QCD & Monte Carlo Event Generators

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Content of lectures

- Introduction: QCD Basics
- Simulating Perturbative QCD
  - Parton Level Event Generation
  - Parton Showers
- Precision Monte Carlo
  - NLO Matching
  - Multijet Merging
- Soft Stuff

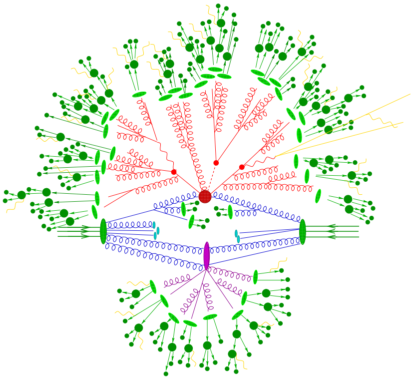
INTRODUCTION

EVENT GENERATORS

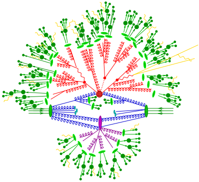
Strategy of event generator

principle: divide et impera

- hard process:
  fixed order perturbation theory
  traditionally: Born-approximation
- bremsstrahlung:
  resummed perturbation theory
- hadronisation:
  phenomenological models
- hadron decays:
  effective theories, data
- "underlying event":
  phenomenological models



... and possible improvements
possible strategies:



- improving the phenomenological models:
  - "tuning" (fitting parameters to data)
  - replacing by better models, based on more physics

    (my hot candidate: "minimum bias" and "underlying event" simulation)

- improving the perturbative description:
  - inclusion of higher order exact matrix elements and correct connection to resummation in the parton shower:

    "NLO-Matching" & "Multijet-Merging"

  - systematic improvement of the parton shower: next-to leading (or higher) logs & colours

Example: QCD precision in Higgs physics

- after discovery: time for precision studies of the newly found boson
  - precision determinartion of Yukawa couplings
  - finding or constraining "Higgs portal" (e.g. to Dark Matter)
  - maybe new particles in loops?
- Higgs signals suffers from different backgrounds, depending on production and decay channel considered in the analysis
- decomposing in bins of different jet multiplicities yields
  - different signal composition (e.g. WBF vs. ggF)
  - different backgrounds (e.g. $t\bar{t}$ in WW final states)
- to this end: must understand Standard Model in detail
  name of the game: uncertainties and their control

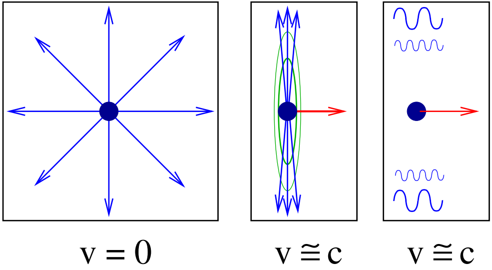
  despite far-reaching claims: analytic resummation and fixed-order calculations will not be sufficient
- same reasoning also true for new resonances/phenomena

QCD BASICS

SCALES & KINEMATICS

Factorization: an electromagnetic analogy

- consider a charge Z moving at constant velocity v



- at $v = 0$: radial E field only
- at $v = c$: B field emerges: $\vec{E} \perp \vec{B}$, $\vec{B} \perp \vec{v}$, $\vec{E} \perp \vec{v}$,
  energy flow $\sim$ Poynting vector $\vec{S} \sim \vec{E} \times \vec{B}$, $\parallel \vec{v}$
- approximate classical fields by "equivalent quanta": photons

- spectrum of photons:

(in dependence on energy ω and transverse distance $b_\perp$)

$$\mathrm{d}n_\gamma = \frac{Z^2\alpha}{\pi}\cdot\frac{\mathrm{d}\omega}{\omega}\cdot\frac{\mathrm{d}b_\perp^2}{b_\perp^2} \xrightarrow{\text{electron}(Z=1)} \frac{\alpha}{\pi}\cdot\frac{\mathrm{d}\omega}{\omega}\cdot\frac{\mathrm{d}b_\perp^2}{b_\perp^2}$$

- Fourier transform to transverse momenta $k_\perp$:

$$\mathrm{d}n_\gamma = \frac{\alpha}{\pi}\cdot\frac{\mathrm{d}\omega}{\omega}\cdot\frac{\mathrm{d}k_\perp^2}{k_\perp^2}$$

note: divergences for $k_\perp \to 0$ (collinear) and $\omega \to 0$ (soft)

- therefore: Fock state for lepton = superposition (coherent):

$$|e\rangle_{\text{phys}} = |e\rangle + |e\gamma\rangle + |e\gamma\gamma\rangle + |e\gamma\gamma\gamma\rangle + \dots$$

photon fluctuations will "recombine"

- lifetime of electron–photon fluctuations: $e(P) \to e(p) + \gamma(k)$
- estimate: use uncertainty relation and Lorentz time dilation
  - $P^2 = (p+k)^2 = M_{\rm virt}^2$ the virtual mass of the incident electron
  - life time = life time in rest frame · time dilation

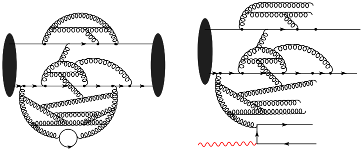
$$\tau \sim \frac{1}{M_{\rm virt}} \cdot \frac{E}{M_{\rm virt}} = \frac{E}{(p+k)^2} \sim \frac{E}{2Ek(1-\cos\theta)} \approx \frac{k}{k^2 \sin^2\theta/2} \approx \frac{\omega}{k_\perp^2}$$

- lifetime larger with smaller transverse momentum

(i.e. with larger transverse distance)

QED Initial and Final State Radiation

- physical interpretation:
  equivalent quanta = quantum manifestation of accompanying fields
- in absence of interaction: recombination enforced by coherence
- but: hard interaction possibly "kicks out" quantum
  $\longrightarrow$ coherence broken
  $\longrightarrow$ equivalent (virtual) quanta become real
  $\longrightarrow$ emission pattern unravels



- alternative idea:
  initial state radiation of photons off incident electron

- consider final state radiation in $\gamma^* \to \ell\bar{\ell}$
  (electron velocities/momenta labelled as v and v'/p and p')
- classical electromagnetic spectrum from radiation function:

(this is from Jackson or any other reasonable book on ED)

$$\frac{\mathrm{d}^2 I}{\mathrm{d}\omega \mathrm{d}\Omega} = \frac{e^2}{4\pi^2}\left|\vec{\epsilon}^{\,*}\cdot\left(\frac{\vec{v}}{1-\vec{v}\cdot\vec{n}} - \frac{\vec{v}'}{1-\vec{v}'\cdot\vec{n}}\right)\right|^2,$$

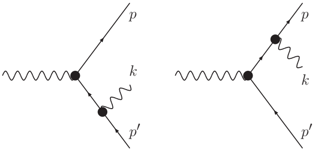
with ϵ the polarisation vector and $\vec{n}(\Omega)$ the direction of the radiation

- recast with four–momenta, equivalent photon spectrum:

$$\begin{aligned}\mathrm{d}N &= \frac{\mathrm{d}^3 k}{(2\pi)^3 2k_0}\,\frac{\alpha}{\pi}\left|\epsilon^*_\mu\left(\frac{p^\mu}{p\cdot k} - \frac{p'^\mu}{p'\cdot k}\right)\right|^2 \\ &= \frac{\mathrm{d}^3 k}{(2\pi)^3 2k_0}\,\frac{\alpha}{\pi}\left|\mathrm{W}_{pp';k}\right|^2\end{aligned}$$

with the eikonal $\mathrm{W}_{pp';k}$

- repeat exercise in QFT, Feynman diagrams:



$$\mathcal{M}_{X\to e^+e^-\gamma} = e\bar{u}(p)\left[\Gamma\,\frac{\not{p}'-\not{k}}{(p'-k)^2}\,\gamma^\mu - \gamma^\mu\,\frac{\not{p}+\not{k}}{(p+k)^2}\,\Gamma\right]u(p')\epsilon^*_\mu(k)$$

$$\stackrel{\text{soft}}{\longrightarrow} \quad e\epsilon^*_\mu(k)\left[\frac{p^\mu}{p\cdot k}-\frac{p'^\mu}{p'\cdot k}\right]\bar{u}(p')\Gamma u(p) = e\mathcal{M}_{X\to e^+e^-\gamma}\cdot \mathrm{W}_{pp';k}$$

- manifestation of Low's theorem:
  soft radiation independent of spin ($\to$ classical)

(radiation decomposes into soft, classical part with logs – i.e. dominant – and hard collinear part)

DGLAP equations for QED

(Dokshitser–Gribov–Lipatov–Altarelli–Parisi Equations)

- define probability to find electron or photon in electron:

$$\begin{aligned} \text{at LO in } \alpha (\text{no emission}): \; \ell(x,\, k_\perp^2) &= \delta(1-x) \\ \text{and} \quad \gamma(x,\, k_\perp^2) &= 0 \end{aligned}$$

(introduced $x =$ energy fraction w.r.t. physical state)

- including emissions:
  - probabilities change
  - energy fraction ξ of lepton parton w.r.t. the physical lepton object reduced by some fraction $z = x/\xi$
  - reminder: differential of photon number w.r.t. $k_\perp^2$:

$$\mathrm{d}n_\gamma \;=\; \frac{\alpha}{\pi}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\frac{\mathrm{d}\omega}{\omega} \;\leftrightarrow\; \frac{\mathrm{d}n_\gamma}{\mathrm{d}\log k_\perp^2} \;=\; \frac{\alpha}{\pi}\frac{\mathrm{d}x}{x}$$

- evolution equations (trivialised)

$$\frac{\mathrm{d}\ell(x,\,k_\perp^2)}{\mathrm{d}\log k_\perp^2} = \frac{\alpha(k_\perp^2)}{2\pi}\int\limits_x^1 \frac{\mathrm{d}\xi}{\xi}\mathcal{P}_{\ell\ell}\left(\frac{x}{\xi},\,\alpha(k_\perp^2)\right)\ell(\xi,\,k_\perp^2)$$

$$\frac{\mathrm{d}\gamma(x,\,k_\perp^2)}{\mathrm{d}\log k_\perp^2} = \frac{\alpha(k_\perp^2)}{2\pi}\int\limits_x^1 \frac{\mathrm{d}\xi}{\xi}\mathcal{P}_{\gamma\ell}\left(\frac{x}{\xi},\,\alpha(k_\perp^2)\right)\ell(\xi,\,k_\perp^2)\,.$$

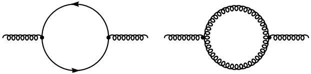
- $k_\perp^2$ plays the role of "resolution parameter"
- the $\mathcal{P}_{ab}(z)$ are the splitting functions, encoding quantum mechanics of the "splitting cross section", for example (at LO)

$$\mathcal{P}_{\ell\ell}(z) = \left(\frac{1+z^2}{1-z}\right)_+ + \frac{3}{2}\delta(1-z)$$

- if $\gamma\to\ell\bar{\ell}$ splittings included, have to add entries/splitting functions into evolution equations above

Running of α_s and bound states

- quantum effect due to loops: couplings change with scale
- running driven by β–function

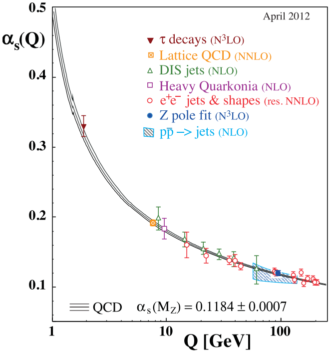


$$\beta(\alpha_s) = \mu_R^2 \frac{\partial \alpha_s(\mu_R^2)}{\partial \mu_R^2} = \frac{\beta_0}{4\pi}\,\alpha_s^2 + \frac{\beta_1}{(4\pi)^2}\alpha_s^3 + \ldots$$

with

$$\beta_0 = \frac{11}{3}\,C_A - \frac{4}{3}\,T_R n_f$$

$$\beta_1 = \frac{34}{3}\,C_A^2 - \frac{20}{3}\,C_A T_R n_f - 4\,C_F T_R n_f$$



- Casimir operators in the fundamental and adjoint representation:

$$C_F = \frac{N_c^2 - 1}{2N_c} \quad \text{and} \quad C_A = N_c$$

with $N_c = 3$ colours and $T_R = 1/2$.
- n_f = the number of (quark) flavours
- the Casimirs correspond to quark and gluon colour charges
- explicit expression for strong coupling

$$\alpha_s(\mu_R^2) \equiv \frac{g_s^2(\mu_R^2)}{4\pi} = \frac{1}{\frac{\beta_0}{4\pi} \log \frac{\mu_R^2}{\Lambda_{\mathrm{QCD}}{}^2}}$$

with Λ_{QCD} the Landau pole of QCD, $\Lambda_{\mathrm{QCD}} \approx 250\mathrm{MeV}$.

Hadrons in initial state: DGLAP equations of QCD

- similar to QED case:
  define probabilities (at LO) to find a parton q – quark or gluon – in hadron h at energy fraction x and resolution parameter/scale Q:
  parton distribution function (PDF) $f_{q/h}(x,\,Q^2)$
- scale-evolution of PDFs: DGLAP equations

$$\frac{\partial}{\partial\log Q^2}\begin{pmatrix} f_{q/h}(x,\,Q^2) \\ f_{g/h}(x,\,Q^2)\end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi}\int\limits_x^1\frac{\mathrm{d}z}{z}\begin{pmatrix}\mathcal{P}_{qq}\left(\frac{x}{z}\right) & \mathcal{P}_{qg}\left(\frac{x}{z}\right) \\ \mathcal{P}_{gq}\left(\frac{x}{z}\right) & \mathcal{P}_{gg}\left(\frac{x}{z}\right)\end{pmatrix}\begin{pmatrix} f_{q/h}(z,\,Q^2) \\ f_{g/h}(z,\,Q^2)\end{pmatrix},$$

- QCD splitting functions:

$$\begin{aligned}
\mathcal{P}^{(1)}_{qq}(x) &= C_F\left[\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\delta(1-x)\right] = \left[P^{(1)}_{qq}(x)\right]_+ + \gamma^{(1)}_q\delta(1-x)\\
\mathcal{P}^{(1)}_{qg}(x) &= T_R\left[x^2+(1-x)^2\right] = P^{(1)}_{qg}(x)\\
\mathcal{P}^{(1)}_{gq}(x) &= C_F\left[\frac{1+(1-x)^2}{x}\right] = P^{(1)}_{gq}(x)\\
\mathcal{P}^{(1)}_{gg}(x) &= 2C_A\left[\frac{x}{(1-x)_+}+\frac{1-x}{x}+x(1-x)\right]\\
&\quad+\frac{11C_A-4n_fT_R}{6}\delta(1-x) = \left[P^{(1)}_{gg}(x)\right]_+ + \gamma^{(1)}_g\delta(1-x)\,.
\end{aligned}$$

- remark: IR regularisation by $+$–prescription & terms $\sim\delta(1-x)$ from physical conditions on splitting functions

(flavour conservation for $q\to qg$ and momentum conservation for $g\to gg,\, q\bar{q}$)

Hadron production: Scales

- consider QCD final state radiation
- pattern for $q \to qg$ similar to $\ell \to \ell\gamma$ in QED:

$$\begin{aligned} \mathrm{d}w^{q\to qg} &= \frac{\alpha_s(k_\perp^2)}{2\pi} C_F \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \frac{\mathrm{d}\omega}{\omega} \left[1 + \left(1 - \frac{\omega}{E}\right)^2\right] \\ &\stackrel{\omega=E(1-z)}{=} \frac{\alpha_s(k_\perp^2)}{2\pi} C_F \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \mathrm{d}z \frac{1+z^2}{1-z} = \frac{\alpha_s(k_\perp^2)}{2\pi} C_F \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \mathrm{d}z\, P_{qg}^{(1)}(z)\,. \end{aligned}$$

- divergent structures for:

  $z \to 1$ (soft divergence) $\longleftrightarrow$ infrared/soft logarithms
  $k_\perp^2 \to 0$ (collinear/mass divergence) $\longleftrightarrow$ collinear logarithms

- cut regularise with cut-off $k_{\perp,\mathrm{min}} \sim 1\mathrm{GeV} > \Lambda_{\mathrm{QCD}}$

- find two perturbative regimes:
  - a regime of jet production, where $k_\perp \sim k_\| \sim \omega \gg k_{\perp,\mathrm{min}}$ and emission probabilities scale like $w \sim \alpha_s(k_\perp) \ll 1$; and
  - a regime of jet evolution, where $k_{\perp,\mathrm{min}} \le k_\perp \ll k_\| \le \omega$ and therefore emission probabilities scale like $w \sim \alpha_s(k_\perp) \log^2 k_\perp^2 \gtrsim 1$.
- in jet production:
  standard fixed–order perturbation theory
- in jet evolution regime,
  perturbative parameter not α_s any more
  but rather towers of $\exp\left[\alpha_s \log k_\perp^2 \log k_\|\right]$
- induces counting of leading logarithms (LL), $\alpha_s L^{2n}$,
  next-to leading logarithms (NLL), $\alpha_s L^{2n-1}$, etc.

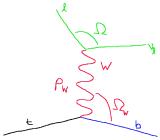
MONTE CARLO FOR

PARTON LEVEL

Simulating hard processes (signals & backgrounds)

- Simple example: $t \to bW^+ \to b\bar{l}\nu_l$:

$$|\mathcal{M}|^2 = \frac{1}{2}\left(\frac{8\pi\alpha}{\sin^2\theta_W}\right)^2 \frac{p_t\cdot p_\nu\;\; p_b\cdot p_l}{(p_W^2-M_W^2)^2+\Gamma_W^2 M_W^2}$$



- Phase space integration (5-dim):

$$\Gamma = \frac{1}{2m_t}\frac{1}{128\pi^3}\int \mathrm{d}p_W^2 \frac{\mathrm{d}^2\Omega_W}{4\pi}\frac{\mathrm{d}^2\Omega}{4\pi}\left(1-\frac{p_W^2}{m_t^2}\right)|\mathcal{M}|^2$$

- 5 random numbers $\Longrightarrow$ four-momenta $\Longrightarrow$ “events”.
- Apply smearing and/or arbitrary cuts.
- Simply histogram any quantity of interest - no new calculation for each observable

Calculating matrix elements efficiently

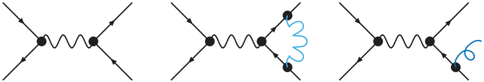
- stating the problem(s):
  - multi-particle final states for signals & backgrounds.
  - need to evaluate $\mathrm{d}\sigma_N$:

$$\int\limits_{\text{cuts}} \left[\prod_{i=1}^{N} \frac{\mathrm{d}^3 q_i}{(2\pi)^3 2E_i}\right] \delta^4\left(p_1 + p_2 - \sum_i q_i\right) \left|\mathcal{M}_{p_1 p_2 \to N}\right|^2 .$$

  - problem 1: factorial growth of number of amplitudes.
  - problem 2: complicated phase-space structure.
  - solutions: numerical methods.

Including higher order corrections

- obtained from adding diagrams with additional:
  loops (virtual corrections) or
  legs (real corrections)



- effect: reducing the dependence on μ_R & μ_F
  NLO allows for meaningful estimate of uncertainties
- additional difficulties when going NLO:
  ultraviolet divergences in virtual correction
  infrared divergences in real and virtual correction

  enforce
  UV regularisation & renormalisation
  IR regularisation & cancellation

(Kinoshita–Lee–Nauenberg–Theorem)

- general structure of NLO calculation for N-body production

$$\begin{aligned}\mathrm{d}\sigma &= \mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}_N(\Phi_{\mathcal{B}}) + \mathrm{d}\Phi_{\mathcal{B}}\mathcal{V}_N(\Phi_{\mathcal{B}}) + \mathrm{d}\Phi_{\mathcal{R}}\mathcal{R}_N(\Phi_{\mathcal{R}}) \\ &= \mathrm{d}\Phi_{\mathcal{B}}\left(\mathcal{B}_N + \mathcal{V}_N + \mathcal{I}_N^{(\mathcal{S})}\right) + \mathrm{d}\Phi_{\mathcal{R}}\left(\mathcal{R}_N - \mathcal{S}_N\right)\end{aligned}$$

- phase space factorisation assumed here ($\Phi_{\mathcal{R}} = \Phi_{\mathcal{B}} \otimes \Phi_1$)

$$\int \mathrm{d}\Phi_1 \mathcal{S}_N(\Phi_{\mathcal{B}} \otimes \Phi_1) = \mathcal{I}_N^{(\mathcal{S})}(\Phi_{\mathcal{B}})$$

- process independent, universal subtraction kernels

$$\begin{aligned}\mathcal{S}_N(\Phi_{\mathcal{B}} \otimes \Phi_1) &= \mathcal{B}_N(\Phi_{\mathcal{B}}) \otimes \mathcal{S}_1(\Phi_{\mathcal{B}} \otimes \Phi_1) \\ \mathcal{I}_N^{(\mathcal{S})}(\Phi_{\mathcal{B}} \otimes \Phi_1) &= \mathcal{B}_N(\Phi_{\mathcal{B}}) \otimes \mathcal{I}_1^{(\mathcal{S})}(\Phi_{\mathcal{B}}),\end{aligned}$$

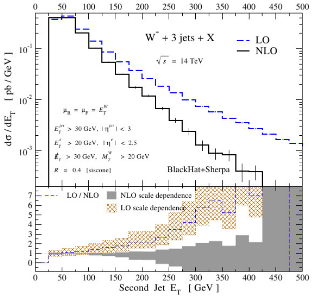
and invertible phase space mapping (e.g. Catani-Seymour)

$$\Phi_{\mathcal{R}} \longleftrightarrow \Phi_{\mathcal{B}} \otimes \Phi_1$$

Aside: choices . . .

- common lore: NLO calculations reduce scale uncertainties
- this is, in general, true. however:

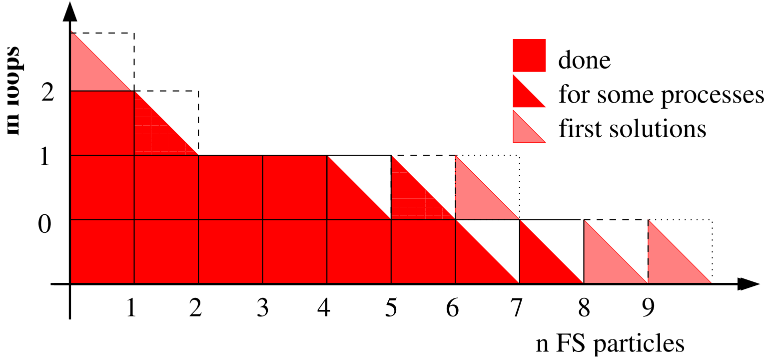
  unphysical scale choices will yield unphysical results



- more ways of botching it at higher orders

Availability of exact calculations (hadron colliders)

- fixed order matrix elements ("parton level") are exact to a given perturbative order. (and often quite a pain!)
- important to understand limitations:
  only tree-level and one-loop level fully automated, beyond: prototyping



GOING MONTE CARLO

PARTON SHOWERS – THE BASICS

An analogy: Radioactive decays

- consider radioactive decay of an unstable isotope with half-life τ.

(and ignore factors of ln 2.)

- "survival" probability after time t is given by

$$\mathcal{S}(t) = \mathcal{P}_{\rm nodec}(t) = \exp[-t/\tau]$$

(note "unitarity relation": $\mathcal{P}_{\rm dec}(t) = 1 - \mathcal{P}_{\rm nodec}(t)$.)

- probability for an isotope to decay at time t:

$$\frac{\mathrm{d}\mathcal{P}_{\rm dec}(t)}{\mathrm{d}t} = -\frac{\mathrm{d}\mathcal{P}_{\rm nodec}(t)}{\mathrm{d}t} = \frac{1}{\tau}\exp(-t/\tau)$$

- now: connect half-life with width $\Gamma = 1/\tau$.
- probability for the isotope to decay at any fixed time t determined by Γ.

- spice things up now: add time–dependence, $\Gamma = \Gamma(t')$
- rewrite

$$\Gamma t \longrightarrow \int_0^t \mathrm{d}t' \Gamma$$

- decay-probability at a given time t is given by

$$\frac{\mathrm{d}\mathcal{P}_{\mathrm{dec}}(t)}{\mathrm{d}t} = \Gamma(t)\,\exp\left[-\int_0^t \mathrm{d}t'\Gamma(t')\right] = \Gamma(t)\,\mathcal{P}_{\mathrm{nodec}}(t)$$

(unitarity strikes again: $\mathrm{d}\mathcal{P}_{\mathrm{dec}}(t)/\mathrm{d}t = -\mathrm{d}\mathcal{P}_{\mathrm{nodec}}(t)/\mathrm{d}t$.)

- interpretation of l.h.s.:
  - first term is for the actual decay to happen.
  - second term is to ensure that no decay before t
    $\Longrightarrow$ conservation of probabilities.
    the exponential is - of course - called the Sudakov form factor.

- differential cross section for gluon emission in $e^+e^- \to$ jets

$$\frac{\mathrm{d}\sigma_{ee\to 3j}}{\mathrm{d}x_1\mathrm{d}x_2} = \sigma_{ee\to 2j}\frac{C_F\alpha_s}{\pi}\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}$$

singular for $x_{1,2} \to 1$.

- rewrite with opening angle θ_{qg} and gluon energy fraction $x_3 = 2E_g/E_{\mathrm{c.m.}}$:

$$\frac{\mathrm{d}\sigma_{ee\to 3j}}{\mathrm{d}\cos\theta_{qg}\mathrm{d}x_3} = \sigma_{ee\to 2j}\frac{C_F\alpha_s}{\pi}\left[\frac{2}{\sin^2\theta_{qg}}\frac{1+(1-x_3)^2}{x_3} - x_3\right]$$

singular for $x_3 \to 0$ ("soft"), $\sin\theta_{qg} \to 0$ ("collinear").

- re-express collinear singularities

$$\frac{2\mathrm{d}\cos\theta_{qg}}{\sin^2\theta_{qg}} = \frac{\mathrm{d}\cos\theta_{qg}}{1-\cos\theta_{qg}} + \frac{\mathrm{d}\cos\theta_{qg}}{1+\cos\theta_{qg}}$$
$$= \frac{\mathrm{d}\cos\theta_{qg}}{1-\cos\theta_{qg}} + \frac{\mathrm{d}\cos\theta_{\bar{q}g}}{1-\cos\theta_{\bar{q}g}} \approx \frac{\mathrm{d}\theta^2_{qg}}{\theta^2_{qg}} + \frac{\mathrm{d}\theta^2_{\bar{q}g}}{\theta^2_{\bar{q}g}}$$

- independent evolution of two jets (q and $\bar{q}$)

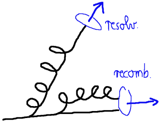
$$\mathrm{d}\sigma_{ee\to 3j} \approx \sigma_{ee\to 2j} \sum_{j\in\{q,\bar{q}\}} \frac{C_F\alpha_s}{2\pi}\frac{\mathrm{d}\theta^2_{jg}}{\theta^2_{jg}}P(z)\,,$$

- note: same form for any $t \propto \theta^2$:
- transverse momentum $k_\perp^2 \approx z^2(1-z)^2E^2\theta^2$
- invariant mass $q^2 \approx z(1-z)E^2\theta^2$

$$\frac{\mathrm{d}\theta^2}{\theta^2} \approx \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \approx \frac{\mathrm{d}q^2}{q^2}$$

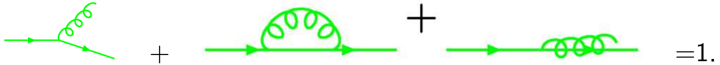
- parametrisation-independent observation: (logarithmically) divergent expression for $t \to 0$.
- practical solution: cut-off Q_0^2.
  $\Longrightarrow$ divergence will manifest itself as $\log Q_0^2$.
- similar for $P(z)$: divergence for $z \to 0$ cured by cut-off.

- what is a parton?
  collinear pair/soft parton recombine!
- introduce resolution criterion $k_\perp > Q_0$.



- combine virtual contributions with unresolvable emissions:
  cancels infrared divergences $\Longrightarrow$ finite at $\mathcal{O}(\alpha_s)$

  (Kinoshita-Lee-Nauenberg, Bloch-Nordsieck theorems)
- unitarity: probabilities add up to one
  $\mathcal{P}(\text{resolved}) + \mathcal{P}(\text{unresolved}) = 1.$



- the Sudakov form factor, once more
- differential probability for emission between q^2 and $q^2 + \mathrm{d}q^2$:

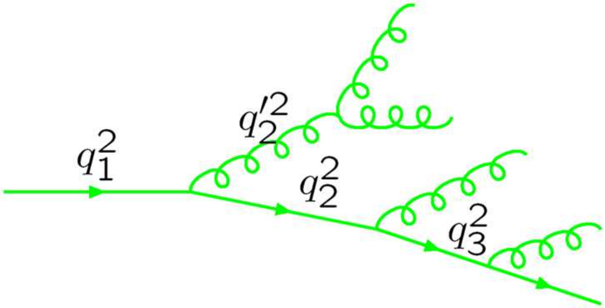
$$\mathrm{d}\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}q^2}{q^2}\int\limits_{z_{\rm min}}^{z_{\rm max}} \mathrm{d}z P(z) =: \mathrm{d}q^2\,\Gamma(q^2)$$

- from radioactive example: evolution equation for Δ

$$-\frac{\mathrm{d}\Delta(Q^2,\,q^2)}{\mathrm{d}q^2} = \Delta(Q^2,\,q^2)\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}q^2} = \Delta(Q^2,\,q^2)\Gamma(q^2)$$

$$\Longrightarrow\ \Delta(Q^2,\,q^2) \ = \ \exp\left[-\int\limits_{q^2}^{Q^2}\mathrm{d}k^2\Gamma(k^2)\right]$$

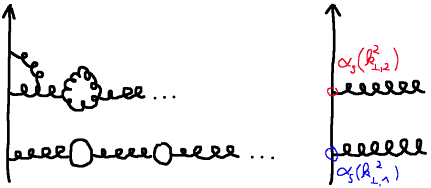
- maximal logs if emissions ordered
- impacts on radiation pattern: in each emission t becomes smaller



$$q_1^2 > q_2^2 > q_3^2,\ q_1^2 > {q_2'}^2$$

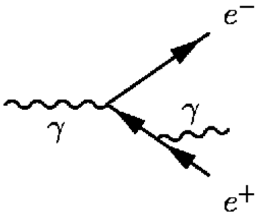
Quantum improvements

- improvement: inclusion of various quantum effects
- trivial: effect of summing up higher orders (loops) $\alpha_s \to \alpha_s(k_\perp^2)$



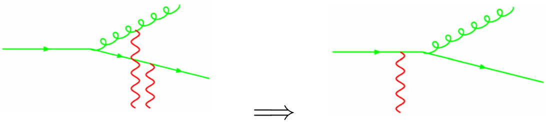
- much faster parton proliferation, especially for small $k_\perp^2$.
- avoid Landau pole: $k_\perp^2 > Q_0^2 \gg \Lambda^2_{\rm QCD} \Longrightarrow Q_0^2 =$ physical parameter.

- soft limit for single emission also universal
- problem: soft gluons come from all over (not collinear!) quantum interference? still independent evolution?
- answer: not quite independent.
- consider case in QED



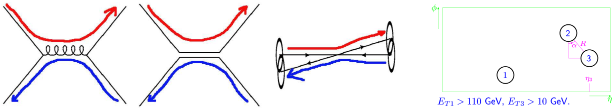
- assume photon into e^+e^- at θ_{ee} and photon off electron at θ photon momentum denoted as k
- energy imbalance at vertex: $k_\perp^\gamma \sim k_\parallel\theta$, hence $\Delta E \sim k_\perp^2/k_\parallel \sim k_\parallel\theta^2$.
- formation time for photon emission: $\Delta t \sim 1/\Delta E \sim k_\parallel/k_\perp^2 \sim 1/(k_\parallel\theta^2)$.
- *ee*-separation: $\Delta b \sim \theta_{ee}\Delta t$
- must be larger than transverse wavelength of photon: $\theta_{ee}/(k_\parallel\theta^2) > 1/k_\perp = 1/(k_\parallel\theta)$
- thus: $\theta_{ee} > \theta$ must be satisfied for photon to form
- angular ordering as manifestation of quantum coherence

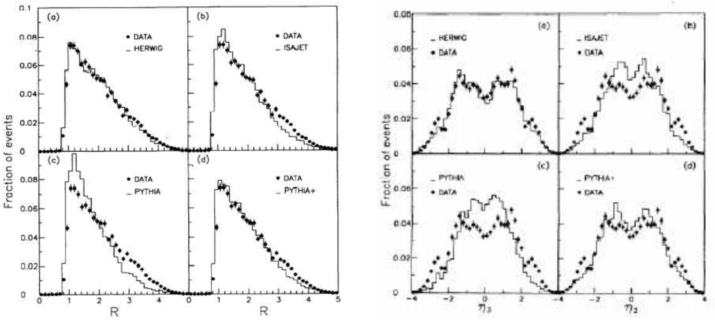
- pictorially:



gluons at large angle from combined colour charge!

- experimental manifestation:
  ΔR of $2^{\rm nd}$ & $3^{\rm rd}$ jet in multi-jet events in pp-collisions





Parametric precision

- analyse connection to Q_T resummation formalism
- consider standard Collins-Soper-Sterman formalism (CSS):

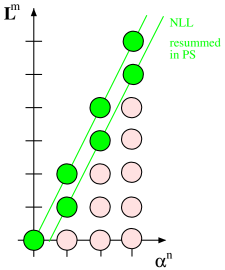
$$\frac{\mathrm{d}\sigma_{AB\to X}}{\mathrm{d}y\mathrm{d}Q_\perp^2} = \mathrm{d}\Phi_X\,\mathcal{B}_{ij}(\Phi_X)\cdot\underbrace{\int\frac{\mathrm{d}^2b_\perp}{(2\pi)^2}\exp(i\vec{b}_\perp\cdot\vec{Q}_\perp)}_{\text{guarantee 4-mom conservation}}\underbrace{\tilde{W}_{ij}(b;\,\Phi_X)}_{\text{higher orders}}$$

with

$$\tilde{W}_{ij}(b;\,\Phi_X) = \overbrace{C_i(b;\,\Phi_X,\,\alpha_s)C_j(b;\,\Phi_X,\,\alpha_s)}^{\text{collinear bits}}\overbrace{H_{ij}(\alpha_s)}^{\text{loops}}$$
$$\underbrace{\exp\left[-\int\limits_{1/b_\perp^2}^{Q_X^2}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\left(A(\alpha_s(k_\perp^2))\log\frac{Q_X^2}{k_\perp^2}+B(\alpha_s(k_\perp^2))\right)\right]}_{\text{Sudakov form factor, } A,\ B \text{ expanded in powers of } \alpha_s}$$

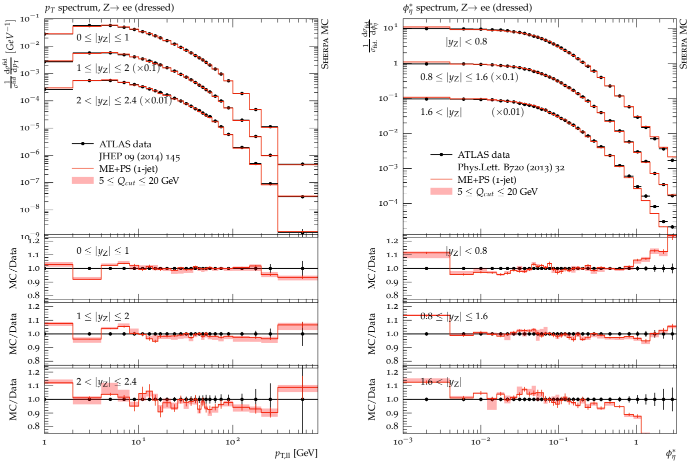
- analyse structure of emissions above
- logarithmic accuracy in $\log \frac{\mu_N}{k_\perp}$ (a la CSS) possibly up to next-to leading log,
  - if evolution parameter $\sim$ transverse momentum,
  - if argument in α_s is $\propto k_\perp$ of splitting,
  - if $K_{ij,k} \to$ terms $A_{1,2}$ and B_1 upon integration

    (OK, if soft gluon correction is included, and if $K_{ij,k} \to$ AP splitting kernels)



- in CSS $k_\perp$ typically is the transverse momentum of produced system, in parton shower of course related to the cumulative effect of explicit multiple emissions
- resummation scale $\mu_N \approx \mu_F$ given by (Born) kinematics – simple for cases like $q\bar{q}' \to V$, $gg \to H$, ... tricky for more complicated cases

Example: achievable precision of shower alone in DY



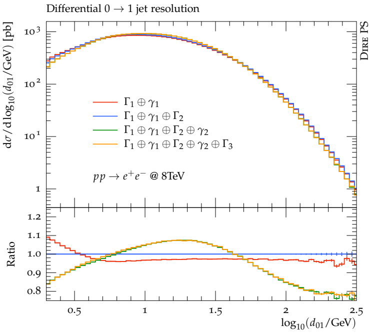
Higher logarithms in parton showering (?)

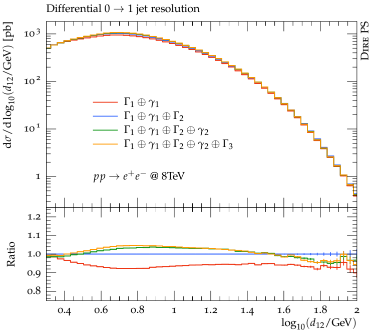
- start including higher orders/log (A_3 & B_2) – see below

(for NNLO the "natural" accuracy is NNLL)

- example below: different orders in DY

(a snapshot from ongoing work within SHERPA)





FIRST IMPROVEMENTS:

ME CORRECTIONS

NLO calculations, in a nutshell

- remember structure of NLO calculation for N-body production

$$\begin{aligned}\mathrm{d}\sigma &= \mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}_N(\Phi_{\mathcal{B}}) + \mathrm{d}\Phi_{\mathcal{B}}\mathcal{V}_N(\Phi_{\mathcal{B}}) + \mathrm{d}\Phi_{\mathcal{R}}\mathcal{R}_N(\Phi_{\mathcal{R}}) \\ &= \mathrm{d}\Phi_{\mathcal{B}}\left(\mathcal{B}_N + \mathcal{V}_N + \mathcal{I}_N^{(\mathcal{S})}\right) + \mathrm{d}\Phi_{\mathcal{R}}\left(\mathcal{R}_N - \mathcal{S}_N\right)\end{aligned}$$

- phase space factorisation assumed here $(\Phi_{\mathcal{R}} = \Phi_{\mathcal{B}} \otimes \Phi_1)$

$$\int \mathrm{d}\Phi_1 \mathcal{S}_N(\Phi_{\mathcal{B}} \otimes \Phi_1) = \mathcal{I}_N^{(\mathcal{S})}(\Phi_{\mathcal{B}})$$

- process independent subtraction kernels

$$\begin{aligned}\mathcal{S}_N(\Phi_{\mathcal{B}} \otimes \Phi_1) &= \mathcal{B}_N(\Phi_{\mathcal{B}}) \otimes \mathcal{S}_1(\Phi_{\mathcal{B}} \otimes \Phi_1) \\ \mathcal{I}_N^{(\mathcal{S})}(\Phi_{\mathcal{B}} \otimes \Phi_1) &= \mathcal{B}_N(\Phi_{\mathcal{B}}) \otimes \mathcal{I}_1^{(\mathcal{S})}(\Phi_{\mathcal{B}})\end{aligned}$$

with universal $\mathcal{S}_1(\Phi_{\mathcal{B}} \otimes \Phi_1)$ and $\mathcal{I}_1^{(\mathcal{S})}(\Phi_{\mathcal{B}})$

Parton showers, compact notation

- Sudakov form factor (**no-decay** probability)

$$\Delta_{ij,k}^{(\mathcal{K})}(t, t_0) = \exp\left[-\int\limits_{t_0}^{t} \frac{\mathrm{d}t}{t} \frac{\alpha_s}{2\pi} \int \mathrm{d}z \frac{\mathrm{d}\phi}{2\pi} \underbrace{\mathcal{K}_{ij,k}(t, z, \phi)}_{\substack{\text{splitting kernel for} \\ (ij)\to ij \text{ (spectator } k)}} \right]$$

- evolution parameter t defined by kinematics

generalised angle (HERWIG ++) or transverse momentum (PYTHIA, SHERPA)

- will replace $\frac{\mathrm{d}t}{t}\mathrm{d}z\frac{\mathrm{d}\phi}{2\pi} \longrightarrow \mathrm{d}\Phi_1$
- scale choice for strong coupling: $\alpha_s(k_\perp^2)$ resums classes of higher logarithms
- regularisation through cut-off t_0

- "compound" splitting kernels $\mathcal{K}_n$ and Sudakov form factors $\Delta_n^{(\mathcal{K})}$ for emission off n-particle final state:

$$\mathcal{K}_n(\Phi_1) = \frac{\alpha_s}{2\pi} \sum_{\text{all}\,\{ij,k\}} \mathcal{K}_{ij,k}(\Phi_{ij,k})\,, \quad \Delta_n^{(\mathcal{K})}(t,t_0) = \exp\left[-\int\limits_{t_0}^{t} \mathrm{d}\Phi_1\, \mathcal{K}_n(\Phi_1)\right]$$

- consider first emission only off Born configuration

$$\mathrm{d}\sigma_B = \mathrm{d}\Phi_N\, \mathcal{B}_N(\Phi_N)$$
$$\cdot \underbrace{\left\{\Delta_N^{(\mathcal{K})}(\mu_N^2, t_0) + \int\limits_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1 \left[\mathcal{K}_N(\Phi_1)\Delta_N^{(\mathcal{K})}(\mu_N^2, t(\Phi_1))\right]\right\}}_{\text{integrates to unity} \longrightarrow \text{"unitarity" of parton shower}}$$

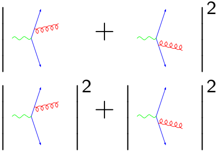
- further emissions by recursion with $Q^2 = t$ of previous emission

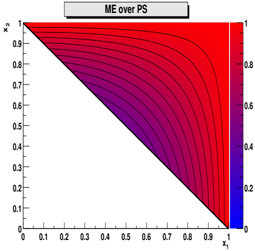
Matrix element corrections

- parton shower ignores interferences typically present in matrix elements
- pictorially

ME :

PS :





- form many processes $\mathcal{R}_N < \mathcal{B}_N \times \mathcal{K}_N$
- typical processes: $q\bar{q}' \to V$, $e^-e^+ \to q\bar{q}$, $t \to bW$
- practical implementation: shower with usual algorithm, but reject first/hardest emissions with probability $\mathcal{P} = \mathcal{R}_N/(\mathcal{B}_N \times \mathcal{K}_N)$

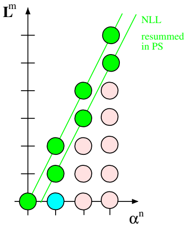
- analyse **first** emission, given by

$$\mathrm{d}\sigma_B = \mathrm{d}\Phi_N\,\mathcal{B}_N(\Phi_N)$$

$$\cdot\underbrace{\left\{\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,t_0)+\int\limits_{t_0}^{\mu_N^2}\mathrm{d}\Phi_1\left[\frac{\mathcal{R}_N(\Phi_N\times\Phi_1)}{\mathcal{B}_N(\Phi_N)}\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,t(\Phi_1))\right]\right\}}_{\text{once more: integrates to unity }\longrightarrow\text{ ``unitarity'' of parton shower}}$$

- radiation given by $\mathcal{R}_N$ (correct at $\mathcal{O}(\alpha_s)$)

  (but modified by logs of higher order in α_s from $\Delta_N^{(\mathcal{R}/\mathcal{B})}$)
- emission phase space constrained by μ_N
- also known as "soft ME correction"
  hard ME correction fills missing phase space
- used for "power shower":
  $\mu_N \to E_{pp}$ and apply ME correction

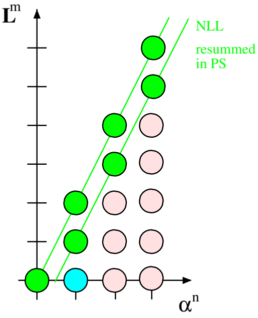


PRECISION MONTE CARLO

NLO MATCHING

NLO matching: Basic idea

- parton shower resums logarithms
  fair description of collinear/soft emissions
  jet evolution (where the logs are large)
- matrix elements exact at given order
  fair description of hard/large-angle emissions
  jet production (where the logs are small)
- adjust ("match") terms:
  - cross section at NLO accuracy &
    correct hardest emission in PS to exactly reproduce ME at order α_s
    ($\mathcal{R}$-part of the NLO calculation)
    (this is relatively trivial)
  - maintain (N)LL-accuracy of parton shower
    (this is not so simple to see)



POWHEG

- reminder: $\mathcal{K}_{ij,k}$ reproduces process-independent behaviour of $\mathcal{R}_N/\mathcal{B}_N$ in soft/collinear regions of phase space

$$\mathrm{d}\Phi_1\,\frac{\mathcal{R}_N(\Phi_{N+1})}{\mathcal{B}_N(\Phi_N)} \stackrel{\text{IR}}{\longrightarrow} \mathrm{d}\Phi_1\,\frac{\alpha_s}{2\pi}\,\mathcal{K}_{ij,k}(\Phi_1)$$

- define modified Sudakov form factor (as in ME correction)

$$\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,t_0) = \exp\left[-\int\limits_{t_0}^{\mu_N^2}\mathrm{d}\Phi_1\,\frac{\mathcal{R}_N(\Phi_{N+1})}{\mathcal{B}_N(\Phi_N)}\right]\,,$$

- assumes factorisation of phase space: $\Phi_{N+1} = \Phi_N \otimes \Phi_1$
- typically will adjust scale of α_s to parton shower scale

- define local K-factors
- start from Born configuration Φ_N with NLO weight:

("local K-factor")

$$
\begin{aligned}
\mathrm{d}\sigma_N^{(\mathrm{NLO})} &= \mathrm{d}\Phi_N\, \bar{\mathcal{B}}(\Phi_N) \\
&= \mathrm{d}\Phi_N \left\{ \mathcal{B}_N(\Phi_N) + \underbrace{\mathcal{V}_N(\Phi_N) + \mathcal{B}_N(\Phi_N) \otimes \mathcal{S}}_{\tilde{\mathcal{V}}_N(\Phi_N)} \right. \\
&\qquad \left. + \int \mathrm{d}\Phi_1 \left[\mathcal{R}_N(\Phi_N \otimes \Phi_1) - \mathcal{B}_N(\Phi_N) \otimes \mathrm{d}S(\Phi_1)\right] \right\}
\end{aligned}
$$

- by construction: exactly reproduce cross section at NLO accuracy
- note: second term vanishes if $\mathcal{R}_N \equiv \mathcal{B}_N \otimes \mathrm{d}S$

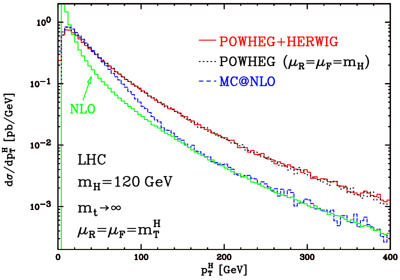
(relevant for MC@NLO)

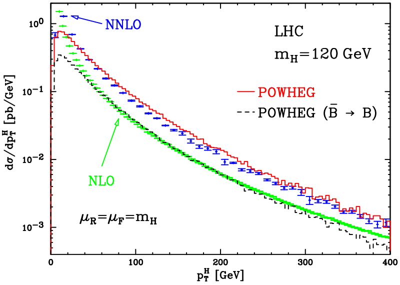
- analyse accuracy of radiation pattern
- generate emissions with $\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,\, t_0)$:

$$\mathrm{d}\sigma_N^{(\mathrm{NLO})} = \mathrm{d}\Phi_N\,\bar{\mathcal{B}}(\Phi_N)$$
$$\times \underbrace{\left\{\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,\, t_0) + \int\limits_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1 \frac{\mathcal{R}_N(\Phi_N\otimes\Phi_1)}{\mathcal{B}_N(\Phi_N)} \Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,\, k_\perp^2(\Phi_1))\right\}}_{\text{integrating to yield 1 - ``unitarity of parton shower''}}$$

- radiation pattern like in ME correction
- pitfall, again: choice of upper scale μ_N^2 (this is vanilla POWHEG!)
- apart from logs: which configurations enhanced by local K-factor

(K-factor for inclusive production of X adequate for $X+$ jet at large $p_\perp$?)

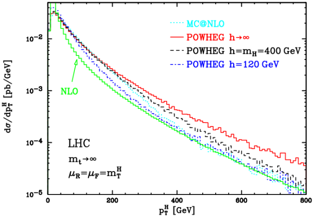




- large enhancement at high $p_{T,h}$
- can be traced back to large NLO correction
- fortunately, NNLO correction is also large $\to \sim$ agreement

- improving POWHEG
- split real-emission ME as

$$\mathcal{R} = \mathcal{R}\left(\underbrace{\frac{h^2}{p_\perp^2 + h^2}}_{\mathcal{R}^{(S)}} + \underbrace{\frac{p_\perp^2}{p_\perp^2 + h^2}}_{\mathcal{R}^{(F)}}\right)$$



- can "tune" h to mimick NNLO - or other (resummation) result

  - differential event rate up to first emission

$$\begin{aligned} \mathrm{d}\sigma \quad &= \quad \mathrm{d}\Phi_B \bar{\mathcal{B}}^{(\mathrm{R}^{(S)})}\left[\Delta^{(\mathcal{R}^{(S)}/\mathcal{B})}(s,\, t_0) + \int\limits_{t_0}^{s} \mathrm{d}\Phi_1 \frac{\mathcal{R}^{(S)}}{\mathcal{B}} \Delta^{(\mathcal{R}^{(S)}/\mathcal{B})}(s,\, k_\perp^2)\right] \\ &\quad + \mathrm{d}\Phi_R\, \mathcal{R}^{(F)}(\Phi_R) \end{aligned}$$

MC@NLO

- MC@NLO paradigm: divide $\mathcal{R}_N$ in soft ("S") and hard ("H") part:

$$\mathcal{R}_N = \mathcal{R}_N^{(S)} + \mathcal{R}_N^{(H)} = \mathcal{B}_N \otimes \mathrm{d}\mathcal{S}_1 + \mathcal{H}_N$$

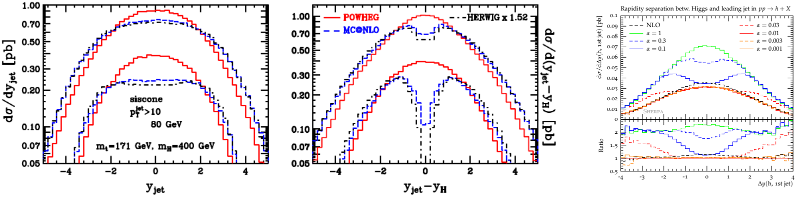
- identify subtraction terms and shower kernels $\mathrm{d}\mathcal{S}_1 \equiv \sum\limits_{\{ij,k\}} \mathcal{K}_{ij,k}$

(modify $\mathcal{K}$ in 1^{st} emission to account for colour)

$$\mathrm{d}\sigma_N = \mathrm{d}\Phi_N \underbrace{\tilde{\mathcal{B}}_N(\Phi_N)}_{\mathcal{B}+\tilde{\mathcal{V}}} \left[\Delta_N^{(\mathcal{K})}(\mu_N^2,\, t_0) + \int\limits_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1\, \mathcal{K}_{ij,k}(\Phi_1)\, \Delta_N^{(\mathcal{K})}(\mu_N^2,\, k_\perp^2) \right]$$
$$+\mathrm{d}\Phi_{N+1}\, \mathcal{H}_N$$

- effect: only resummed parts modified with local K-factor

- phase space effects: shower vs. fixed order



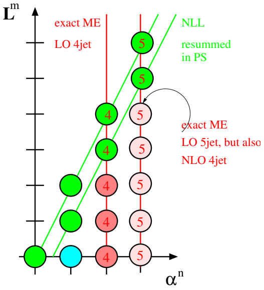
- problem: impact of subtraction terms on local K-factor (filling of phase space by parton shower)
- studied in case of $gg \to H$ above
- proper filling of available phase space by parton shower paramount

PRECISION MONTE CARLO

MULTIJET MERGING

Multijet merging: basic idea

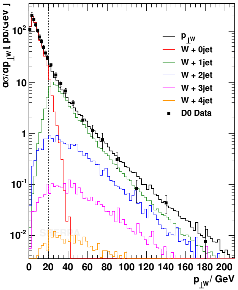
- parton shower resums logarithms
  fair description of collinear/soft emissions
  jet evolution (where the logs are large)
- matrix elements exact at given order
  fair description of hard/large-angle emissions
  jet production (where the logs are small)
- combine ("merge") both:
  result: "towers" of MEs with increasing number of jets evolved with PS
  - multijet cross sections at Born accuracy
  - maintain (N)LL accuracy of parton shower



- separate regions of jet production and jet evolution with jet measure Q_J

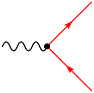
  ("truncated showering" if not identical with evolution parameter)

- matrix elements populate hard regime
- parton showers populate soft domain

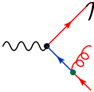


Why it works: jet rates with the parton shower

- consider jet production in $e^+e^- \to$ hadrons
  Durham jet definition: relative transverse momentum $k_\perp > Q_J$
- fixed order: one factor α_S and up to $\log^2 \frac{E_{\rm c.m.}}{Q_J}$ per jet
- use Sudakov form factor for resummation &
  replace approximate fixed order by exact expression:



$$\mathcal{R}_2(Q_J) = \left[\Delta_q(E^2_{\rm c.m.},\, Q_J^2)\right]^2$$



$$\mathcal{R}_3(Q_J) = 2\Delta_q(E^2_{\rm c.m.},\, Q_J^2) \int\limits_{Q_J^2}^{E^2_{\rm c.m.}} \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \left[\frac{\alpha_s(k_\perp^2)}{2\pi} \mathrm{d}z \mathcal{K}_q(k_\perp^2, z) \right.$$

$$\left. \times \Delta_q(E^2_{\rm c.m.},\, k_\perp^2)\Delta_q(k_\perp^2,\, Q_J^2)\Delta_g(k_\perp^2,\, Q_J^2)\right]$$

Multijet merging at LO

- expression for first emission

$$\mathrm{d}\sigma = \mathrm{d}\Phi_N\,\mathcal{B}_N\left[\Delta_N^{(\mathcal{K})}(\mu_N^2,\,t_0) + \int\limits_{t_0}^{\mu_N^2}\mathrm{d}\Phi_1\,\mathcal{K}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,\,t_{N+1})\Theta(Q_J - Q_{N+\ldots}\right.$$

$$+\mathrm{d}\Phi_{N+1}\,\mathcal{B}_{N+1}\,\Delta_N^{(\mathcal{K})}(\mu_{N+1}^2,\,t_{N+1})\Theta(Q_{N+1} - Q_J)$$

- note: $N+1$-contribution includes also $N+2,\ N+3,\ \ldots$

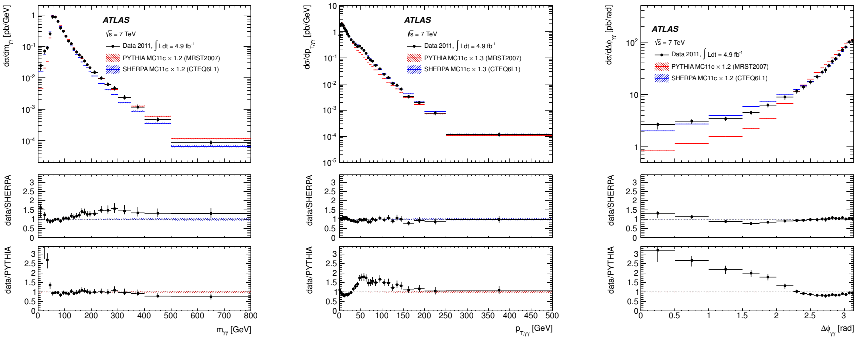
(no Sudakov suppression below t_{n+1}, see further slides for iterated expression)

- potential occurence of different shower start scales: $\mu_{N,N+1,\ldots}$
- "unitarity violation" in square bracket: $\mathcal{B}_N\mathcal{K}_N \longrightarrow \mathcal{B}_{N+1}$

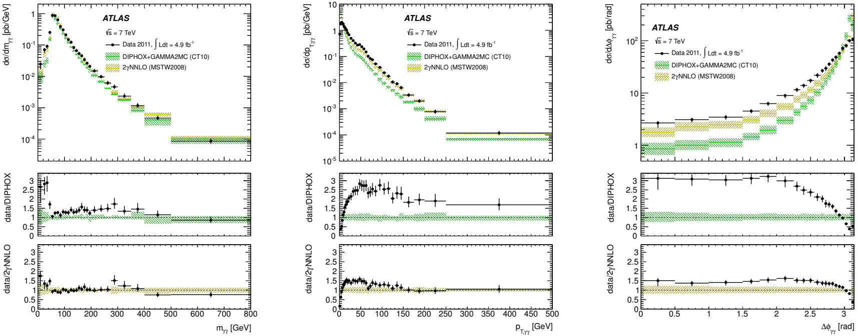
(cured with UMEPS formalism, L. Lonnblad & S. Prestel, JHEP 1302 (2013) 094 & S. Platzer, arXiv:1211.5467 [hep-ph] & arXiv:1307.0774 [hep-ph])

Di-photons @ ATLAS: $m_{\gamma\gamma}$, $p_{\perp,\gamma\gamma}$, and $\Delta\phi_{\gamma\gamma}$ in showers

(arXiv:1211.1913 [hep-ex])



Aside: Comparison with higher order calculations



Multijet-merging at NLO: MEPS@NLO

- basic idea like at LO: towers of MEs with increasing jet multi (but this time at NLO)
- combine them into one sample, remove overlap/double-counting

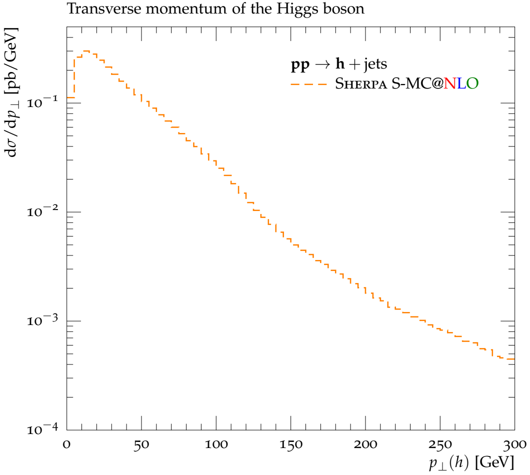
maintain NLO and (N)LL accuracy of ME and PS

- this effectively translates into a merging of MC@NLO simulations and can be further supplemented with LO simulations for even higher final state multiplicities

First emission(s), once more

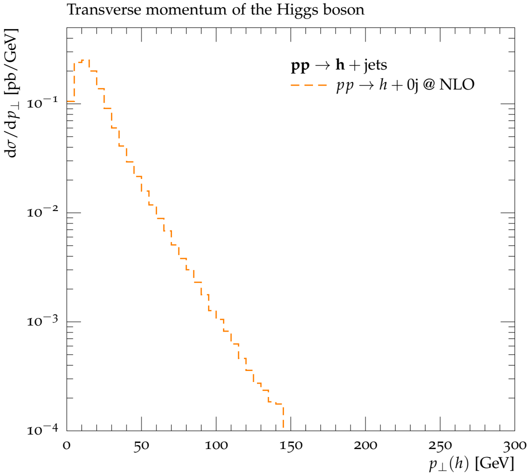
$$
\begin{aligned}
\mathrm{d}\sigma \;=\;& \mathrm{d}\Phi_N\,\tilde{\mathcal{B}}_N\left[\Delta_N^{(\mathcal{K})}(\mu_N^2,t_0)+\int\limits_{t_0}^{\mu_N^2}\mathrm{d}\Phi_1\,\mathcal{K}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Theta(Q_J-Q_{N+1})\right]\\
&+\mathrm{d}\Phi_{N+1}\,\mathcal{H}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Theta(Q_J-Q_{N+1})\\
&+\mathrm{d}\Phi_{N+1}\,\tilde{\mathcal{B}}_{N+1}\left(1+\frac{\mathcal{B}_{N+1}}{\tilde{\mathcal{B}}_{N+1}}\int\limits_{t_{N+1}}^{\mu_N^2}\mathrm{d}\Phi_1\,\mathcal{K}_N\right)\Theta(Q_{N+1}-Q_J)\\
&\quad\cdot\left[\Delta_{N+1}^{(\mathcal{K})}(t_{N+1},t_0)+\int\limits_{t_0}^{t_{N+1}}\mathrm{d}\Phi_1\,\mathcal{K}_{N+1}\Delta_{N+1}^{(\mathcal{K})}(t_{N+1},t_{N+2})\right]\\
&+\mathrm{d}\Phi_{N+2}\,\mathcal{H}_{N+1}\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Delta_{N+1}^{(\mathcal{K})}(t_{N+1},t_{N+2})\Theta(Q_{N+1}-Q_J)+\ldots
\end{aligned}
$$

$p_\perp^H$ in MEPS@NLO



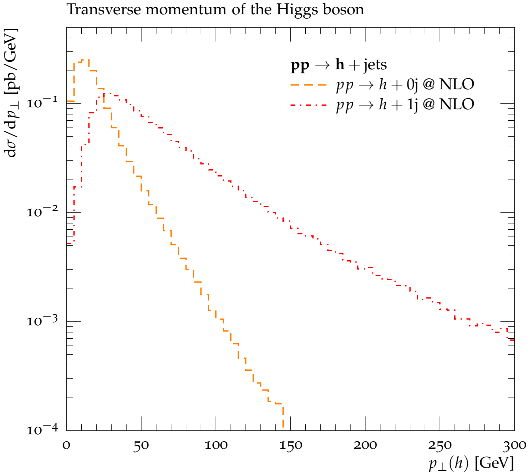
- first emission by MC@NLO

$p_\perp^H$ in MEPS@NLO



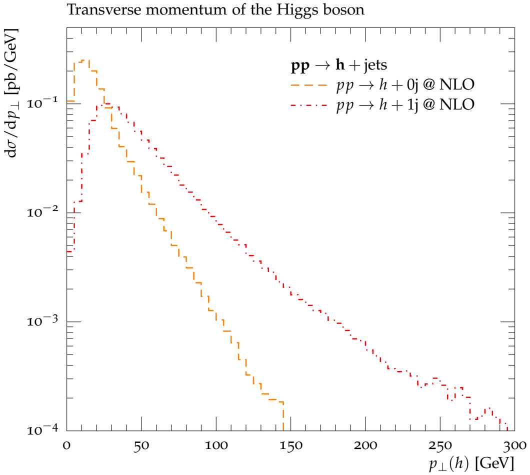
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\text{cut}}$

$p_\perp^H$ in MEPS@NLO



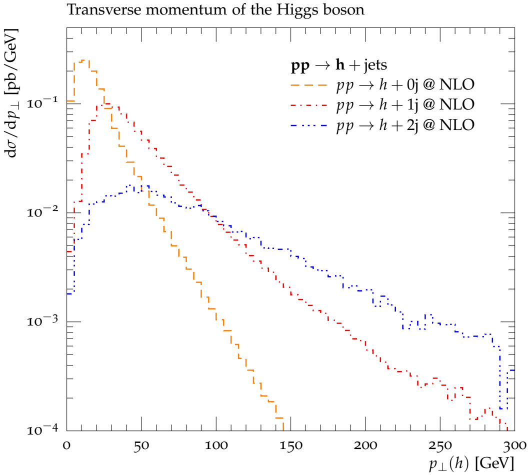
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\rm cut}$
- MC@NLO $pp \to h + \text{jet}$ for $Q_{n+1} > Q_{\rm cut}$

$p_\perp^H$ in MEPS@NLO



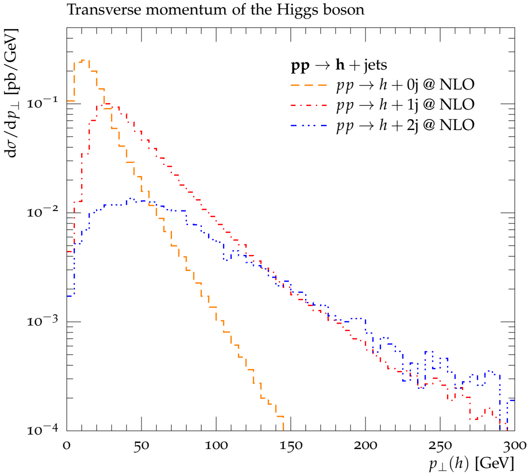
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\text{cut}}$
- MC@NLO $pp \to h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \to h + \text{jet}$ to $Q_{n+2} < Q_{\text{cut}}$

$p_\perp^H$ in MEPS@NLO



- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\rm cut}$
- MC@NLO $pp \to h +$ jet for $Q_{n+1} > Q_{\rm cut}$
- restrict emission off $pp \to h +$ jet to $Q_{n+2} < Q_{\rm cut}$
- MC@NLO $pp \to h +$ 2jets for $Q_{n+2} > Q_{\rm cut}$

$p_\perp^H$ in MEPS@NLO



- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\rm cut}$
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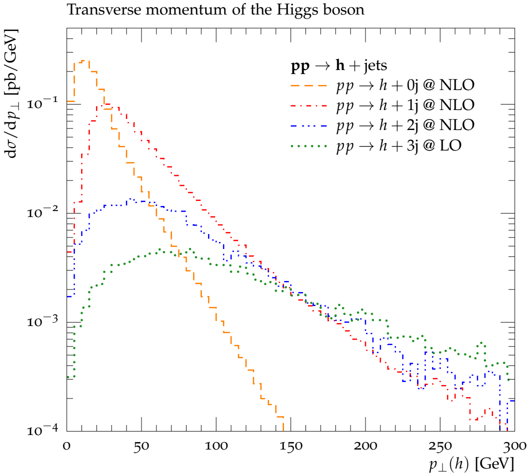
- restrict emission off $pp \to h +$ jet to $Q_{n+2} < Q_{\rm cut}$
- MC@NLO $pp \to h +$ 2jets for $Q_{n+2} > Q_{\rm cut}$
- iterate

$p_\perp^H$ in MEPS@NLO



- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\rm cut}$
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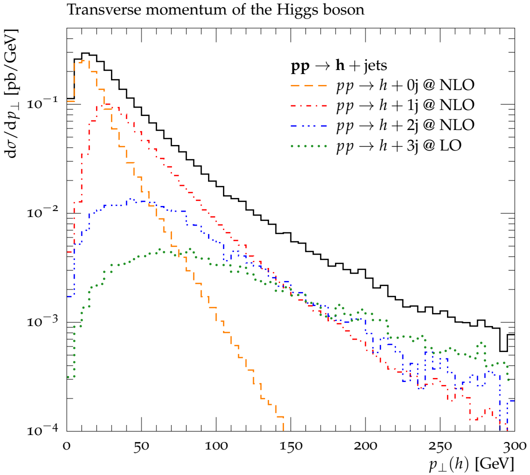
- restrict emission off $pp \to h + \text{jet}$ to $Q_{n+2} < Q_{\rm cut}$
- MC@NLO $pp \to h + 2\text{jets}$ for $Q_{n+2} > Q_{\rm cut}$
- iterate

$p_\perp^H$ in MEPS@NLO



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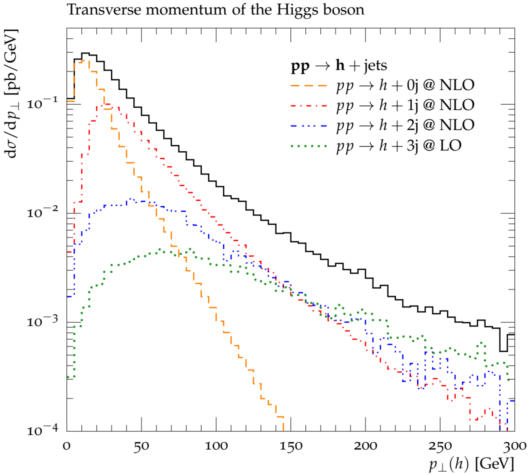
- MC@NLO $pp \to h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \to h + \text{jet}$ to $Q_{n+2} < Q_{\text{cut}}$
- MC@NLO $pp \to h + 2\text{jets}$ for $Q_{n+2} > Q_{\text{cut}}$
- iterate
- sum all contributions

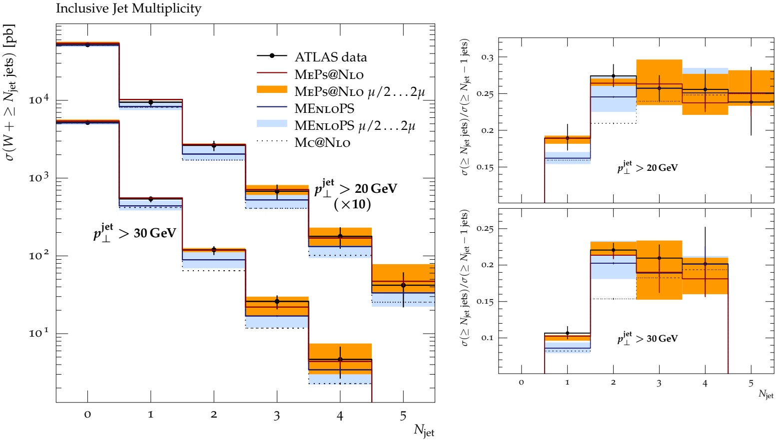
$p_\perp^H$ in MEPS@NLO

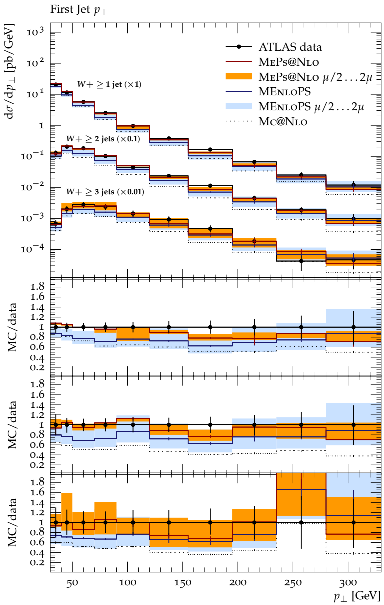


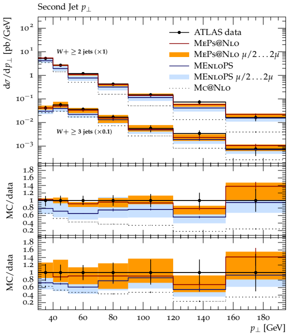
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\rm cut}$
- MC@NLO $pp \to h+$jet for $Q_{n+1} > Q_{\rm cut}$
- restrict emission off $pp \to h+$jet to $Q_{n+2} < Q_{\rm cut}$
- MC@NLO $pp \to h+$2jets for $Q_{n+2} > Q_{\rm cut}$
- iterate
- sum all contributions
- eg. $p_\perp(h)$>200 GeV has contributions fr. multiple topologies

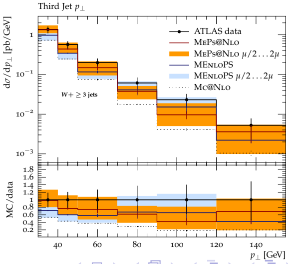
Example: MEPS@NLO for W+jets

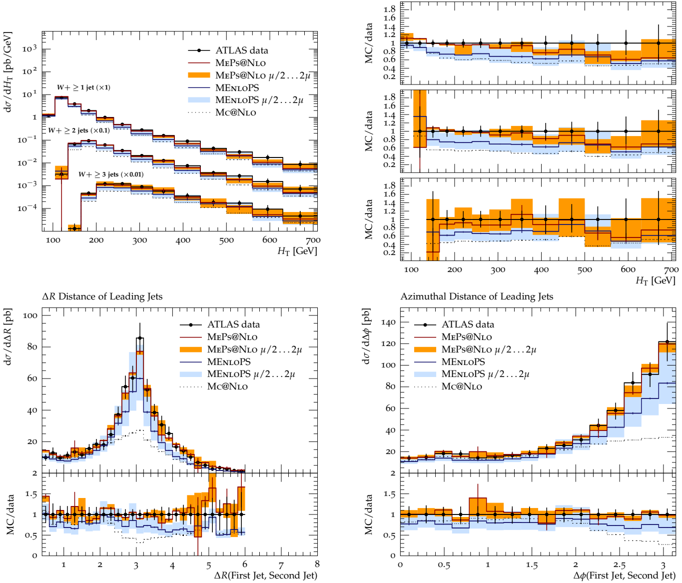
(up to two jets @ NLO, from BLACKHAT, see arXiv: 1207.5031 [hep-ex])




First Jet $p_\perp$
ATLAS data
MEPs@NLO
MEPs@NLO $\mu/2\ldots2\mu$
MENLOPS
MENLOPS $\mu/2\ldots2\mu$
MC@NLO
$W+\geq 1$ jet ($\times 1$)
$W+\geq 2$ jets ($\times 0.1$)
$W+\geq 3$ jets ($\times 0.01$)
$d\sigma/dp_\perp$ [pb/GeV]
MC/data
$p_\perp$ [GeV]


Second Jet $p_\perp$
ATLAS data
MEPs@NLO
MEPs@NLO $\mu/2\ldots2\mu$
MENLOPS
MENLOPS $\mu/2\ldots2\mu$
MC@NLO
$W+\geq 2$ jets ($\times 1$)
$W+\geq 3$ jets ($\times 0.1$)
$d\sigma/dp_\perp$ [pb/GeV]
MC/data
$p_\perp$ [GeV]


Third Jet $p_\perp$
ATLAS data
MEPs@NLO
MEPs@NLO $\mu/2\ldots2\mu$
MENLOPS
MENLOPS $\mu/2\ldots2\mu$
MC@NLO
$W+\geq 3$ jets
$d\sigma/dp_\perp$ [pb/GeV]
MC/data
$p_\perp$ [GeV]


ATLAS data
MEPs@NLO
MEPs@NLO μ/2...2μ
MENLOPS
MENLOPS μ/2...2μ
MC@NLO
W+ ≥ 1 jet (×1)
W+ ≥ 2 jets (×0.1)
W+ ≥ 3 jets (×0.01)
dσ/dH_T [pb/GeV]
H_T [GeV]
MC/data
ΔR Distance of Leading Jets
dσ/dΔR [pb]
ΔR(First Jet, Second Jet)
Azimuthal Distance of Leading Jets
dσ/dΔφ [pb]
Δφ(First Jet, Second Jet)

SIMULATING SOFT QCD

HADRONISATION

QCD radiation, once more

- remember QCD emission pattern

$$\mathrm{d}w^{q\to qg} = \frac{\alpha_s(k_\perp^2)}{2\pi}\,C_F\,\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\,\frac{\mathrm{d}\omega}{\omega}\,\left[1+\left(1-\frac{\omega}{E}\right)\right].$$

- spectrum cut-off at small transverse momenta and energies by onset of hadronization, at scales $R \approx 1\,\mathrm{fm}/\Lambda_{\mathrm{QCD}}$
- two (extreme) classes of emissions: gluons and gluers determined by relation of formation and hadronization times

- gluers formed at times R, with momenta $k_\parallel \sim k_\perp \sim \omega \gtrsim 1/R$
- assuming that hadrons follow partons,

$$\mathrm{d}N_{\rm (hadrons)} \sim \int\limits_{k_\perp > 1/R}^{Q} \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \frac{C_F\,\alpha_s(k_\perp^2)}{2\pi} \left[1 + \left(1 - \frac{\omega}{E}\right)\right] \frac{\mathrm{d}\omega}{\omega}$$

$$\sim \frac{C_F \alpha_s(1/R^2)}{\pi} \log(Q^2 R^2) \frac{\mathrm{d}\omega}{\omega}$$

or - relating their energyn with that of the gluers -

$$\mathrm{d}N_{\rm (hadrons)}/\mathrm{d}\log\epsilon = \mathrm{const.}\,,$$

a plateau in log of energy (or in rapidity)

- impact of additional radiation
- new partons must separate before they can hadronize independently
- therefore, one more time

$$\begin{array}{llllll} t^{\rm form} & \sim & \frac{k_\parallel}{k_\perp^2} & & \\ t^{\rm sep} & \sim & R\theta & \sim & t^{\rm form}\,(Rk_\perp) \\ t^{\rm had} & \sim & k_\parallel R^2 & \sim & t^{\rm form}\,(Rk_\perp)^2\,. \end{array}$$

- for gluers $Rk_\perp \approx 1$: all times the same
- naively; new & more hadrons following new partons
- but: colour coherence
  primary and secondary partons not separated enough in

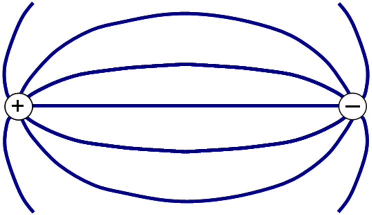
$$1/R \lesssim \omega_{\rm (hadron)} \lesssim 1/(R\theta)$$

  and therefore no independent radiation

Hadronisation: General thoughts

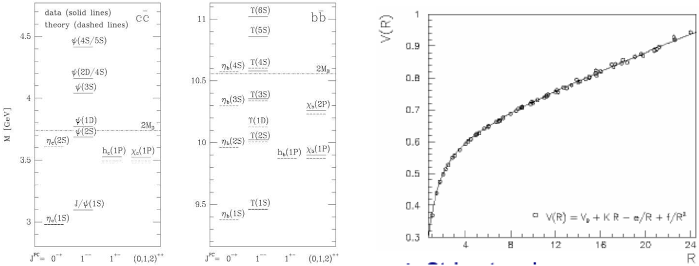
- confinement the striking feature of low–scale sotrng interactions
- transition from partons to their bound states, the hadrons
- the Meissner effect in QCD

QED:

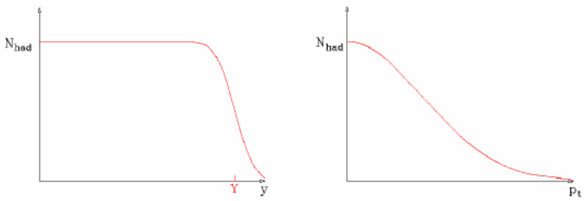


QCD:

- linear QCD potential in Quarkonia – like a string



- combine some experimental facts into a naive parameterisation
- in $e^+e^- \to$ hadrons: exponentially decreasing $p_\perp$, flat plateau in y for hadrons



- try "smearing": $\rho(p_\perp^2) \sim \exp(-p_\perp^2/\sigma^2)$

- use parameterisation to "guesstimate" hadronisation effects:

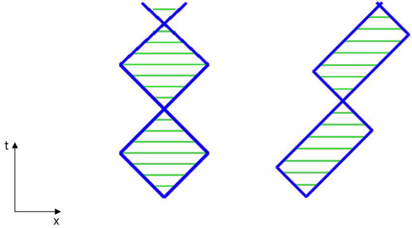
$$\begin{aligned}
E &= \int_0^Y \mathrm{d}y\mathrm{d}p_\perp^2\rho(p_\perp^2)p_\perp \cosh y = \lambda \sinh Y\\
P &= \int_0^Y \mathrm{d}y\mathrm{d}p_\perp^2\rho(p_\perp^2)p_\perp \sinh y = \lambda(\cosh Y - 1) \approx E - \lambda\\
\lambda &= \int \mathrm{d}p_\perp^2\rho(p_\perp^2)p_\perp = \langle p_\perp\rangle\,.
\end{aligned}$$

- estimate $\lambda \sim 1/R_{\rm had} \approx m_{\rm had}$, with $m_{\rm had}$ 0.1-1 GeV.
- effect: jet acquire non-perturbative mass $\sim 2\lambda E$ ($\mathcal{O}(10\mathrm{GeV})$ for jets with energy $\mathcal{O}(100\mathrm{GeV})$).

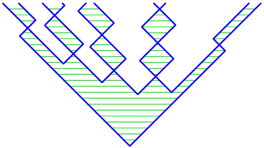
- similar parametrization underlying Feynman-Field model for independent fragmentation
- recursively fragment $q \to q' +$ had, where
  - transverse momentum from (fitted) Gaussian;
  - longitudinal momentum arbitrary (hence from measurements);
  - flavour from symmetry arguments + measurements.
- problems: frame dependent, "last quark", infrared safety, no direct link to perturbation theory, ...

The string model

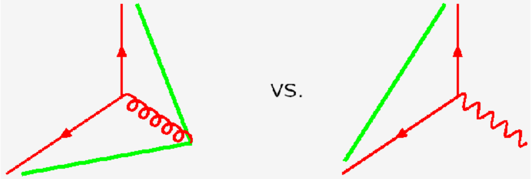
- a simple model of mesons: yoyo strings
  - light quarks ($m_q = 0$) connected by string, form a meson
  - area law: $m^2_{\rm had} \propto$ area of string motion
  - L=0 mesons only have 'yo-yo' modes:



- turn this into hadronisation model $e^+e^- \to q\bar{q}$ as test case
- ignore gluon radiation: $q\bar{q}$ move away from each other, act as point-like source of string
- intense chromomagnetic field within string:
  more $q\bar{q}$ pairs created by tunnelling and string break-up
- analogy with QED (Schwinger mechanism):
  $\mathrm{d}\mathcal{P} \sim \mathrm{d}x\mathrm{d}t \exp\left(-\pi m_q^2/\kappa\right)$, $\kappa =$ "string tension".



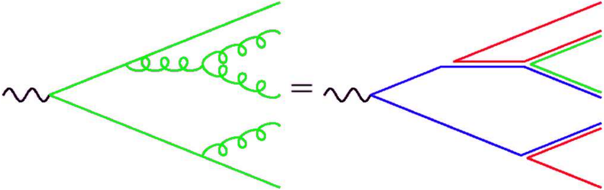
- string model = well motivated model, constraints on fragmentation (Lorentz-invariance, left-right symmetry, ...)
- how to deal with gluons?
- interpret them as kinks on the string $\Longrightarrow$ the string effect



- infrared-safe, advantage: smooth matching with PS.

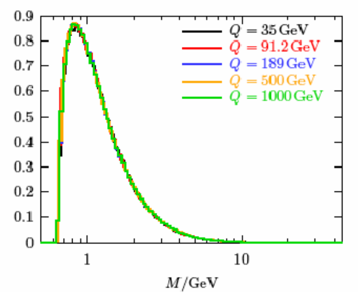
The cluster model

- underlying idea: preconfinement/LPHD
  - typically, neighbouring colours will end in same hadron
  - hadron flows follow parton flows $\longrightarrow$ don't produce any hadrons at places where you don't have partons
  - works well in large–N_c limit with planar graphs
- follow evolution of colour in parton showers



- paradigm of cluster model: clusters as continuum of hadron resonances
- trace colour through shower in $N_c \to \infty$ limit
- force decay of gluons into $q\bar{q}$ or $\bar{d}d$ pairs, form colour singlets from neighbouring colours, usually close in phase space
- mass of singlets: peaked at low scales $\approx Q_0^2$
- decay heavy clusters into lighter ones or into hadrons (here, many improvements to ensure leading hadron spectrum hard enough, overall effect: cluster model becomes more string-like)
- if light enough, clusters will decay into hadrons
- naively: spin information washed out, decay determined through phase space only $\to$ heavy hadrons suppressed (baryon/strangeness suppression)

- self–similarity of parton shower will end with roughly the same **local** distribution of partons, with roughly the same invairant mass for colour singlets
- adjacent pairs colour connected, form colourless (white) clusters.
- clusters ("≈ excited hadrons) decay into hadrons



Practicalities

- practicalities of hadronisation models: parameters
  - kinematics of string or cluster decay: 2-5 parameters
  - must "pop" quark or diquark flavours in string or cluster decay — cannot be completely democratic or driven by masses alone
    $\longrightarrow$ suppression factors for strangeness, diquarks 2-10 parameters
  - transition to hadrons, cannot be democratic over multiplets
    $\longrightarrow$ adjustment factors for vectors/tensors etc. 2-6 parameters
- tuned to LEP data, overall agreement satisfying
- validity for hadron data not quite clear

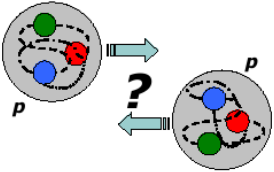
  (beam remnant fragmentation not in LEP.)
- there are some issues with inclusive strangeness/baryon production

SIMULATING SOFT QCD

UNDERLYING EVENT

Multiple parton scattering

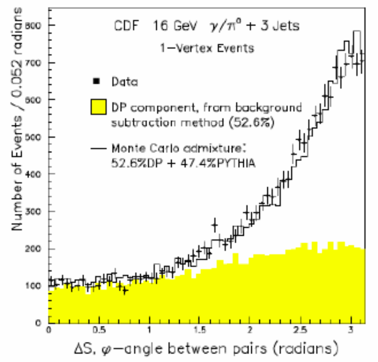
- hadrons = extended objects!
- no guarantee for one scattering only.
- running of α_S
  $\Longrightarrow$ preference for soft scattering.



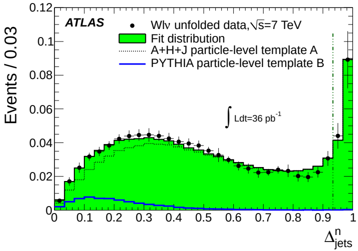
- first experimental evidence for double–parton scattering: events with $\gamma + 3$ jets:
  - cone jets, $R = 0.7$, $E_T > 5$ GeV; $|\eta_j| < 1.3$;
  - "clean sample": two softest jets with $E_T < 7$ GeV;
- cross section for DPS

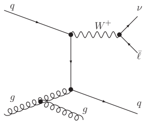
$$\sigma_{\rm DPS} = \frac{\sigma_{\gamma j}\sigma_{jj}}{\sigma_{\rm eff}}$$

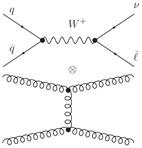
$\sigma_{\rm eff} \approx 14 \pm 4$ mb.

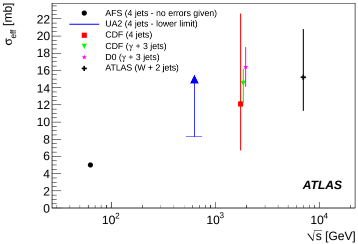


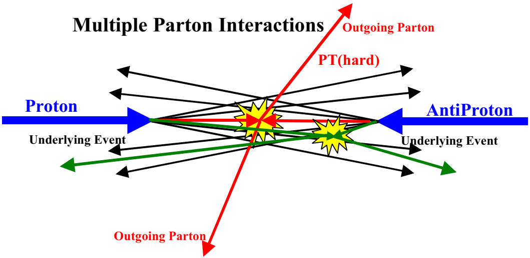
- more measurements, also at LHC
- ATLAS results from $W+2$ jets











but: how to define the underlying event?

1. everything apart from the hard interaction, but including IS showers, FS showers, remnant hadronisation.
2. remnant-remnant interactions, soft and/or hard.
3. lesson: hard to define

- origin of MPS: parton–parton scattering cross section exceeds hadron–hadron total cross section

$$\sigma_{\rm hard}(p_{\perp,\rm min}) = \int\limits_{p^2_{\perp,\rm min}}^{s/4} {\rm d}p_\perp^2 \frac{{\rm d}\sigma(p_\perp^2)}{{\rm d}p_\perp^2} > \sigma_{pp,\rm total}$$

  for low $p_{\perp,\rm min}$

- remember

$$\frac{{\rm d}\sigma(p_\perp^2)}{{\rm d}p_\perp^2} = \int\limits_0^1 {\rm d}x_1{\rm d}x_2 f(x_1,\, q^2) f(x_2,\, q^2)\,\frac{{\rm d}\hat\sigma_{2\to2}}{{\rm d}p_\perp^2}$$

- $\langle \sigma_{\rm hard}(p_{\perp,\rm min})/\sigma_{pp,\rm total}\rangle \geq 1$
- depends strongly on cut-off $p_{\perp,\rm min}$ (energy-dependent)!

Modelling the underlying event

- take the old PYTHIA model as example:
  - start with hard interaction, at scale $Q^2_{\rm hard}$.
  - select a new scale $p_\perp^2$ from

$$\exp\left[-\frac{1}{\sigma_{\rm norm}}\int\limits_{p_\perp^2}^{Q^2_{\rm hard}} {\rm d}p_\perp{}'^2 \frac{{\rm d}\sigma(p_\perp^2)}{{\rm d}p'^2_\perp}\right]$$

    with constraint $p_\perp^2 > p^2_{\perp,\rm min}$
  - rescale proton momentum ("proton-parton = proton with reduced energy").
  - repeat until no more allowed $2 \to 2$ scatter

Modelling the underlying event

- possible refinements:
  - may add impact-parameter dependence $\longrightarrow$ more fluctuations
  - add parton showers to UE
  - "regularisation" to dampen sharp dependence on $p_{\perp,\mathrm{min}}$: replace $1/\hat{t}$ in MEs by $1/(t+t_0)$, also in α_s.
  - treat intrinsic $k_\perp$ of partons ($\to$ parameter)
  - model proton remnants ($\to$ parameter)

Practicalities

- see some data comparison in Minimum Bias
- practicalities of underlying event models: parameters
  - profile in impact parameter space — 2-3 parameters
  - IR cut-off at reference energy, its energy evolution, dampening paramter and normalisation cross section — 4 parameters
  - treating colour connections to rest of event — 2-5 parameters
- tuned to LHC data, overall agreement satisfying
- energy extrapolation not exactly perfect, plus other process categories such as diffraction etc..

STATE OF THE ART

State of the art: fixed order

- NLO (QCD) "revolution" consolidated:
  - lots of routinely used tools for large FS multis (4 and more)
  - incorporation in MC tools done, need comparisons, critical appraisals and a learning curve in their phenomenological use
  - to improve: description of loop–induced processes
- amazing success in NNLO (QCD) calculations:
  - emergence of first round of $2 \to 2$ calculations
  - next revolution imminent (with question marks)
  - first MC tools for simple processes ($gg \to H$, DY), more to be learnt by comparison etc. (see above)
- first N^{3}LO calculation in $gg \to H$, more to come (?)
- attention turning to NLO (EW)
  - first benchmarks with new methods (V+3j)
  - calculational setup tricky
  - need maybe faster approximation for high-scales (EW Sudakovs)

Limitations: fixed order

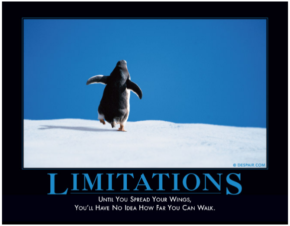
- practical limitations/questions to be overcome:
  - dealing with IR divergences at NNLO: slicing vs. subtracting

    (I'm not sure we have THE solution yet)
  - how far can we push NNLO? are NLO automated results stable enough for NNLO at higher multiplicity?
  - users of codes: higher orders tricky $\rightarrow$ training needed

    (MC = black box attitude problematic - a new brand of pheno/experimenters needed?)
- limitations of perturbative expansion:
  - breakdown of factorisation at HO (Seymour et al.)
  - higher-twist: compare $(\alpha_s/\pi)^n$ with $\Lambda_{\rm QCD}/M_Z$
- limitations in analytic resummation: process- and observable-dependent
  - first attempts at automation (CAESAR and some others) – checks/cross-comparison necessary
- showering needs to be improved (for NNLO the "natural" accuracy is NNLL)

State of the art: event generation

- Systematic improvement of event generators by including higher orders has been at the core of QCD theory and developments in the past decade:
  - multijet merging ("CKKW", "MLM")
  - NLO matching ("MC@NLO", "POWHEG")
  - MENLOPS NLO matching & merging
  - MEPS@NLO ("SHERPA", "UNLOPS", "MINLO", "FxFx")



- multijet merging an important tool for many relevant signals and backgrounds - pioneering phase at LO & NLO over
- complete automation of NLO calculations done
  $\longrightarrow$ must benefit from it!

(it's the precision and trustworthy & systematic uncertainty estimates!)

Vision

- we have constructed lots of tools for precision physics at LHC
  but we did not cross-validate them careful enough (yet)
  but we did not compare their theoretical foundations (yet)
- we also need unglamorous improvements:
  - systematically check advanced scale-setting schemes (MINLO)
  - automatic (re-)weighting for PDFs & scales (ME: ✓, PS: -)
  - scale compensation in PS is simple (implement and check)
  - PDFs: to date based on FO vs. data — will we have to move to resummed/parton showered?

(reminder: LO* was not a big hit, though)

- . . . and maybe we will have to go to the "dirty" corners:
  higher-twist, underlying event, hadronization, . . .

(many of those driven by experiment)