Exercise 12

1. **QCD colour factors**

   The generators of the $SU(3)$ symmetry group of Quantum-Chromo-Dynamics (QCD) are the eight 3×3 matrices T^a. They satisfy the algebra

$$\left[T^a, T^b\right] = i f_{abc} T^c \,, \tag{1}$$

   with f_{abc} the real and antisymmetric $SU(3)$ structure constants. Imposing an implicit sum w.r.t. $a = 1, \ldots, 8$ one observes

$$T^a_{ij} T^a_{kl} = \frac{1}{2}\left[\delta_{il}\delta_{jk} - \frac{1}{3}\delta_{ij}\delta_{kl}\right] . \tag{2}$$

   Using the above relations calculate

   (a) $T^a_{ij} T^b_{jl} T^b_{lk} T^a_{ki}$

   (b) $T^a_{ij} T^b_{jl} T^a_{lk} T^b_{ki}$

   (c) $f_{abc} f_{abd} T^c_{ij} T^d_{ji}$.

2. **QCD pair production**

   Following the lecture notes, compute the differential cross section $\frac{\mathrm{d}\sigma}{\mathrm{d}t}$ for the process

$$g(p_1)\, g(p_2) \to Q(k_1)\, \bar{Q}(k_2)$$

   to lowest order in QCD. To simplify the calculation the quarks shall be considered as massless.

   (a) According to the QCD Feynman rules draw all possible diagrams contributing to the process in lowest order of the interaction.

   (b) Write down the expressions for the corresponding matrix elements.

   (c) Determine the invariant matrix element squared of the process where the colour and spin indices are averaged (summed) over initial (final) states.

   (d) From the transition amplitude squared determine the differential cross section $\frac{\mathrm{d}\sigma}{\mathrm{d}t}$, with $t = (p_1 - k_1)^2$.