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Quantum Field Theory WS 05/06

Exercise 11

1. **Dirac algebra**

   Prove that the traces over gamma matrices satisfy

$$\begin{aligned}
\mathrm{Tr}\,[\,\gamma^\mu\gamma^\nu\,] &= 4g^{\mu\nu}\ ,\\
\mathrm{Tr}\,[\,\gamma^{\mu_1}\dots\gamma^{\mu_{2n}}\,] &= \sum_{i=2}^{2n}(-1)^i g^{\mu_1\mu_i}\,\mathrm{Tr}\,[\,\gamma^{\mu_2}\dots\gamma^{\mu_{i-1}}\gamma^{\mu_{i+1}}\dots\gamma^{\mu_{2n}}\,]\ ,\\
\mathrm{Tr}\,[\,\gamma^{\mu_1}\dots\gamma^{\mu_{2n+1}}\,] &= 0\ .
\end{aligned}$$

   What follows for $\mathrm{Tr}\,[\,\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\,]$?

2. **Bhabha scattering**

   Compute the unpolarized lowest-order differential cross section $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$ for the Bhabha scattering process,

$$e^-(p_1)e^+(p_2)\to e^-(k_1)e^+(k_2)\,,$$

   assuming all particles to be massless.

   Note that the relative sign of the two contributing Feynman diagrams is negative. In the center-of-mass frame of the two incoming particles, where

$$\begin{aligned}
p_1 &= (E,0,0,E)\\
p_2 &= (E,0,0,-E)\\
k_1 &= (E,E\sin\theta,0,E\cos\theta)\\
k_2 &= (E,-E\sin\theta,0,-E\cos\theta)\,,
\end{aligned}$$

   the final formula reads

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\frac{\alpha^2}{8E^2}\left(\frac{1+\cos^4\theta/2}{\sin^4\theta/2}+\frac{1}{2}\left(1+\cos^2\theta\right)-2\frac{\cos^4\theta/2}{\sin^2\theta/2}\right)\ .$$