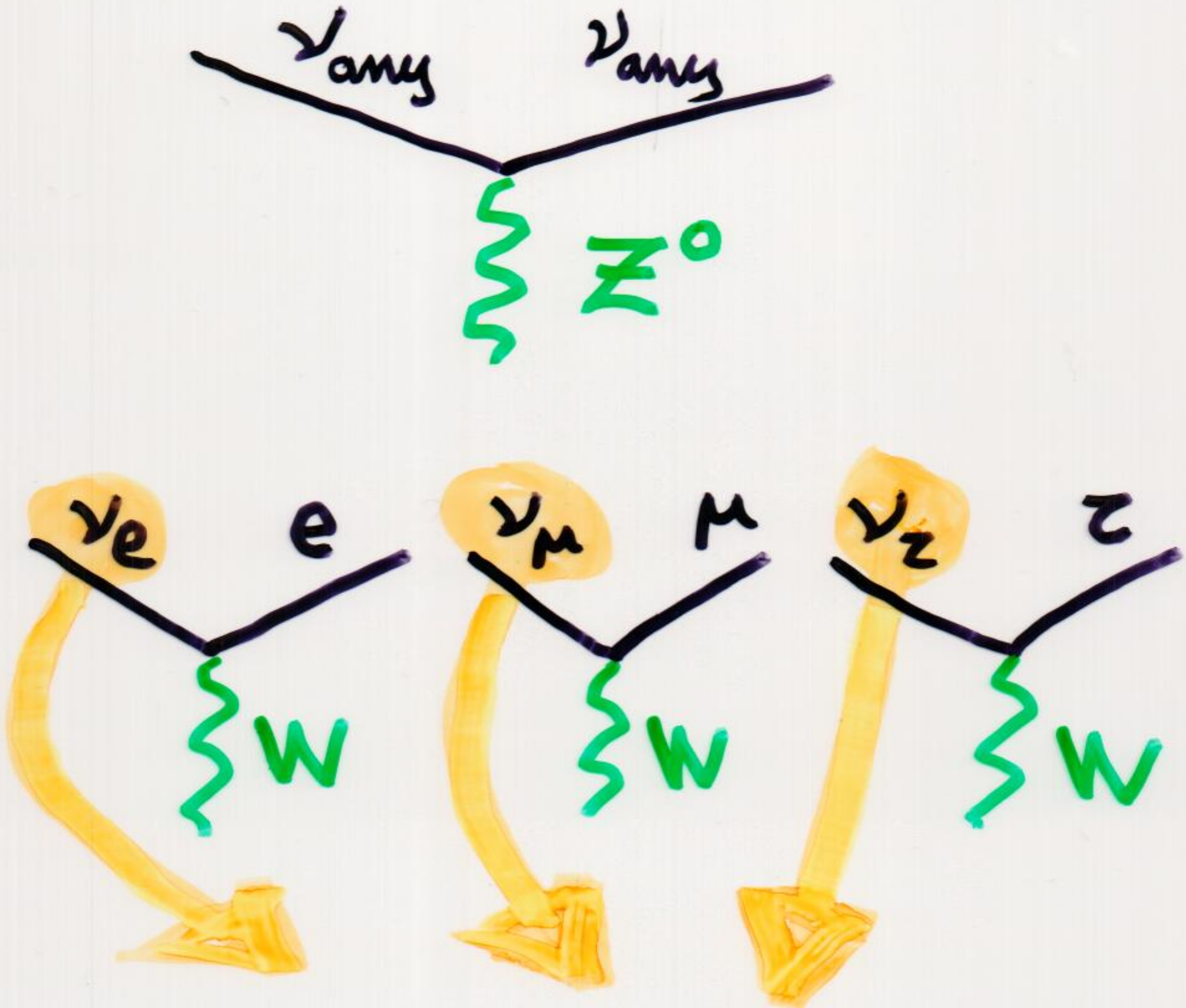
NUCLEOSYNTHESICERS

CLAIM VICTORY

$10 \longleftrightarrow 110$

FECUND = ~~STERILE~~

HAS WEAK INTERACTIONS



DEFINITION

eg $\nu_e = U_{e1}\nu_1 + U_{e2}\nu_2 + U_{e3}\nu_3$

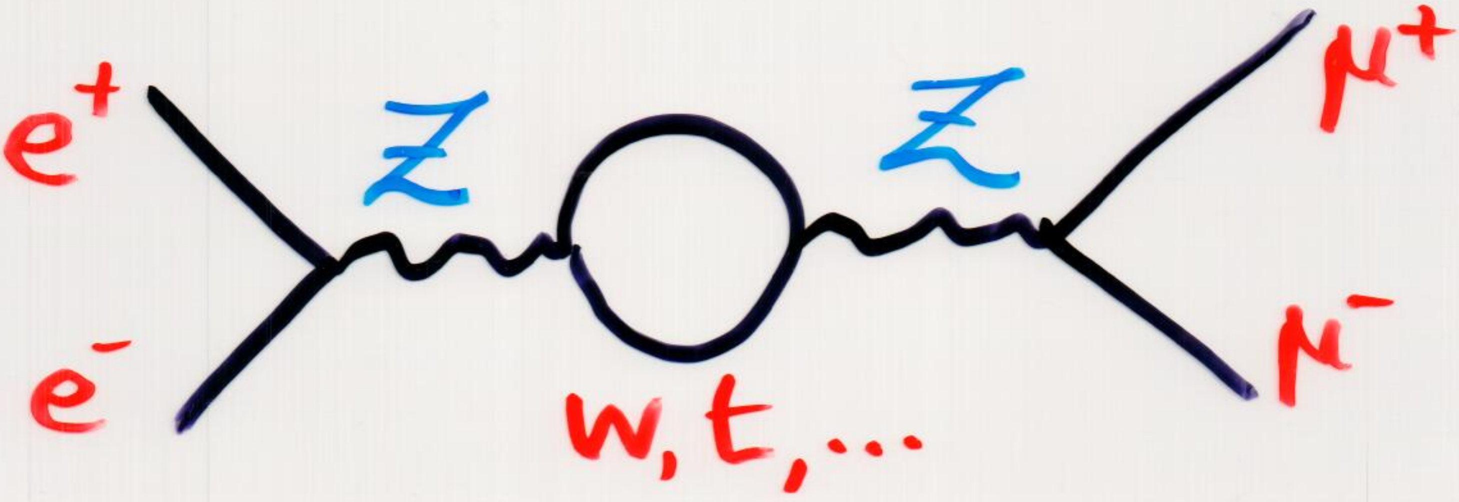
$+ U_{es}\nu_s$

GWS ELECTROWEAK MODEL is a THEORY

QED $g-2$ LAMB shift

CONSISTENT AT THE (1-LOOP) QUANTUM LEVEL

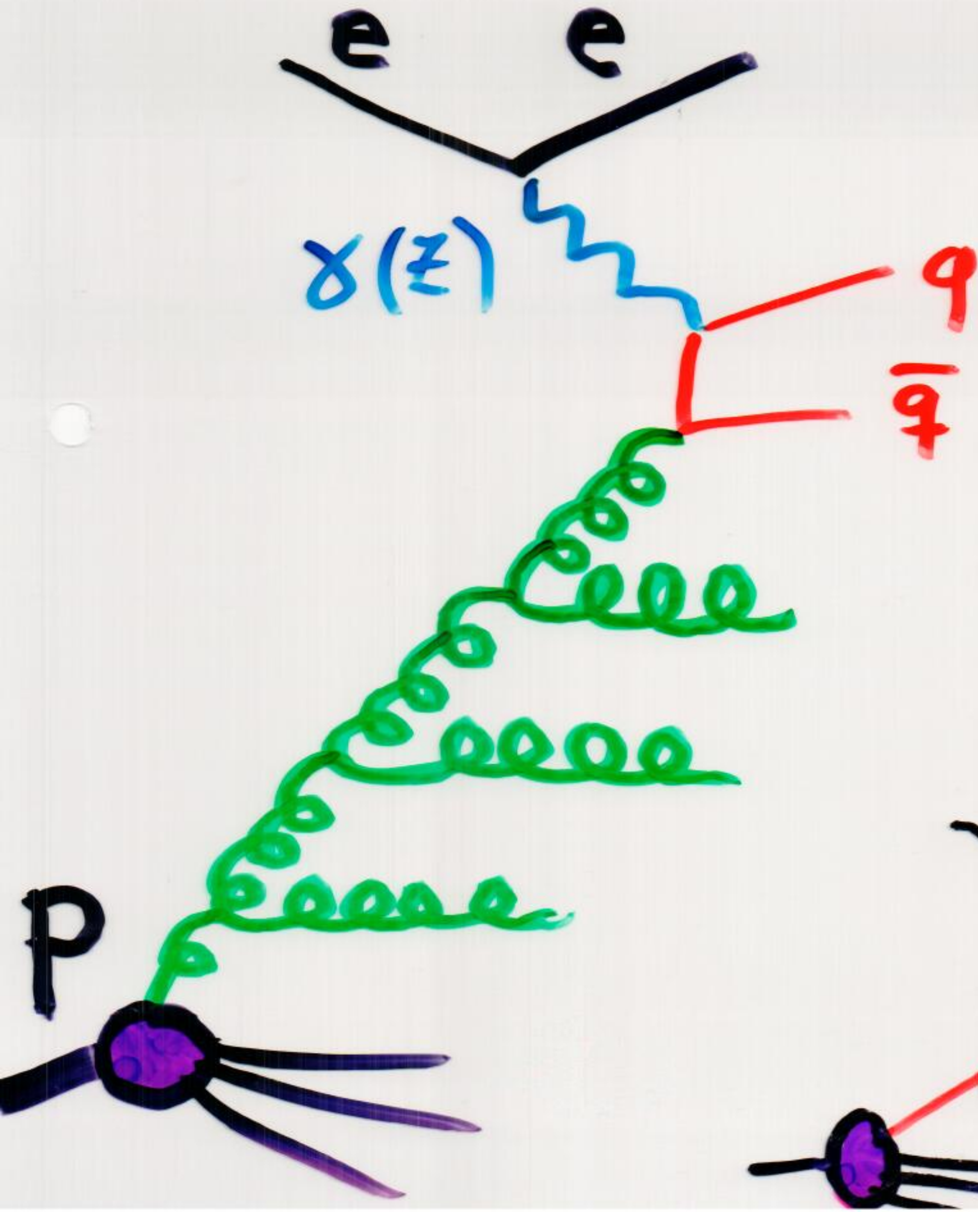
PRECISION LOW-ENERGY TESTS (eg: μ-DECAY)
+ LEP EXPERIMENTS

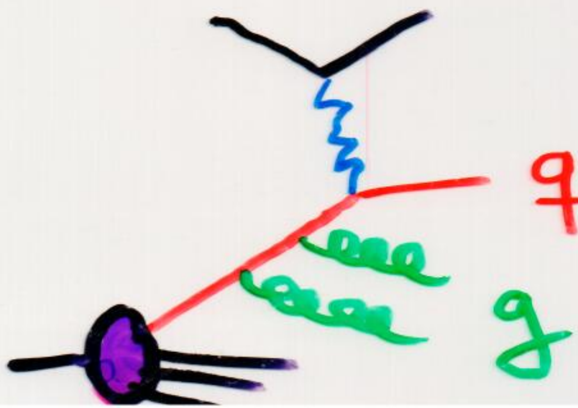


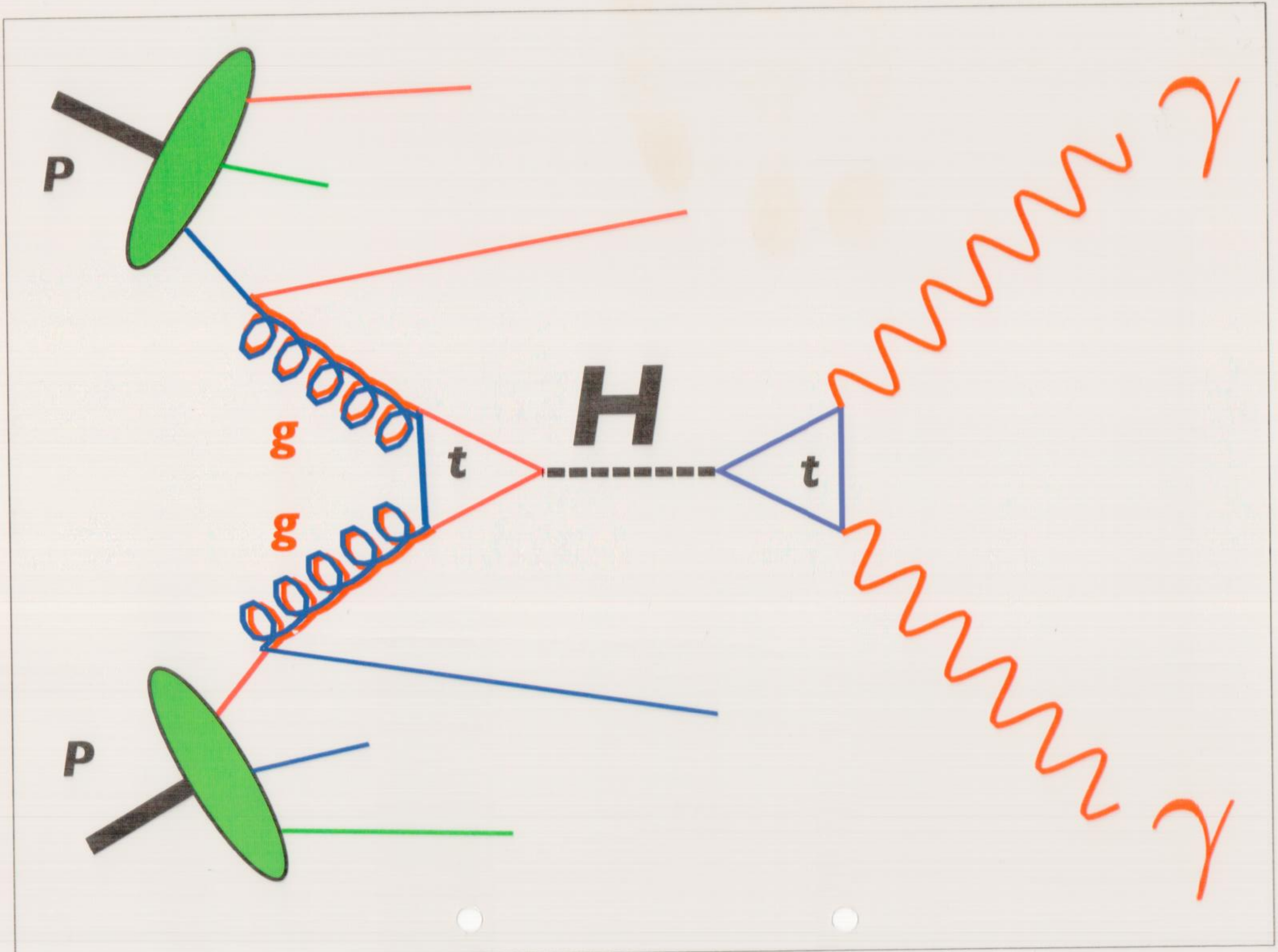
QCD FIRMLY CONFIRMED AS A NONABELIAN GAUGE TH.

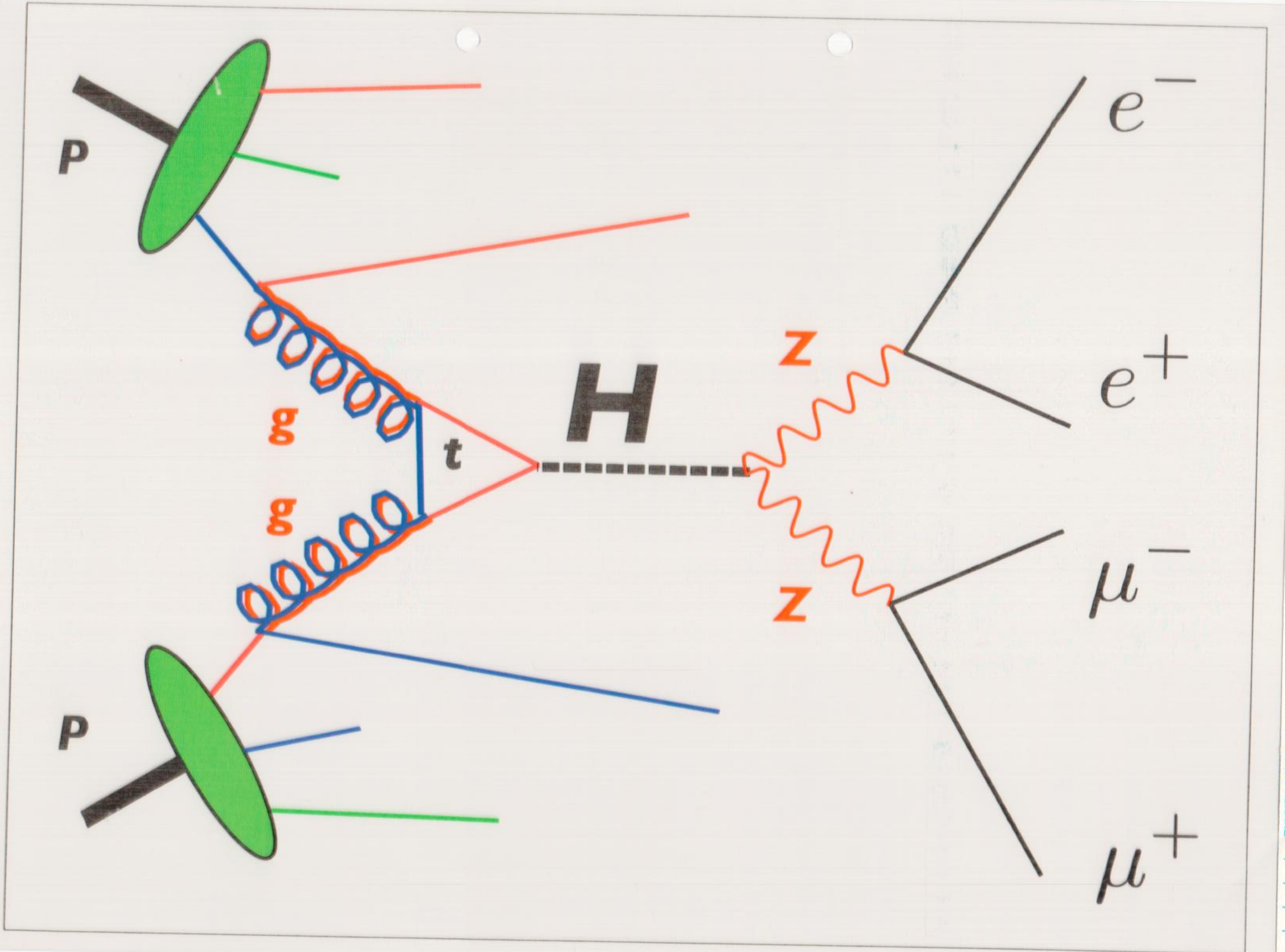
RECENT : HERA $ep \to e \ldots$
LHC $pp \to \ldots$
KINEMATICAL DOMAIN $\ni$

$x \ll 1$






P
P
g
g
t
H
t
γ
γ



P
P
g
g
t
H
Z
Z
e^-
e^+
μ^-
μ^+

ASTRO → PARTICLE - PHYSICS

IN THE LAST 20 YEARS
OF THE XXth C

- LIMITS ON AXIONS — RED GIANTS, SN-1987A — QUASI-STANDARD
- ON m_ν

- SOLAR
  ATMOSPHERIC

m_ν

1" BEYOND THE SM

IS NON-ACCELERATOR

"PARTICLE
ASTROPHYSICS"

AN (OBSERVATIONAL)

(NEW) SCIENCE

YES:

RESULTS, NOT ONLY LIMS.

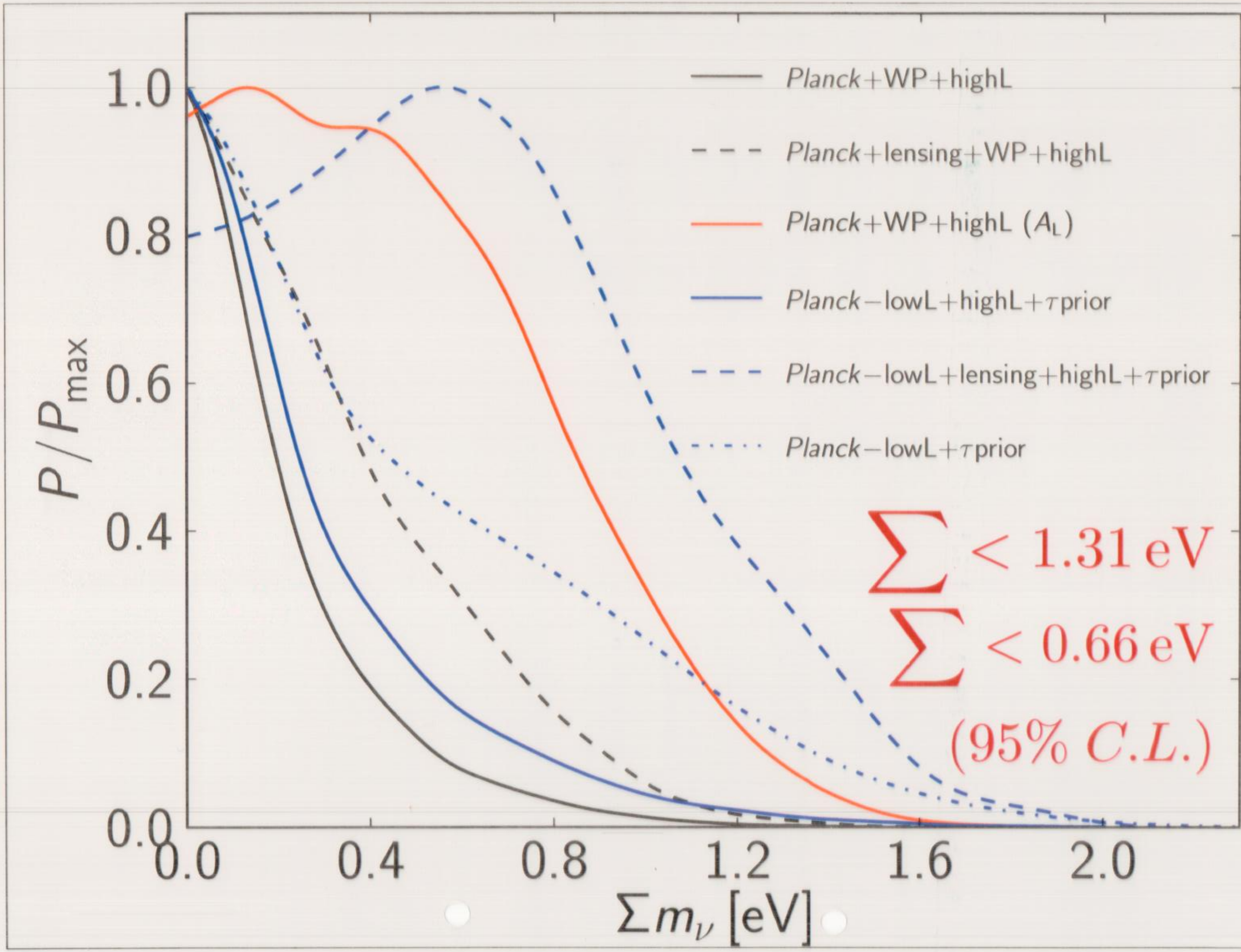
NO

CENTURIES OLD

SOME PAST LESSONS ON ~PARTICLE PHYSICS FROM ASTROPHYSICS

- **ROEMER, 1675** MEAS. c FROM DELAYED JUPITER SATELL. ECLIPSE (REDONE LAB 100 YEARS LATER)
- **NEWTON, 1683** GRAV. LAW AND [COUPLING CONSTANT] × M (LAB: CAVENDISH 1798)
- **GRAVIT. WAVES** INDIRECT EVID. TAYLOR et al 79
- **PHOTON MASS LIMIT** DAVIS et al, 1975 (ORDER MAG. BETTER THAN LAB. WILLIAM et al, 71)
- **NUCL. TRANSMUTS.** EDDINGTON 1900 H → He STARS
- **EXCITED LEVEL C** HOYLE. TO BURN 3 He → Heavies
- **1/2 DOZEN PARTS FROM COSMIC RAYS** (μ, π, Λ, ...)
- **BARYON # NON CONS.** (PROTON DECAY) SAKHAROV 67 (FROM FACT HE WAS THERE) (MOSCOW)
- **m(ν) LIMITS** GERSTEIN ZELDOVICH, 66
- **# DIFF. NEUTRINOS**, SHVARTSMAN, 69 (FROM EFFECT IN NUCLEOSYNTHESIS) ≤ 5 GENS.

WE ARE ALL IN THE GUTTER (OR THE LEP TUNNEL)
BUT SOME OF US ARE LOOKING AT THE STARS
(Adapt. from Oscar Wilde, Lady Windermere's Fan)



Planck+WP+highL
Planck+lensing+WP+highL
Planck+WP+highL (A_L)
Planck−lowL+highL+τprior
Planck−lowL+lensing+highL+τprior
Planck−lowL+τprior
$\sum < 1.31\,\mathrm{eV}$
$\sum < 0.66\,\mathrm{eV}$
(95% C.L.)
P/P_{max}
Σm_ν [eV]

V

MASSES AND OSCIL-LATIONS

ν_e

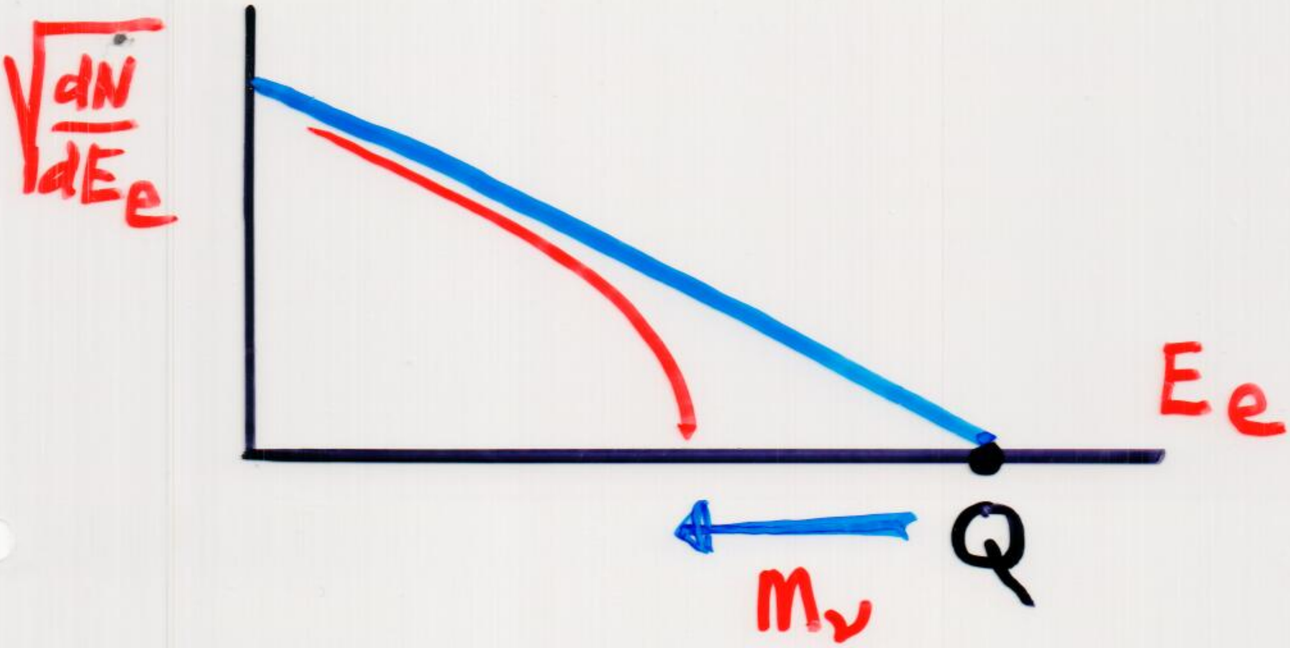
1933 PERRIN NOTED

1934 WORKED OUT by FERMI

n
p

$E_e \sim E_{MAX}$

$E_\nu \sim m_\nu$



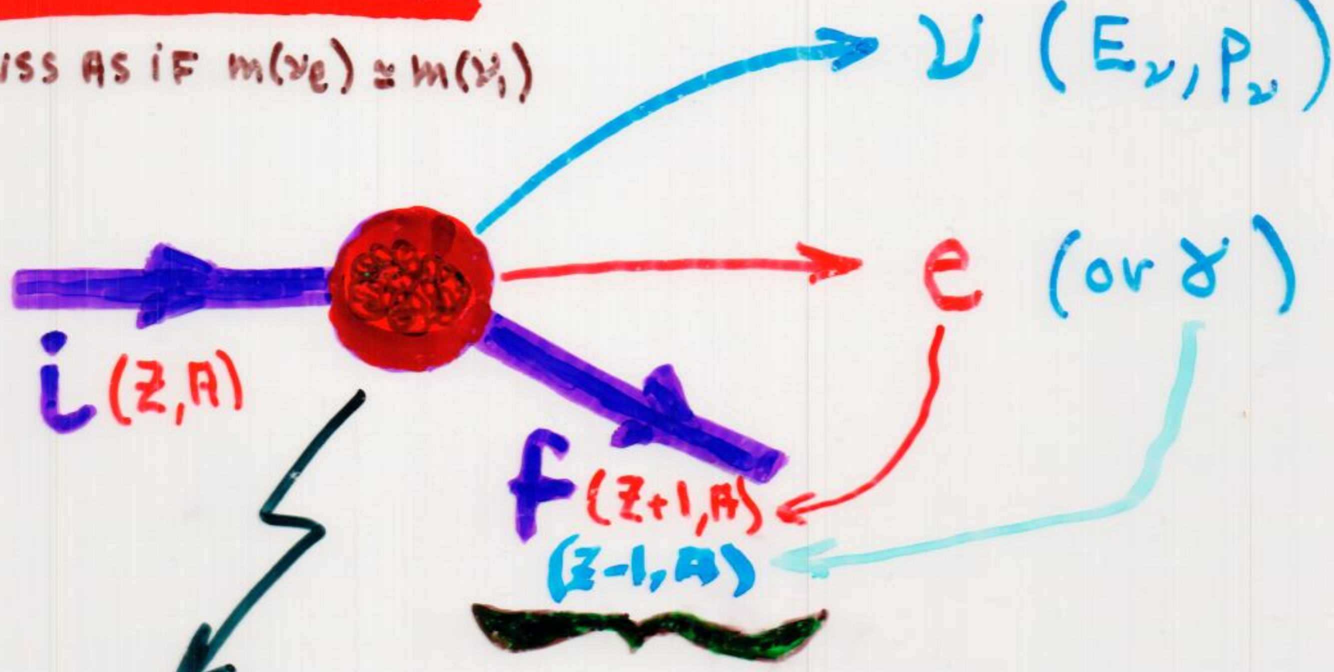
$m(\bar{\nu}_e)$: T = ^3H β-decay

$m(\nu_e)$: ^{163}Ho IBEC

m_{ν_e} EFFECTS IN 3-BODY DECAYS

ENPOINT ANALYSIS

- DISCUSS AS IF $m(\nu_e) \simeq m(\nu_1)$



$|M_{if}|^2$

$\Phi_\nu \propto \underbrace{p_\nu E_\nu}$

$$Q = E_i - E_f \simeq m_i - m_f$$

(recoil effects neglig.)

$$m_i \cong m_f + \underbrace{E}_{e/\gamma} + E_\nu$$

(energy conservat.)

$$(Q-E)\sqrt{(Q-E)^2 - m_\nu^2}$$

FERMI-PERRIN

$E \lesssim Q$

max. sensitivity

"ENDPOINT"

ALLOWED β-DECAYS $i \to f^+ + e^- + \bar{\nu}_e$

$$\phi_e \, |\mathcal{M}_{if}|^2 = p_e E_e \, F_c(E_e)$$

NUCLEAR $|\mathcal{M}|^2$ ~ CONSTANT, SLOWLY VARYING

$F_c(E_e)$ → COULOMB ATTRAC $(e^- f^+)$

$p_e E_e$ → ELECT. PHASE SPACE

"KURIE" ENDPOINT PLOT

$$K = \sqrt{\frac{1}{p_e F_c} \frac{dW}{dE_e} \frac{1}{E_e}}$$

$$\propto \sqrt{E_\nu \, p_\nu}$$

$$\equiv \left[(Q - E_e) \sqrt{(Q - E_e)^2 - m_\nu^2} \right]^{\frac{1}{2}}$$

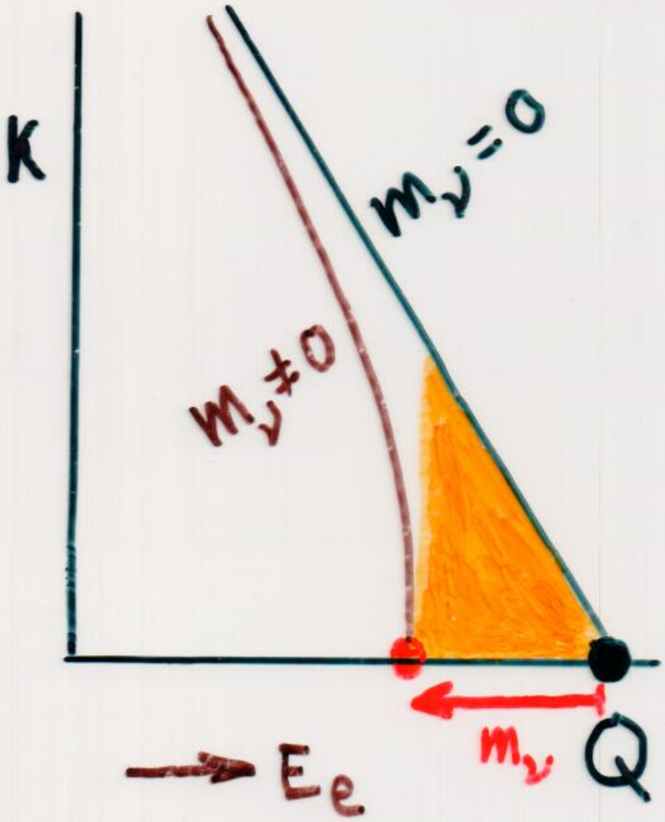


FIGURE OF MERIT

$\frac{1}{W} \Big\{$  $\Big\}$ $= g(Q, m_\nu) = \frac{1}{W} \int_{Q-m_\nu}^{Q} \frac{dW(m_\nu = 0)}{dE_e} \, dE_e$

$$\simeq \left(\frac{m_\nu}{Q} \right)^3$$

SMALL Q IS GOOD

EXPLICITLY:

$$dW \tilde{\propto} \int \frac{d^3p_e}{2E_e} \frac{d^3p_\nu}{2E_\nu} \frac{d^3p_N}{2M_N} \delta^4(\mathbb{P}_f - \mathbb{P}_i) |\mathcal{M}_{if}|^2$$

IN ALLOWED β-DECAY

~CONSTANT

$$|\mathcal{M}_{if}|^2 = F_c(E_e) E_e E_\nu |\mathcal{M}_N|^2$$

FERMI'S GOLDEN RULE:

$$dW \propto \int p_e^2 dp_e \; p_\nu^2 dp_\nu \; \delta^1(Q - E_e - E_\nu) F_c$$

$E_\nu = Q - E_e$

$E^2 = \bar{p}^2 + m^2$

$$\underbrace{p_e E_e}_{\Phi_e} dE_e \quad \underbrace{p_\nu E_\nu}_{\Phi_\nu} dE_\nu$$

$$\frac{dW}{p_e E_e dE_e} \propto F_c(E_e) \underbrace{(Q - E_e)}_{E_\nu} \underbrace{\sqrt{(Q - E_e)^2 - m_\nu^2}}_{p_\nu}$$

TRITIUM : BEST β-DECAY NUCLIDE

$$\boxed{{}^3H \to {}^3He^+ + e^- + \bar{\nu}_e}$$

$$\begin{cases} Q \cong 18.6 \text{ keV} \\ T_{1/2} \simeq 12 \text{ years} \end{cases}$$

(TYP : MeV's)

$$\boxed{g({}^3H) = 8\ 10^{-9} \left(\frac{m_\nu}{30\text{eV}}\right)^3}$$

★ TRITIUM DOESN'T MAKE IT TO G.B.R.

$$\boxed{{}^{187}_{75}Re \to {}^{187}_{76}Os^+ + e^- + \bar{\nu}_e}$$

$$\begin{cases} Q \simeq 2.6 \text{ keV} \\ T_{1/2} \simeq 4\ 10^{10} \text{ years} \end{cases}$$

(P. REMEMBER THIS CONST. NATURE)

ONLY KNOWN AVENUE IN β-DECAY : IMPROVE THE (RESOLUTION) IN TRITIUM EXPERIMENTS
(vs COUNT RATE) RHENIUM

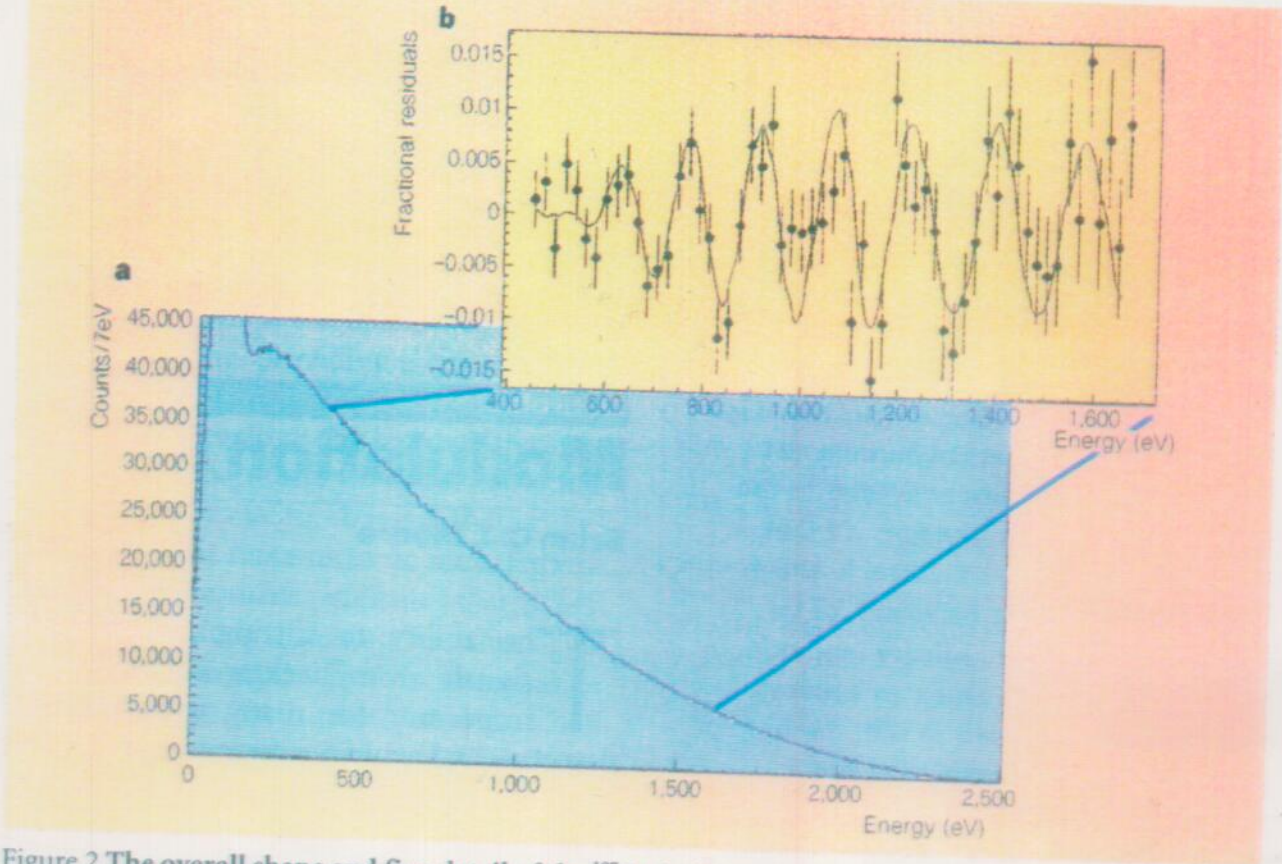
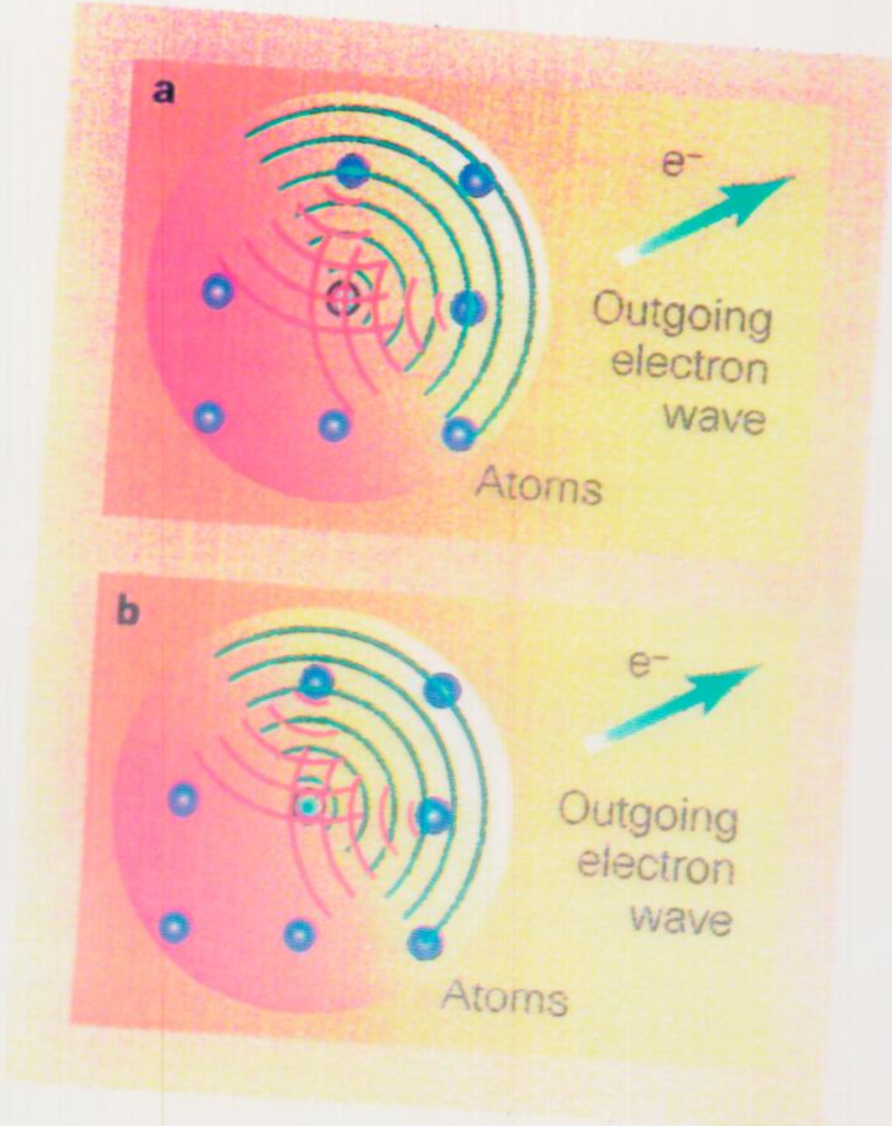


Figure 2 **The overall shape and fine detail of the ^{187}Re β-decay spectrum measured by Gatti *et al.*[1]. a, The complete β-spectrum of ^{187}Re. b, β-environmental fine structure, which is expressed as a fine ripple on top of the standard β-spectrum. The smooth line shows the predicted shape of the BEFS spectrum. At low energies, the ripples are too close together to be resolved by the detector, and at high energies the oscillations become negligible.**

^{187}Re β decay

$Q \sim 2.5$ keV


a
e⁻
Outgoing
electron
wave
Atoms
b
e⁻
Outgoing
electron
wave
Atoms

S. KOONIN (91)

$$\boxed{{}^3H \rightarrow {}^3He\ e^-\ \bar{\nu}_e}$$

PDB 1995, m^2 [eV^2]

$-39 \pm 34 \pm 15$	WEINHAMER
$-24 \pm 48 \pm 61$	HOLTSCHUH
$-65 \pm 85 \pm 65$	KAWASAMI
$-147 \pm 68 \pm 41$	ROBERTSON

ETC

1996

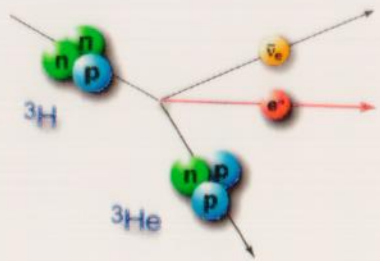
$$\overline{m^2} < -96 \pm 21\ eV^2$$

ν_e is a

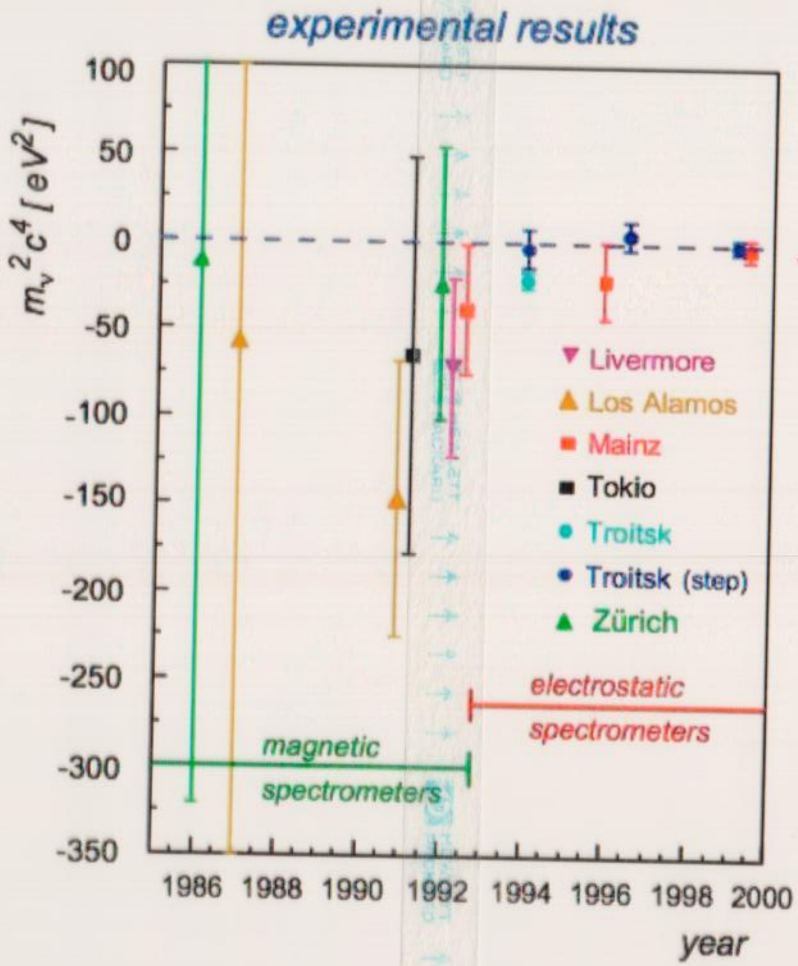
TACHYON

4.6 σ EVIDENT

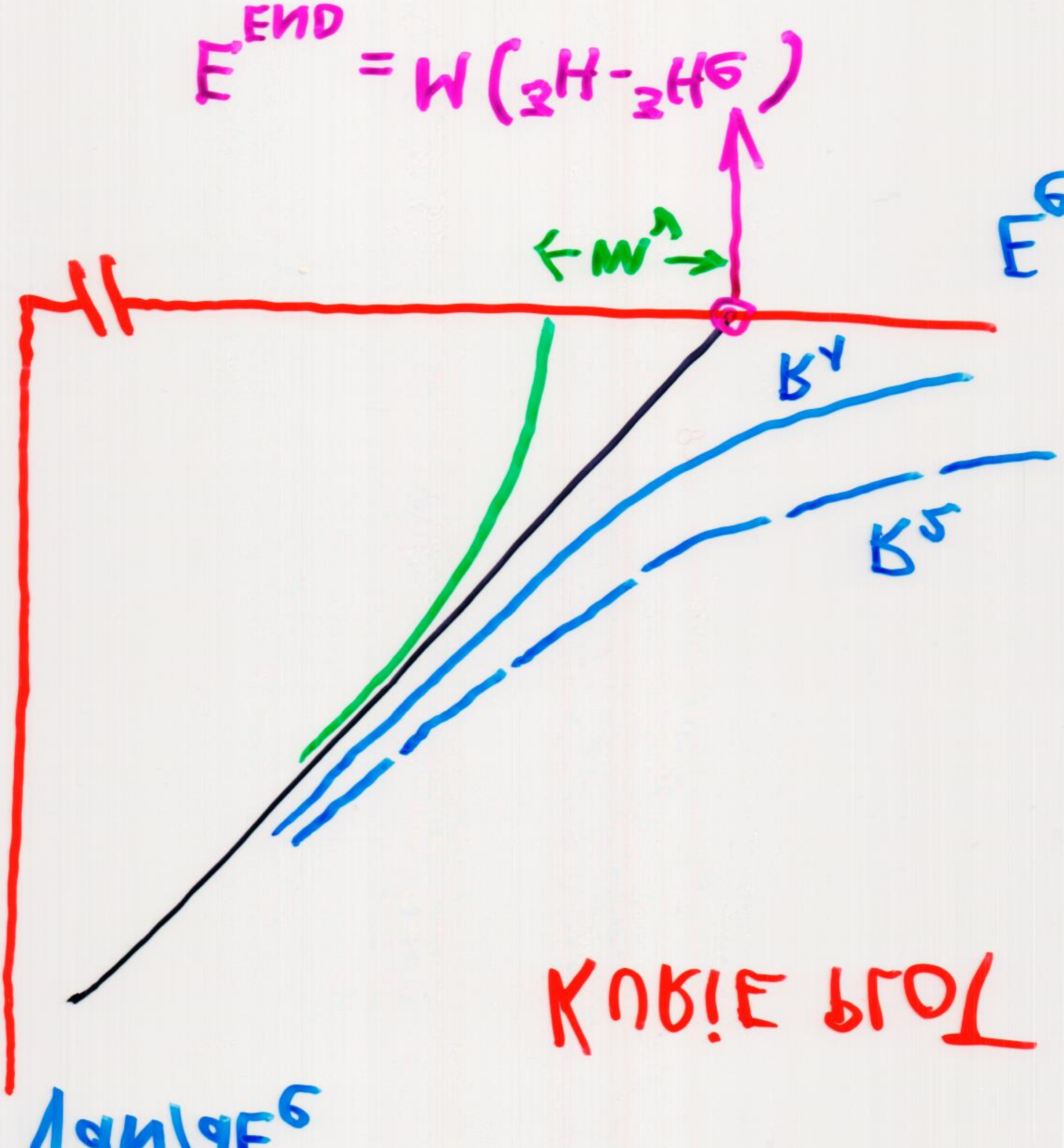
Past Tritium Beta Decay Experiments



	m_ν
ITEP *T_2 in complex molecule* *magn. spectrometer (Tret'yakov)*	17-40 eV
Los Alamos *gaseous T_2 - source* *magn. spectrometer (Tret'yakov)*	< 9.3 eV
Tokio *T - source* *magn. spectrometer (Tret'yak*	< 13.1 eV
Livermore *gaseous T_2 - source* *magn. spectrometer (Tret'yak*	< 7.0 eV
Zürich *T_2 - source impl. on carrier* *magn. spectrometer (Tret'yakov)*	< 11.7 eV
Troitsk (1994-2001) *gaseous T_2 - source* *electrostat. spectrometer*	< 2.2 eV
Mainz (1994-2001) *frozen T_2 - source* *electrostat. spectrometer*	< 2.2 eV



$m_\nu = \left(\sum |U_{ei}|^2 m_i\right)^{1/2}$

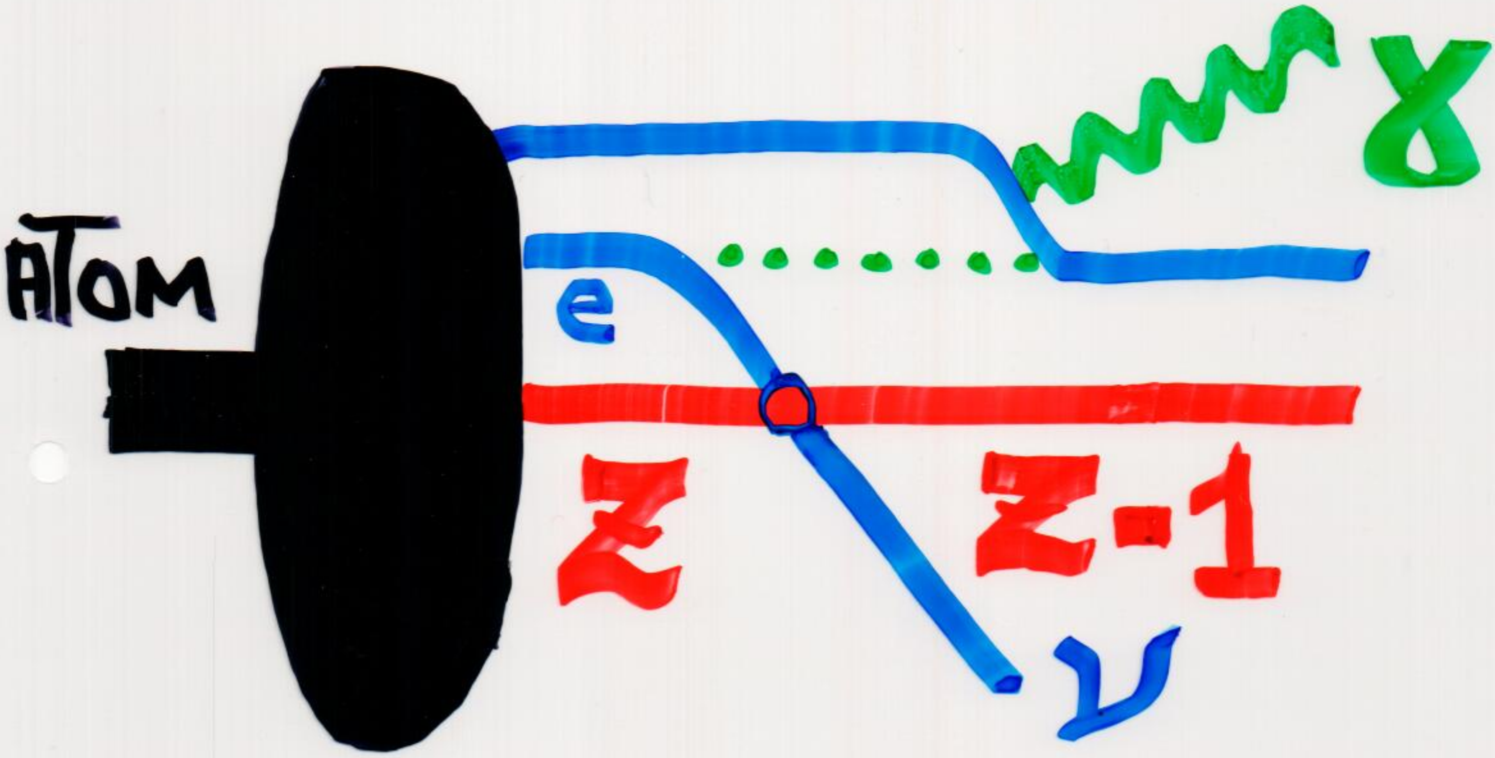


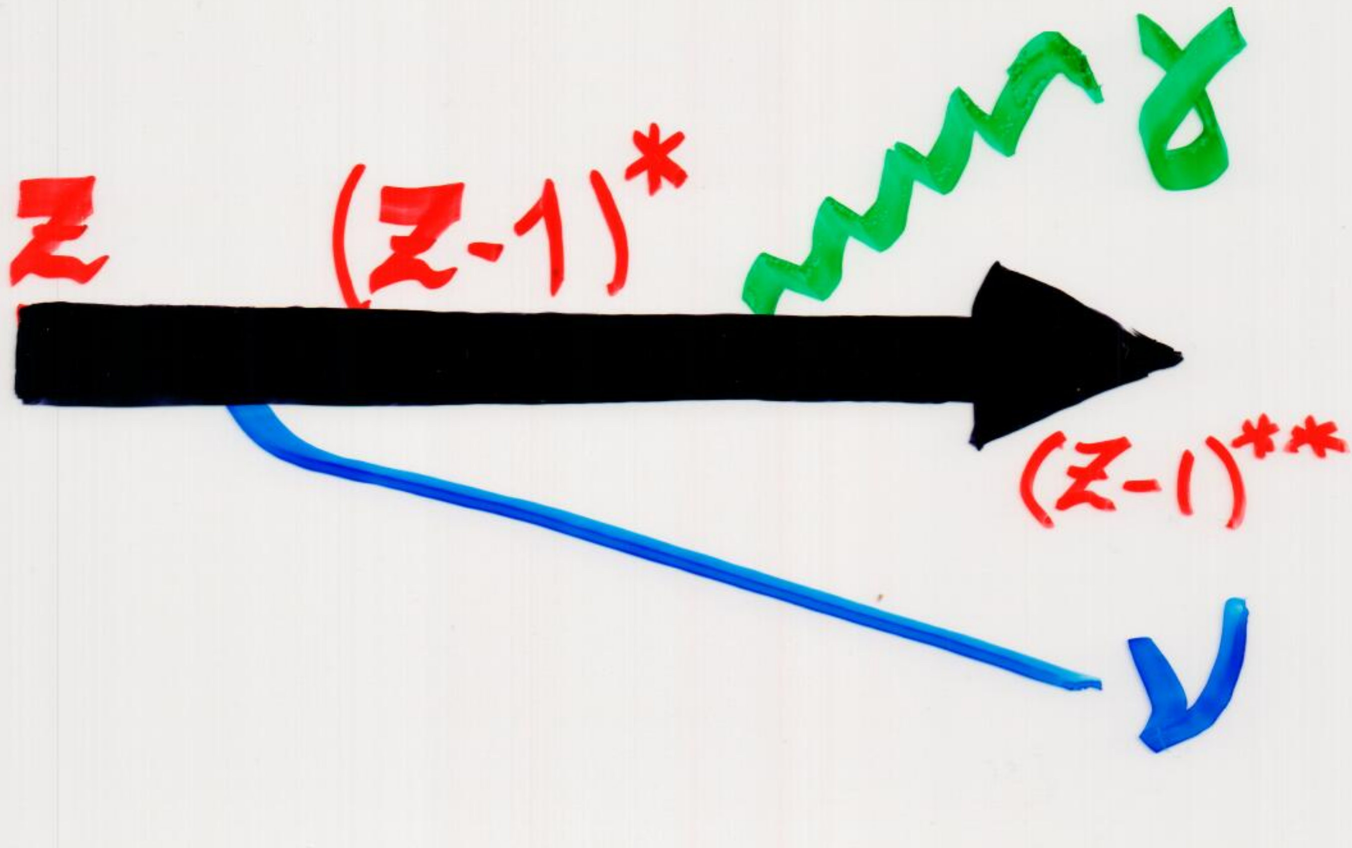
E_END = M(3H - 3He)
m_ν
E_e
dN/dE_e
KURIE PLOT

↑ $m(\bar{\nu}_e)$

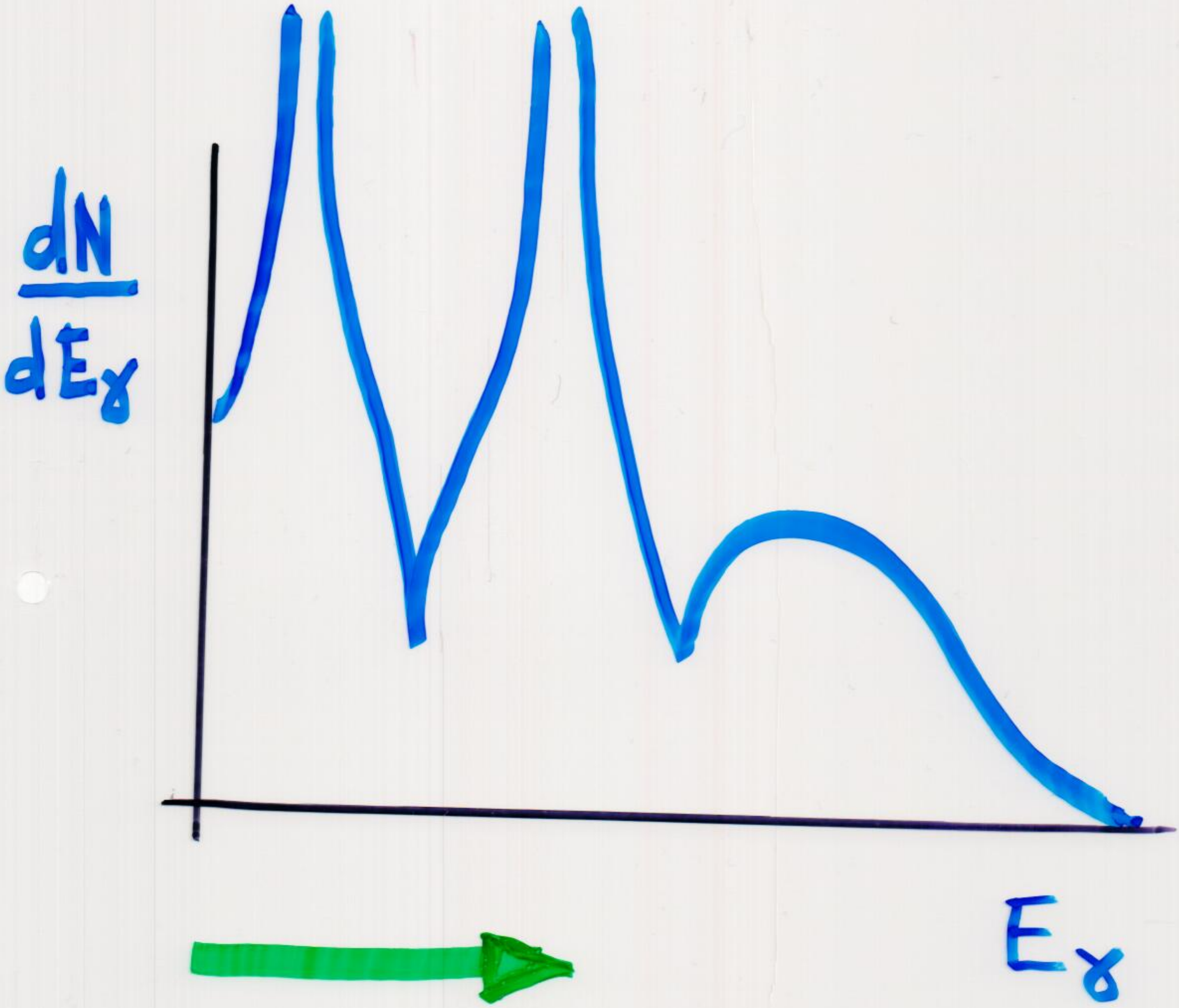
$m(\nu_e)$?

IBEC





3-BODY ; $E_\gamma \approx E_x$ RESONANT [TWO BODY]



$Q \approx E_\nu^{max} \sim E_X$

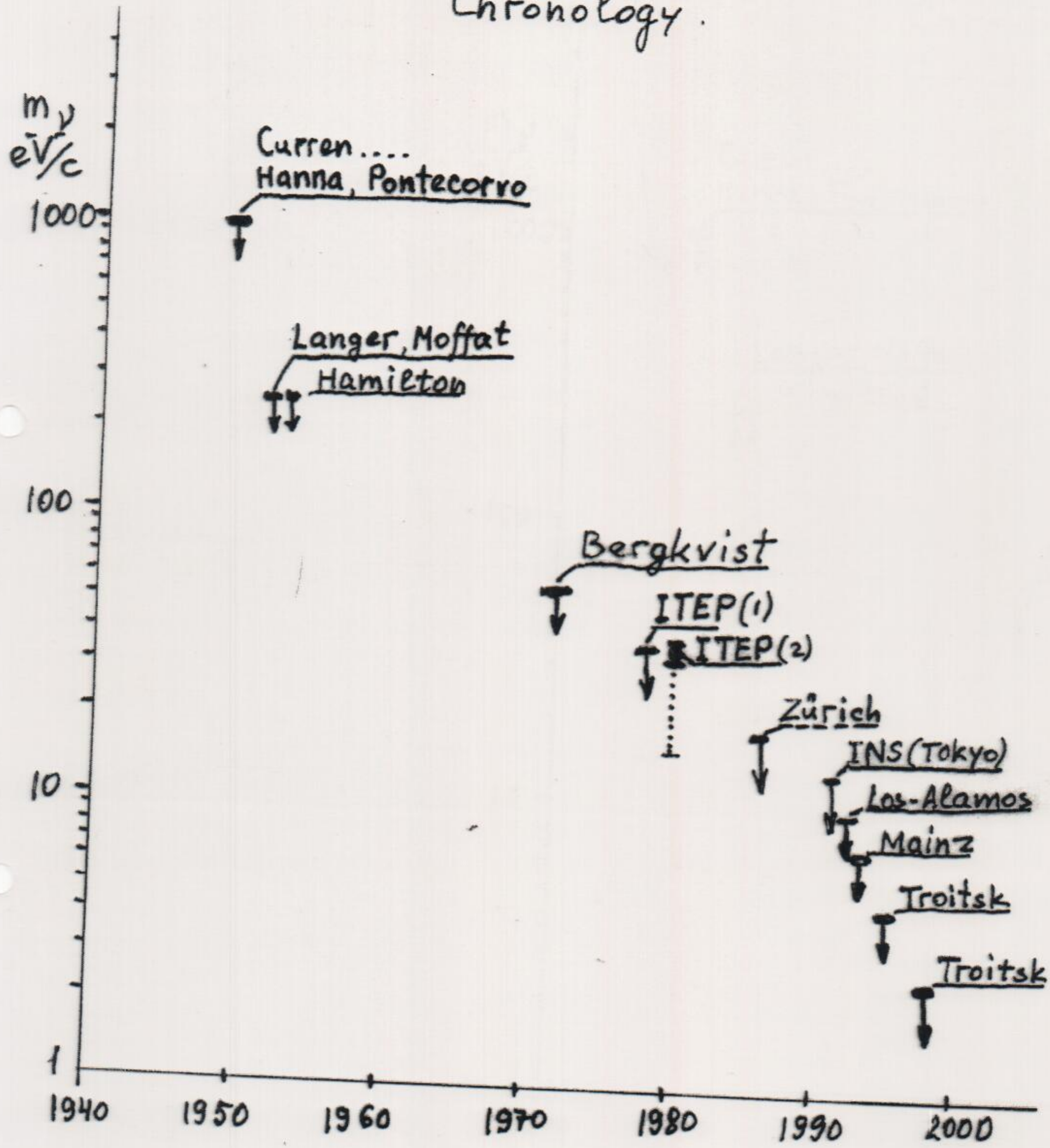
^{163}Ho BEST HOPE

$Q \sim 2 \leftrightarrow 10$ keV (1981)

$\longrightarrow Q \cong 2560\ eV$

LOBASHEV 99

The neutino mass in ^{3}T decay.
Chronology.



ISOLDE

^{193}Pt

^{163}Ho

95%CL

$< 225\,eV$

SPRINGER
BENNET
BAISDEN

LIMITS ON $m(\nu_e)$ AS OPPOSED TO $m(\bar{\nu}_e)$

ν_μ

π^+ AT REST IN A TARGET

$\pi^+ \to \mu^+ \nu_\mu$

$$m^2(\nu_\mu) = m_\pi^2 + m_\mu^2 - 2m_\pi(m_\mu^2 + \vec{p}_\mu^{\,2})^{1/2}$$

$|\vec{p}_\mu|$: MEASURED

CPT : $m(\pi^+) = m(\pi^-)$

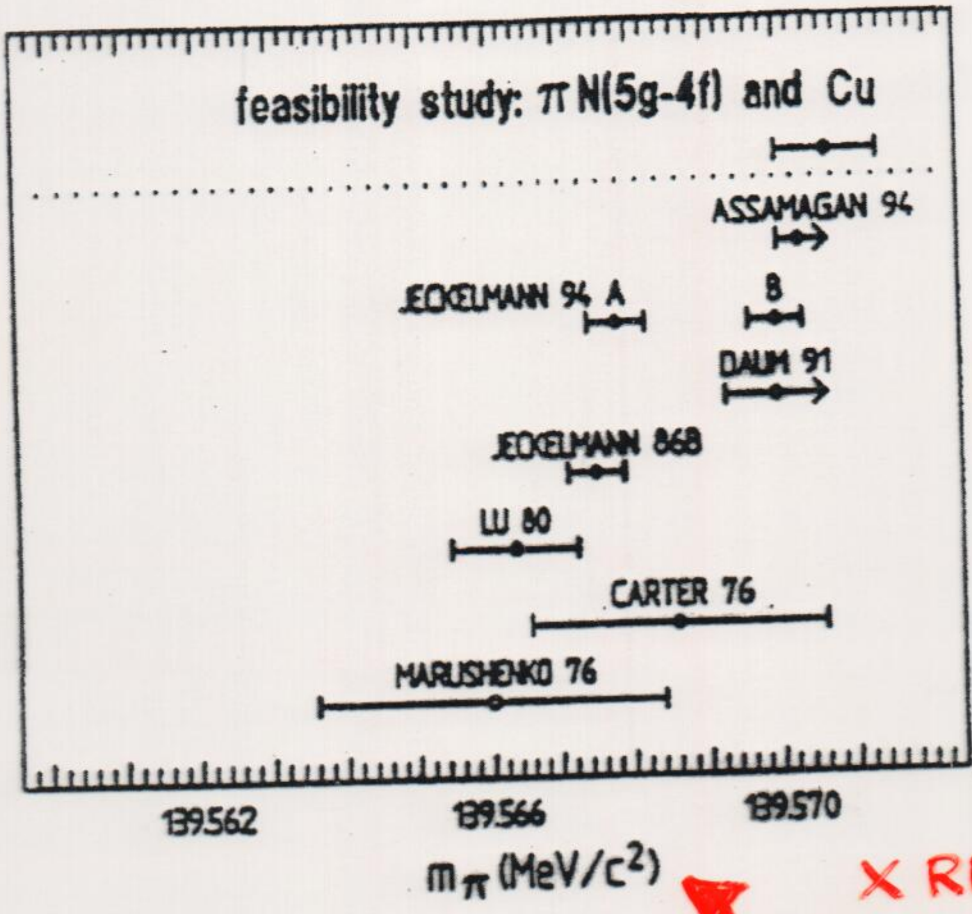
m_{π^-} : X-RAY ENERGIES IN π^--ATOMS

m_μ : m_e , m_μ / m_e

HF Spilting in (μe)

LOBASHEV
VENICE 99

ν_μ



X RAYS FROM
π^--ATOMS

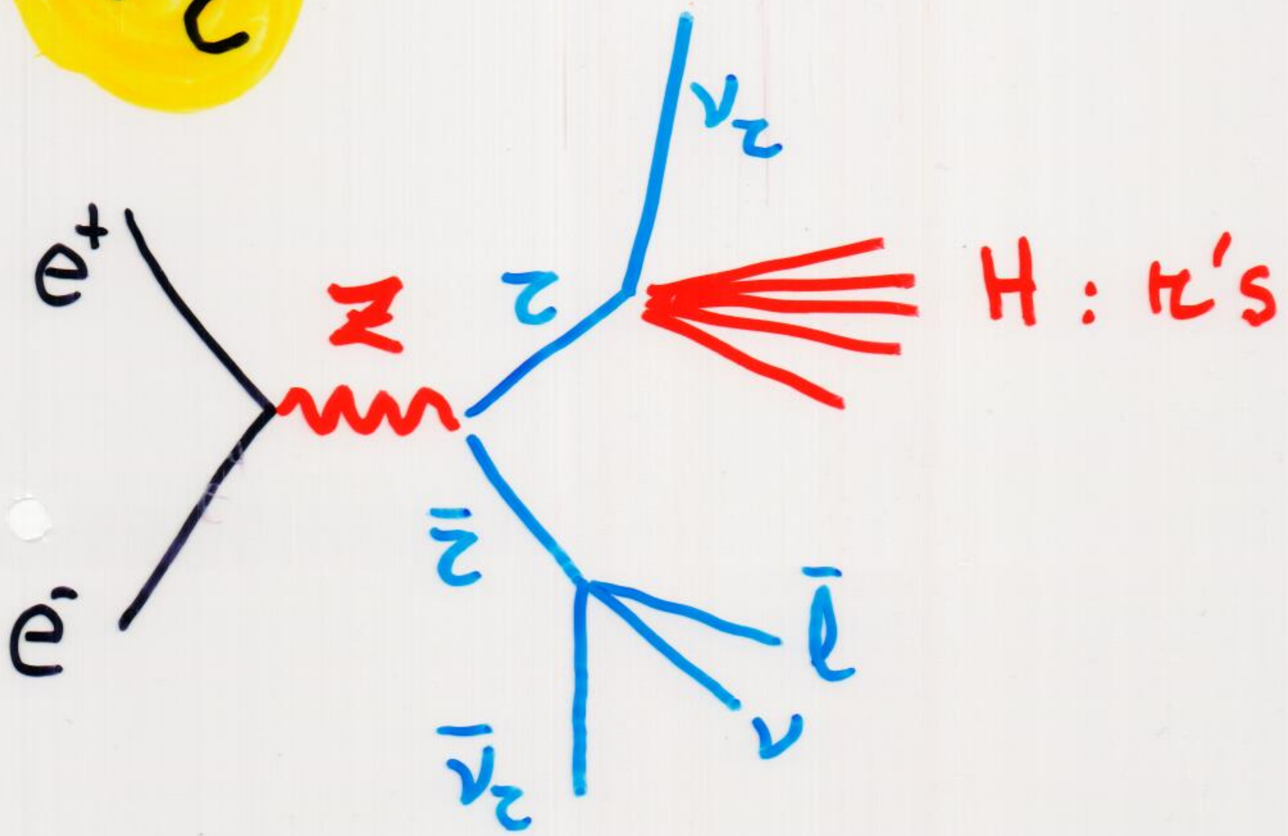
Discard the solution A (Jeckelmann, 94)

Remain only: $\pi_{REST} \to \mu \nu_\mu$ ASSAMAGAN 96

$$m_{\nu_\mu}^2 = (-0.016 \pm 0.023) MeV^2$$

$$m_{\nu_\mu} \begin{cases} < 0.17 MeV & (90\% C.L.) \\ < 0.195 MeV & (95\% C.L.) \end{cases}$$

ν_τ



$$X = \frac{E_H}{E_{BEAM}} < 1$$

$$M < m_\tau$$

ALLOWED DOMAIN IN

$\{x, M\}$ PLANE

ν_τ

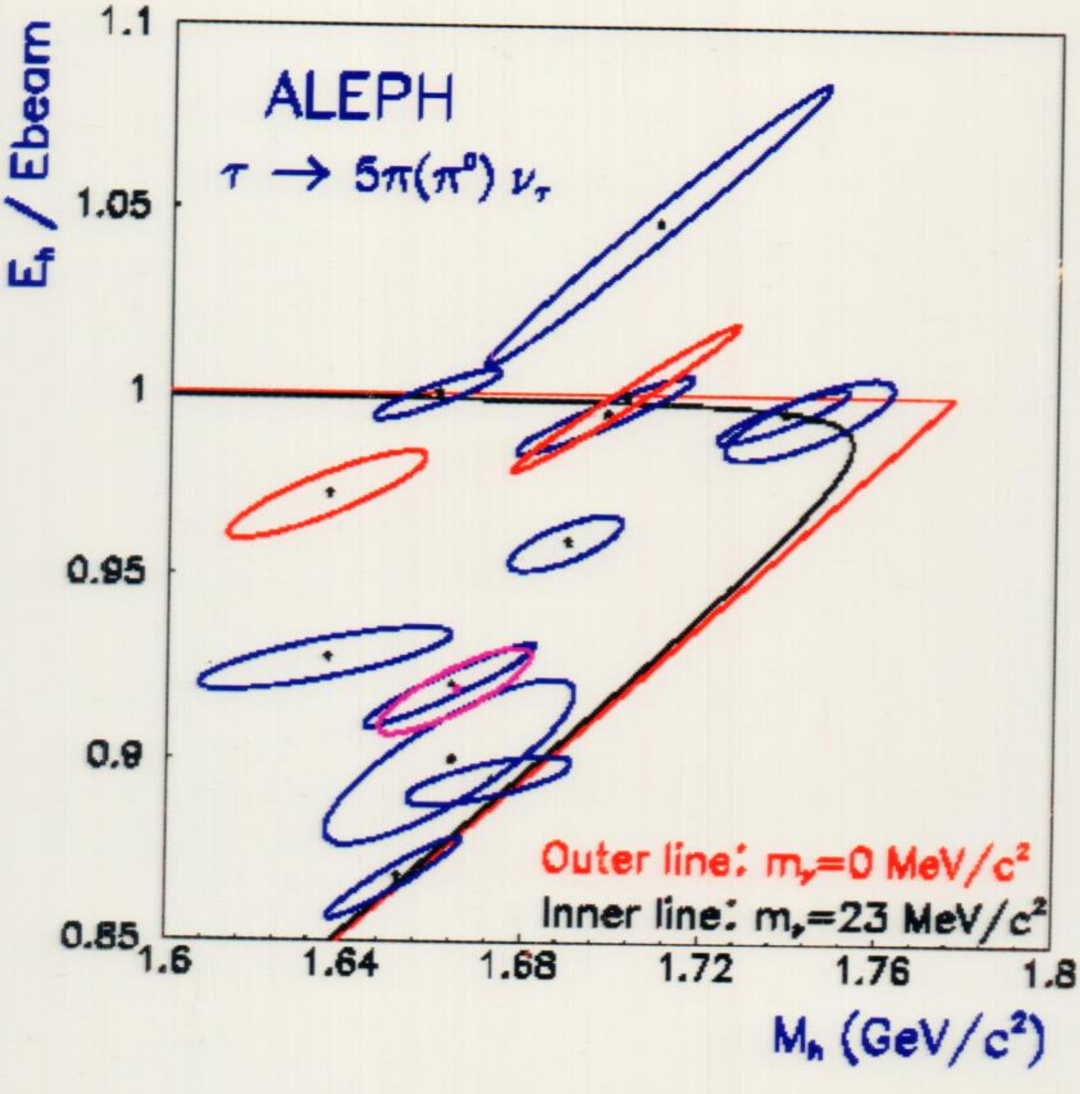


Fig. 4. Distribution in the upper part of the (m_h, E_h) plane for $\tau^- \to 3\pi^- 2\pi^+ (\pi^0)\nu_\tau$ candidates in the data. The *two lines* show the allowed region for a massless and for a 23MeV/c^2 neutrino. The only $\tau^- \to 3\pi^- 2\pi^+ \pi^0 \nu_\tau$ event in the plot is the one with the largest hadronic energy

$\nu_{e,\mu,\tau}$ KINEMATICAL MASS LIMITS

$$\begin{cases} {}^3H \to {}^3He + e^- + \bar{\nu}_e \\ \pi^+ \to \mu^+ + \nu_\mu \\ Z \to \tau\tau \end{cases}$$

$$\tau \to \nu_\tau + \begin{matrix} e\ \nu_e \\ \mu\ \nu_\mu \end{matrix}$$

$$\tau \to 5\pi + \nu_\tau$$

$$m(\nu_e) \lesssim 2\ eV$$

$$m(\nu_\mu) \lesssim 195\ keV$$

$$m(\nu_\tau) \lesssim 23\ MeV$$

PROPERTY OF

SPIN FIELDS, PARTICLES

HELICITY PARTICLES

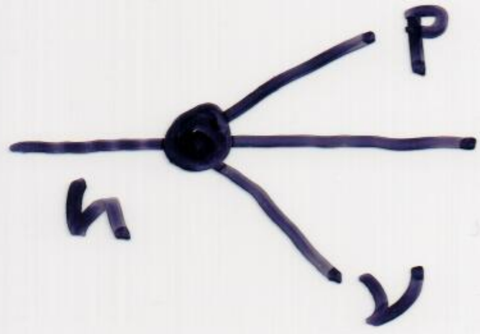
(SPIN ALONG $\vec{p}$) (NOT $\mathcal{L}$-INVARIANT)

CHIRALITY $\doteq$ **HANDEDNESS** FIELDS

V-A

$$W^{+}_{\mu}\,\bar{\nu}\,\gamma_{\mu}\underbrace{(1+\gamma_5)\,e}_{\text{LH FIELD } e_L}$$

ν_L → THIS PARTICLE, IF MASSLESS HAS HELICITY −1



e^- → THIS PARTICLE HAS HELICITY

$$-|\vec{v}| = -\frac{|\vec{p}|}{E} \xrightarrow[m_e \to 0]{} -1$$

HELICITY, HANDEDNESS $S=\frac{1}{2}$ PART.

'DIRAC' 4-SPINOR

$g_{\mu\nu} = \mathrm{DIAG}(1,-1,-1,-1)$

D.E.: $\boxed{(i\partial\!\!\!/ - m)\psi(x) = 0}$ $\quad \partial\!\!\!/ \equiv \gamma_\mu \frac{\partial}{\partial x_\mu}$

F: $\psi(x) = \int \frac{d^4p}{(2\pi)^2} e^{-ipx} \psi(p)$

M.E.: $\boxed{(p\!\!\!/ - m)\psi(p) = 0}$ $\quad p\!\!\!/ \equiv \gamma_\mu p^\mu$

$*\ (p\!\!\!/ + m) \Rightarrow (p^2 - m^2)\psi(p) = 0$

$\psi(p) = 0$ EXCEPT AT $p^2 = m^2$

CONSTRUCT ALL SOLUTIONS:

$\boxed{\psi(p) = (p\!\!\!/ + m)\,\delta(p^2 - m^2)\,\chi(p)}$

$\chi(p)$ ENTIRELY ARBITRARY FOUR-COLUMN SPINOR

HAS 4 NORMALIZED SOLS: $\begin{pmatrix}1\\0\\0\\0\end{pmatrix}\begin{pmatrix}0\\1\\0\\0\end{pmatrix}\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\begin{pmatrix}0\\0\\1\\0\end{pmatrix}$

2: $p_0 = \pm\sqrt{m^2 + |\vec{p}|^2}$ ($p_0 > 0$ OF $(p\!\!\!/ + m)\psi(p) = 0$)
in field theory

2: SPIN "UP", "DOWN"

EXERCISE:

PROVE THAT $\Psi(p)$ BEHAVES AS SPIN 1/2 UNDER ROTATIONS

SPIN (of a NON-RELATIVISTIC "PAULI" 2-SPINOR)

$\xi = \begin{pmatrix} x \\ x' \end{pmatrix}$ $\boxed{\sigma_3 \equiv \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}}$

$\sigma_3 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\sigma_3 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = -\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

"3-DIRECT": SPIN 'UP' SPIN 'DOWN'

HELICITY (of a LORENTZ-COVARIANT "DIRAC" 4-SPR)

$\psi = \begin{pmatrix} x \\ * \\ \circ \\ \bullet \end{pmatrix}$ $\boxed{\vec{\Sigma} \equiv \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}}$ $\left(\vec{\Sigma} = i \vec{\gamma} \times \vec{\gamma} \text{ in Bj-D} \right)$

$P_\pm^2 = P_\pm$

$P_+ P_- = 0$

$$\boxed{P_\pm \equiv \frac{1}{2}\left(1 \pm \frac{\vec{\Sigma} \cdot \vec{p}}{|\vec{p}|}\right)}$$

HELICITY 'PROJECTORS' ALONG (AGAINST) DIRECT. OF MOTION

$P_\pm \, \psi(p)$: $(\pm)$ HELICITY

HAVE SPIN $\begin{matrix} \text{ALONG} \\ \text{AGAINST} \end{matrix}$ $\vec{p}$

BRING THEM TO REST $\begin{pmatrix} x \\ * \\ \circ \\ \bullet \end{pmatrix} \to \begin{pmatrix} \bar{x} \\ \bar{x}' \\ 0 \\ 0 \end{pmatrix}$

$\vec{p} = (0, 0, p_3)$ $P_\pm \to \frac{1}{2}(1 \pm \sigma_3)$ $\nearrow \uparrow \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $\searrow \downarrow \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

HANDEDNESS (CHIRALITY)

$$\boxed{\mathcal{L}_W = g_W \left(W_\mu \bar{e} \gamma^\mu (1+\gamma_5) \nu + W_\mu^\dagger \bar{\nu} \gamma^\mu (1+\gamma_5) e \right)}$$

$$P_{L/R} \equiv \frac{1}{2}(1 \pm \gamma_5) \qquad P_i^2 = P_i \qquad P_L P_R = 0$$

L AND R HAND CHIRAL PROJECTORS

$$\nu_L \equiv \frac{1}{2}(1+\gamma_5)\nu, \qquad e_L \equiv \frac{1}{2}(1+\gamma_5) e$$

ONLY LEFT HANDED LEPTONS AND QUARKS (OR THEIR RIGHT-HANDED ANTIPARTICLES) PARTICIPATE IN CHARGED WEAK INTERACS.

(FOR SPINORS SATISFYING DIRAC EQ.)

HANDEDNESS = HELICITY + $O(m/E)$

(THUS THE MISNOMER)

EXERCISE : PROVE IT

AND QUANTIFY THE ADMIXTURE

FOR THE SOPHISTICATED

d_μ IS A SPACELIKE DIRECTION $(d^2=-1, d\cdot p=0)$

WHICH IS PURE $\vec{d}$ IN p_μ REST SYSTEM

SPIN PROJECTOR ALONG d_μ IS

$$P_\pm(d) = \frac{1}{2}(1 \pm \not{d}\gamma_5)$$

$\vec{d} \parallel \vec{p}$ WOULD BE HELICITY

THESIS IS :

$$(\not{p}-m)\, \boxed{P_{-H}}\, (\not{p}+m)$$

$$= (\not{p}-m) \boxed{\left[P_L + O\left(\frac{m}{E}\right) P_R \right]} (\not{p}+m)$$

INSURES WE ARE IN D-SOL.

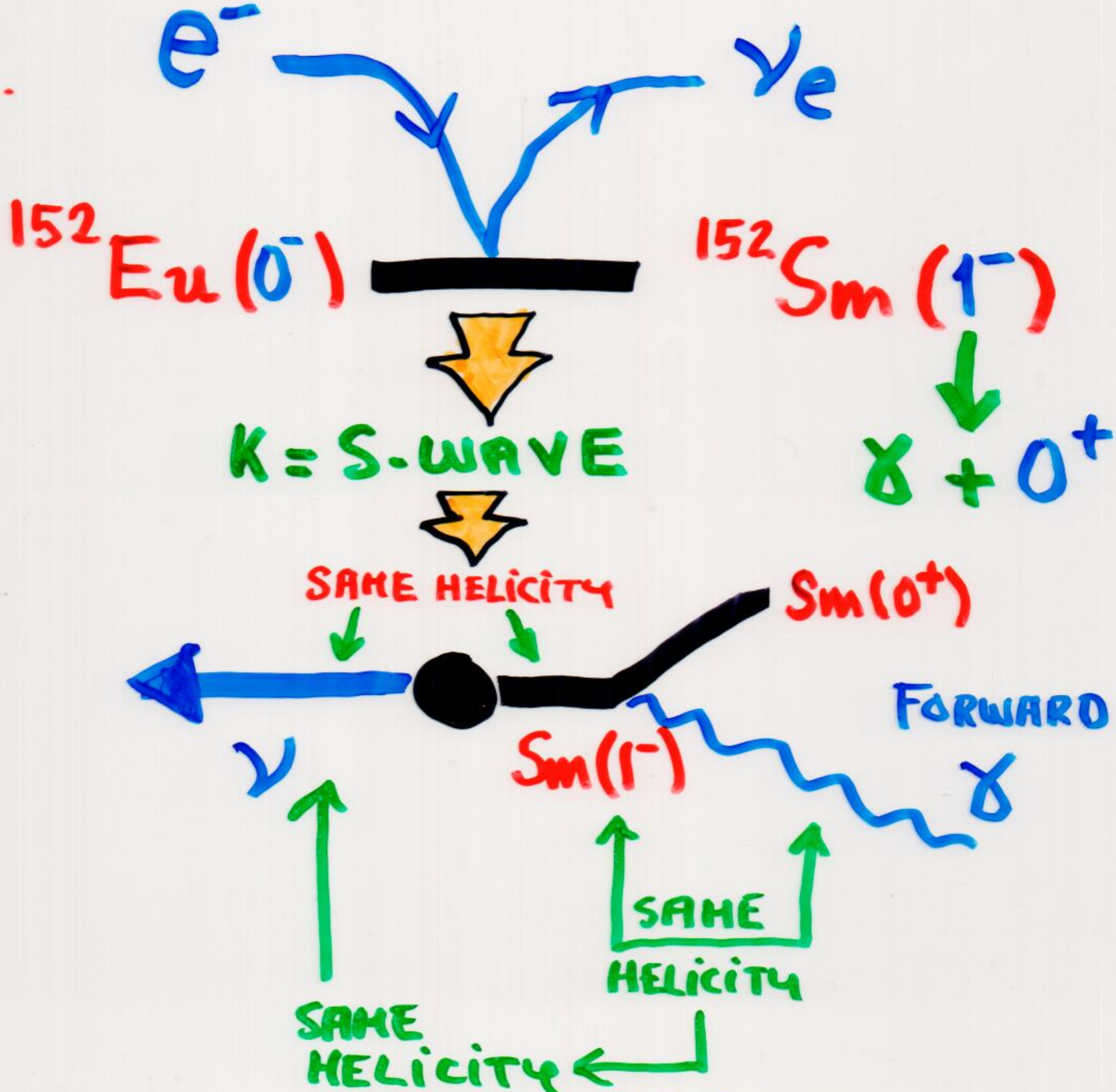
COMPUTE

FOR THE UNSOPHISTICATED

$$\psi(x) = \sqrt{\frac{E+m}{2E}}\, e^{-ipx} \begin{pmatrix} \chi \\ \frac{\vec{\sigma}\vec{p}}{E+m}\chi \end{pmatrix}$$

ν_e MADE IN K-CAPTURE HAS HELICITY $\simeq -1$

[GOLDHABER - GOODRINS - SUNYAR]



$m_\nu \to 0$: LH

HELICITY

$\bar{\nu}$ $\approx +1$

ν ≈ -1

e^+ (μ^+- decay $\frac{E}{m_e} \gg 1$) $+1$

e^- (β decay) $\approx -\left|\frac{v_e}{c}\right|$

LH!

CHARGED WEAK

V-A CURRENTS $\gamma_\mu(1+\gamma_5)$

FECUND FIELDS

$$(1+\gamma_5)\psi_e \quad (1+\gamma_5)\psi_\nu$$

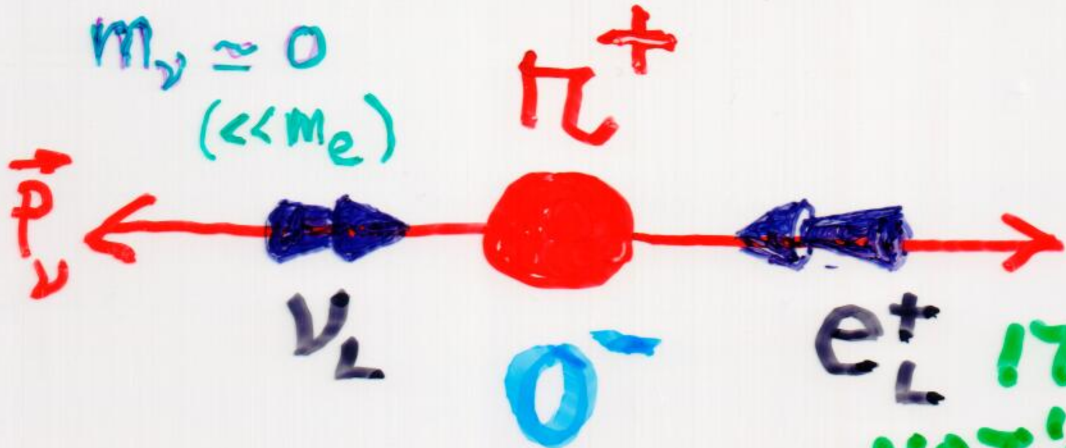
HAVE NO OTHER COUPLINGS

iF $m_\nu = 0$ LEFT-HANDED "WEYL SPINORS"

TWO CRUCIAL EXPERIMENTS

(1) SEMILEPTONIC

$$\boxed{\pi \to e\nu \,/\, \pi \to \mu\nu}$$



IT DOES NOT 'WANT' TO

$$R = \frac{\Gamma(\pi \to e\nu)}{\Gamma(\pi \to \mu\nu)}$$

$$\simeq \boxed{\left(\frac{m_e}{m_\mu}\right)^2} \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2$$

↳ HELICITY SUPPRESSION

(2) PURELY LEPTONIC

MEASUREMENT OF THE e^+ POLARISATION IN MUON DECAY BY MEANS OF BHABHA SCATTERING

J. DUCLOS *, J. HEINTZE, A. DE RUJULA ** and V. SOERGEL
CERN, Geneva, Switzerland

Received 12 February 1964

DIRAC

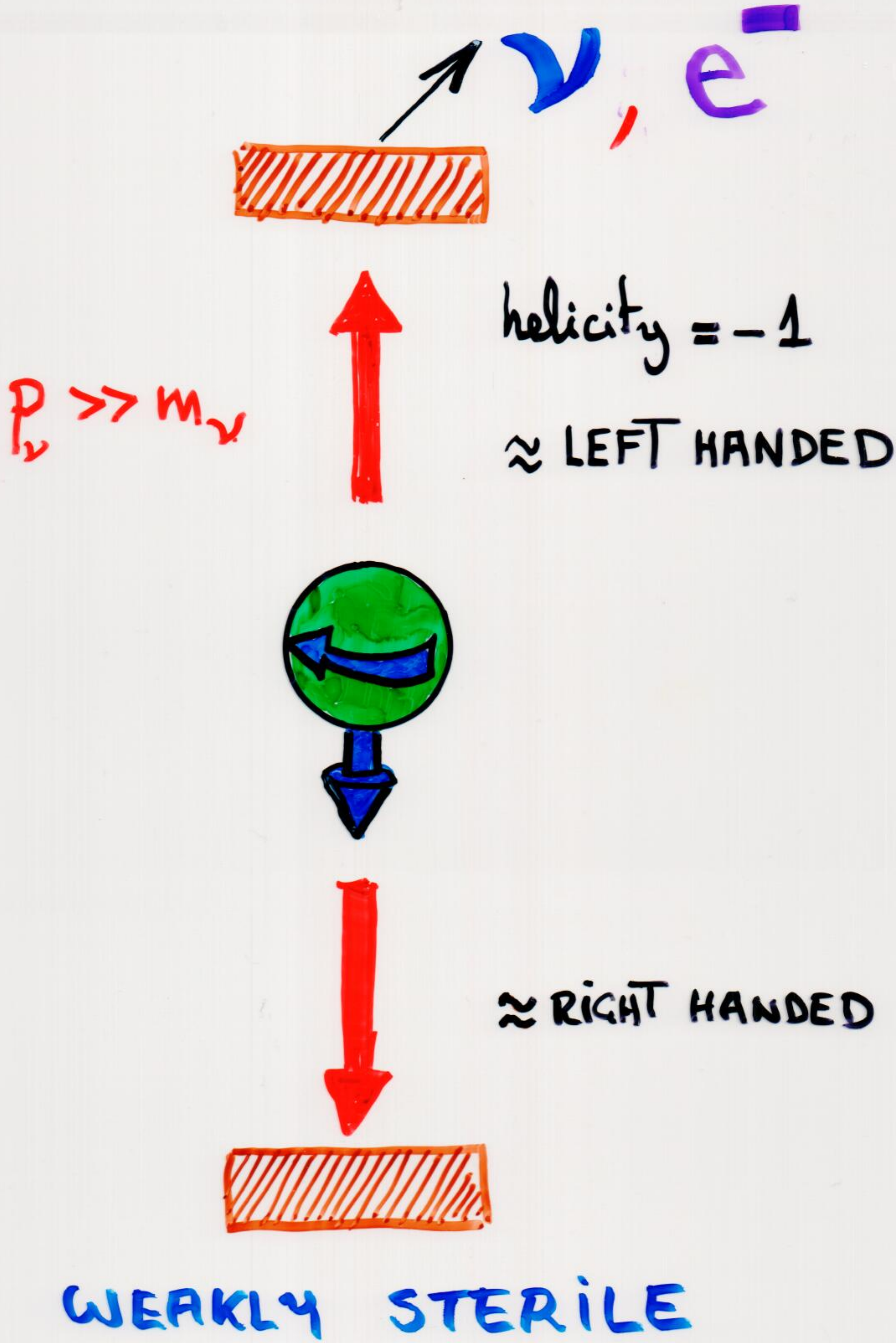
and

MAJORANA

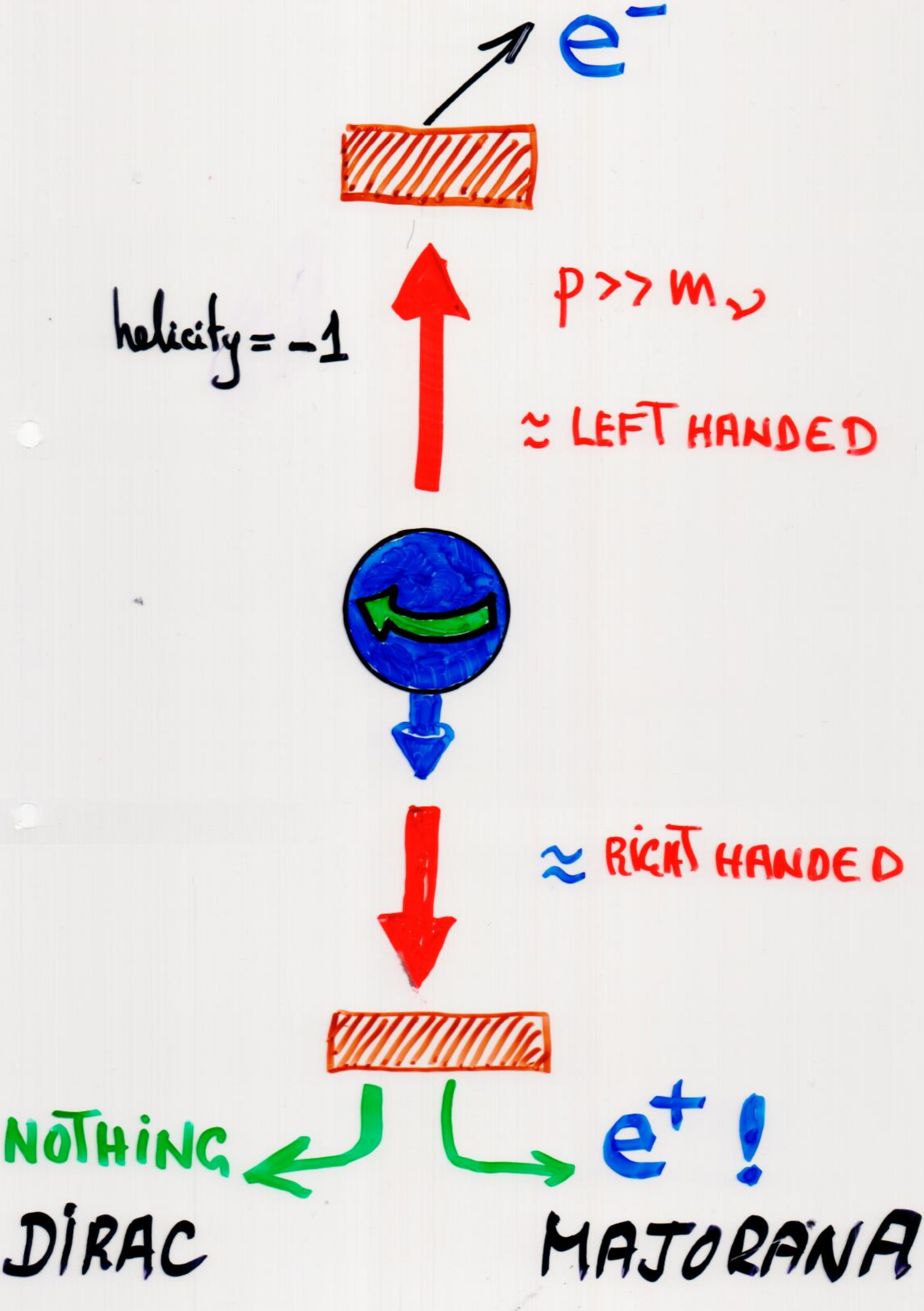
Fermions

$m \neq 0$

ν_e



ν, e^-
helicity = −1
$p_\nu \gg m_\nu$
≈ LEFT HANDED
≈ RIGHT HANDED
WEAKLY STERILE



e^-
$p >> m_\nu$
helicity = −1
≈ LEFT HANDED
≈ RIGHT HANDED
NOTHING
e^+ !
DIRAC
MAJORANA

DIRAC, MAJORANA NEUTRINOS

NATURE IS QUITE CREATIVE WITH (ELECTRICALLY NEUTRAL) BOSONS

$\pi^0, \rho^0, \gamma \ldots$ ARE THEIR OWN ANTIPARTICLE

$J/\psi, \Upsilon, Z, H$ (BOSON NUMBER NOT CONSERVED)

$K^0, D^0, \ldots$ ARE DISTINCT FROM THEIR

B^0 ANTIPARTICLES $(\overline{K^0}, \overline{D^0}, \ldots)$

BOSON NUMBER ~ CONSERVED (S, C, ...)

¿ COULD NATURE BE EQUALLY CREATIVE WITH (ELECTRICALLY NEUTRAL) FERMIONS [i.e. ν's]

THEORETICALLY POSSIBLE

$$\begin{cases} \nu \neq \bar{\nu} & (\text{'DIRAC'}) \\ \nu = \bar{\nu} & (\text{'MAJORANA'}) \end{cases}$$

DIRAC-QUANTUM CONSERVATION

$\psi = \psi_e$

$(i\not\partial - m)\psi = 0 \qquad \bar{\psi} \equiv \psi^\dagger \gamma_0 = \psi^{*,T}\gamma_0$

EQS. OF MOTION $\xleftarrow{\text{E.L.}}$ $\mathcal{L}(x) = \bar{\psi}(i\not\partial - m)\psi$

$\psi \to e^{i\alpha}\psi \;:\; \mathcal{L} \to \mathcal{L}$

NOETHER'S THEOREM

$\exists\, j_\mu \ni \partial^\mu j_\mu(x) = 0$

HERE $j_\mu = i\bar{\psi}\gamma_\mu\psi$

$j \cdot A \Rightarrow j_\mu = \bar{e}(p')\left[(p+p')_\mu + \frac{i\sigma_{\mu\nu}(p-p')^\nu}{2m}\right]e(p)$

NO FLOW TO/FROM INFINITY $\quad \frac{dQ}{dt} = 0 \qquad Q = \int d^3x\, j_0(x) = N(e^+) - N(e^-)$

FREE $\mathcal{L}$: IN F.T. JUST #S, DID NOT INTROD E-CHARGE

PROJECTING $\mathcal{L}_D$ ON CHIRAL EIGENSTATES

$$P_{L,R} = \frac{1}{2}(1 \pm \gamma_5) \; ; \quad P_i^2 = P_i \; , \; P_L P_R = 0$$

$$\psi = \frac{1}{2}(1+\gamma_5)\psi + \frac{1}{2}(1-\gamma_5)\psi \equiv \psi_L + \psi_R$$

$$\mathcal{L} = \bar{\psi}(i\not\partial - m)\psi = (\bar{\psi}_L + \bar{\psi}_R)(i\not\partial - m)(\psi_L + \psi_R)$$

$$\gamma_\mu (1+\gamma_5) = (1-\gamma_5)\gamma_\mu \qquad 2\bar{\psi}_L = \bar{\psi}(1-\gamma_5)$$

$$2\bar{\psi}_R = \bar{\psi}(1+\gamma_5)$$

$$\boxed{\mathcal{L} = \psi_L i\not\partial \psi_L + \psi_R i\not\partial \psi_R - m\bar{\psi}_L \psi_R - m\bar{\psi}_R \psi_L}$$

- $m = 0$ TWO INDEP: $i\not\partial \psi_L = 0 = i\not\partial \psi_R$
  2-DIM. "WEYL"
- m "COUPLING CONSTANT" 

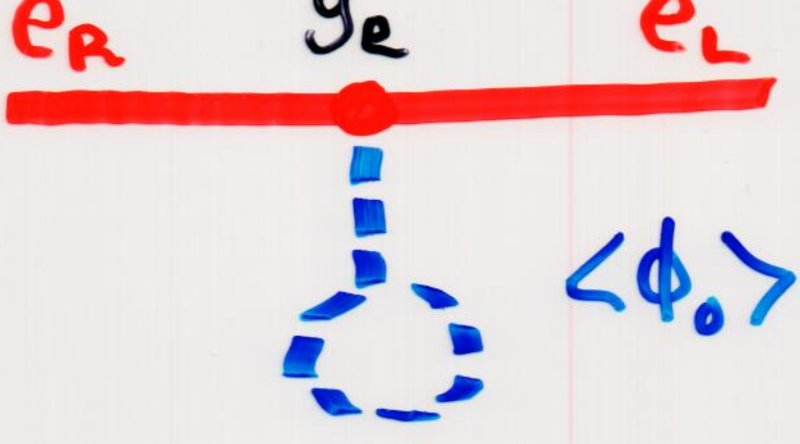
  OF AN INTERACTION $\psi_L \leftrightarrow \psi_R$

THAT IS WHAT FERMION MASSES **ARE**

IN THE STANDARD MODEL !!

$$\mathcal{L}_H = g_e \bar{e}_R (\phi^+, \phi^0) \begin{pmatrix} \nu \\ e \end{pmatrix}_L + h.c.$$

$$\xrightarrow{\text{SSB}} \ni \underbrace{g_e \langle \phi^0 \rangle}_{m_e} \bar{e}_R e_L + h.c.$$



BACKWARDS :

IF YOU TAKE TWO 2-COMP 'WEYL' SPINORS

$$i\not{\partial} e_R = 0$$

$$i\not{\partial} e_L = 0$$

AND ADD A COUPLING

$$m \bar{e}_R e_L + h.c.$$

YOU GET A 4-COMPONENT 'DIRAC' SPINOR

$$e = e_R + e_L$$

WITH A 'DIRAC' MASS m

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi - m \bar{\psi} \psi$$

$\exists \neq$ POSSIBILITY : MAJORANA

$\mathcal{L}_D(\nu)$

DESCRIBES 4 GUYS

$\nu_L \quad \nu_R$

$\bar{\nu}_L \quad \bar{\nu}_R$

ONLY TWO INTERACT

WOULD IT NOT BE
MORE NATURAL IF
ONLY 2 EXISTED ?

GUIDANCE (NO SPIN) NEUTRAL MESONS

K

$$K_0 = \frac{1}{\sqrt{2}}\left[K_1 + iK_2\right] \qquad \bar{K}_0 = K_0^* = \frac{1}{\sqrt{2}}\left[K_1 - iK_2\right]$$

$$\mathcal{L} = \partial_\mu K_0^* \partial^\mu K_0 - m^2 K_0^* K_0 + \text{S.I.}$$

$(\Box + m^2)\varphi = 0$

$(p^2 - m^2)\varphi_p = 0$

$\mathcal{L} \to \mathcal{L}$ as $K_0 \to e^{i\varphi} K_0$ IF K_0S ARE THE ONLY S-PARTICLES IN A PROCESS $pp \to pp + n(K_0) + \bar{n}(\bar{K}_0)$

THEIR "Q" IS CONSERVED $(n = \bar{n})$

π

$\pi_0^* = \pi_0$ "IS ITS OWN ANTIPARTICLE"

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \pi_0 \partial^\mu \pi_0 - m^2 \pi_0^2\right) + \text{ETC}$$

$\mathcal{L} \not\to \mathcal{L}$ as $\pi_0 \to e^{i\alpha}\pi_0$ $\qquad pp \to pp + \pi_0$ OK

ALL WE WANT IS TO IMITATE THE $\pi_0 = \pi_0^*$ 'NEUTRALITY' FOR A CHIRAL FIELD

- THE CONVENTIONALLY DEFINED CHARGE CONJUGATION OPERATION FLIPS CHARGES [e.g. ELECTRIC Q, LEPTON, BARYON #] LEAVING EVERYTHING ELSE [eg CHIRALITY] UNCHANGED.

$$CC[\nu_L] \rightarrow \bar{\nu}_L$$ WHICH, IF $\exists$, IS STERILE

- RECALL 'DIRAC SEA' : e^+ = ABSENCE OF e^-

IT WOULD HAVE BEEN MORE NATURAL TO SAY

e^+_L = ABSENCE (ANTIPART.) OF e^-_R

- IT IS THIS TYPE OF PART-ANTIPART "MAJORANA CONJUGATION" THAT ONE NEEDS TO WORKOUT

(BOOKS OFTEN CONFUSING/CONFUSED)

"MAJORANA" CONJUGATION

NOTICE *

$$\text{MC}: \ \psi \rightarrow \psi^c = C\bar{\psi}^T = C\gamma_0^T \psi^*$$

$C = i\gamma_0\gamma_2$ [YOU MAY CHECK $C^T = C^{-1} = C^\dagger = -C$
$\overline{\psi^c} = \psi^T C$]

*

MC: JUST COMPLEX CONJ. BUT FOR (CRUCIAL) COSMETICS:

$$\boxed{\psi_L \xrightarrow{MC} (\psi_L)^c = (\psi^c)_R}$$

[i.e. $((1+\gamma_5)\psi)^c = (1-\gamma_5)\psi^c$]

IN DIRAC CASE $\psi = \psi_L + \psi_R$ ASSUMED INDEPENDENT

"MAJORANA" $\boxed{\psi_R = (\psi_L)^c}$ CONSTRUCT. WITH SAME OBJECT

$$\psi = \psi_L + (\psi_L)^c$$

[* ? MEANINGFUL IF ∃ ν MIXING]

$\Rightarrow$ $\boxed{\psi^c = \psi}$ (ANALOGOUS TO $\pi_0 = \pi_0^*$)

MAJORANA MASS

$\nu_L \quad m \quad \nu_R$

$= \nu_L^c$

$$\mathcal{L}_m = \frac{1}{2}\overline{(\psi_L)^c}\, m\, \psi_L + h.c.$$

$$= \frac{1}{2}\psi_L^T C\, m\, \psi_L + h.c.$$

$$\mathcal{L} \not\to \mathcal{L} \qquad \psi_L \to e^{i\alpha}\psi_L$$

LEPTON # VIOLATED BY MASS TERM

(LIKE FOR m_0)

BUT NOT BY KINETIC TERM

$$\overline{\psi}_L\, i\not\partial\, \psi_L$$

(UNLIKE FOR m_0)

L-VIOLATING AMPLITUDES $\propto M$

IF YOU HAVEN'T UNDERSTOOD THIS IN THE FIRST ROUND, IT IS THE FAULT OF THE "COSMETICS" ($i\gamma_0\gamma_2 \cdots$)

THE PRINCIPLES ARE TRIVIAL:

- A PARTICLE NOT CARRYING ANY "CONSERVED CHARGE" IS THE SAME AS ITS ANTIPARTICLE
- IT HAS ½ AS MANY DEGREES OF FREEDOM / ($P \neq \bar{P}$ PAIR)
- IT CAN BE DESCRIBED BY A REAL, AS OPPOSED TO A COMPLEX FIELD

MAJORANA SUMMARY

- THE LH GUY THAT MAKES e^-

AND

THE RH GUY THAT MAKES e^+

ARE THE TWO HELICITY STATES OF THE SAME GUY

- (IT IS OFTEN WRITTEN THAT ONE IS THE ANTIPARTICLE OF THE OTHER BUT!
THEY DO NOT HAVE OPPOSITE ν-NUMBER,
THEY HAVE OPPOSITE CHIRALITY)

- SINCE FOR SPINORS THE CHIRALITY-FLIP 'OPERATOR' IS THE MASS
GOING FROM ONE GUY TO THE OTHER REQUIRES A "MASS INSERTION"

DIRAC vs MAJORANA

$\exists$ "e" IN 4 "DIRAC" STATES

$e^-_{L,R}$ + antiparticles $e^+_{L,R}$

WE ONLY KNOW $\exists$ 2 "ν_e" STATES

ν^e_L : $\nu^e_L + n \to p + e^-$

"ν^e_R" : "ν^e_R" $+ p \to n + e^+$

WE DO **NOT** KNOW **WHETHER**

$\exists$ 4 ν_e "DIRAC" STATES

ν^e_L, $\bar{\nu}^e_R$ WITH WEAK INTERACS

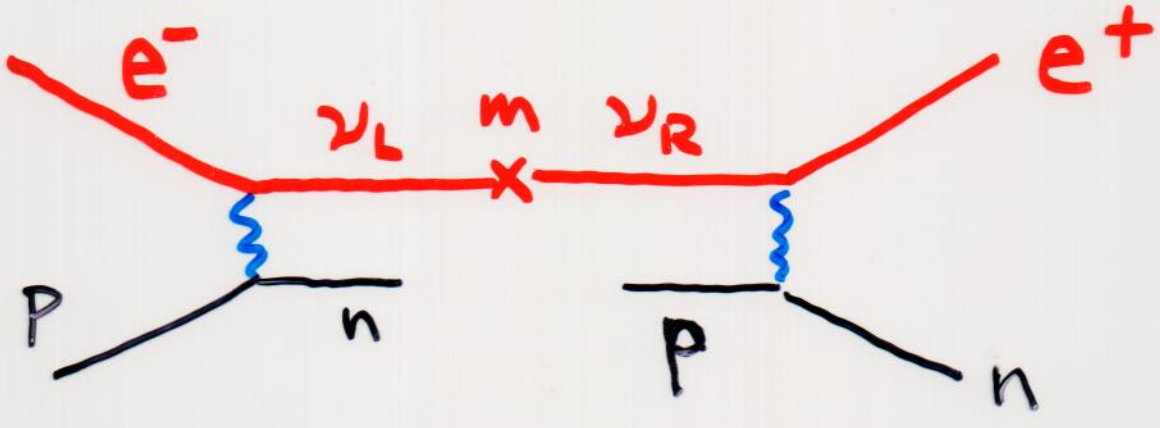
$\bar{\nu}^e_L$, ν^e_R WITHOUT THEM

OR $\exists$ ONLY 2 "MAJORANA" STATES

ν^e_L, ν^e_R : ~~A PART.-ANTIPART~~

CHIRAL PAIR

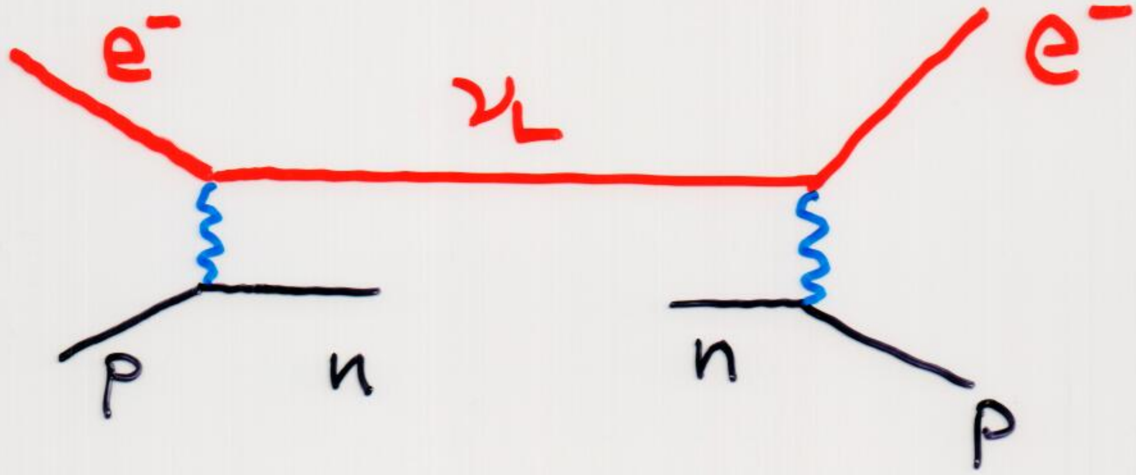
YOU CAN HAVE:



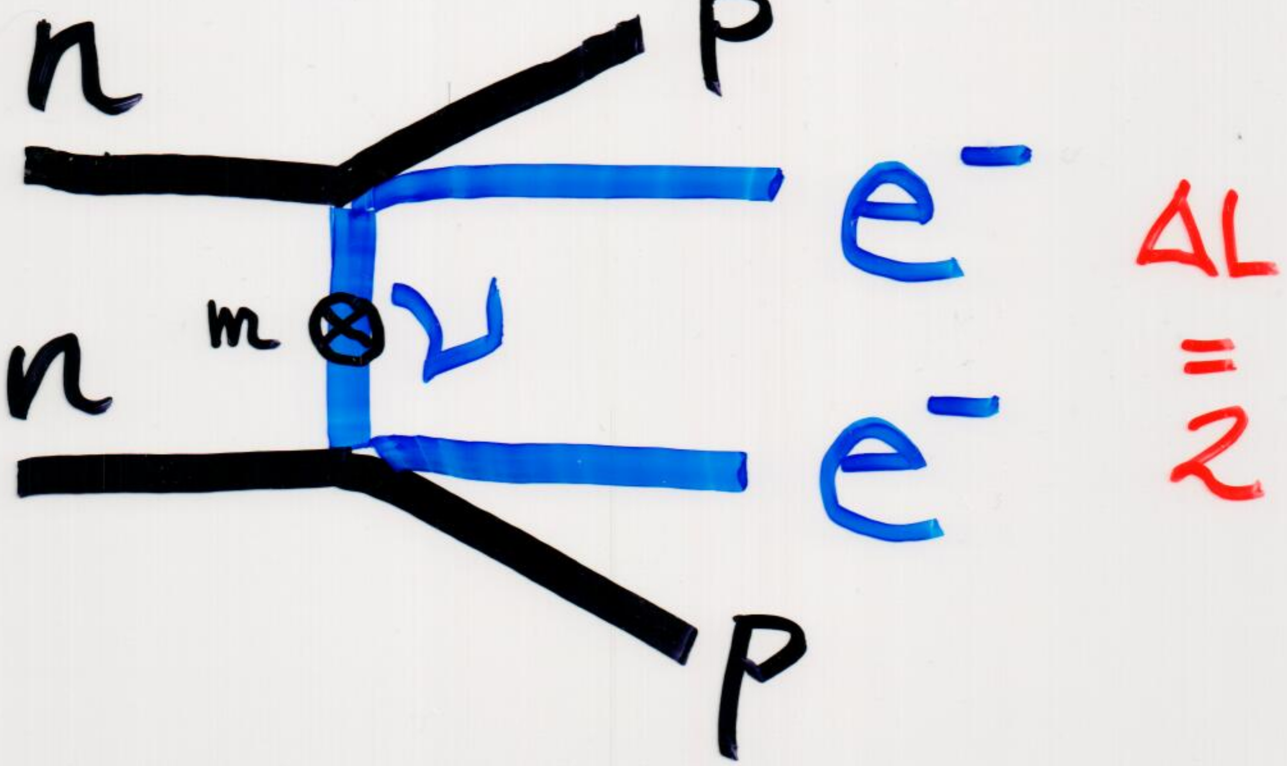
FOR MAJORANA NEUTRINOS

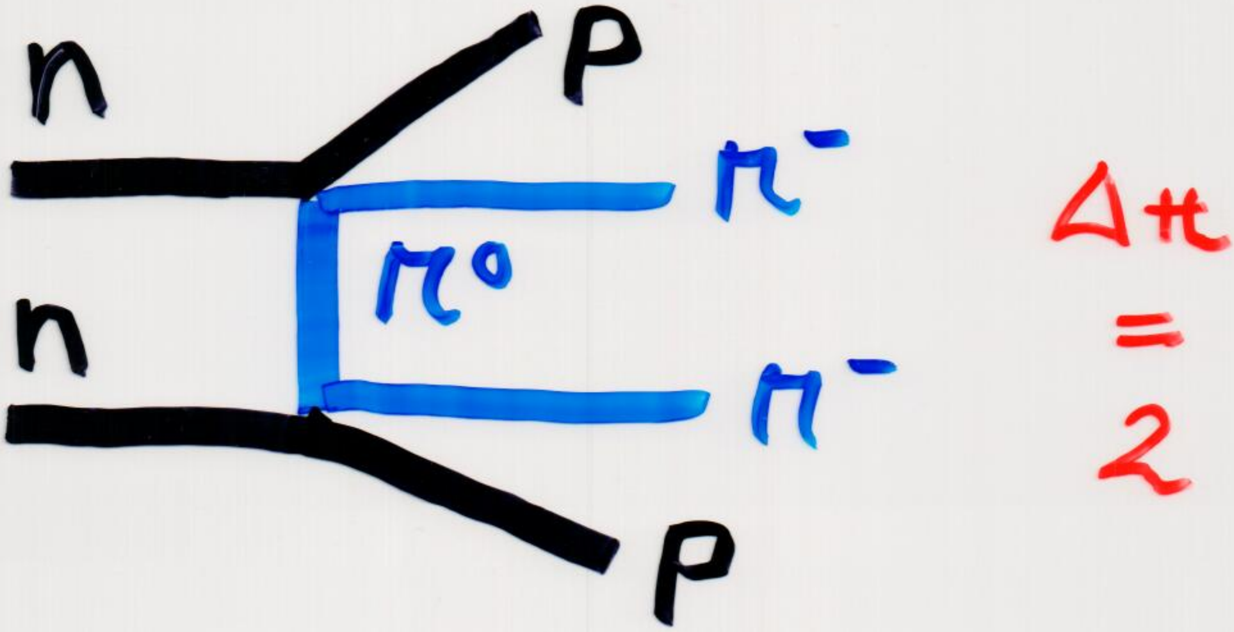
BUT IT IS ONLY m/E IN AMPLITUDE

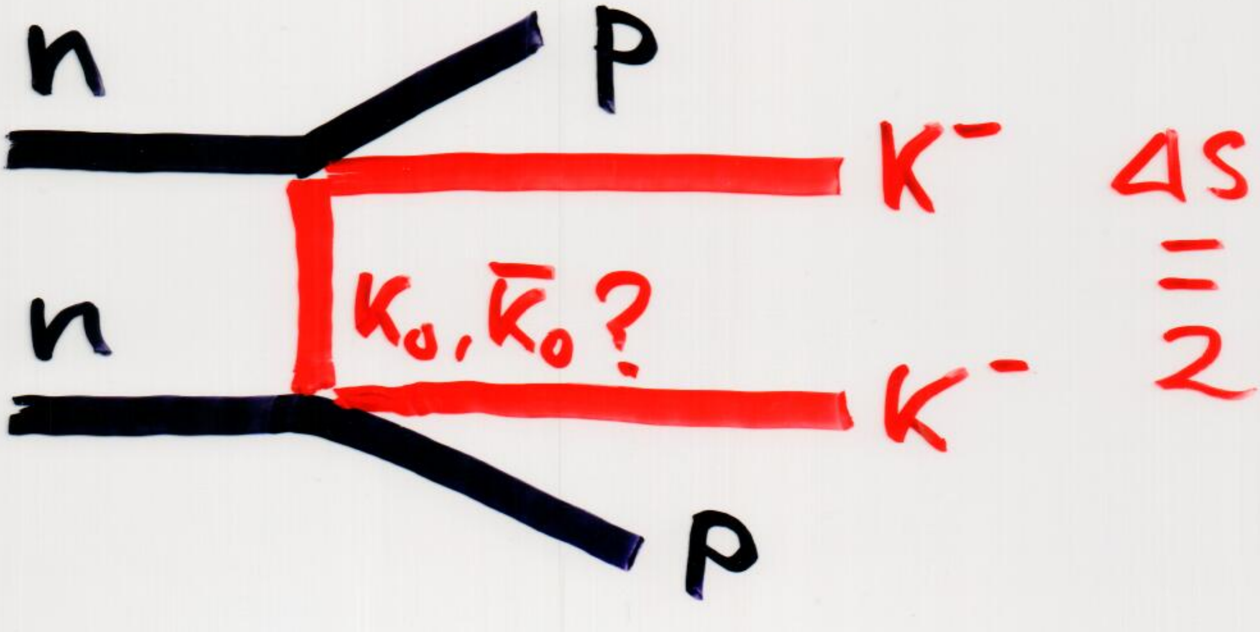
OF



THAT IS WHY IT IS SO IMPORTANT TO TRY AND DO THIS (AT VERY SHORT DISTANCES AND) VERY LOW ENERGIES

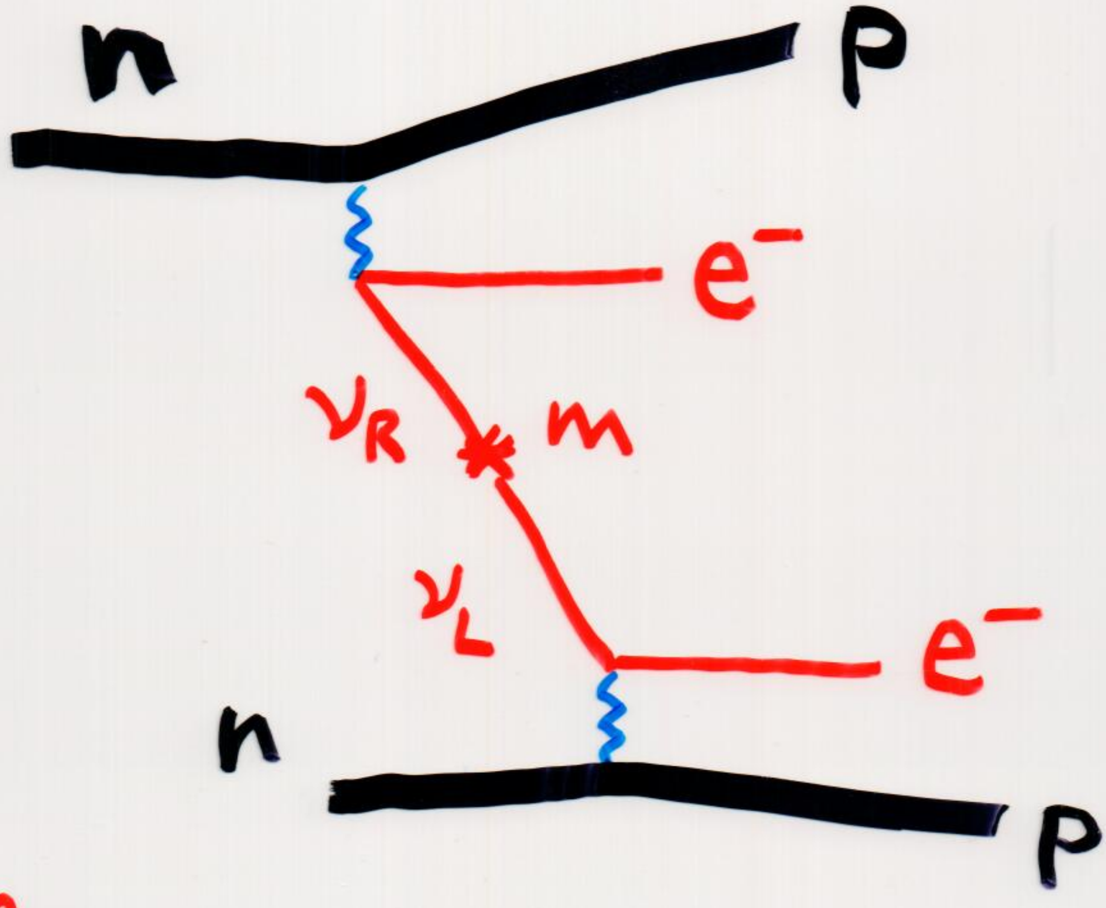

n
p
e^-
m
ν
n
e^-
p
$\Delta L = 2$


n
p
π^-
π^0
n
π^-
p
$\Delta \pi = 2$


n
p
K^-
$K_0, \bar{K}_0$?
n
K^-
p
$\Delta S = 2$

ν-LESS $\beta\beta$-DECAY

$(N, P) \rightarrow (N-2, P+2)$



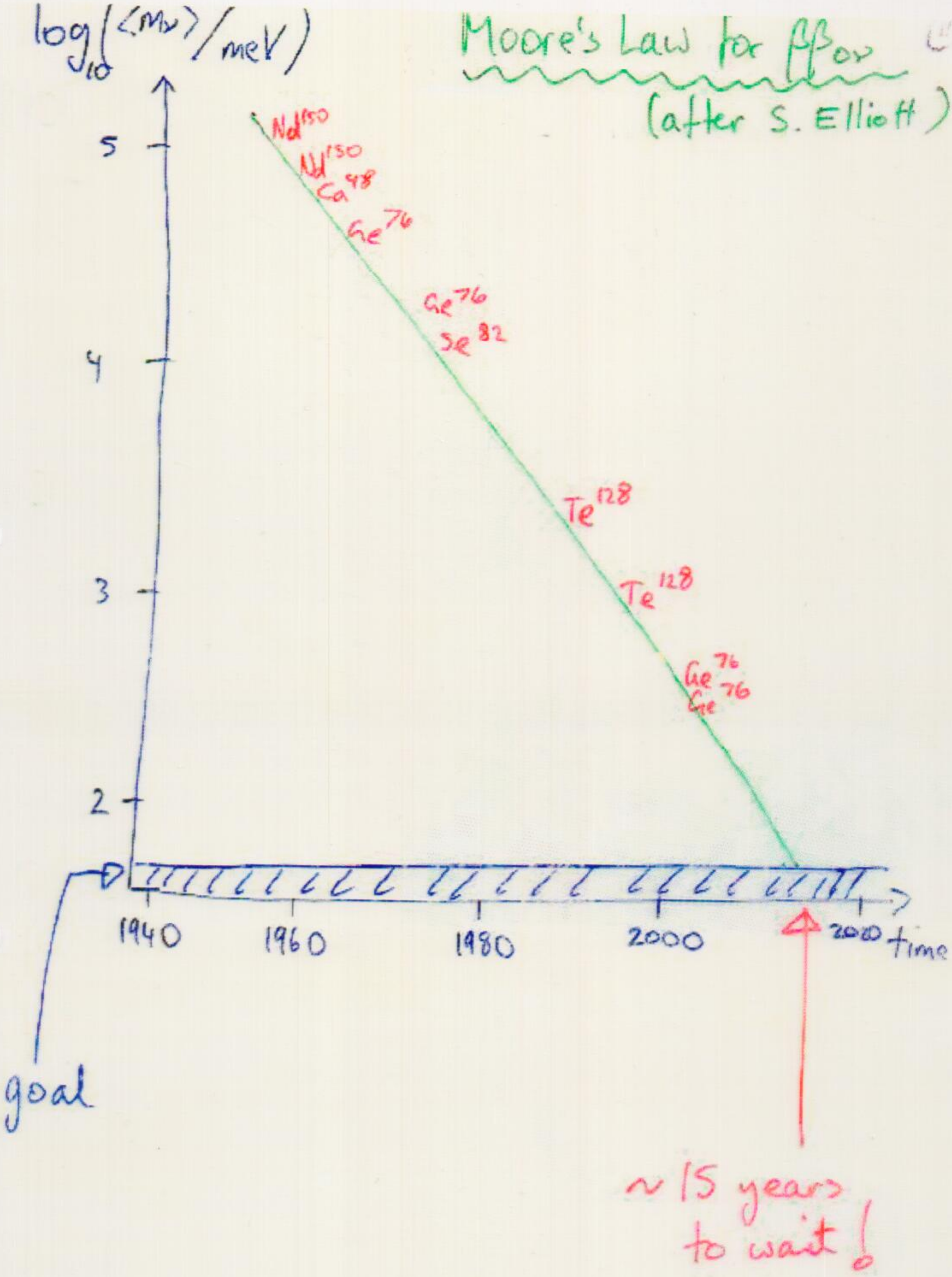
e.g.

$\tau_{1/2}(^{76}Ge) > 2\,10^{25}\,y$

$\langle m_\nu [MAJ] \rangle \leq 0.4\,eV$

Moore's Law for $\beta\beta_{0\nu}$

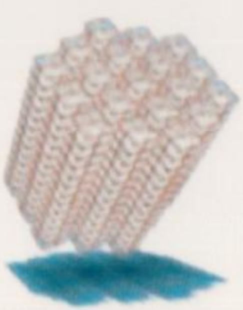
(after S. Elliott)



$$\langle m_\nu \rangle \equiv \sum |U_{ei}|^2 m_i e^{2i\delta_{ei}}$$

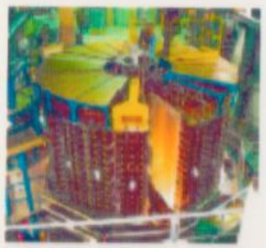
$0\nu\beta\beta$ decay Experiments - Efforts Underway

CUORE



EXO200

NEMO



Collaboration	Isotope	Technique	mass ($0\nu\beta\beta$ isotope)	Status
CANDLES	Ca-48	305 kg CaF_2 crystals - liq. scint	0.3 kg	Construction
CARVEL	Ca-48	$^{48}CaWO_4$ crystal scint.	~ tonne	R&D
GERDA I	Ge-76	Ge diodes in LAr	15 kg	Operating
II		Point contact Ge in LAr	30-35 kg	Construction
MAJORANA DEMONSTRATOR	Ge-76	Point contact Ge	30 kg	Construction
1TGe (GERDA & MAJORANA)	Ge-76	Best technology from GERDA and MAJORANA	~ tonne	R&D
NEMO3	Mo-100 Se-82	Foils with tracking	6.9 kg 0.9 kg	Complete
SuperNEMO Demonstrator	Se-82	Foils with tracking	7 kg	Construction
SuperNEMO	Se-82	Foils with tracking	100 kg	R&D
LUCIFER	Se-82	ZnSe scint. bolometer	18 kg	R&D
AMoRE	Mo-100	CaMoO4 scint. bolometer	50 kg	R&D
MOON	Mo-100	Mo sheets	200 kg	R&D
COBRA	Cd-116	CdZnTe detectors	10 kg 183 kg	R&D
CUORICINO	Te-130	TeO_2 Bolometer	10 kg	Complete
CUORE-0	Te-130	TeO_2 Bolometer	11 kg	Commissioning
CUORE	Te-130	TeO_2 Bolometer	206 kg	Construction
KamLAND-ZEN	Xe-136	2.7% in liquid scint.	380 kg	Operating
NEXT-100	Xe-136	High pressure Xe TPC	80 kg	Construction
EXO200	Xe-136	Xe liquid TPC	160 kg	Operating
nEXO	Xe-136	Xe liquid TPC	~ tonne	R&D
DCBA	Nd-150	Nd foils & tracking chambers	20 kg	R&D
SNO+	Nd-150	0.1% ^{nat}Nd suspended in Scint	55 kg	Construction

GERDA

MAJORANA

CANDLES

Wilkerson 2013

"THEORY"

of

ν

MASSES

SM

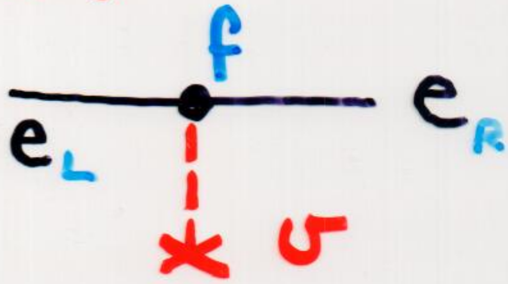
$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$ DOUB. e_R SINGLET

ν_R USELESS

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ v \end{pmatrix}$$ SSB

$$\Delta\mathcal{L} = f\,\bar{L}\,\phi\, e_R$$

$$\Rightarrow f v\, \bar{e}_L e_R = m_e\, \bar{e}_L e_R$$



GUTS

$$SO(10) \supset SU(3) \otimes SU(2)_L \otimes U(1)$$

COMPLETE q, ℓ FAMILY $(+N_R) \in 16$

Inevitably $$\Delta\mathcal{L}' = g\,\bar{L}\,\phi^\dagger N_R$$

$$m_\nu = \frac{(gv)^2}{M_N}$$

$$M_N = \frac{(gv)^2}{m_\nu} \sim \frac{(250\,\mathrm{GeV})^2}{0.01\,\mathrm{eV}}$$

$M_N \sim 6\; 10^{15}\,\mathrm{GeV}$ IS THE SCALE OF (SS) SO(10) GRAND UNIF. !!

Δm_ν^2 OBSERVED BY ATMOSPHERIC, SOLAR ν-DETECTORS ARE AT THE SCALE $\sim$ EXPECTED FOR m_ν's IN (SUPERSYMETRIC) GRAND UNIFIED THEORIES

MECHANISM 1979

MINKOWSKI, GLASHOW...

GELLMANN, RAMOND,

SLANSKI ; YANAGIDA

MORE RECENT IDEAS ON

m_ν's OR LEPTONIC WEAK

MIXING ANGLES ARE ALL

"HOW TO..."

or

"WHY NOT..."

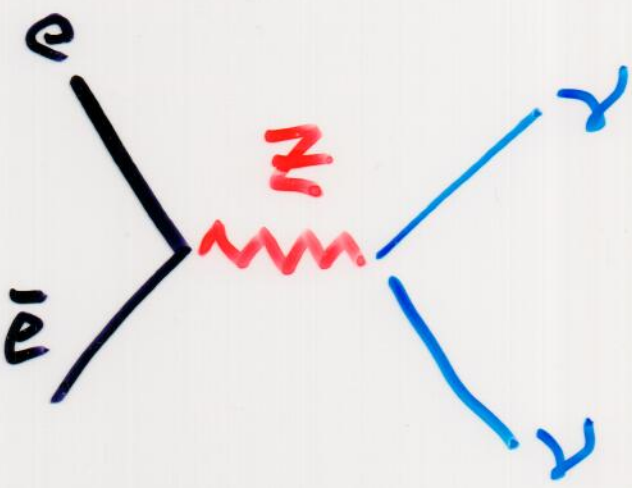
... DO THIS OR THAT

eg : MAXIMAL ν-MIXING

RECENT EXCEPTION ?

✓ LIMITS

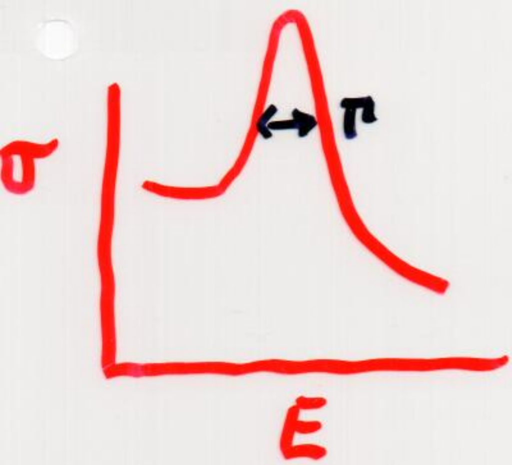
✓ NUMBERS



"LIGHT ν":

$Z \to 2\nu$

POSSIBLE



$$\Gamma_{\mathrm{INV}} = \Gamma - \Gamma_{\mathrm{vis}}(q, \ell)$$

+ OVERALL S.M. FIT

$$\mathcal{N}\left[\begin{matrix}\text{LIGHT}\\\text{FECUND}\end{matrix}\right] = 2.99 \pm 0.01$$

QED is a "GAUGE" theory

$$\psi$$

$$\psi \to \psi e^{i\alpha}$$ $N[e^+ + e^-]$ CONSERVED

$$\psi \to \psi e^{i\alpha(x)}$$

$$\to \exists\, \gamma : A_\mu$$

$$m(\gamma) = 0$$

AND

$$\mathcal{L}_{int} = -ie\,\bar{\psi}\,\gamma_\mu\,\psi\,A^\mu$$

The SM : GAUGE th. ALSO OF STRONG, WEAK INTERACS.

e.g : WEAK INTERACTIONS OF QUARKS, LEPTONS SPECIFIED

in particular :

" FLAVOUR MIXING IN WEAK INTERACTIONS "

GAUGE THEORIES $\Longleftrightarrow$ D

e.g. TWO FAMILIES

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

WHY $\mathcal{M}$[MIXING] IS UNITARY ?

(WHY DID I WRITE IT REAL (ORTHOGONAL))

FASCINATING HISTORICAL DEV.

- M.E.$(\mu \to e\nu\nu) \simeq$ M.E$(n \to pe\nu)$

  "UNIVERSALITY" ($\equiv$ WHO KNOWS WHY)

- $|\text{M.E}(\Lambda \to pe\nu)|^2 \sim \frac{1}{20} |\text{M.E.}(n \to pe\nu)|^2$

  "CABIBBO UNIV." $\bar{u}_L \gamma_\mu (d \cos\theta + s \sin\theta)_L$

- GELL.MANN GLASHOW $\to$ SCNCs!!

- BJORKEN.GLASHOW : WROTE SOLUTION $\bar{c}_L \gamma_\mu (-d \sin + s \cos)$

- GLASHOW / ILIOPOULOS / MAIANI

  REALIZED THE SOLUTION WAS THE SOLUTION

QED $\mathcal{L}_e = \bar{\Psi}(i\not{\partial} - m)\Psi$

BY DECREE, IMPOSE "LOCAL G. INV."

$\mathcal{L} \to \mathcal{L} : \Psi \to e^{i\alpha(x)}\Psi(x)$

$\exists A_\mu : A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \alpha$

$\mathcal{L} = \bar{\Psi}(i\not{D} - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

$D_\mu = \partial_\mu - ieA_\mu$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

F.F : $\exists\ \gamma \ni q^2 = 0 ; S = 1 ; \epsilon_\mu^{1,2}$

$iej_\mu A^\mu$:

$\bar{\Psi}\gamma_\mu\Psi$

WHY A GAUGE TH?

— THEOREM:

THE ONLY LOCAL CONSISTENT Q.F.Th. (R+Q.M) DESCRIBING S,W,EM INTS AS THEY ARE IS THE STANDARD (GAUGE) MODEL

- OR AN EXTENSION THEREOF
- OR A # THEORY OF WHICH THE SM IS AN "EFFECTIVE" LOW. ENERGY REALIZATION

(AS IN GUTS OR STRINGS)

$\boxed{SU(2)_L} \supset SU(3) \otimes SU(2)_L \otimes U(1)$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \equiv D_L$$

$$D_L \to e^{i\vec{\alpha}(x)\frac{\vec{\sigma}}{2}} D_L$$

BY DECREE : $\mathcal{L}$ INVARIANT NON-ABELIAN LOCAL G.T.

$\Rightarrow \exists \vec{B}_\mu \qquad \mathcal{L}_B = -\frac{1}{2}\vec{B}_{\mu\nu}\vec{B}^{\mu\nu}$

$$\vec{B}_{\mu\nu} = \partial_\mu \vec{B}_\nu - \partial_\nu \vec{B}_\mu + g\,\vec{B}_\mu \times \vec{B}_\nu$$

$\mathcal{L}_B$:

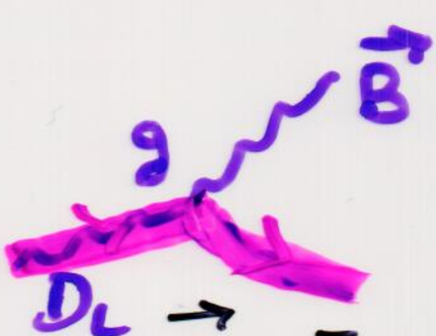
 g^0

 g^1

 g^2

$\mathcal{L}_D$: $\bar{D}_L\, i\partial\!\!\!/\, D_L \to \bar{D}_L\, i D\!\!\!\!/\, D_L$

$$D_\mu = \partial_\mu - ig\frac{\vec{\sigma}}{2}\vec{B}_\mu$$



$$\vec{j}_\mu = \bar{D}_L \gamma_\mu \frac{\vec{\sigma}}{2} D_L$$

BONUS FROM A GAUGE THEORY:

UNIVERSALITY AUTOMATIC:

(SAME REPRESENTATION

⇒ SAME CHARGE)

$$\bar{D}_L \left[i\not\partial - g \frac{\vec{\tau}}{2} \vec{\not B} \right] D_L$$

$$\bar{D}'_L \left[i\not\partial - g' \frac{\vec{\tau}}{2} \vec{\not B} \right] D'_L$$

$$\Downarrow$$

YOU CANNOT MAKE LAGRANGIAN

GAUGE INVARIANT FOR $g \neq g'$

SINCE THE GAUGE TRANSF. OF $\vec{B}_\mu$

CONTAINS g (OR g')

TWO DOUBLETS : $\begin{pmatrix} u_0 \\ d_0 \end{pmatrix}_L \begin{pmatrix} c_0 \\ s_0 \end{pmatrix}_L$

WHO IS WHO ? : NAME $\Leftrightarrow$ MASS VALUE

INTRODUCE THE MOST GENERAL MASS TERM

(ONLY CONSTRAINT : Q - CONSERVATION

$SU(2)_L$ SPONT. BROKEN $\phi \to \phi_0$)

$$(\bar{u}_0, \bar{c}_0)_R \, M_U \begin{pmatrix} u_0 \\ c_0 \end{pmatrix}_L + h.c.$$

$$(\bar{d}_0, \bar{s}_0)_R \, M_D \begin{pmatrix} d_0 \\ s_0 \end{pmatrix}_L + h.c.$$

DIAGONALIZE : $\mathbb{U}_R, \mathbb{U}_L, \mathbb{D}_R, \mathbb{D}_L$ UNITARY

$$M_U = \mathbb{U}_R^\dagger \begin{pmatrix} m_u & 0 \\ 0 & m_c \end{pmatrix} \mathbb{U}_L$$

$$M_D = \mathbb{D}_R^\dagger \begin{pmatrix} m_d & 0 \\ 0 & m_s \end{pmatrix} \mathbb{D}_L$$

$$\vec{j}_\mu = (\bar{u}_0, \bar{d}_0)_L \gamma_\mu \frac{\vec{\sigma}}{2} \begin{pmatrix} u_0 \\ d_0 \end{pmatrix}_L$$
$$+ (u_0 \to c_0, d_0 \to s_0)$$

$\sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $\quad j^+_\mu = j^{1+i2}_\mu$ COUPLES TO W^+_μ

$$j^+_\mu = (\bar{u}, \bar{c})_L \gamma_\mu K \begin{pmatrix} d \\ s \end{pmatrix}_L$$

j^0_μ FLAVOUR-DIAG. GIM

$K = U^\dagger_L D_L$ UNITARY

2 gen-case : ALL PHASES CAN BE REABSORBED IN INOCUOUS REDEF. OF q-FIELDS : K ORTHOGONAL

$$K = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

PARAMETERS

$$\left.\begin{aligned} U_{n\times n} &\to 2n^2 \\ UU^\dagger = 1 &\to -n^2 \end{aligned}\right\} n^2 \text{ params}$$

UP + DOWN FIELD REDEF

eg $c \to e^{i\varphi_c}$

$$\left.\begin{aligned} &-2n \text{ parameters} \\ &[\text{BUT ONE IS GLOBAL}] \end{aligned}\right\} -(2n-1)$$

$$n^2 - 2n + 1 = (n-1)^2$$

ROTATIONS : $\frac{1}{2} n(n-1)$ ANGLES

$(n-1)^2 - \frac{n(n-1)}{2}$ CP-viol. PHASES

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & 4c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}$$

$$U_M = U\,D$$

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

$$\tau_{\beta\beta} \propto 1/|m_{\beta\beta}|^2 \qquad m_{\beta\beta} = \sum_i U_{ei}^2\, m_i$$

V

OSCI-
LLATI-
ONS

$c, u, t \ (Q = 2/3)$

$s, d, b \ (Q = -1/3)$

(OF OLD)

NAME ≡ FLAVOUR ≡ MASS ≡ Q NUMB.

THE WEAK INTS. "MIX FLAVOURS"

$$W \cdot (\bar{u}, \bar{c}) \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$P(c \to d) \propto \sin^2\theta$$

$$P(c \to s) \propto \cos^2\theta$$

A TWO-FAMILY EXAMPLE

$$\nu_e = \cos\theta\, \nu_1 + \sin\theta\, \nu_2$$

$$\nu_\mu = -\sin\theta\, \nu_1 + \cos\theta\, \nu_2$$

(OR $\nu_\mu \to \nu_S$, IF DESIRED)

$\nu_{1,2}$ MASS EIGENSTATES

$$m_1 \neq m_2$$

$$\mathcal{L}_W^\ell = g_W W^\mu \left[\bar{e}_L \gamma_\mu \nu_{e,L} + \bar{\mu}_L \gamma_\mu \nu_{\mu,L} \right] + \text{h.c}$$

L WILL PLAY NO ROLE

DIRAC/MAJORANA

WILL PLAY NO ROLE

(m_ν / E_ν NEGLIGIBLE IN ALL CASES TO BE DISCUSSED ANON)

ν OSCILLATIONS

AS I MAKE A ν OF SPECIFIC FLAVOUR [COUPLED TO A PARTICULAR CHARGED LEPTON] IT IS IN A SPECIFIC STATE (PURE 'FLAVOUR')

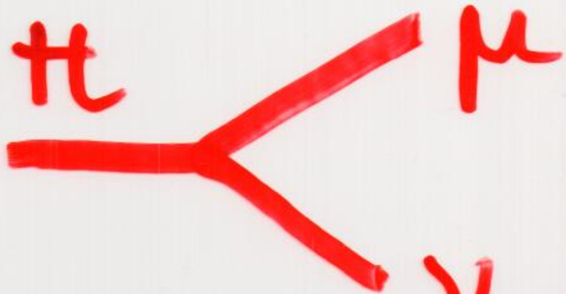
$$\nu_\ell = \cos\theta\, \nu_1 + \sin\theta\, \nu_2 \perp \nu_{\ell'}$$

AS THE STATES OF # MASS PROPAGATE DIFFERENTLY

$$\nu_\ell(x) \neq \nu_\ell(0) \equiv \nu_\ell$$

$|\langle \nu_\ell(x) | \nu_\ell \rangle|^2 < 1$ ℓ-DISAPPEARCE

$|\langle \nu_\ell(x) | \nu_{\ell'} \rangle|^2 > 0$ ℓ' APPEARANCE



$$\nu_\mu = -\nu_1 \sin\theta + \nu_2 \cos\theta$$
$$\equiv \nu_\mu(x=0)$$

$$\nu_\mu(x) = -\nu_1 \sin\theta\, e^{ip_1 x} + \nu_2 \cos\theta\, e^{ip_2 x}$$

$$|\vec{p}_i| = \sqrt{E^2 - m_i^2} \quad : |\bar{p}_1| \neq |\bar{p}_2|$$

$$\boxed{\langle \nu_\mu(x) | \nu_e \rangle \neq 0}$$

$$\downarrow$$

$\nu_1 \cos\theta + \nu_2 \sin\theta$: MAKES ELECTRONS

ORDER $\uparrow\downarrow$

NOTICE EXACT PARALLELISM

$$d_W = \cos\theta_c\, d + \sin\theta_c\, s$$

$$s_W = -\sin\theta_c\, d + \cos\theta_c\, s$$

$$\mathcal{L}_W^q = g_W W^\mu \left[\bar{u}_L \gamma_\mu d_{W,L} + \bar{c}_L \gamma_\mu s_{W,L} \right] + \text{h.c.}$$

In matrix notation

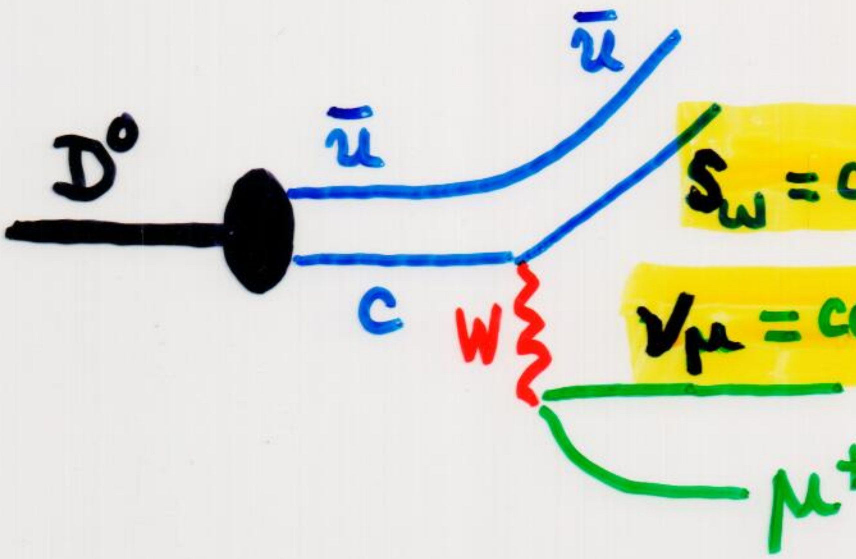
$$(\bar{u}, \bar{c})_L \gamma_\mu \underbrace{\begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}}_{M_{GIM}} \begin{pmatrix} d \\ s \end{pmatrix}_L$$

IS THE CHARGED WEAK CURRENT

M_{GIM} NECESSARILY UNITARY IN A GAUGE THEORY SUCH AS THE S.M.

PUZZLE # 1

$D^0 \to (K^-/\pi^-)\,\mu^+\nu_\mu$, THAT IS :



$s_w = \cos\theta_c\, s - \sin\theta_c\, d$

$\nu_\mu = \cos\theta\, \nu_2 - \sin\theta\, \nu_1$

EVERYBODY SAYS : ν_μ MAY OSCILLATE

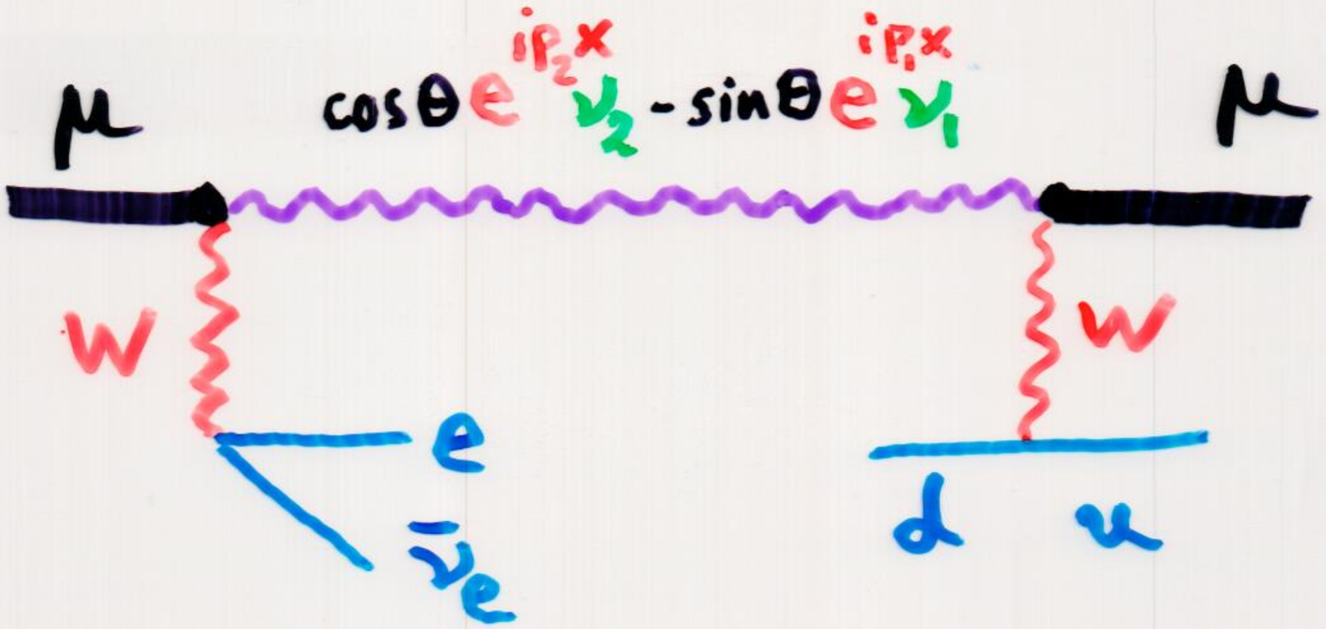
NOBODY SAYS : s_w MAY ($K^-\pi^-$ OSCILLATS)

WHY ! ?

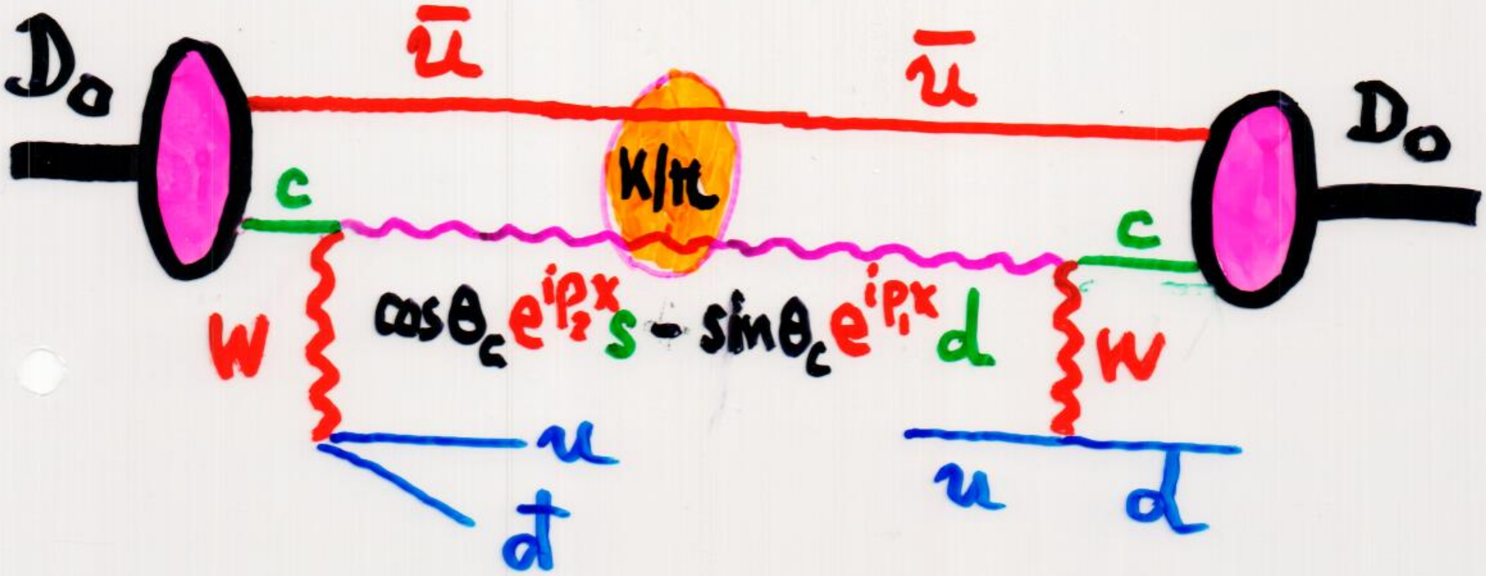
EVERYBODY SAYS
$$\begin{cases} BR(D \to K) \propto \cos^2\theta_c \\ BR(D \to \pi) \propto \sin^2\theta_c \end{cases}$$

NOBODY SAYS
$$\begin{cases} BR(D \to \nu_2) \propto \cos^2\theta \\ BR(D \to \nu_1) \propto \sin^2\theta \end{cases}$$

WHY ! ?



A ν_μ OSCILLATION (DISAPP.) EXPERIMENT



D0
ū
ū
D0
K/π
c
c
W
cosθc e^ip2x s - sinθc e^ip1x d
W
u
d̄
u
d

WHY SAY COMPLETELY DIFF. THINGS ?

FOR $D \to K, \pi$ DECAY
K/π OSCILLATS

OR $D \to \mu \nu_\mu$ DECAY
ν_μ / ν_e OSCILLATS

ANSWERS ?

ν OSCILLS : WHAT ALL BOOKS SAY

e.g. 2-GENERATION MAXIMAL MIX.

$$\nu_e = (\nu_1 + \nu_2)/\sqrt{2}$$
$$\nu_\mu = (\nu_1 - \nu_2)/\sqrt{2}$$

$t=0$ $\xrightarrow{t\text{-EVOL}}$

$$\mu^+ \to e^+ + \bar{\nu}_\mu \rightsquigarrow \begin{cases} \text{NOT ONLY } \bar{\nu}_\mu \to \mu^+ \\ \text{BUT ALSO } \bar{\nu}_e \to e^+ \end{cases}$$

$$\nu_e \rightsquigarrow \begin{cases} \text{NOT ONLY } \nu_e \to e^- \\ \text{BUT ALSO } \nu_\mu \to \mu^- \end{cases}$$

— x DIST →

$$\langle \nu_{e,\mu} | \nu_e(t) \rangle = \frac{1}{2}\Big(\langle \nu_1 | \pm \langle \nu_2 | \Big) \Big(e^{iE_1 t} |\nu_1\rangle + e^{iE_2 t} |\nu_2\rangle \Big)$$

$$= \frac{1}{2}\left(E^{iE_1 t} \pm E^{iE_2 t} \right)$$

$$P_{e\to\mu} = \left| \langle \nu_\mu | \nu_e(t) \rangle \right|^2$$

$$= \boxed{\sin^2 \frac{(E_2 - E_1)}{2} t}$$

BUT t IS NOT MEASURED !

$$P_{e\to\mu} = \sin^2 \frac{(E_2 - E_1)}{2} t$$

AS HARRY LIPKIN PUTS IT:

$$x = v t = \frac{p}{E} t \quad : t = \frac{E}{p} x \simeq x$$

$$E_2 - E_1 = \sqrt{p^2 + m_2^2} - \sqrt{p^2 + m_1^2}$$

$$\simeq \frac{1}{2} \frac{1}{p} (m_2^2 - m_1^2) \left[1 + 0 \frac{m^2}{p^2}\right]$$

$$\boxed{P_{e\to\mu} = \sin^2 \frac{\Delta m^2}{4p} x} \quad \mathrm{I}$$

IF $\nu_{1,2}$ TRAVEL AT $\neq v$

(SAME p, DIFFERENT E)

THEY SHOULD ARRIVE AT THE SAME x AT SOMEWHAT $\neq t$

$$x = v_1 t_1 = \frac{p_1}{E_1} t_1 \qquad x = \frac{p_2}{E_2} t_2$$

$$\mathrm{II} \quad P_{e\to\mu} = \sin^2 \frac{E_2 t_2 - E_1 t_1}{2} = \sin^2 \frac{\Delta m^2}{2p} x$$

THE SECOND RESULT IS CONSISTENT WITH "THE BOOKS'" ARGUMENT. IT IS WRONG

- WHY ARE ALL BOOKS INCONSISTENT?

BECAUSE THEY READ ANSWER #I
IN THE PREVIOUS BOOKS

- WHY DID BOOK #1 USE ANSWER #I?

BECAUSE BRUNO PONTECORVO DIXIT
(AFTER P-GRIBOV!)

- IS THE INCONSISTENTLY DERIVED ANSWER #I THE RIGHT ANSWER?

SURPRISINGLY, YES

- WHAT IS WRONG WITH THE BOOKS?

ABSOLUTELY
EVERYTHING

(BUT THE FINAL
RESULT)

$$\frac{1}{2} m v^2$$

THM:

ν OSCILLATIONS

are

IMPOSSIBLE

due to Q.M.E

QUANTUM-MECHANICAL ENTANGLEMENT

ν QME:

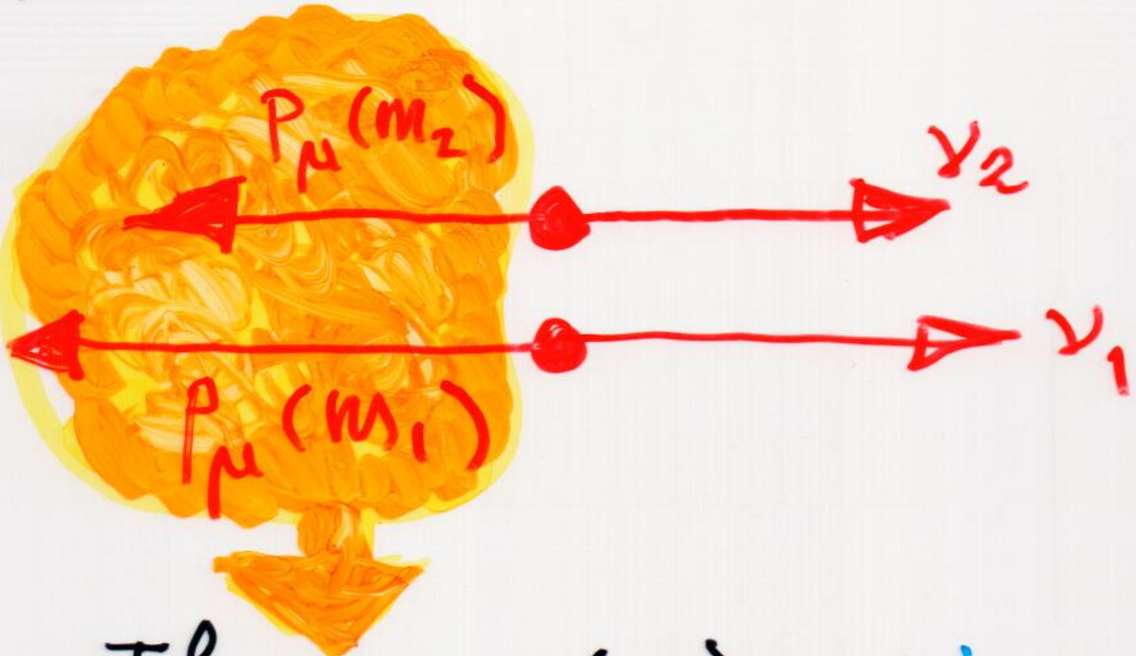
NAUENBERG arXiv 9812441
KAYSER PRD 24, 110 (1981)
OHEN, GLASHOW, LIGETI arXiv 0810.4602

π (rest) $\rightarrow \mu \nu_\mu$

$$\nu_\mu = -s_\theta |\nu_1\rangle + c_\theta |\nu_2\rangle$$

$$|\vec{p}_\mu| \equiv$$

$$\equiv p_\mu = \frac{1}{2m_\pi} \sqrt{(m_\pi^2 - (m_\mu + m_\nu)^2)(m_\pi^2 - (m_\mu - m_\nu)^2)}$$



If $p_\mu = p_\mu(m_2)$ $\quad \nu_\mu \Rightarrow \nu_1$

If $p_\mu = p_\mu(m_1)$ $\quad \nu_\mu \Rightarrow \nu_2$

AND $\langle \mu(p \leftarrow m_2) | \mu(p \leftarrow m_1) \rangle = 0$

NO COHERENCE !!

p-diff DUE TO ≠ ν masses

$$\Delta p_m = p_\mu(m_1) - p_\mu(m_2)$$

eg $m_1 = m_\nu \quad m_2 = 0$

$$\Delta p_m \cong \frac{m_\nu^2}{2m_\pi} \frac{m_\pi^2 + m_\mu^2}{m_\pi^2 - m_\mu^2}$$

$$\sim 1.3\ 10^{-8} eV \left(\frac{m_\nu}{1eV}\right)^2$$

p-UNCERTAINTY DUE TO $\tau_\pi = \frac{1}{\Gamma} = 2.6\ 10^{-8} s$

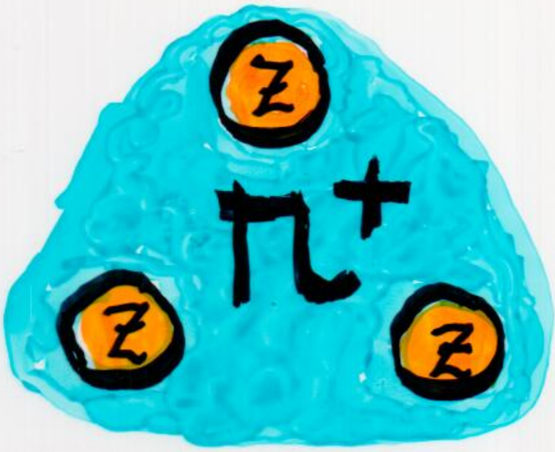
$$\Delta p_\Gamma \cong p(m_\pi + \Gamma) - p(m_\pi)$$

$$\simeq \left(1 + \frac{m_\mu^2}{m_\pi^2}\right) \frac{\Gamma}{2}$$

$$\sim 2\ 10^{-8} eV$$

BOTH TINY !!

π at rest ?



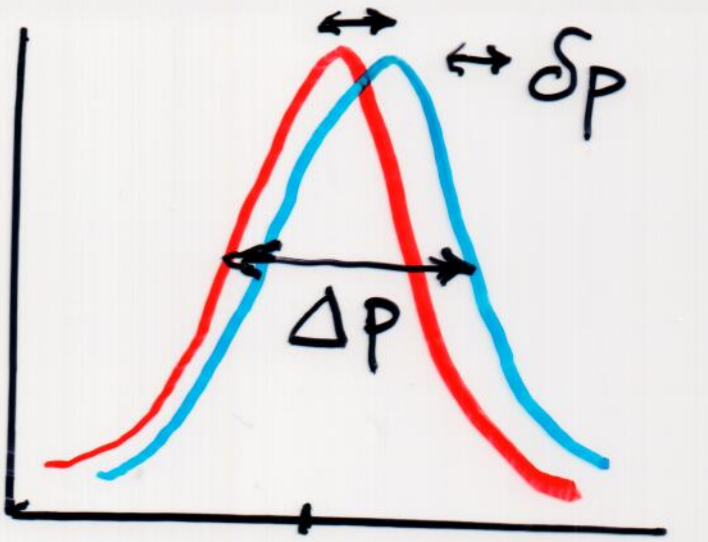
$$\Delta p_L \sim \frac{1}{1\,\text{Å}} \sim 2\,10^3\,\text{eV}$$

$$\Delta p_A \sim \text{“}v\text{”}\, m_\pi \sim Z\alpha\, m_\mu \sim Z\, 1\,\text{MeV}$$

EXTENT OF WAVE PACKET

$$\Delta p_{L,A} \ggg \delta p_{\mu,\pi}$$

m_ν-INDUCED DIFFERENCE OF MOMENTA



GREAT COHERENCE $\langle \mu(p \leftarrow m_2) | \mu(p \leftarrow m_1) \rangle \simeq 1$!

DOING IT RIGHT : QUESTION # 1

(A) STATES OF THE SAME p DIFF. E ?

(AS IN THE BOOKS)

$$E_1^2 = p^2 + m_1^2 \; ; \quad E_2^2 = p^2 + m_2^2$$

(B) STATES OF THE SAME E DIFF. p ?

$$p_1^2 = E^2 - m_1^2 \; ; \quad p_2^2 = E^2 - m_2^2$$

(C) SOMETHING IN BETWEEN, SINCE

$\Delta E \neq 0$ $\Delta p \neq 0$ BECAUSE OF

THE WAY THE SOURCE IS MADE ?

2x2 ν OSCILLATIONS

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$\nu_\mu(0) = \cos\theta\, \nu_2 - \sin\theta\, \nu_1 \equiv \nu_\mu$$

$$\nu_\mu(x) = \cos\theta\, e^{-ip_2 \cdot x} |\nu_2\rangle - \sin\theta\, e^{-ip_1 \cdot x} |\nu_1\rangle$$

$$\boxed{p_i \cdot x = p_i^\mu x_\mu}$$

$$\langle \nu_e | \nu_\mu(x) \rangle = \left(\cos\theta \langle \nu_1 | + \sin\theta \langle \nu_2 |\right) |\nu_\mu(x)\rangle$$

$$= \frac{1}{2} \sin 2\theta \left(e^{-ip_1 \cdot x} - e^{-ip_2 \cdot x} \right)$$

$$P(\nu_\mu \to \nu_e) = |\langle \nu_e | \nu_\mu(x) \rangle|^2$$

$$= \sin^2(2\theta) \sin^2\left[\frac{(p_1 - p_2)\cdot x}{2}\right]$$

"APPEARANCE PROBABILITY"

$$P(\nu_\mu \to \nu_\mu) = 1 - P(\nu_\mu \to \nu_e)$$

"DISAPPEARANCE PROB."

$(P_1 - P_2)^\alpha x_\alpha$? CAREFUL WITH DIFFS. VIGILANT WITH SUMS

1-d motion ⇒ ≃ plane waves arrive

$$(P_1 - P_2)_\alpha = (E_1 - E_2, P_1 - P_2) \; ; \; P_i = |\bar{P}_i|$$

$$x_\alpha = (t, x) \; ; \; x = |\bar{x}|$$

$$= x\left(\frac{1}{v_g}, 1\right)$$ v_g = COMMON GROUP v

$$\frac{E_1}{P_1} = \frac{E_2}{P_2}$$ WHEN SUMMED

ANSATZ NOT TO BE FORGOTTEN

$$x_\alpha = x\left(\frac{E_1 + E_2}{P_1 + P_2}, 1\right) = \frac{x}{2p}(E_1 + E_2, P_1 + P_2)$$

$$\boxed{(P_1 - P_2)^\alpha \cdot x_\alpha \simeq \frac{x}{2p} \Delta m^2}$$

$$\boxed{P_{\mu\to e} = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2}{4p}x\right)}$$

- CORRECT AND EXACT EXPRESSION
- NO APPROXIMATIONS BUT v_g !!
- NO CHEATING WITH VELOCITIES ...
- NO REFERENCE TO TIME

(WHICH IS NEVER MEASURED!)

$$\boxed{L = \frac{4p}{\Delta m^2}}$$

1-RAD OSCILLATION LENGTH

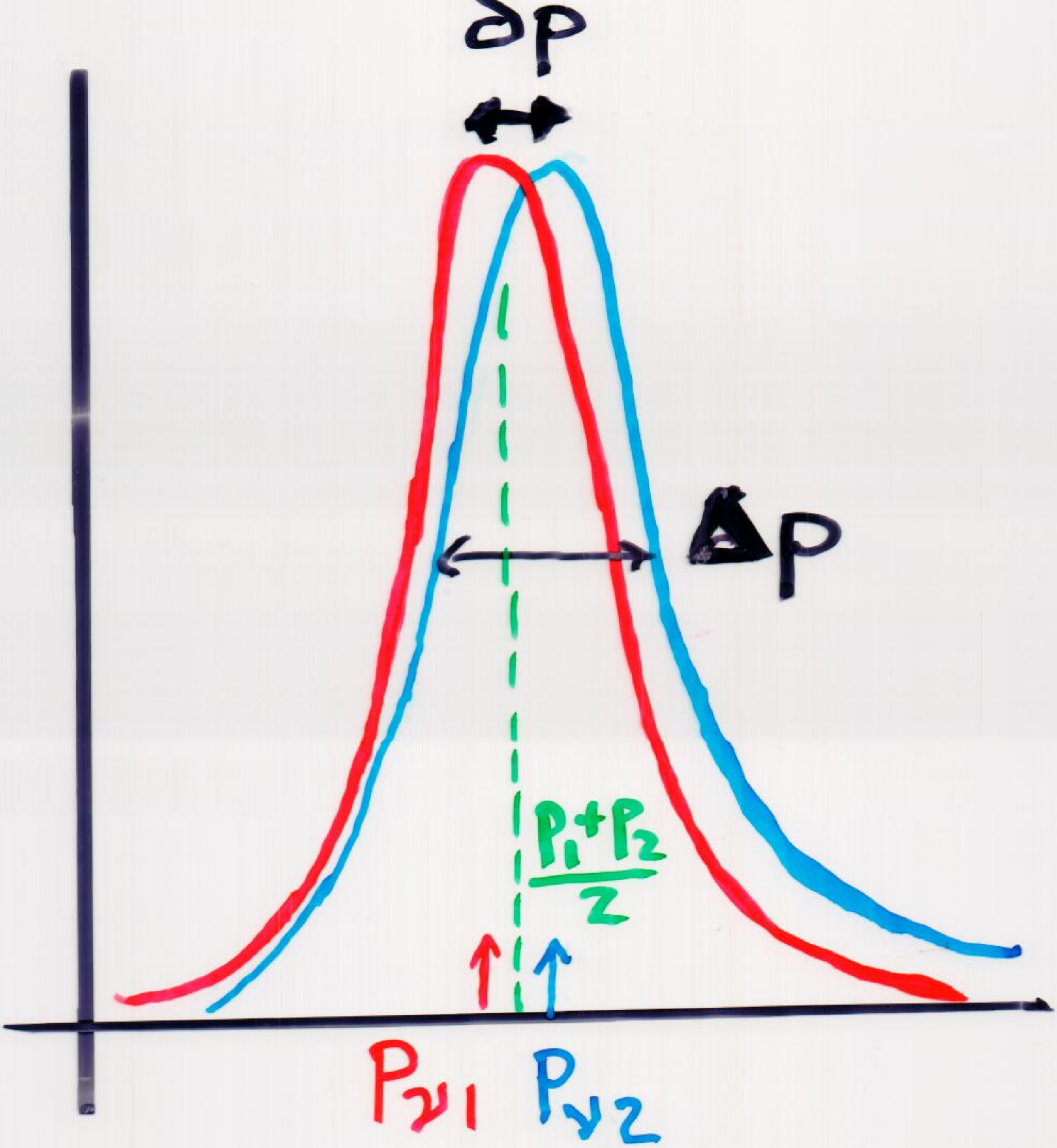
$(x = L \rightarrow \sin^2(x/L) = \sin^2(1))$

IN 'ENGINEER'S' UNITS

$$\boxed{L \simeq 1.27\,\mathrm{km}\left(\frac{p}{\mathrm{GeV}}\right)\left(\frac{\mathrm{eV}^2}{\Delta m^2}\right)}$$

v_g ?

$\vec{p}_{\nu i} = -\vec{p}_{\mu i}$



$\delta p \lll \Delta p$

CONSIDERATION SUCH AS THE ABOVE INDUCE PEOPLE TO DO **SPECIF. CASUISTICS**

- BEAM DUMPS
- DECAY IN FLIGHT: TUNNELS, ATMOSPHERE
- PRODUCTION IN $\odot$'S EXTENSIVE CORE
- K_L/K_S DECAY IN FLIGHT, REGENERAT.
- $B_0/\bar{B}_0$ IN e^+e^- COLLIDER: SIZE OF BEAMS, BEAMCROSSING Δt

CONSTRUCT
WAVE-PACKETS
WITH SPECIFIC $\Delta E, \Delta p$

A LOT OF EFFORT PUT IN THIS

BUT...

$$\Delta m^2 \equiv m_2^2 - m_1^2$$

$$\boxed{P(\nu_\mu \to \nu_e) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2}{4 p_\nu} x\right)}$$

PARAMS

$\theta, \Delta m^2$

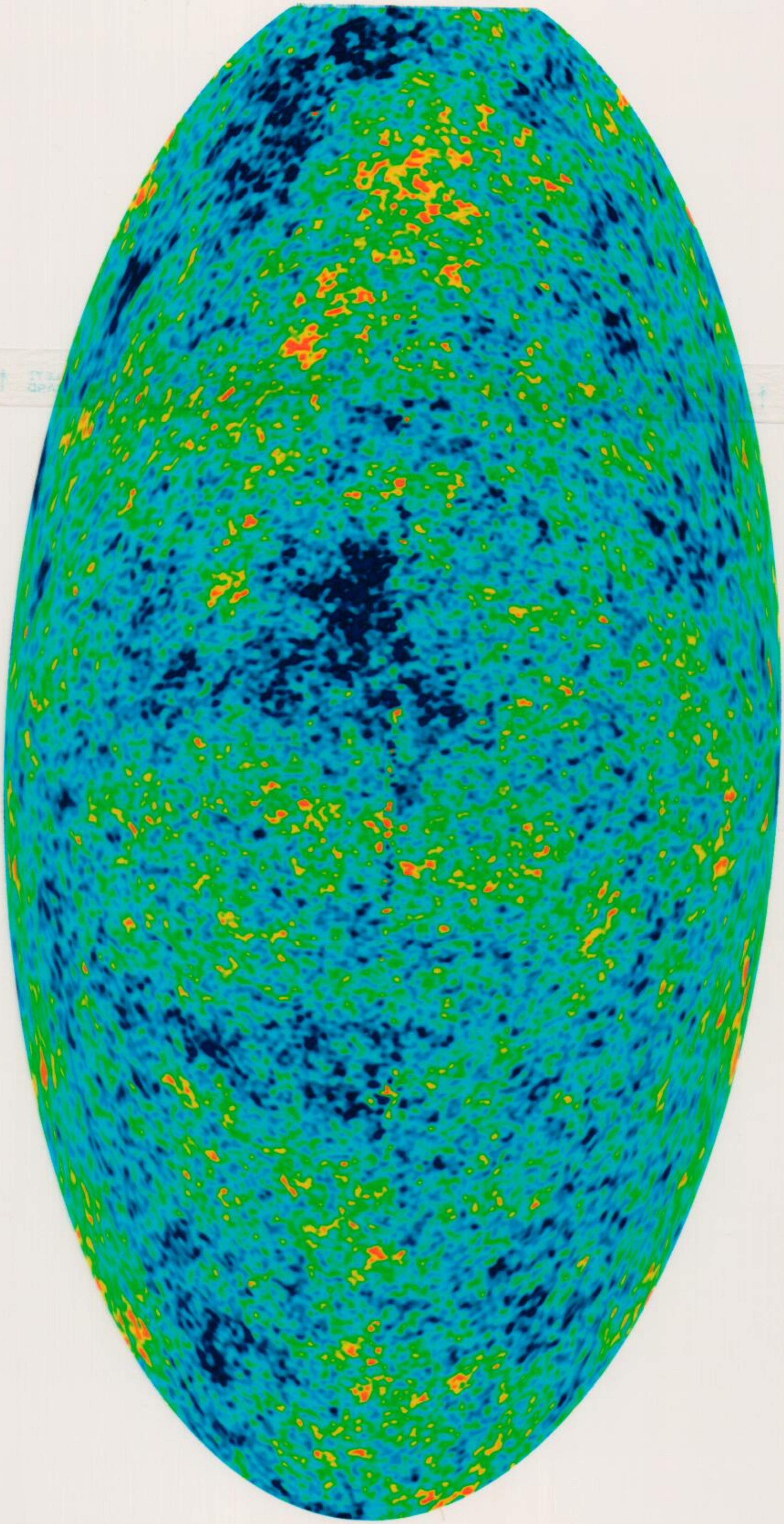
$\downarrow$

x/L

$$L \equiv 1.27\,\mathrm{km}\,\frac{p_\nu}{\mathrm{GeV}}\,\frac{\mathrm{eV}^2}{\Delta m^2}$$

VERY SMALL Δm^2 MEASURABLE

MACRO-SCOPIC QM



NOT
ALWAYS
ENOUGH

TWO MORE DETAILS:

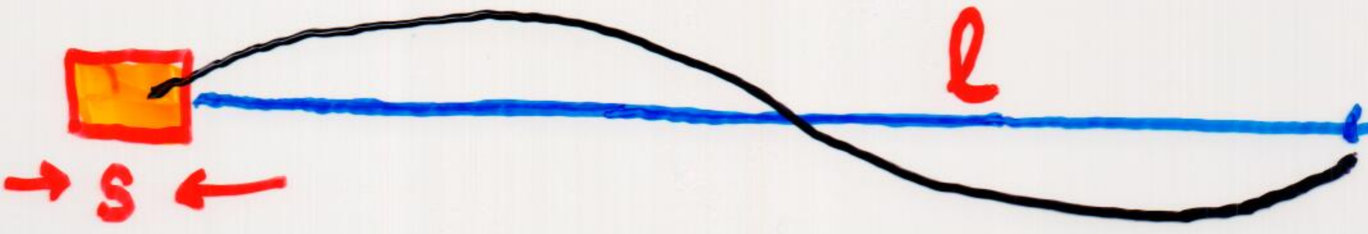
EXTENDED SOURCES

MATTER EFFECTS

AND

K/π OSCILLATION vs
ν_μ/ν_e OSCILLATION
PUZZLE STILL UNRESOLVED?

TAKE SOURCE-SIZE TO BE MUCH SMALLER THAN ν-OSCILLATION LENGTH

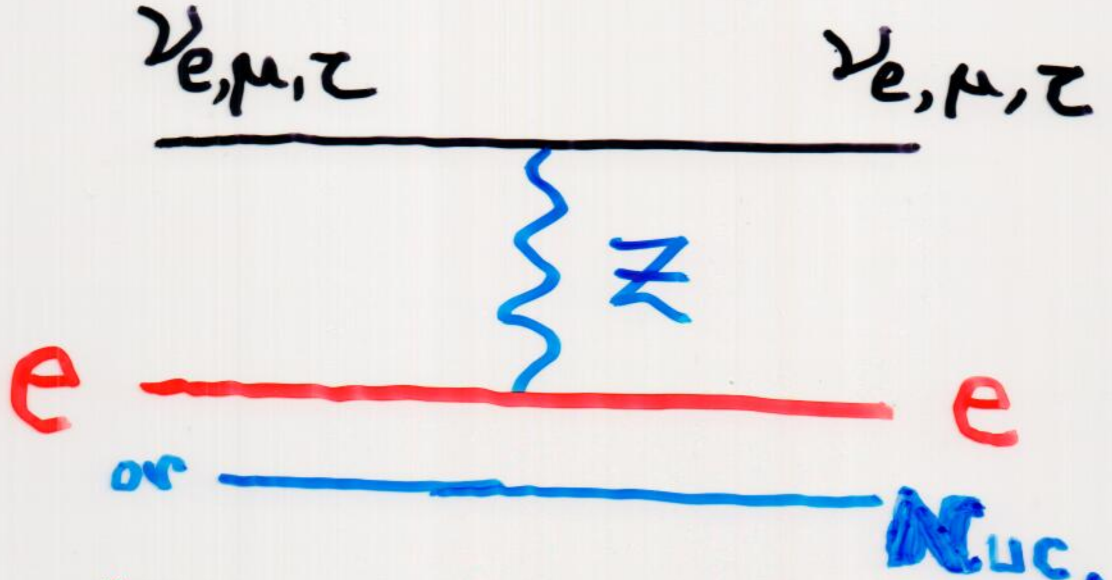


[OTHERWISE INTEGRATE OVER SOURCE LOCATIONS AT THE END]

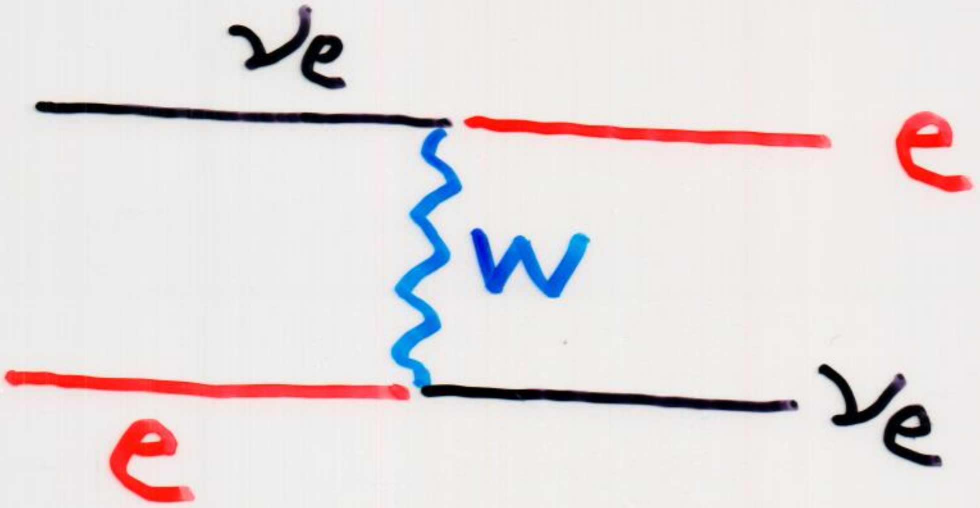
e.g: SK

$2R_{\oplus} > \ell\,[SK] > h\,[ATMS]$

ν MATTER-EFFECTS

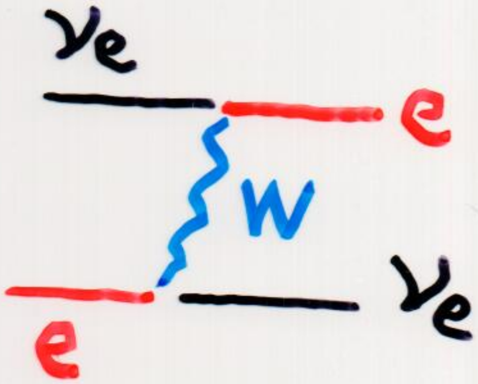


FOR ALL FLAVOURS



FOR ν_e ONLY

NONE OF THE ABOVE FOR STERILE NEUTRINOS



$$\frac{G_F}{\sqrt{2}}\,\bar{\nu}_e\,\gamma^\mu(1+\gamma_5)\,e\;\bar{e}\,\gamma_\mu(1+\gamma_5)\,\nu_e$$

$$=$$

$$\frac{G_F}{\sqrt{2}}\,\bar{\nu}_e\,\gamma_\mu(1+\gamma_5)\,\nu_e\;\cdot\;\bar{e}\,\gamma^\mu(1+\gamma_5)\,e$$

FORWARD SCATTERING

$$\propto\;\bar{\nu}_e\,\gamma_0\,\bar{\nu}_e\;\underbrace{\bar{e}\,\gamma_0\,e}$$

$$\langle M|\cdots|M\rangle = n_e$$

$$\mathcal{L}_{eff} = \bar{\nu}_e(i\not{\partial}-m)\nu_e + \bar{\nu}_e^L\gamma_0\nu_e^L\,\frac{n_e G_F}{\sqrt{2}}$$

$$\Delta m^2(\nu_e)\Big|_M = 2\sqrt{2}\,G_F\,E_\nu\,n_e$$

$\Longleftrightarrow$ γ INDEX OF

BACK TO OSCILLIS IN $D \to K/\pi + \dots$

EXACT ANALOGY TO ν'S :

$\nu_\mu(0)$ ~~~ ν_1/ν_2 MIXTURE ~~~ $\nu_\mu(x)$?

$$L = 1.27 \, \text{km} \, \frac{P}{\text{GeV}} \frac{\text{eV}^2}{m_K^2 - m_\pi^2} \sim 6\,10^{-11} \text{cm} \, \frac{P}{\text{GeV}}$$

OSCILLATING $\sin^2$ IMMED. AVERAGE $\frac{1}{2}$

$(\sin^2[x/L])$

$$P(D_0 \to D_0) = 1 - \frac{1}{2} \sin^2(2\theta_c)$$

$$\equiv P[\text{osc}]$$

RESULT FOR THE "OSCILLATION INTERPRETATION"

BUT! DEEP IN OUR 

WE KNOW THAT WHAT REALLY HAPPENS IS QUITE DIFFERENT

$D \to K \ldots \quad P = \cos^2\theta_c$

$D \to \pi \ldots \quad P = \sin^2\theta_c$

K, π TRAVEL INCOHERENTLY

$K \to D$ (SINGLY, WEAKLY) $P = \cos^2\theta_c$

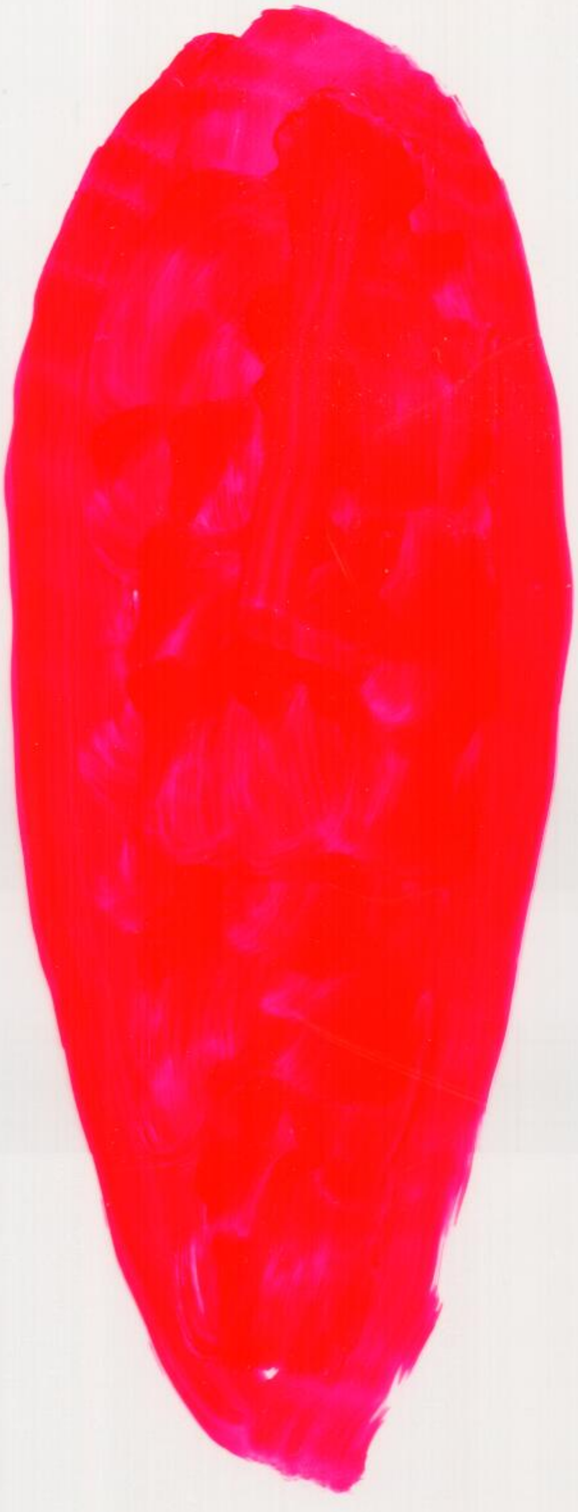
$\pi \to D$ (SINGLY, WEAKLY) $P = \sin^2\theta_c$

$$P(D_0 \to D_0) =$$

$$\cos^4(\theta_c) + \sin^4(\theta_c)$$

$$\equiv P[\text{TRUE}]$$

RESULT FOR THE "BRANCHING RATIO" INTERPRET



P[osc.]

$$\equiv$$

P[TRUE]

$$1-\frac{1}{2}\sin^2(2\theta)$$

$$\equiv$$

$$\cos^4(\theta)+\sin^4(\theta)$$

OUF!!!

WHY

DO WE KNOW

IN OUR HEARTS

THAT ONLY ONE

DERIVATION IS

CORRECT

!¿?

THE LOSS OF COHERENCE

$$P_{osc} \propto \sin^2\left(\frac{\Delta m^2}{4p} x\right)$$

BLURRED

IF THE Δp UNCERTAINTY $\exists$ AT $x = x_D$

$$x_D \left[\frac{\Delta m^2}{4p} - \frac{\Delta m^2}{4p \pm \Delta p}\right] \sim O\left(\frac{\pi}{2}\right) = 90^\circ$$

THE COHERENCE OF THE WAVES IS LOST

$$x_D = 2\pi \frac{p^2}{\Delta m^2} \frac{1}{\Delta p} = 2\pi \frac{p^2}{\Delta m^2} \Delta x$$

UNCERTAINTY IN LOCALIZATION OF THE ν-GENERATING PARTICLE

(APPARENTLY REQUIRING A CASE BY CASE STUDY)

$\frac{1}{\Delta p} = \Delta x$: $(\Delta p)^{-1}$ IS THE LENGTH OF THE WAVE PACKET

SIMPLEST EXAMPLE

$\mu^+ \to e^+ \nu \bar{\nu}$ μ^+ AT REST IN A SOLID

$\Delta x \sim 1 \text{Å}$ $\left(\Delta p = \frac{1}{\Delta x} = 2\,\text{keV}\right.$: IT WOULD BE DIFFICULT TO PREPARE A COHERENT μ^+ BEAM WITH SMALLER $\left.\Delta p\right)$

$$x_D(\nu_1/\nu_2) = 2\pi \frac{(30\,\text{MeV})^2}{3\,10^{-3}\,\text{eV}^2}\,10^{-8}\,\text{cm} = 1.9\,10^{10}\,\text{cm}$$

~ 20000 km

$D^+ \to K/\pi$ ... D^+ EQUALLY LOCALIZED

$$x_D(K/\pi) = 2\pi \frac{(1\,\text{GeV})^2}{m_K^2 - m_\pi^2}\,10^{-8}\,\text{cm} = 2\,10^{-7}\,\text{cm}$$

~ 20 ATOMS

K/π TRAVEL INCOHERENTLY

FINGER
TO FINGER

$\lambda \sim 2\pi/p$

$\longleftrightarrow$

NUMBER OF
FINGERS

$p/\Delta p$

WHY INDULGE IN THESE K/π CONSIDERATIONS?

① BECAUSE THE DECOHERENCE DISCUSSION IS RELEVANT TO $\nu_\odot$

$$x_D = 2\pi \frac{p_\nu^2}{\Delta m_\nu^2} \boxed{\Delta x_\odot}$$

EFFORT IN ESTIMATING

$x_D < x_\odot$ IN SOME CASES

(^{7}Be NEUTRINOS, LARGE $\Delta m^2_\odot$ SOLUTS.)

WE MAY GET ν_1 AND ν_2 (AS K AND π) ARRIVING SEPARATE FROM THE ⊙, BUT...

IT DOES NOT MATTER!

: THE RESPONSE OF AN EXPERIMENT [THAT DOES NOT MEASURE T.O.F.] IS THE SAME WHETHER OR NOT ν_1/ν_2 GET DECOHERENT IN FLIGHT

⇒ NO NEED TO FIGURE OUT $\Delta x_\odot$

2

SLOW

PRECISE

USELESS

2 TO UNDERSTAND ... WHICH IS WHAT WE ARE HERE FOR

(2) $A \to B\cos\theta + C\sin\theta$

B.R. vs OSC. INTERPRETATIONS

DEPENDS ON $x \gtrless x_D$ DECOHERENCE LENGTH OF WAVE PKT.

(3) IF $x > L_{osc}$ RESULTS (EXP. COUNT RATES)

DO NOT DEPEND ON "INTERPRETATION"

(4) FOR $x < L_{osc}$ YOU MAY OBSERVE

OSCILLATIONS : $P =$

(AS IN e.g. $K^0, \bar{K}^0$ SYSTEM)

(5) in e.g. $K \to \pi\, \mu\, \nu_\mu$ $\begin{pmatrix} e \\ \mu \end{pmatrix} \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$

$\to \pi\, e\, \nu_e$

OSCILLATE OVER TINY DISTANCE, THEN NO LONGER $\cos\theta\,\mu + \sin\theta\, e$

OSCILLATE OVER LONG DISTANCE, THEN ν_1 , ν_2

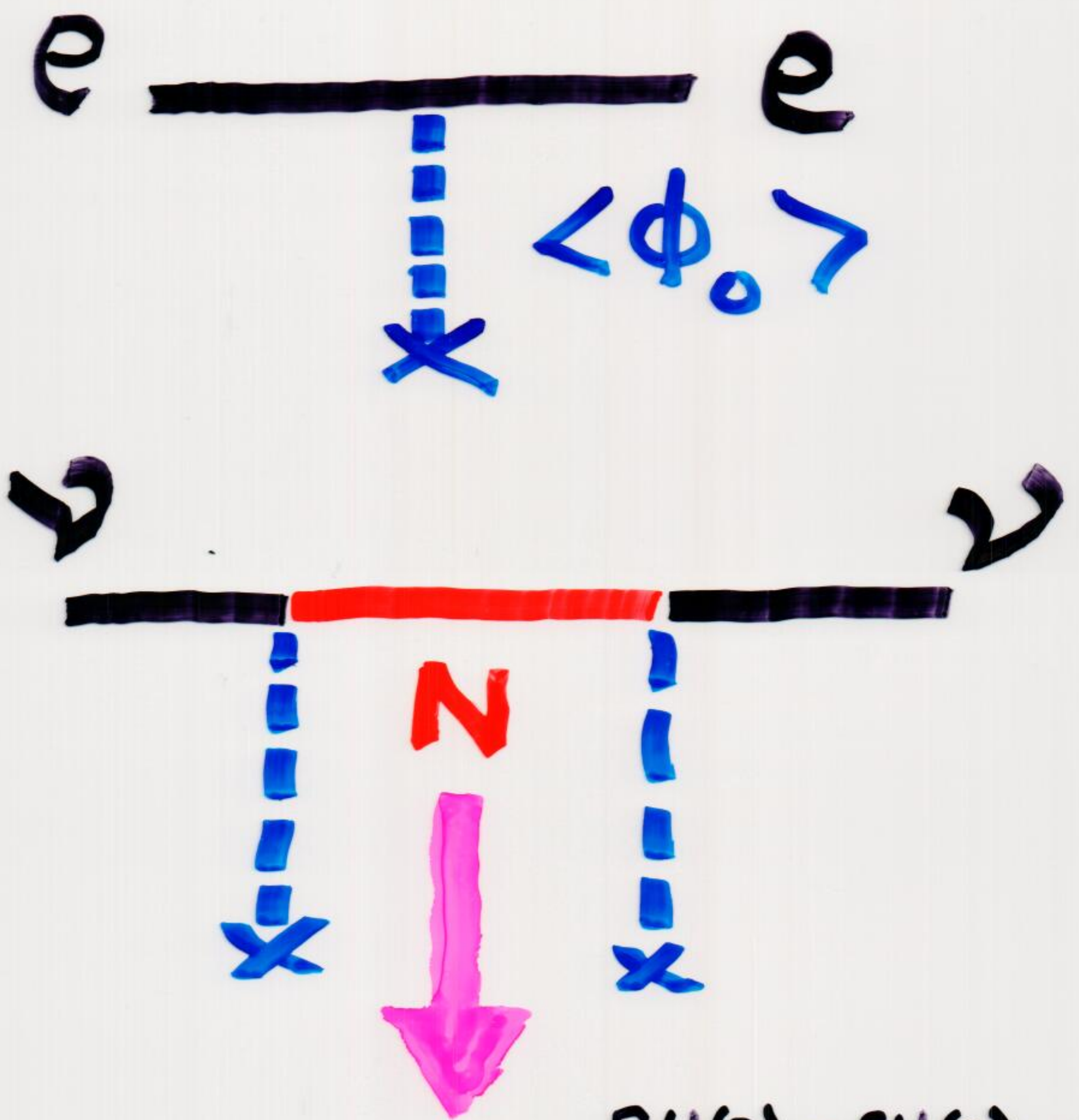
WHAT HAVE WE LEARNED?

0. MISTRUST ALL BOOKS, SAVE $

1. "FIXED $^{E}_{P}$ PRESCRIPTS!" WRONG

ν masses :

WINDOW BEYOND THE STANDARD MODEL ?



GU SCALE

$SU(3) \times SU(2) \times U(1)$

$\subset O(10)$

WHAT WE DID NOT COVER

ELECTRON CAPTURE

ν-LESS $\beta\beta$ DECAY

STERILE NEUTRINOS

νS IN COSMOLOGY

SOLAR MODEL

HEROIC $\begin{matrix}\text{RIGHT}\\ \text{WRONG}\end{matrix}$ EXPERIMENTS

CURRENT RESULTS ON ν MASSES, MIXINGS

MODELS OF THE ABOVE

MY GODESS!