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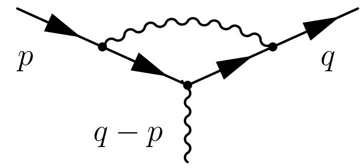
Quantum Field Theory WS 05/06

Exercise 14

1. **QED Ward identity**

   Consider the QED vertex correction at one-loop level (with momenta p incoming and q outgoing)

   $$\Gamma^\mu(p, q, q-p) =$$

   (a) Calculate the pole part of the amplitude in dimensional regularization, i.e. $D = 4 - 2\eta$.

   (b) Convince yourself that the pole parts of the QED vertex correction and the QED fermion self-energy can be related according to

   $$(q-p)_\mu \Gamma^\mu = -ie\mu^\eta \left[\Sigma(q) - \Sigma(p)\right] .$$

   Does this relation hold true for the full result as well? Can you guess the origin of this symmetry?

2. **One-loop renormalization of QED**

   The QED Lagrangian can be reformulated in terms of bare quantities ψ_0, A_0^μ, m_0, e_0 and α_0 by adding the counter-term Lagrangian

   $$\mathcal{L}^{c.t.} = -\frac{1}{4}K_3 F_{\mu\nu}F^{\mu\nu} - \frac{K_\alpha}{2\alpha}(\partial_\mu A^\mu)^2 + iK_2\bar{\psi}\partial\!\!\!/\psi - mK_m\bar{\psi}\psi - e\mu^\eta K_1\bar{\psi}A\!\!\!/\psi \ ,$$

   (a) Find the connection between the physical and the bare quantities.

   (b) From your results for the QED self-energy, vacuum polarization and vertex correction read off the counter-term coefficients K_1, K_2, K_3, K_α and K_m with the finite parts being undetermined.

   (c) Find the scaling behaviour of the running coupling constant $\alpha_{\rm QED}(\mu) = \dfrac{e^2(\mu)}{4\pi}$ from the β function of the theory.