Lectures on Supersymmetry

Georg Weiglein

IPPP Durham

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Lecture 1:

- Limitations of the Standard Model
- The SUSY algebra
- Consequences of the SUSY algebra, SUSY multiplets

The Standard Model (SM) cannot be the ultimate theory

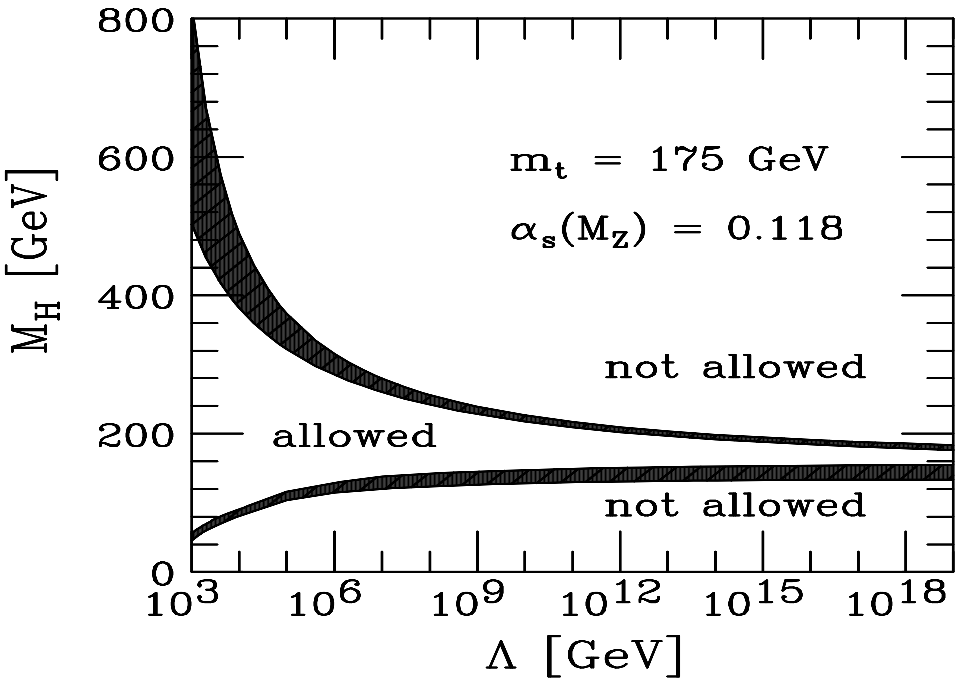
– doesn't contain gravity

– further problems: "hierarchy problem", . . .

Up to which energy scale Λ can it be valid?

– $\Lambda < M_{\mathrm{Pl}} \approx 10^{19}$ GeV (inclusion of gravity effects necessary)

– stability of Higgs potential:
[*T. Hambye, K. Riesselmann '97*]



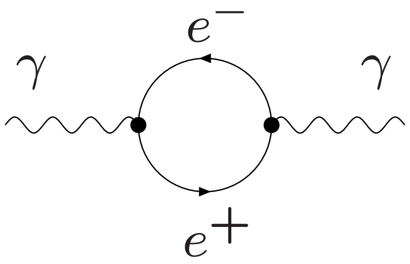
– hierarchy problem: Higgs mass unstable w.r.t. large quantum corrections
$\Rightarrow \delta M_{\mathrm{H}}^2 \sim \Lambda^2$

The hierarchy problem

Consider loop corrections to propagators $\Leftrightarrow$ corrections to particle masses

$$\Delta(p^2) \sim \frac{1}{p^2 - m^2 + \Sigma(p^2)}$$

Photon self-energy in QED:

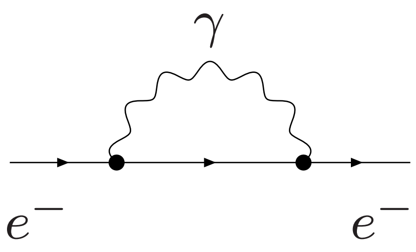


$$\Sigma^{\gamma\gamma}(0) = 0$$

consequence of U(1) gauge invariance of QED $\Rightarrow$ photon stays massless

$$\Delta^{-1}_{\gamma\gamma}(p^2) \to 0 \text{ for } p^2 \to 0$$

Electron self-energy in QED:



for $\Lambda \to \infty$: $\quad \Sigma^{ee} \sim m_{\mathrm{e}} \int^{\Lambda} \frac{dk}{k} \to \ln \Lambda$

$\Rightarrow$ logarithmically divergent correction to electron mass, δm_{e}

Within QED: divergence can be removed via renormalization
$\Rightarrow k \to \infty$ possible

QED as effective theory, underlying more fundamental theory at scale Λ
$\Rightarrow$ cutoff scale Λ

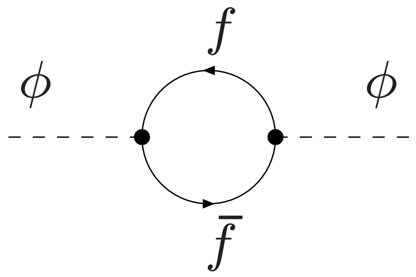
For $\Lambda = M_{\mathrm{Pl}}$: $\delta m_{\mathrm{e}} \approx 2\,\frac{\alpha}{\pi}\, m_{\mathrm{e}} \log(M_{\mathrm{Pl}}/m_{\mathrm{e}}) \approx 0.2\, m_{\mathrm{e}}$

$\Rightarrow$ modest correction, proportional to m_{e}

reason: chiral symmetry in limit $m_{\mathrm{e}} \to 0$, $\psi_{\mathrm{e}} \to e^{i\gamma_5\theta}\psi_{\mathrm{e}}$

$\Rightarrow$ breaking proportional to m_{e} $\Rightarrow$ symmetry "protects" m_{e}

Contribution of heavy fermions to Higgs self-energy:



$$\Sigma_f^{\phi\phi} \sim -2\, N(f)\, \lambda_f^2 \int d^4k \left(\frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{(k^2 - m_f^2)^2} \right)$$

for $\Lambda \to \infty$:
$$\Sigma_f^{\phi\phi} \sim -2\, N(f)\, \lambda_f^2 \left(\underbrace{\int \frac{d^4k}{k^2}}_{\sim \Lambda^2} + 2m_f^2 \underbrace{\int \frac{dk}{k}}_{\sim \ln \Lambda} \right)$$

$\Rightarrow$ quadratically divergent!

For $\Lambda = M_P$: $\delta M_\phi^2 \sim M_P^2 \Rightarrow \delta M_\phi^2 \approx 10^{30}\, M_\phi^2$ ($M_\phi \lesssim 1$ TeV)

no additional symmetry for $M_\phi = 0$, no protection against large corrections

$\Rightarrow$ in general: scalar masses tend to be near highest mass scale of the theory

$\Rightarrow$ hierarchy problem, extreme fine-tuning necessary to get small M_ϕ

⇒ Hierarchy problem is instability of small Higgs mass to large corrections in a theory with a large mass scale in addition to the weak scale

E.g.: Grand Unified Theory (GUT): $\delta M_\phi^2 \approx \lambda \langle v_{\mathrm{GUT}} \rangle^2$

Even if $\lambda = 0$ at tree level: non-zero coupling regenerated by radiative corrections: $\delta M_\phi^2 \approx \mathcal{O}(\alpha/\pi) \langle v_{\mathrm{GUT}} \rangle^2$ (cf. 'Little Higgs' models)

Would need a symmetry to suppress *many* orders of perturbation theory

Hierarchy problem is not just a problem of the Higgs mass;

problem: why is $M_{\mathrm{W}} \ll M_{\mathrm{GUT}}, M_{\mathrm{Pl}}$, why is $V_{\mathrm{Coulomb}} \gg V_{\mathrm{Newton}}$?

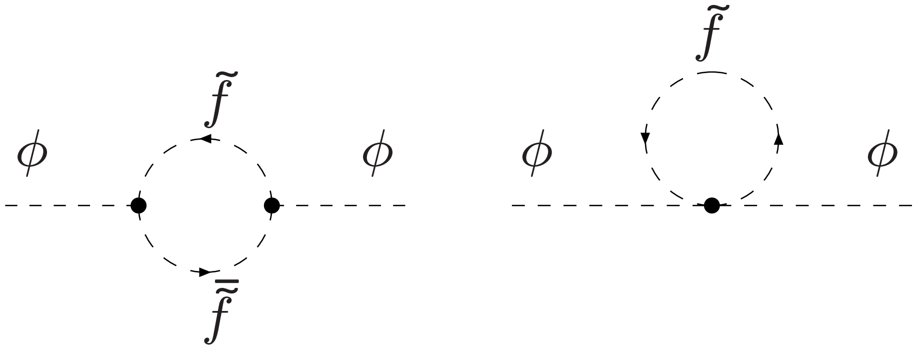
Note however: there is another fine-tuning problem in nature, for which we have no clue so far – cosmological constant

Supersymmetry:

Symmetry between fermions and bosons

$$\begin{aligned} Q|\text{boson}\rangle &= |\text{fermion}\rangle \\ Q|\text{fermion}\rangle &= |\text{boson}\rangle \end{aligned}$$

SUSY: additional contributions from scalar fields:



$$\Sigma_{\tilde{f}}^{\phi\phi} \sim N(\tilde{f})\, \tilde{\lambda}_f \int d^4k \left(\frac{1}{k^2 - m_{\tilde{f}_L}^2} + \frac{1}{k^2 - m_{\tilde{f}_R}^2} \right) + \text{terms without quadratic div.}$$

for $\Lambda \to \infty$: $\Sigma_{\tilde{f}}^{\phi\phi} \sim 2\, N(\tilde{f})\, \tilde{\lambda}_f\, \Lambda^2$

$\Rightarrow$ quadratic divergencies cancel for

$$N(\tilde{f}_L) = N(\tilde{f}_R) \;=\; N(f)$$
$$\tilde{\lambda}_f \;=\; \lambda_f^2$$

complete correction vanishes if furthermore

$$m_{\tilde{f}} = m_f$$

For $m^2_{\tilde{f}} = m^2_f + \Delta^2, \quad \tilde{\lambda}_f = \lambda^2_f$, "soft SUSY breaking"

$$\Rightarrow \; \Sigma^{\phi\phi}_{f+\tilde{f}} \sim N(f)\,\lambda_f^2\,\Delta^2 + \ldots$$

$\Rightarrow$ correction stays acceptably small if mass splitting is of weak scale

$\Rightarrow$ realized if mass scale of SUSY partners

$$M_{\text{SUSY}} \lesssim 1\,\text{TeV}$$

$\Rightarrow$ SUSY at TeV scale provides attractive solution of hierarchy problem

Other recent proposals for solving hierarchy problem in models with extra spatial dimensions:

E.g.: Gravity in large extra dimensions

[*N. Arkani-Hamed, S. Dimopoulos, G. Dvali '98*]

Gravity propagates in δ large extra spatial dimensions

fundamental Planck scale in $4+\delta$-dimensional space, M_*, is of order 1 TeV

$$M_{\mathrm{Pl}}^2 = V_\delta \, M_*^{2+\delta}$$

$\Rightarrow$ large value of M_{Pl} related to relatively large volume V_δ (10^{-3} m–10^{-15} m for $\delta = 2$–6) of extra dimensions

The SUSY algebra

Symmetry: a group of transformations that leave Lagrangian invariant

generators of the group fulfill certain algebra

Noether's theorem: symmetries $\Leftrightarrow$ conservation laws

Unification of fundamental interactions?

electroweak and strong interactions:

described by gauge theories: internal symmetries

γ, Z, $W^{\pm}$: spin 1

gravity:

described by general relativity: invariance under space–time transformations

graviton G: spin 2

Space–time symmetries: Poincaré group

Lorentz transformations:

$M^{\rho\nu} = -M^{\nu\rho}$ (6 generators), translations P^{ρ} (4 generators)

continous group: Lie-group

particle states characterized by eigenvalues of mass and spin

For later use it is convenient to describe spin $\frac{1}{2}$ particles not in terms of 4-component ‘Dirac spinors’ but in terms two 2-component ‘Weyl spinors’

Spinor representation of the Lorentz group

Form (complex) linear combinations of the generators of rotations and Lorentz boosts such that commutation relations become

$$\left[J_i^{\pm}, J_j^{\pm}\right] = i\epsilon_{ijk} J^{\pm,k}; \quad \left[J_i^{+}, J_j^{-}\right] = 0$$

⇒ two SU(2) algebras, commute with each other

⇒ Lorentz group is equivalent to a (complexified) version of SU(2) $\times$ SU(2) ($\rightarrow$ SL(2, C))

$\Rightarrow$ Representations of Lorentz group are labelled by two 'spins', j_1, j_2, where $j_1, j_2 = 0, \frac{1}{2}, 1, \ldots$ as usual for SU(2)

Basic representations:

$(\frac{1}{2}, 0)$: LEFT-handed 2-component Weyl spinor, ψ_α

$(0, \frac{1}{2})$: RIGHT-handed 2-component Weyl spinor, $\bar{\psi}^{\dot{\alpha}}$

The two component Weyl spinors ψ_α (left-handed) and $\bar{\psi}^{\dot{\alpha}}$ (right-handed) transform under Lorentz transformations as follows:

$$
\begin{aligned}
\psi'_\alpha &= M_\alpha{}^\beta \psi_\beta; & \bar{\psi}'_{\dot{\alpha}} &= (M^*)_{\dot{\alpha}}{}^{\dot{\beta}} \bar{\psi}_{\dot{\beta}} \\
\psi'^\alpha &= (M^{-1})_\beta{}^\alpha \psi^\beta; & \bar{\psi}'^{\dot{\alpha}} &= (M^{*-1})_{\dot{\beta}}{}^{\dot{\alpha}} \bar{\psi}^{\dot{\beta}}
\end{aligned}
$$

where $M = \exp(i\frac{\vec{\sigma}}{2}(\vec{\vartheta} - i\vec{\varphi}))$ and $\vec{\vartheta}$ and $\vec{\varphi}$ are the three rotation angles and boost parameters, respectively

$\Rightarrow$ spinors with undotted indices (first two components of Dirac spinor) transform according to $(\frac{1}{2}, 0)$-representation of Lorentz group,

spinors with dotted indices (last two components of Dirac spinor) transform according to $(0, \frac{1}{2})$-representation

Internal symmetries, e.g. gauge symmetries

QED: $\psi \to e^{iQ\lambda}\psi$, $A_\mu \to A_\mu + \partial_\mu \lambda$, generator: Q, group U(1)

QCD: SU(3), SM: SU(2) $\times$ U(1)

generators T^a of internal symmetries satisfy Lie algabra, $\left[T^a, T^b\right] = i f^{abc} T_c$

'Trivial' extension of Poincaré group, T^a commute with Poincaré generators

$[P^\mu, T^a] = 0$, $[M^{\mu\nu}, T^a] = 0$

$\Rightarrow$ direct product

(Poincaré group) $\otimes$ (internal symmetry group)

Particle states characterized by maximal set of commuting observables:

$$|\underbrace{m, s; \vec{p}, s_3}_{\text{space–time}};\ \underbrace{Q, I, I_3, Y, \ldots}_{\text{internal}}\rangle$$

quantum numbers

Direct product $\Rightarrow$ no irreducible multiplets containing particles with different mass or different spin

$\Rightarrow$ Look for extension of space–time symmetry with new generators Q_α such that
$[Q_\alpha, M^{\mu\nu}] \neq 0$ or $[Q_\alpha, P^\mu] \neq 0$

But: general "no go theorems" of QFT

Coleman–Mandula theorem '67:

Any Lie-group containing Poincaré group P and internal symmetry group G must be direct product $P \otimes G$

No go theorem can be evaded if instead of Lie-group (generators fulfill commutator relations):
[*Gol'fand, Likhtman '71*] [*Volkov, Akulov '72*] [*Wess, Zumino '73*]

$$[\ldots,\ldots] \to \{\ldots,\ldots\}$$

Anticommutator: $\{A, B\} = AB + BA$

$\Rightarrow$ 'graded Lie algebra', 'superalgebra'

Haag, Lopuszanski, Sohnius theorem '75:

no direct symmetry transformations between fields with different integer spins

$\Rightarrow$ particles with different spin in the same multiplet only possible for SUSY theories, $Q|\text{boson}\rangle = |\text{fermion}\rangle$, $Q|\text{fermion}\rangle = |\text{boson}\rangle$

symmetry generator Q: fermionic operator, needs to have spin 1/2

E.g.:

$$\begin{array}{ccccc} \text{spin } 2 & \longrightarrow & \text{spin } \frac{3}{2} & \longrightarrow & \text{spin } 1 \\ \text{graviton} & & \text{gravitino} & & \text{photon} \end{array}$$

Q changes spin (behavior under spatial rotations) by $\frac{1}{2}$

$\Rightarrow$ SUSY transformation influences in general both space–time and internal quantum numbers

Simplest case:
only one fermionic generator Q_α (and conjugate $\bar{Q}_{\dot\beta}$, $\alpha = 1, 2$, $\dot\beta = 1, 2$)

$\Rightarrow$ $N = 1$ SUSY algebra:

$$\begin{aligned}
[Q_\alpha, P_\mu] = \left[\bar{Q}_{\dot\beta}, P_\mu\right] &= 0 \\
[Q_\alpha, M^{\mu\nu}] &= i\,(\sigma^{\mu\nu})_\alpha{}^\beta\, Q_\beta \\
\left\{Q_\alpha, Q_\beta\right\} = \left\{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\right\} &= 0 \\
\{Q_\alpha, \bar{Q}_{\dot\alpha}\} &= 2(\sigma^\mu)_{\alpha\dot\alpha} P_\mu
\end{aligned}$$

Haag, Lopuszanski, Sohnius theorem $\Rightarrow$ only possibility

$P_0 = H$, $[Q_\alpha, H] = 0$ $\Rightarrow$ conserved charge

$\Rightarrow$ SUSY: symmetry that relates bosons to fermions

unique extension of Poincaré group of symmetries of $D = 4$ relativistic QFT

'generalizes classical notion of a dimension to quantum domain'

'so beautiful it must be true . . . '

Consequences of the SUSY algebra

Global SUSY transformation:

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = \underbrace{2(\sigma^\mu)_{\alpha\dot{\alpha}} P_\mu}_{\text{constant translation in space–time}}$$

If SUSY transformations are made local
⇒ space–time transformation differing from point to point

Invariance under local SUSY transformations:

⇒ invariance under local coordinate change

⇒ general relativity

⇒ local SUSY includes gravity, called "supergravity"

In the following: mostly global SUSY transformations considered (flat space–time)

Q_α changes spin of particle by $\frac{1}{2}$

$Q_\alpha|\text{boson}\rangle = |\text{fermion}\rangle$, $Q_\alpha|\text{fermion}\rangle = |\text{boson}\rangle$

Consider fermionic state $|f\rangle$ with mass m: bosonic state $|b\rangle = Q_\alpha|f\rangle$

$P^2|f\rangle = m^2|f\rangle$

$\Rightarrow P^2|b\rangle = P^2 Q_\alpha|f\rangle = Q_\alpha P^2|f\rangle = Q_\alpha m^2|f\rangle = m^2|b\rangle$

$\Rightarrow$ for each fermionic state there is a bosonic state with the same mass

$\Rightarrow$ states are paired bosonic $\leftrightarrow$ fermionic

$$\left\{Q_\alpha, \bar{Q}_{\dot\beta}\right\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$$

$$\Rightarrow \left\{Q_\alpha, \bar{Q}_{\dot\beta}\right\} \bar\sigma_\nu^{\alpha\dot\beta} = 2 \underbrace{\sigma^\mu_{\alpha\dot\beta}\bar\sigma_\nu^{\alpha\dot\beta}}_{2g^\mu{}_\nu} P_\mu = 4P_\nu$$

$\nu = 0 \Rightarrow H = P_0 = \frac{1}{4}\left\{Q_\alpha, \bar{Q}_{\dot\beta}\right\} \bar\sigma_0^{\alpha\dot\beta} = \frac{1}{4}\left(\left\{Q_1, Q_1^\dagger\right\} + \left\{Q_2, Q_2^\dagger\right\}\right)$

where $(\sigma^\mu)_{\alpha\dot\alpha} \equiv \{1, \sigma^1, \sigma^2, \sigma^3\}_{\alpha\dot\alpha}$; $\quad (\sigma_\mu)_{\alpha\dot\alpha} = g_{\mu\nu}(\sigma^\nu)_{\alpha\dot\alpha}$; $\quad (\bar\sigma^\mu)_{\dot\alpha\alpha} \equiv (\sigma^\mu)_{\alpha\dot\alpha}$

and $\bar{Q}_{\dot\alpha} = (Q_\alpha)^\dagger$

$\left\{Q_i, Q_i^\dagger\right\} = Q_i Q_i^\dagger + Q_i^\dagger Q_i$: hermitean operator, eigenvalues ≥ 0

$\Rightarrow$ for any state $|\alpha\rangle$: $\langle\alpha|H|\alpha\rangle \geq 0$

spectrum of H is bounded from below, ≥ 0

$\Rightarrow$ no negative eigenvalues

State with lowest energy: vacuum state $|0\rangle$

if vacuum state is symmetric, i.e. $Q|0\rangle = 0$, $Q^\dagger|0\rangle = 0$ for all Q

$\Rightarrow$ vacuum has zero energy, $\langle 0|H|0\rangle = E_{\text{vac}} = 0$

For spontaneous symmetry breaking: vacuum state is not invariant

$\Rightarrow$ If SUSY is spontaneously broken, i.e. $Q_\alpha|0\rangle \neq 0$,
then $\langle 0|H|0\rangle = E_{\text{vac}} > 0$

$\Rightarrow$ non-vanishing vacuum energy

SUSY multiplets:

Particularly important: massless representations

go to reference frame where $P_\mu = (E, 0, 0, E)$

$\Rightarrow Q_1, Q_{\dot{1}} = 0$ as operators; thus left with $Q_2, Q_{\dot{2}}$ only

define $a = \frac{1}{\sqrt{4E}} Q_2,\ a^\dagger = \frac{1}{\sqrt{4E}} \bar{Q}_{\dot{2}}$

$\Rightarrow$ annihilation and creation operators: $\{a, a^\dagger\} = 1, \{a, a\} = 0, \{a^\dagger, a^\dagger\} = 0$

start with state of lowest helicity λ_0

application of $a^\dagger \Rightarrow$ one additional state with helicity $\lambda_0 + \frac{1}{2}$

$\Rightarrow$ one fermionic + one bosonic state

(N SUSY generators $\Rightarrow 2^{N-1}$ bosonic and 2^{N-1} fermionic states)

$\Rightarrow$ equal number of bosonic and fermionic states in supermultiplet

Most relevant multiplets (possess also CPT conjugate 'mirrors'):

- chiral supermultiplet: $-\frac{1}{2}, 0$

  Weyl fermion (quark, lepton, ...) $+$ complex scalar (squark, slepton)

- vector supermultiplet: $-1, -\frac{1}{2}$

  Gauge boson (massless vector) $+$ Weyl fermion (gaugino)

- graviton supermultiplet: $-2, -\frac{3}{2}$

  graviton $+$ gravitino

More on spinors:

The components of the spinors are Grassmann variables, i.e. anticommuting c-numbers

Raising and lowering of indices through the totally antysymmetric ϵ-tensor:

$$\epsilon^{\alpha\beta} = \epsilon^{\dot{\alpha}\dot{\beta}} \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \quad \epsilon_{\alpha\beta} = \epsilon_{\dot{\alpha}\dot{\beta}} \equiv \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\Rightarrow \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta; \quad \psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta; \quad \bar{\psi}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\psi}^{\dot{\beta}}; \quad \bar{\psi}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\psi}_{\dot{\beta}};$$

In particular

$$\epsilon^{\alpha}{}_{\beta} = \epsilon^{\alpha\gamma}\epsilon_{\gamma\beta} = \delta^{\alpha}{}_{\beta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The product of two Grassmann spinors is defined through

$$\theta\zeta \equiv \theta^\alpha\zeta_\alpha = \theta^\alpha\epsilon_{\alpha\beta}\zeta^\beta = -\epsilon_{\alpha\beta}\zeta^\beta\theta^\alpha = \zeta^\beta\epsilon_{\beta\alpha}\theta^\alpha = \zeta^\beta\theta_\beta = \zeta\theta$$

$$\bar{\theta}\bar{\zeta} \equiv \bar{\theta}_{\dot{\alpha}}\bar{\zeta}^{\dot{\alpha}} = -\bar{\zeta}^{\dot{\alpha}}\bar{\theta}_{\dot{\alpha}} = \bar{\zeta}\bar{\theta}$$

The γ-matrices are defined by

$$\gamma^\mu \equiv \begin{pmatrix} 0 & (\sigma^\mu)_{\alpha\dot{\alpha}} \\ (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} & 0 \end{pmatrix}; \quad \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

have the usual commutation relation, $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$; follows from

$$(\sigma^\mu)_{\alpha\dot{\alpha}}(\bar{\sigma}^\nu)^{\dot{\alpha}\alpha} = \mathrm{Tr}(\sigma^\mu\bar{\sigma}^\nu) = 2g^{\mu\nu}$$

The Lorentz covariant expressions involving 4-component Dirac spinors can be written in two-component notation using

$$\Phi = \begin{pmatrix} \lambda_\alpha \\ \bar{\phi}^{\dot{\alpha}} \end{pmatrix}; \quad \overline{\Psi} = \begin{pmatrix} \chi^\alpha & \bar{\psi}_{\dot{\alpha}} \end{pmatrix}$$

$$\begin{aligned}
\overline{\Psi}\Phi &= \chi\lambda + \bar{\psi}\bar{\phi} = \chi^\alpha\lambda_\alpha + \bar{\psi}_{\dot{\alpha}}\bar{\phi}^{\dot{\alpha}} \\
\overline{\Psi}\gamma^5\Phi &= \bar{\psi}\bar{\phi} - \chi\lambda = \bar{\psi}_{\dot{\alpha}}\bar{\phi}^{\dot{\alpha}} - \chi^\alpha\lambda_\alpha \\
\overline{\Psi}\gamma^\mu\Phi &= \chi\sigma^\mu\bar{\phi} - \lambda\sigma^\mu\bar{\psi} = \chi^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\phi}^{\dot{\alpha}} - \lambda^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} \\
\overline{\Psi}\gamma^\mu\gamma^5\Phi &= \chi\sigma^\mu\bar{\phi} + \lambda\sigma^\mu\bar{\psi} = \chi^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\phi}^{\dot{\alpha}} + \lambda^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} \\
\overline{\Psi}\gamma^\mu\gamma^\nu\Phi &= \chi\sigma^\mu\bar{\sigma}^\nu\lambda + \bar{\psi}\bar{\sigma}^\mu\sigma^\nu\bar{\phi} = \chi^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}(\bar{\sigma})^{\dot{\alpha}\beta}\lambda_\beta + \bar{\psi}_{\dot{\alpha}}(\bar{\sigma}^\mu)^{\dot{\alpha}\beta}(\sigma^\nu)_{\beta\dot{\beta}}\bar{\phi}^{\dot{\beta}}
\end{aligned}$$

Summary of Lecture 1:

- Standard Model cannot be the ultimate theory

  Hierarchy problem, . . .
  $\Rightarrow$ strong motivation for low-energy (TeV scale) SUSY

- SUSY: relates bosons to fermions

  SUSY algebra involves anticommutators: 'graded Lie algebra'

  SUSY is unique extension of Poincaré group of space–time symmetries

  local SUSY includes gravity: "supergravity"

- Exact SUSY $\Leftrightarrow E_{\mathrm{vac}} = 0$

  Spontaneous breaking of global SUSY $\Rightarrow E_{\mathrm{vac}} > 0$

- SUSY multiplets: states are paired bosonic $\leftrightarrow$ fermionic

  chiral, vector, graviton supermultiplet, . . .

Lectures on Supersymmetry

Georg Weiglein

IPPP Durham

Torino, 07/2003

<u>Lecture 2:</u>

- Construction of SUSY field theories
- $N > 1$ SUSY
- Soft SUSY breaking

Superspace and superfields

For compact representation of SUSY transformations

⇒ 'fermionic variables' θ^α, $\bar{\theta}^{\dot{\alpha}}$ introduced: anticommuting c-numbers (Grassmann variables)

E.g.: translations, generators P_μ, parameters of transformation x^μ

$N = 1$ SUSY, generators Q_α, $\bar{Q}_{\dot{\alpha}}$, parameters of transformation θ^α, $\bar{\theta}^{\dot{\alpha}}$

⇒ Extension of 4-dim. space–time by coordinates θ^α, $\bar{\theta}^{\dot{\alpha}}$: superspace

Point in superspace: $X = (x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$

Superfield: $\phi(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$

Taylor expansion in Grassmann variable: $\theta^\alpha \theta^\beta \theta^\gamma = 0$

$\Rightarrow$ Taylor expansion terminates after second term, i.e. $\phi(\theta) = a + \theta\psi + \theta\theta\, f$ $(\theta\theta \equiv \theta^\alpha \theta_\alpha)$

Integration: $\int d\theta = 0$, $\int d\theta\, \theta = 1$

$d^2\theta = -\frac{1}{4}\epsilon_{\alpha\beta} d\theta^\alpha d\theta^\beta$

$$\Rightarrow \quad \int d^2\theta\, \phi(\theta) = \int d^2\theta (a + \theta\psi + \theta\theta\, f) = f$$

With Grassmann variables:

SUSY algebra can be written in terms of commutators only

$$\begin{aligned}
[\theta Q, \bar\theta \bar Q] &= 2\theta\sigma^\mu \bar\theta P_\mu \\
[\theta Q, \theta Q] = [\bar\theta \bar Q, \bar\theta \bar Q] &= 0 \\
[P^\mu, \theta Q] = [P^\mu, \bar\theta \bar Q] &= 0
\end{aligned}$$

$\Rightarrow$ can be treated like Lie-group with anticommuting parameters

SUSY transformations

Group element of finite SUSY transformation:

$$S(y,\xi,\bar\xi) = \mathrm{exp}\, i\left(\xi Q + \bar\xi\bar Q - y^\mu P_\mu\right)$$

in analogy to group elements for Lie-groups

ξ, $\bar\xi$ are independent of x^μ: global SUSY transformation

Transformation of superfield: $S(y,\xi,\bar\xi)\phi(x,\theta,\bar\theta)$

Use Hausdorff's formula ($e^A e^B = \ldots$) and SUSY algebra

$$\Rightarrow \quad S(y,\xi,\bar\xi)\phi(x,\theta,\bar\theta) = \phi(x^\mu + y^\mu - i\xi\sigma^\mu\bar\theta + i\theta\sigma^\mu\bar\xi, \xi+\theta, \bar\xi+\bar\theta)$$

representations of generators obtained from infinitesimal transformation of superfield

$$\Rightarrow \quad P_\mu = i\partial_\mu, \quad Q_\alpha = -i\,\partial_\alpha + (\sigma^\mu\bar\theta)_\alpha\partial_\mu, \quad \bar Q_{\dot\alpha} = i\,\bar\partial_{\dot\alpha} - (\theta\sigma^\mu)_{\dot\alpha}\partial_\mu$$

Definition of covariant derivatives:

$$D_\alpha = -i\,\partial_\alpha - (\sigma^\mu\bar\theta)_\alpha\partial_\mu, \qquad \bar D_{\dot\alpha} = i\,\bar\partial_{\dot\alpha} + (\theta\sigma^\mu)_{\dot\alpha}\partial_\mu$$

Anticommutation relations of Q_α, D_α:

$$\begin{aligned}
\{Q_\alpha, D_\beta\} &= \{\bar Q_{\dot\alpha}, \bar D_{\dot\beta}\} = \{Q_\alpha, \bar D_{\dot\beta}\} = 0\\
\{Q_\alpha, Q_\beta\} &= \{\bar Q_{\dot\alpha}, \bar Q_{\dot\beta}\} = 0\\
\{D_\alpha, D_\beta\} &= \{\bar D_{\dot\alpha}, \bar D_{\dot\beta}\} = 0\\
\{Q_\alpha, \bar Q_{\dot\alpha}\} &= 2i(\sigma^\mu)_{\alpha\dot\alpha}\partial_\mu = 2(\sigma^\mu)_{\alpha\dot\alpha}P_\mu\\
\{D_\alpha, \bar D_{\dot\alpha}\} &= -2i(\sigma^\mu)_{\alpha\dot\alpha}\partial_\mu = -2(\sigma^\mu)_{\alpha\dot\alpha}P_\mu
\end{aligned}$$

$\Rightarrow$ D_α, $\bar D_{\dot\alpha}$ anticommute with SUSY generators

$\Rightarrow$ are invariant under SUSY transformations

$$[(\xi Q + \bar\xi\bar Q), D_\alpha] = 0, \qquad [(\xi Q + \bar\xi\bar Q), \bar D_{\dot\alpha}] = 0$$

General superfield in component form

Most general form of field depending on x, θ, $\bar{\theta}$:

$$\begin{aligned}\Phi(x,\theta,\bar{\theta}) \;&=\; \varphi(x)+\theta\psi(x)+\bar{\theta}\bar{\chi}(x)+\theta\theta F(x)+\bar{\theta}\bar{\theta}H(x)+\theta\sigma^{\mu}\bar{\theta}A_{\mu}(x)\\ &\quad+(\theta\theta)\bar{\theta}\bar{\lambda}(x)+(\bar{\theta}\bar{\theta})\theta\xi(x)+(\theta\theta)(\bar{\theta}\bar{\theta})D(x)\end{aligned}$$

Further terms vanish because of $\theta\theta\theta = 0$

Components (can be complex):

φ, F, H, D: scalar fields

A_μ: vector field

ψ, $\bar{\chi}$, $\bar{\lambda}$, ξ: Weyl-spinorfields

$\Rightarrow$ Too many components in 4-dim. for irreducible representation of SUSY with spin ≤ 1 (chiral or vector multiplet)

$\Rightarrow$ representation is reducible
(not all component fields mix with each other under SUSY transf.)

$\Rightarrow$ Irreducible superfields (smallest building blocks) from imposing conditions on general superfield
conditions need to be invariant under SUSY transformations:

$\bar{D}_{\dot{\alpha}}\Phi = 0$: left-handed chiral superfield (LHχSF)

$D_{\alpha}\Phi = 0$: right-handed chiral superfield (RHχSF)

$\Phi = \Phi^{\dagger}$: vector superfield

Usefulness of two-component spinors:

SM fermions: left-handed and right-handed components transform differently

$\Rightarrow$ need superfields with only two fermionic degrees of freedom

$\Rightarrow$ chiral superfields describe left- or right-handed component of SM fermion + scalar partner

LHχSF in components:

$$\begin{aligned}\phi(x,\theta,\bar\theta) \;=\; & \varphi(x)+\sqrt{2}\theta\psi(x)-i\theta\sigma^\mu\bar\theta\partial_\mu\varphi(x)+\frac{i}{\sqrt{2}}(\theta\theta)(\partial_\mu\psi(x)\sigma^\mu\bar\theta)\\ & -\frac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\partial^\mu\partial_\mu\varphi(x)-(\theta\theta)F(x)\end{aligned}$$

φ, F: scalar fields, ψ: Weyl-spinor field

F: auxiliary field, unphysical (has mass dimension 2)

RHχSF:

$$\bar\phi(x,\theta,\bar\theta)=\Big(\phi(x,\theta,\bar\theta)\Big)^\dagger$$

Transformation of component fields under infinitesimal SUSY transf.:

$$\delta\phi(x,\theta,\bar\theta) = i(\xi Q + \bar\xi\bar Q)\phi(x,\theta,\bar\theta)$$

Comparison with

$$\delta\phi(x,\theta,\bar\theta) = \delta\varphi + \sqrt{2}\theta\delta\psi - \ldots - (\theta\theta)\delta F$$

$\Rightarrow$ determination of $\delta\varphi$, $\delta\psi$, δF:

$$\begin{aligned} \delta\varphi &= \sqrt{2}\xi\psi && \text{boson} \to \text{fermion} \\ \delta\psi_\alpha &= -\sqrt{2}F\xi_\alpha - i\sqrt{2}(\sigma^\mu\bar\xi)_\alpha\partial_\mu\varphi && \text{fermion} \to \text{boson} \\ \delta F &= \partial_\mu(-i\sqrt{2}\psi\sigma^\mu\bar\xi) && F \to \text{total derivative} \end{aligned}$$

Vector superfield in components:

$$\begin{aligned}
V(x,\theta,\bar\theta) &= c(x) + i\theta\chi(x) - i\bar\theta\bar\chi(x) + \theta\sigma^\mu\bar\theta v_\mu(x) \\
&+ \frac{i}{2}(\theta\theta)(M(x)+iN(x)) - \frac{i}{2}(\bar\theta\bar\theta)(M(x)-iN(x)) \\
&+ i(\theta\theta)\bar\theta\left(\bar\lambda(x) + \frac{i}{2}\partial_\mu\chi(x)\sigma^\mu\right) - i(\bar\theta\bar\theta)\theta\left(\lambda(x) - \frac{i}{2}\sigma^\mu\partial_\mu\bar\chi(x)\right) \\
&+ \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)\left(D(x) - \frac{1}{2}\partial^\mu\partial_\mu c(x)\right)
\end{aligned}$$

$v_\mu(x)$: real vector field $\rightarrow$ describes gauge boson
(V must transform as adjoint representation of gauge group)

Number of components can be reduced by SUSY gauge transformation:

$$e^{gV} \longrightarrow e^{-ig\Lambda^\dagger} e^{gV} e^{ig\Lambda},$$

where $\Lambda(x,\theta,\bar\theta)$ is a chiral superfield and g is the gauge coupling

can perform a transformation such that $\chi(x) = c(x) = M(x) = N(x) \equiv 0$:
"Wess–Zumino gauge"

Wess–Zumino gauge removes many unphysical degrees of freedom

still leaves “ordinary” gauge freedom, e.g. $A_\mu(x) \longrightarrow A_\mu(x) + \partial_\mu \lambda(x)$ for abelian theory

Wess–Zumino gauge not preserved under SUSY transformations

Transformation of component fields under infinitesimal SUSY transf.:

$$\delta\lambda = \ldots, \quad \delta v_\mu = \ldots, \ldots, \quad \delta D = -\xi\sigma^\mu\partial_\mu\bar{\lambda}(x) - \partial_\mu\lambda(x)\sigma^\mu\bar{\xi}$$

$\Rightarrow$ D transforms into a total derivative

Supersymmetric Lagrangians

Aim: construct an action that is invariant under SUSY transformations:

$$\delta \int d^4x \mathcal{L}(x) = 0$$

Satisfied if $\mathcal{L} \longrightarrow \mathcal{L} +$ total derivative

F and D terms (the terms with the largest number of θ and $\bar{\theta}$ factors) of chiral and vector superfields transform into a total derivative under SUSY transformations

$\Rightarrow$ Use F-terms and D-terms to construct an invariant action:

$$S = \int d^4x \left(\int d^2\theta \mathcal{L}_f + \int d^2\theta d^2\bar{\theta} \mathcal{L}_d \right)$$

If Φ is a LHχSF $\Rightarrow$ Φ^n is also a LHχSF (since $\bar{D}_{\dot{\alpha}}\Phi^n = 0$ for $\bar{D}_{\dot{\alpha}}\Phi = 0$)

$\Rightarrow$ products of chiral superfields are chiral superfields, products of vector superfields are vector superfields

F-term Lagrangian:

$$\mathcal{L}_F = \int d^2\theta \sum_{ijk} \left(a_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} \lambda_{ijk} \Phi_i \Phi_j \Phi_k \right) + \text{ h.c.}$$

Terms of higher order in Φ_i lead to non-renormalizable Lagrangians

$\Rightarrow$ F-term Lagrangian contains mass terms, scalar–fermion interactions ($\rightarrow$ superpotential), but no kinetic terms

D-term Lagrangian:

$$\mathcal{L}_D = \int d^2\theta d^2\bar{\theta} V$$

$\Rightarrow$ D-term Lagrangian contains kinetic terms

Example: the Wess–Zumino Lagrangian

Construction of Lagrangian from chiral superfields Φ_i

$\Rightarrow$ Φ_i, $\Phi_i\Phi_j$, $\Phi_i\Phi_j\Phi_k$

$\Phi_i^\dagger\Phi_i$: vector superfield, $(\Phi_i^\dagger\Phi_i)^\dagger = \Phi_i^\dagger\Phi_i$

$$\left[\Phi_i^\dagger\Phi_i\right]_{\theta\theta\bar\theta\bar\theta} = F^\dagger F + (\partial_\mu\varphi^*)(\partial^\mu\varphi) + \frac{i}{2}(\psi\sigma^\mu\partial_\mu\bar\psi - \partial_\mu\psi\sigma^\mu\bar\psi) + \partial_\mu(\ldots)$$

Auxiliary field F can be eliminated via equations of motion

$$\begin{aligned}\Rightarrow \quad \mathcal{L} \;=\; & \frac{i}{2}(\psi_i\sigma^\mu\partial_\mu\bar\psi_i - (\partial_\mu\psi_i)\sigma^\mu\bar\psi_i) - \frac{1}{2}m_{ij}(\psi_i\psi_j + \bar\psi_i\bar\psi_j) \\ & + (\partial_\mu\varphi_i^*)(\partial^\mu\varphi_i) - \sum_i \left|a_i + \frac{1}{2}m_{ij}\varphi_j + \frac{1}{3}\lambda_{ijk}\varphi_j\varphi_k\right|^2 \\ & - \lambda_{ijk}\varphi_i\psi_j\psi_k - \lambda_{ijk}^\dagger\varphi_i^*\bar\psi_j\bar\psi_k\end{aligned}$$

Lagrangian for scalar fields φ_i and spinor fields ψ_i with the same mass m_{ii} contains couplings of type $hf\bar{f}$ and $\tilde{h}\bar{\tilde{h}}\tilde{f}$ with the same strength

$\Rightarrow$ SUSY implies relations between masses and couplings

$\mathcal{L}$ can be rewritten as kinetic part + contribution of superpotential $\mathcal{V}$:

$$\mathcal{V}(\varphi_i) = a_i\varphi_i + \frac{1}{2}m_{ij}\varphi_i\varphi_j + \frac{1}{3}\lambda_{ijk}\varphi_i\varphi_j\varphi_k$$

$$\begin{aligned}\Rightarrow \quad \mathcal{L} \; = \; & \frac{i}{2}(\psi_i\sigma^\mu\partial_\mu\bar{\psi}_i - (\partial_\mu\psi_i)\sigma^\mu\bar{\psi}_i) + (\partial_\mu\varphi_i^*)(\partial^\mu\varphi_i) \\ & -\sum_i\left|\frac{\partial\mathcal{V}}{\partial\varphi_i}\right|^2 - \frac{1}{2}\frac{\partial^2\mathcal{V}}{\partial\varphi_i\partial\varphi_j}\psi_i\psi_j - \frac{1}{2}\frac{\partial^2\mathcal{V}^*}{\partial\varphi_i^*\partial\varphi_j^*}\bar{\psi}_i\bar{\psi}_j\end{aligned}$$

$\mathcal{V}$ determines all interactions and mass terms

Special case $a_i = 0$: Wess–Zumino model

In order to include vector bosons need to construct Lagrangian containing also vector superfields

Further requirement: gauge invariance

$$\int d^2\theta d^2\bar{\theta}\Phi^\dagger\Phi \longrightarrow \int d^2\theta d^2\bar{\theta}\Phi^\dagger e^{2gV}\Phi$$

Kinetic terms for gauge fields from field strength

$\Rightarrow$ Supersymmetric Lagrangian in the Wess–Zumino gauge for chiral superfields ϕ_i (with component fields $\varphi_i,\ \psi_i$) and vector superfields V^a (with component fields $v^a_\mu,\ \lambda^a$):

$$\begin{aligned}
\mathcal{L} \;=\; & (D_\mu\varphi)^*_i(D^\mu\varphi)_i + \frac{i}{2}\psi_i\sigma^\mu(D_\mu\bar{\psi})_i - \frac{i}{2}(D_\mu\psi)_i\sigma^\mu\bar{\psi}_i \\
& - \frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \frac{i}{2}\lambda^a\sigma^\mu(D_\mu\bar{\lambda})^a - \frac{i}{2}(D_\mu\lambda)^a\sigma^\mu\bar{\lambda}^a \\
& + \sqrt{2}ig\,\bar{\psi}_i\lambda^a T^a_{ij}\varphi_j - \sqrt{2}ig\,\varphi^*_i T^a_{ij}\psi_j\lambda^a \\
& - \frac{1}{2}\frac{\partial^2\mathcal{V}}{\partial\varphi_i\partial\varphi_j}\psi_i\psi_j - \frac{1}{2}\frac{\partial^2\mathcal{V}^*}{\partial\varphi^*_i\partial\varphi^*_j}\bar{\psi}_i\bar{\psi}_j - V(\varphi_i,\varphi^*_j)
\end{aligned}$$

$\mathcal{V}(\varphi_i) = a_i\varphi_i + \frac{1}{2}m_{ij}\varphi_i\varphi_j + \frac{1}{3}\lambda_{ijk}\varphi_i\varphi_j\varphi_k$: holomorphic function of chiral sf.

The potential V is the sum of the F-terms and D-terms and reads

$$V(\varphi_i, \varphi_j^*) = \sum_i \left| \frac{\partial \mathcal{V}}{\partial \varphi_i} \right|^2 + \frac{1}{2} g^2 \sum_a (\varphi_i^* T_{ij}^a \varphi_j + \xi^a)^2$$

All couplings are determined either by gauge couplings or the superpotential

E.g.: D-term in potential V: quartic scalar interaction is given by the gauge coupling (in contrast to the SM case)

The ξ^a-term (called the Fayet-Iliopoulos term) can be present only for $U(1)$ gauge fields

The requirement of gauge invariance imposes constraints on the coefficients a_i, m_{ij} and λ_{ijk} of the superpotential $\mathcal{V}$

The (gauge) covariant derivatives are (f^{abc} are the structure constants of the gauge group)

$$(D_\mu \varphi)_i = \partial_\mu \varphi_i + ig\, v_\mu^a T_{ij}^a \varphi_j, \; (D_\mu \psi)_i = \partial_\mu \psi_i + ig\, v_\mu^a T_{ij}^a \psi_j, \; (D_\mu \lambda)^a = \partial_\mu \lambda^a - g f^{abc} v_\mu^b \lambda^c$$

Summary on construction of SUSY Lagrangians:

Construct Lagrangians for $N = 1$ SUSY from chiral superfields and vector superfields:

multiplets containing pairs of fields with the same mass, spin differs by $\frac{1}{2}$

Fermion Yukawa interactions and scalar quartic self-interactions are determined by the superpotential

Gauge interactions determine couplings of the gauge fields

$\Rightarrow$ Many relations between couplings

$N > 1$ SUSY

So far: $N = 1$ SUSY, simplest case, only one fermionic generator and its hermitean adjoint: Q_α, $\bar{Q}^{\dot{\beta}}$

$\Rightarrow$ one superpartner for photon: photino

N-extended SUSY: N generators Q^A_α, $\bar{Q}^{\dot{\beta}}_B$, $A = 1, \ldots, N$

$\Rightarrow$ N superpartners for the photon, . . .

Generalization of anticommutator relation:

$$\{Q^A_\alpha, Q^B_\beta\} = \epsilon_{\alpha\beta} X^{AB}$$

$X^{AB} = -X^{BA}$: "central charges"

Problem:

helicity $+\frac{1}{2}$ and helicity $-\frac{1}{2}$ fermions are in same supermultiplet
(e.g.: hypermultiplet for $N = 2$ SUSY)

$\Rightarrow$ helicity $+\frac{1}{2}$ and $-\frac{1}{2}$ fermions need to transform in the same way under gauge transformations

not possible for chiral fermions of electroweak theory

$\Rightarrow$ $N > 1$ SUSY theories are 'non-chiral'

$\Rightarrow$ $N = 1$ SUSY theories are the best candidates for a realistic low-energy theory (extension of the SM)

However: $N > 1$ SUSY have interesting properties

e.g.: $N = 4$ SUSY field theory (flat space) is finite

Seiberg–Witten solution in $N = 2$ SUSY, . . .

Soft SUSY breaking

Exact SUSY: $m_f = m_{\tilde{f}}, \ldots$

$\Rightarrow$ in a realistic model: SUSY must be broken

Only satisfactory way for model of SUSY breaking:
spontaneous SUSY breaking

Specific SUSY-breaking schemes (see below) in general yield effective Lagrangian at low energies, which is supersymmetric except for explicit soft SUSY-breaking terms

Soft SUSY-breaking terms: do not alter dimensionless couplings
(i.e. dimension of coupling constants of soft SUSY-breaking terms > 0)

$\Rightarrow$ no quadratic divergences (in all orders of perturbation theory)

scale of SUSY-breaking terms: $M_{\mathrm{SUSY}} \lesssim 1$ TeV

Classification of possible soft breaking terms:

[*L. Girardello, M. Grisaru '82*]

- scalar mass terms: $m^2_{\phi_i}\,|\phi_i|^2$
- trilinear scalar interactions: $A_{ijk}\phi_i\phi_j\phi_k + \text{h.c.}$
- gaugino mass terms: $\frac{1}{2}m\bar{\lambda}\lambda$
- bilinear terms: $B_{ij}\phi_i\phi_j + \text{h.c.}$
- linear terms: $C_i\phi_i$

$\Rightarrow$ relations between dimensionless couplings unchanged

no additional mass terms for chiral fermions

Unconstrained MSSM:

no particular SUSY breaking mechanism assumed, parameterization of possible soft SUSY-breaking terms

$\Rightarrow$ relations between dimensionless couplings unchanged
no quadratic divergencies

most general case:

$\Rightarrow$ 105 new parameters: masses, mixing angles, phases

Good phenomenological description for universal breaking terms (FCNC, . . .)

Scenarios for SUSY breaking $\Rightarrow$ prediction for soft SUSY-breaking terms in terms of small set of parameters

Experimental determination of SUSY parameters

$\Rightarrow$ Patterns of SUSY breaking

Summary of Lecture 2:

- Lagrangians for $N = 1$ SUSY are constructed from chiral superfields and vector superfields (multiplets containing pairs of fields with spin differing by $\frac{1}{2}$)

- $N = 1$ SUSY theories are the best candidates for a realistic low-energy theory (extension of the SM)

  $N > 1$ SUSY theories are 'non-chiral'

- Soft SUSY-breaking terms: do not alter dimensionless couplings, $M_{\text{SUSY}} \lesssim 1$ TeV

  $\Rightarrow$ no quadratic divergences

Lectures on Supersymmetry

Georg Weiglein

IPPP Durham

Torino, 07/2003

<u>Lecture 3:</u>

- The Minimal Supersymmetric Standard Model (MSSM)
- Models for SUSY breaking

The Minimal Supersymmetric Standard Model (MSSM)

MSSM: superpartners for SM fields

SM matter fermions are in different representation of gauge group than gauge bosons

$\Rightarrow$ need to be placed in different superfields

$\Rightarrow$ no SM fermion is a gaugino

Fermions, sfermions:

use definition of chiral superfields via left-handed fermions
($\Rightarrow$ the conjugates of right-handed ones appear)

LHχSF Q: quark, squark SU(2) doublets

LHχSF U: up-type quark, squark singlets

LHχSF D: down-type quark, squark singlets

LHχSF L: lepton, slepton SU(2) doublets

LHχSF E: lepton, slepton singlets

$\Rightarrow$ one generation of SM fermions and their superpartners described by five LHχSFs

Gauge bosons, gauginos:

Vector superfields:

gluons g and gluinos $\tilde{g}$

W bosons $W^{\pm}$, W^{0} and winos $\tilde{W}^{\pm}$, $\tilde{W}^{0}$

B boson B^{0} and bino $\tilde{B}^{0}$

Higgs bosons, higgsinos:

LHχSF

In MSSM: two Higgs doublets needed $\Rightarrow$ two LHχSFs

Comparison with SM case:

$$\mathcal{L}_{\mathsf{SM}} = \underbrace{m_d \bar{Q}_L H d_R}_{\text{d-quark mass}} + \underbrace{m_u \bar{Q}_L \tilde{H} u_R}_{\text{u-quark mass}}$$

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \tilde{H} = i\sigma_2 H^\dagger, \quad H \to \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \tilde{H} \to \begin{pmatrix} v \\ 0 \end{pmatrix}$$

In SUSY: term $\bar{Q}_L H^\dagger$ not allowed

Superpotential is holomorphic function of chiral superfields, i.e. depends only on φ_i, not on φ_i^*

No soft SUSY-breaking terms allowed for chiral fermions

$\Rightarrow$ H_d and H_u needed to give masses to down- and up-type fermions

Furthermore: two doublets also needed for cancellation of anomalies, quadratic divergences

Chiral supermultiplets of the MSSM:

		spin 0	spin $\frac{1}{2}$	$(\mathrm{SU}(3)_c,\ \mathrm{SU}(2),\ \mathrm{U}(1)_Y)$
squarks and quarks	Q	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$(3, 2, \frac{1}{6})$
	U	$\tilde{u}_R^*$	u_R^+	$(\bar{3}, 1, -\frac{2}{3})$
	D	$\tilde{d}_R^*$	d_R^+	$(\bar{3}, 1, \frac{1}{3})$
sleptons and leptons	L	$(\tilde{\nu}, \tilde{e}_L)$	(ν, e_L)	$(1, 2, -\frac{1}{2})$
	E	$\tilde{e}_R^*$	e_R^+	$(1, 1, 1)$
higgs and higgsinos	H_u	(h_u^+, h_u^0)	$(\tilde{h}_u^+, \tilde{h}_u^0)$	$(1, 2, \frac{1}{2})$
	H_d	(h_d^0, h_d^-)	$(\tilde{h}_d^0, \tilde{h}_d^-)$	$(1, 2, -\frac{1}{2})$

Vector supermultiplets:

	spin $\frac{1}{2}$	spin 1	(SU(3)$_c$, SU(2), U(1)$_Y$)
gluinos and gluons	$\tilde{g}$	g	$(8, 1, 0)$
winos and W-bosons	$\widetilde{W}^{\pm}, \widetilde{W}^0$	$W^{\pm}, W^0$	$(1, 3, 0)$
bino and B-boson	$\widetilde{B}$	B	$(1, 1, 0)$

Superpotential:

$$\mathcal{V}_{\text{MSSM}} = U\mathbf{y_u}QH_u + D\mathbf{y_d}QH_d + E\mathbf{y_e}LH_d + \mu H_u H_d$$

$\mathbf{y_i}, \mathbf{a_i}$: Yukawa couplings, 3×3 matrices in family space

All terms have to be invariant under all gauge groups, SU(3)$_c$, SU(2), U(1)$_Y$

$\Rightarrow$ need $Y = 0$ for all terms

$H_u H_d \equiv (H_u)_a (H_d)_b \epsilon^{ab}$ (a, b: weak isospin indices), . . .

$\mathcal{V}_{\text{MSSM}}$ is not the most general gauge-invariant superpotential

contains only terms that are necessary to build a realistic model

Gauge interactions introduce only terms with even number of superpartners

The same holds for minimal version of $\mathcal{V}_{\text{MSSM}}$

$\Rightarrow$ MSSM has further symmetry: "R-parity"

all SM-particles and Higgs bosons: even R-parity, $P_R = +1$

all superpartners: odd R-parity, $P_R = -1$

Soft breaking terms:

$$
\begin{aligned}
\mathcal{L}_{\text{soft}} \;=\; & -\frac{1}{2}\left(M_1 \widetilde{B}\widetilde{B} + M_2 \widetilde{W}\widetilde{W} + M_3 \tilde{g}\tilde{g}\right) + \text{h.c.} \\
& - m_{H_u}^2 H_u^+ H_u - m_{H_d}^2 H_d^+ H_d - (b H_u H_d + \text{h.c.}) \\
& - \left(\tilde{u}_R \mathbf{a_u} \widetilde{Q} H_u - \tilde{d}_R \mathbf{a_d} \widetilde{Q} H_d - \tilde{e}_R \mathbf{a_e} \widetilde{L} H_d\right) + \text{h.c.} \\
& - \widetilde{Q}^+ \mathbf{m_Q^2} \widetilde{Q} - \widetilde{L}^+ \mathbf{m_L^2} \widetilde{L} - \tilde{u}_R \mathbf{m_u^2} \tilde{u}_R^* - \tilde{d}_R \mathbf{m_d^2} \tilde{d}_R^* - \tilde{e}_R \mathbf{m_e^2} \tilde{e}_R^*
\end{aligned}
$$

Most general parameterization of SUSY-breaking terms that keep relations between dimensionless couplings unchanged $\Rightarrow$ no quadratic divergences

Gaugino mass terms, scalar mass terms, and terms like in superpotential for all scalars

$\mathbf{m_i^2}$: 3×3 matrices in family space

$\Rightarrow$ many new parameters

Particle content of the MSSM:

Superpartners for Standard Model particles:

$$\Big[u, d, c, s, t, b\Big]_{L,R} \quad \Big[e, \mu, \tau\Big]_{L,R} \quad \Big[\nu_{e,\mu,\tau}\Big]_L \qquad \text{Spin } \tfrac{1}{2}$$

$$\Big[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}\Big]_{L,R} \quad \Big[\tilde{e}, \tilde{\mu}, \tilde{\tau}\Big]_{L,R} \quad \Big[\tilde{\nu}_{e,\mu,\tau}\Big]_L \qquad \text{Spin } 0$$

$$g \quad \underbrace{W^{\pm}, H^{\pm}} \quad \underbrace{\gamma, Z, H_1^0, H_2^0} \qquad \text{Spin 1 / Spin 0}$$

$$\tilde{g} \qquad \tilde{\chi}^{\pm}_{1,2} \qquad \tilde{\chi}^0_{1,2,3,4} \qquad \text{Spin } \frac{1}{2}$$

Enlarged Higgs sector:
Two Higgs doublets, physical states: $h^0, H^0, A^0, H^{\pm}$

Breaking of SU(2) $\times$ U(1)$_Y$ (electroweak symmetry breaking)

$\Rightarrow$ fields with different SU(2) $\times$ U(1)$_Y$ quantum numbers can mix if they have the same SU(3)$_c$, U(1)$_{\text{em}}$ quantum numbers

Squark mixing:

Stop, sbottom mass matrices ($X_\mathrm{t} = A_\mathrm{t} - \mu/\tan\beta$, $X_\mathrm{b} = A_\mathrm{b} - \mu\tan\beta$):

$$\mathcal{M}^2_{\tilde{\mathrm{t}}} = \begin{pmatrix} m^2_{\tilde{t}_L} + m^2_\mathrm{t} + DT_1 & m_\mathrm{t} X_\mathrm{t} \\ m_\mathrm{t} X_\mathrm{t} & m^2_{\tilde{t}_R} + m^2_\mathrm{t} + DT_2 \end{pmatrix} \quad \Rightarrow \quad m_{\tilde{\mathrm{t}}_1}, m_{\tilde{\mathrm{t}}_2}, \theta_{\tilde{\mathrm{t}}}$$

$$\mathcal{M}^2_{\tilde{\mathrm{b}}} = \begin{pmatrix} m^2_{\tilde{b}_L} + m^2_\mathrm{b} + DB_1 & m_\mathrm{b} X_\mathrm{b} \\ m_\mathrm{b} X_\mathrm{b} & m^2_{\tilde{b}_R} + m^2_\mathrm{b} + DB_2 \end{pmatrix} \quad \Rightarrow \quad m_{\tilde{\mathrm{b}}_1}, m_{\tilde{\mathrm{b}}_2}, \theta_{\tilde{\mathrm{b}}}$$

off-diagonal element prop. to mass of partner quark ($\tan\beta \equiv v_u/v_d$)

$\Rightarrow$ mixing important in stop sector (also in sbottom sector for large $\tan\beta$)

gauge invariance $\Rightarrow m_{\tilde{t}_L} = m_{\tilde{b}_L}$

$\Rightarrow$ relation between $m_{\tilde{\mathrm{t}}_1}, m_{\tilde{\mathrm{t}}_2}, \theta_{\tilde{\mathrm{t}}}, m_{\tilde{\mathrm{b}}_1}, m_{\tilde{\mathrm{b}}_2}, \theta_{\tilde{\mathrm{b}}}$

Neutralinos and charginos:

Higgsinos and electroweak gauginos mix

charged:

$$\tilde{W}^+, \tilde{h}_u^+ \to \tilde{\chi}_1^+, \tilde{\chi}_2^+, \qquad \tilde{W}^-, \tilde{h}_d^- \to \tilde{\chi}_1^-, \tilde{\chi}_2^-$$

$\Rightarrow$ charginos: mass eigenstates

mass matrix given in terms of M_2, μ, $\tan\beta$

neutral:

$$\underbrace{\tilde{\gamma}, \tilde{Z},}_{\tilde{W}^0, \tilde{B}^0} \tilde{h}_u^0, \tilde{h}_d^0 \to \tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$$

mass matrix given in terms of M_1, M_2, μ, $\tan\beta$

$\Rightarrow$ only one new parameter

$\Rightarrow$ MSSM predicts mass relations between neutralinos and charginos

Models for SUSY breaking

Phenomenological constraints on soft-breaking terms:

E.g.: $(m^2)_{ij}\varphi_j^*\varphi_i = \tilde{e}_R m_{ee}^2 \tilde{e}_R^\dagger + \tilde{e}_R m_{e\mu}^2 \tilde{\mu}_R^\dagger + \ldots$

$(m^2)_{ij}$: 3×3 matrix in family (flavor) space

no a priori reason why off-diagonal terms $m_{e\mu}^2$ should be much smaller than diagonal terms m_{ee}^2 , $m_{ij}^2 \sim 1$ TeV^2

but: if off-diagonal terms are large

$\Rightarrow$ observable lepton number violation (e.g. $\mu \to e\gamma$)

Strong experimental constraints:

if $m_{e\mu}^2 \approx m_{ee}^2$
$\Rightarrow$ SUSY contribution 5–6 orders of magnitude above experimental limit

$\Rightarrow$ need $m_{e\mu}^2 \ll m_{ee}^2$

Similarly for squark mass matrices; also strong constraints on $\mathcal{CP}$-violating complex phases

$\Rightarrow$ experimental constraints satisfied if SUSY breaking is suitably 'universal', $\approx$ diagonal in flavour space

$\Rightarrow$ small flavour-changing neutral currents (FCNC)
no new complex phases in this limit

$\Rightarrow$ hints of an "organizing principle" for soft breaking terms?

Simplest ansatz:

Assume universality at high energy scale (GUT scale, M_{Pl}, ...)

- All scalar masses are the same: $\tilde{m}^2 = m_0^2$ (assumption of "universality")
- The gaugino masses are the same: $M_i = m_{1/2}$ ("GUT relation")
- Soft-breaking trilinear terms are universal:
  $\mathcal{L}_{\mathrm{tri}} = A_0(H_U Q y_u \bar{u} + H_D Q y_d \bar{d} + H_D L y_l \bar{e})$, y_u, y_d, ... are the same matrices which appear in Yukawa couplings (assumption of "proportionality")

Results in five parameters, if possible phases are ignored:

$$m_0^2,\ m_{1/2},\ A_0,\ b,\ \mu$$

Require correct value of M_Z

$\Rightarrow$ $|\mu|$, b given in terms of $\tan\beta$, $\mathrm{sign}\,\mu$

$\Rightarrow$ Scenario characterized by

$$m_0^2,\ m_{1/2},\ A_0,\ \tan\beta,\ \mathrm{sign}\,\mu$$

Usually called 'CMSSM' (constrained MSSM) or 'mSUGRA' (see below)

In agreement with all phenomenological constraints (see below)

Spontaneous breaking of global SUSY

If global SUSY is spontaneously broken $\Rightarrow$ $\langle 0|H|0\rangle = E_{\text{vac}} > 0$

$\Rightarrow$ $V > 0$ required

$$V(\varphi_i, \varphi_j^*) = \underbrace{\sum_i \left|\frac{\partial \mathcal{V}}{\partial \varphi_i}\right|^2}_{F\text{-term}} + \underbrace{\frac{1}{2} g^2 \sum_a (\varphi_i^* T_{ij}^a \varphi_j + \xi^a)^2}_{D\text{-term}}$$

$\Rightarrow$ either $\langle F\rangle > 0$ (F-term breaking) or $\langle D\rangle > 0$ (D-term breaking)

$\Rightarrow$ requires that $F_i = 0$, $D^a = 0$ cannot be simultaneously satisfied for any values of the fields

F-term breaking:

need linear term in superpotential $\mathcal{V}(\phi_i) = a_i\phi_i + \frac{1}{2}m_{ij}\phi_i\phi_j + \frac{1}{3}\lambda_{ijk}\phi_i\phi_j\phi_k$

$\Rightarrow$ requires a chiral superfield that is a singlet under all gauge groups

$\Rightarrow$ not possible within the MSSM

D-term breaking:

If $\langle\varphi_i\rangle = 0 \Rightarrow \xi^a$-term (Fayet-Iliopoulos term) leads to spontaneous SUSY breaking

Does not work in the MSSM (leads to charge and color-breaking minima)

Further problems of spontaneous breaking of global SUSY:

- Mass sum rules for SM particles and superpartners (at tree level), e.g.

$$\sum_i (-1)^f m_i^2 = 0$$

  $(-1)^f = 1$ for bosons, $(-1)^f = -1$ for fermions

  $\Rightarrow$ not all superpartners can be heavier than SM particles

  $\Rightarrow$ spectra not realistic

- difficult to give masses to gauginos (SUSY doesn't allow scalar–gaugino–gaugino couplings)

- hierarchy problem, if characteristic SUSY-breaking scale enters as parameter in the Lagrangian

- cosmological constant problem

$\Rightarrow$ Very difficult to build realistic model

Problems can be overcome if SUSY breaking happens in a 'hidden sector', i.e. by fields which have only very small couplings to ordinary matter

SUSY breaking in the hidden sector: tree-level (like F- and D-term breaking) or dynamical breaking (similar to chiral symmetry breaking in QCD), . . .

SUSY-breaking terms in the MSSM arise radiatively via interaction that communicates SUSY breaking rather than through tree-level couplings to SUSY breaking v.e.v.s

$\Rightarrow$ phenomenology depends mainly on mechanism for communicating SUSY breaking rather than on SUSY-breaking mechanism itself

If mediating interactions are $\approx$ flavor-diagonal
$\Rightarrow$ universal soft-breaking terms

“Hidden sector” : $\longrightarrow$ Visible sector:

SUSY breaking MSSM

“Gravity-mediated” : mSUGRA

“Gauge-mediated” : GMSB

“Anomaly-mediated” : AMSB

“Gaugino-mediated”

...

SUGRA: mediating interactions are gravitational

GMSB: mediating interactions are ordinary electroweak and QCD gauge interactions

AMSB, Gaugino-mediation: SUSY breaking happens on a different brane in a higher-dimensional theory

Gravity-mediated SUSY breaking

Local SUSY $\Leftrightarrow$ supergravity:

Spontaneous breaking of global SUSY $\longrightarrow$ massless neutral Weyl fermion: "goldstino"

Spontaneous breaking of local SUSY $\longrightarrow$ goldstino absorbed ("eaten") by gravitino

$\Rightarrow$ gives longitudinal components to gravitino

$\Rightarrow$ gravitino acquires a mass, graviton stays massless

"Super-Higgs mechanism"

only known consistent way of breaking local SUSY

It is possible to have spontaneous breaking of local SUSY and also $\langle V \rangle = 0$

Quantum field theory of supergravity:

QFT with spin 2 (and spin $\frac{3}{2}$) field is not renormalizable

$\Rightarrow$ cannot be extended to arbitrarily high energies

$\Rightarrow$ QFT of supergravity has to be interpreted as effective theory

contains non-renormalizable terms prop. to inverse powers of M_{Pl}

Best candidate for fundamental theory: string theory

SUSY breaking in hidden sector:

$\Rightarrow$ supergravity Lagrangian contains non-renormalizable terms that communicate between hidden and visible sector $\sim 1/M_{\mathsf{Pl}}^n$

Dimensional analysis:

SUSY breaking in hidden sector by v.e.v. $\langle F \rangle$, coupling $\sim 1/M_{\mathrm{Pl}}$

require $m_{\mathrm{soft}} \to 0$ for $\langle F \rangle \to 0$ (no SUSY breaking) and for $M_{\mathrm{Pl}} \to \infty$ (vanishing gravitational interaction)

$$\Rightarrow \quad m_{\mathrm{soft}} \approx \frac{\langle F \rangle}{M_{\mathrm{Pl}}}$$

For $m_{\mathrm{soft}} \lesssim 1$ TeV (hierarchy problem)

$\Rightarrow \sqrt{\langle F \rangle} \approx 10^{11}$ GeV: scale of SUSY breaking in hidden sector

In general: $m_{\frac{3}{2}} \approx \frac{\langle F \rangle}{M_{\mathrm{Pl}}}$

$\Rightarrow m_{\frac{3}{2}} \approx m_{\mathrm{soft}}$, gravitational interactions

$\Rightarrow$ gravitino not important for collider phenomenology

Non-renormalizable terms in supergravity Lagrangian:

$$\mathcal{L}_{\text{NR}} = -\frac{1}{M_{\text{Pl}}} F_X \sum_a \frac{1}{2} f_a \lambda^a \lambda^a + \text{h.c.} - \frac{1}{M_{\text{Pl}}^2} F_X F_X^* \, k^i_j \varphi_i \varphi^{*j}$$
$$- \frac{1}{M_{\text{Pl}}} F_X \left(\frac{1}{6} y'^{ijk} \varphi_i \varphi_j \varphi_k + \frac{1}{2} \mu'^{ij} \varphi_i \varphi_j \right) + \text{h.c.}$$

F_X: auxiliary field for a chiral supermultiplet X in the hidden sector

φ_i, λ^a: scalar and gaugino fields in the MSSM

If $\langle F_X \rangle \sim 10^{10}$–$10^{11}$ GeV

$\Rightarrow$ yields soft SUSY-breaking terms of MSSM with $m_{\text{soft}} \approx 10^2$–$10^3$ GeV

Assumption of a "minimal" form of the supergravity Lagrangian (theoretically difficult to justify; supergravity doesn't require flavor-universal terms)

$\Rightarrow$ soft-breaking terms which obey "universality" and "proportionality"

$\Rightarrow$ "supergravity inspired scenario", "mSUGRA", characterized by five parameters

$$m_0^2,\ m_{1/2},\ A_0,\ \tan\beta,\ \text{sign}\,\mu$$

m_0: universal scalar mass parameter

$m_{1/2}$: universal gaugino mass parameter

A_0: universal trilinear coupling

$\tan\beta$: ratio of Higgs vacuum expectation values

sign(μ): sign of supersymmetric Higgs parameter

m_0, $m_{1/2}$, A_0: GUT scale parameters

$\Rightarrow$ particle spectra from renormalization group running to weak scale

Lightest SUSY particle (LSP) is usually lightest neutralino

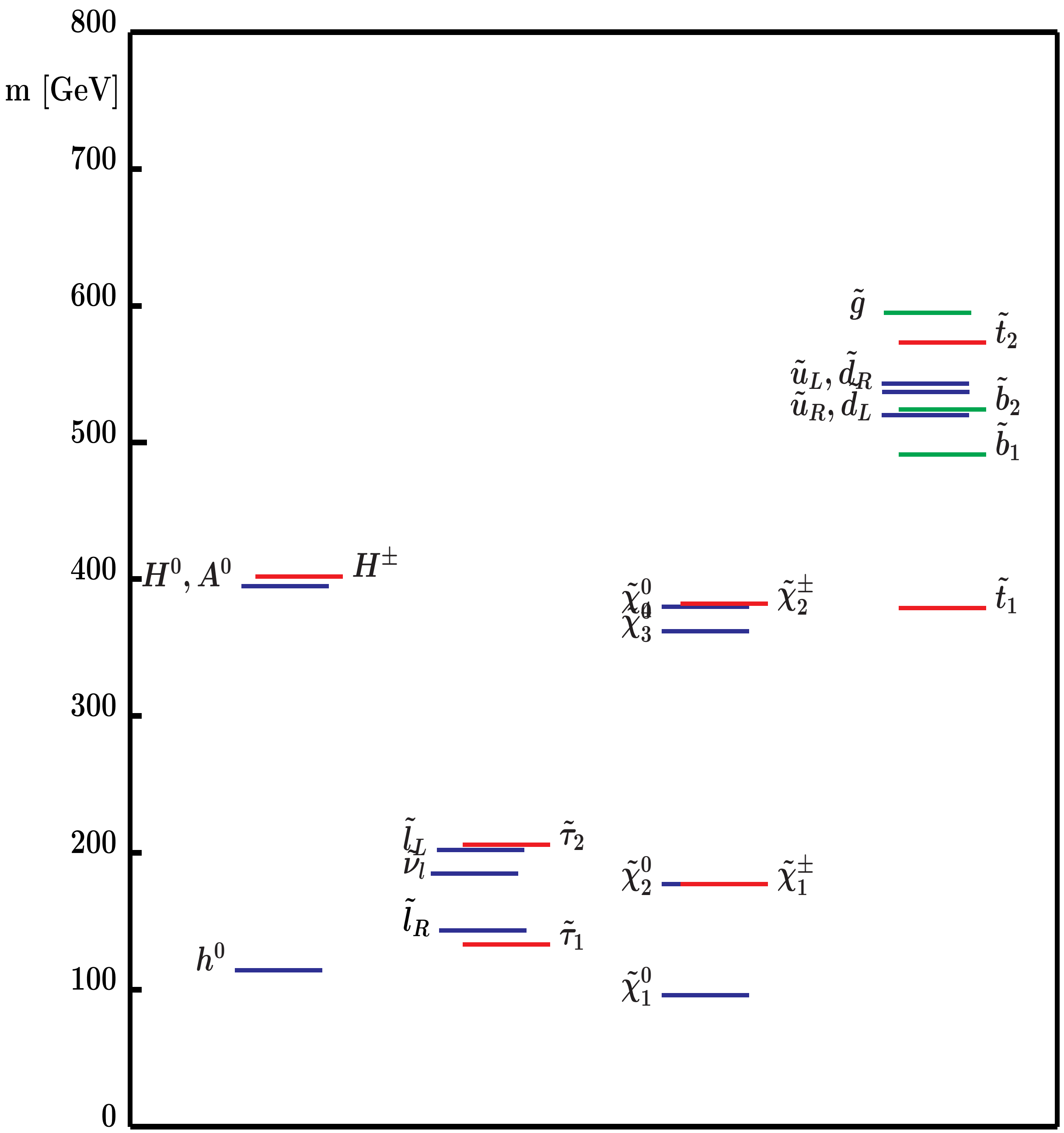
gaugino masses run in same way as gauge couplings

$\Rightarrow$ gluino heavier than charginos, neutralinos

“Typical” mSUGRA scenario

(SPS 1a benchmark scenario):

www.ippp.dur.ac.uk/∼georg/sps



Gauge-mediated SUSY breaking (GMSB)

New chiral supermultiplets, "messengers", couple to SUSY breaking in hidden sector

Couple indirectly to MSSM fields via gauge interactions

$\Rightarrow$ mediation of SUSY breaking via electroweak and QCD gauge interactions

$\Rightarrow$ $\approx$ flavour-diagonal

SUSY breaking in messenger spectrum

$\Rightarrow$ masses of SUSY particles from loop diagrams with messenger particles, gauge-interaction strength

$$\Rightarrow \quad m_{\text{soft}} \approx \frac{\alpha_i}{4\pi} \frac{\langle F \rangle}{M_{\text{mes}}}, \quad M_{\text{mes}} \sim \sqrt{\langle F \rangle}$$

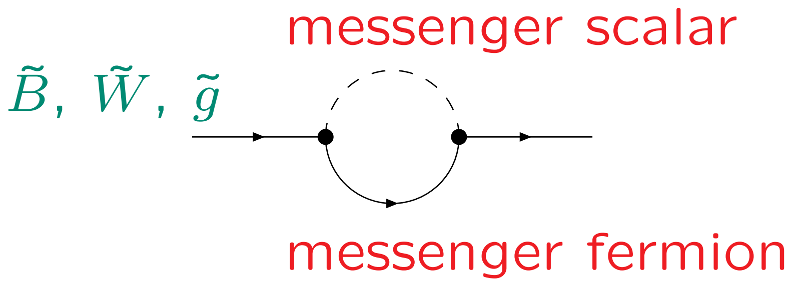
For $m_{\text{soft}} \lesssim 1$ TeV $\Rightarrow \sqrt{\langle F \rangle} \approx 10^4$–$10^5$ GeV

$\Rightarrow$ scale of SUSY breaking in hidden sector much lower than in SUGRA

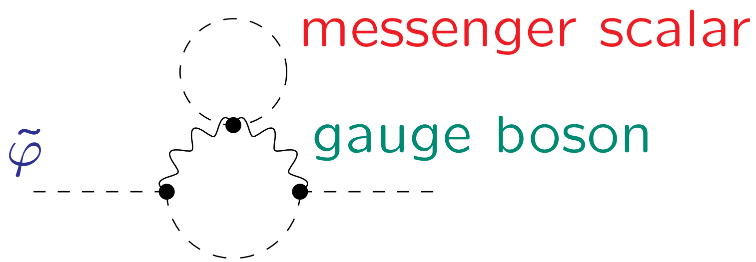
Gravitino mass: $m_{\frac{3}{2}} \approx \frac{\langle F \rangle}{M_{\mathsf{Pl}}} \approx 10^{-9}$ GeV

$\Rightarrow$ Gravitino is always the lightest SUSY particle (LSP)

Gaugino masses generated at one-loop order, $m_\lambda \approx \frac{\alpha_i}{4\pi}$



Scalar masses generated at two-loop order, $m_\varphi^2 \approx \left(\frac{\alpha_i}{4\pi}\right)^2$



$\Rightarrow$ Typical mass hierarchy in GMSB scenario between strongly interacting and weakly interacting particles $\sim \alpha_3/\alpha_2/\alpha_1$

GMSB scenario characterized by

$$M_{\text{mes}}, N_{\text{mes}}, \Lambda, \tan\beta, \text{sign}(\mu)$$

M_{mes}: messenger mass scale

N_{mes}: messenger index (number of messenger multiplets)

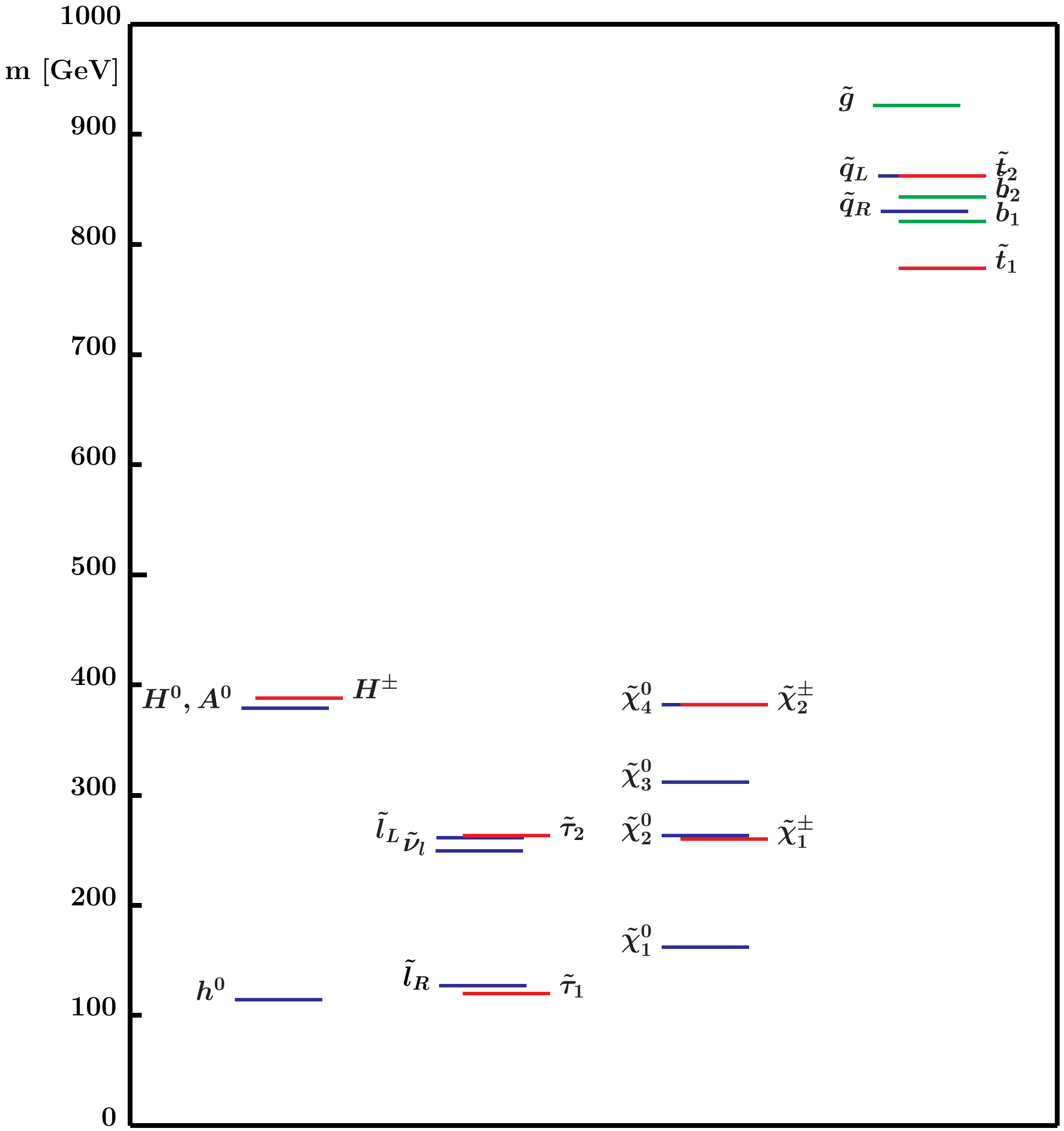
Λ: universal soft SUSY breaking mass scale felt by low-energy sector

LSP is always the gravitino, next-to-lightest SUSY particle (NLSP):
$\tilde{\chi}_1^0$ or $\tilde{\tau}_1$

can decay into LSP inside or outside the detector

GMSB scenario with $\tilde{\tau}$ NLSP

(SPS 7 benchmark scenario):



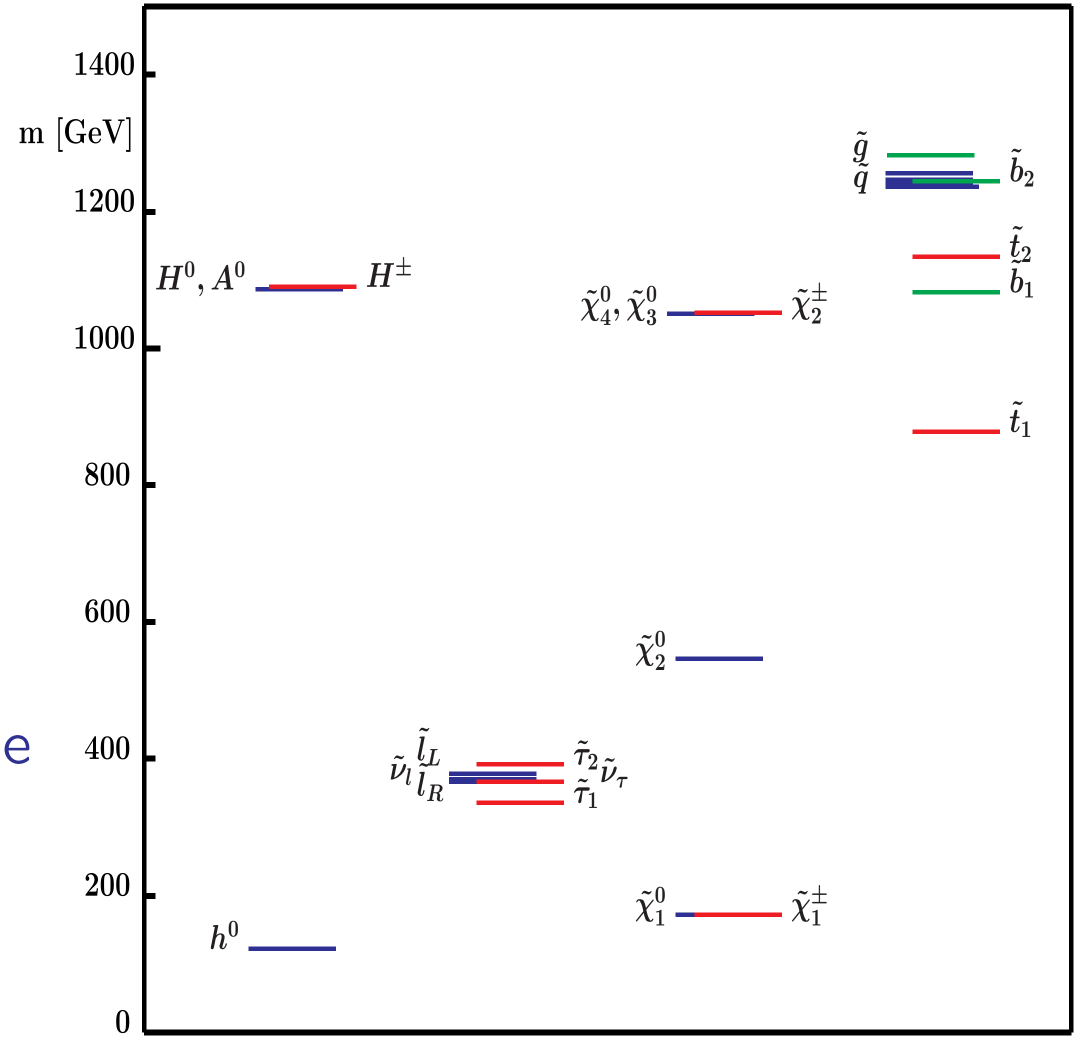
AMSB scenario characterized by

$$m_{\text{aux}}, m_0, \tan\beta, \text{sign}(\mu)$$

m_{aux}: overall scale of SUSY particle masses

m_0: phenomenological parameter: universal scalar mass term

AMSB spectrum (SPS 9):



typical feature: very small neutralino–chargino mass difference

Summary of Lecture 3:

- Unconstrained MSSM: no particular SUSY breaking mechanism assumed, parameterization of possible soft SUSY-breaking terms

- SUSY breaking mechanism not well understood

  Most attractive framework: SUSY breaking happens in ‘hidden sector’, can be communicated to MSSM fields in different ways

- Phenomenology depends mainly on mechanism for communicating SUSY breaking:

  Gravity mediation, gauge mediation, . . .

Lectures on Supersymmetry

Georg Weiglein

IPPP Durham

Torino, 07/2003

Lecture 4:

- Properties of SUSY theories

- Higgs mass bound in SUSY theories

- The LSP as a dark matter candidate

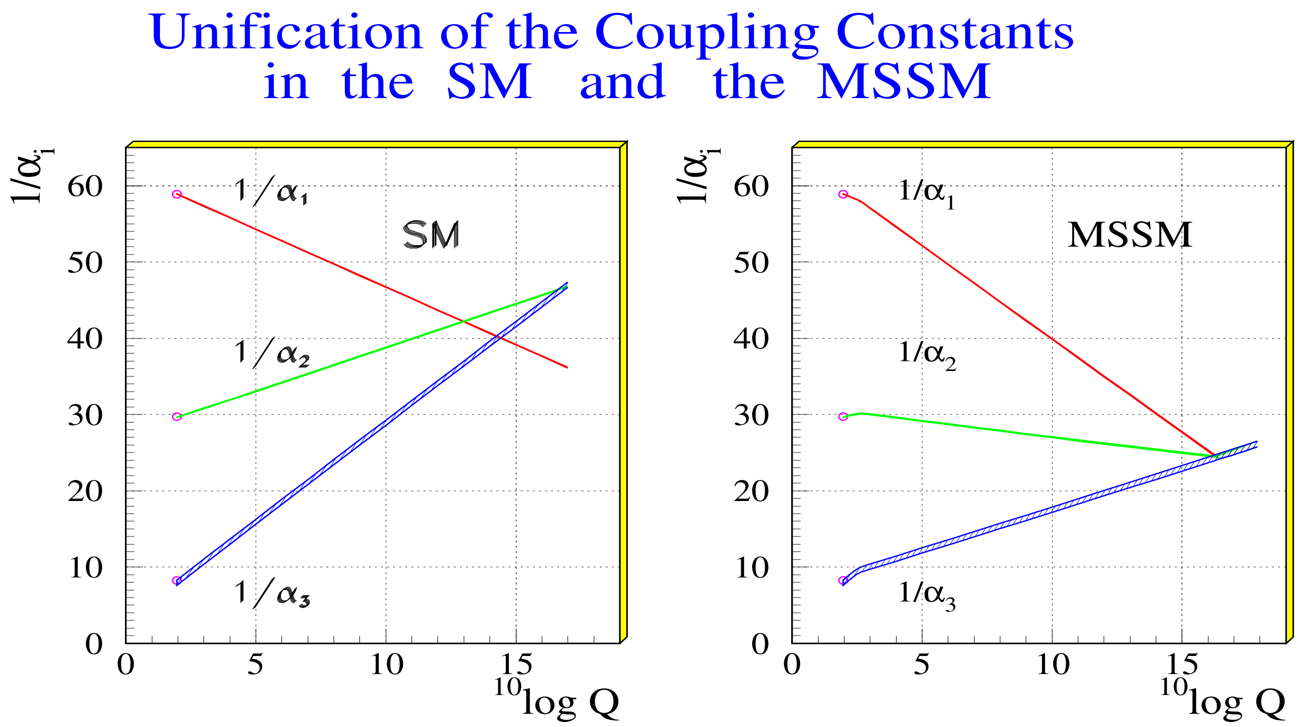
Properties of SUSY theories:

Gauge coupling unification:

Running of gauge couplings: $$\frac{1}{g^2(\mu^2)} = \frac{1}{g^2(\mu_0^2)} + \beta \ln\left(\frac{\mu^2}{\mu_0^2}\right)$$

Unification of couplings at high scale ⇔ "Grand unified theories" (GUTs)

E.g.: SO(10) GUTs, can naturally accomodate right-handed neutrinos

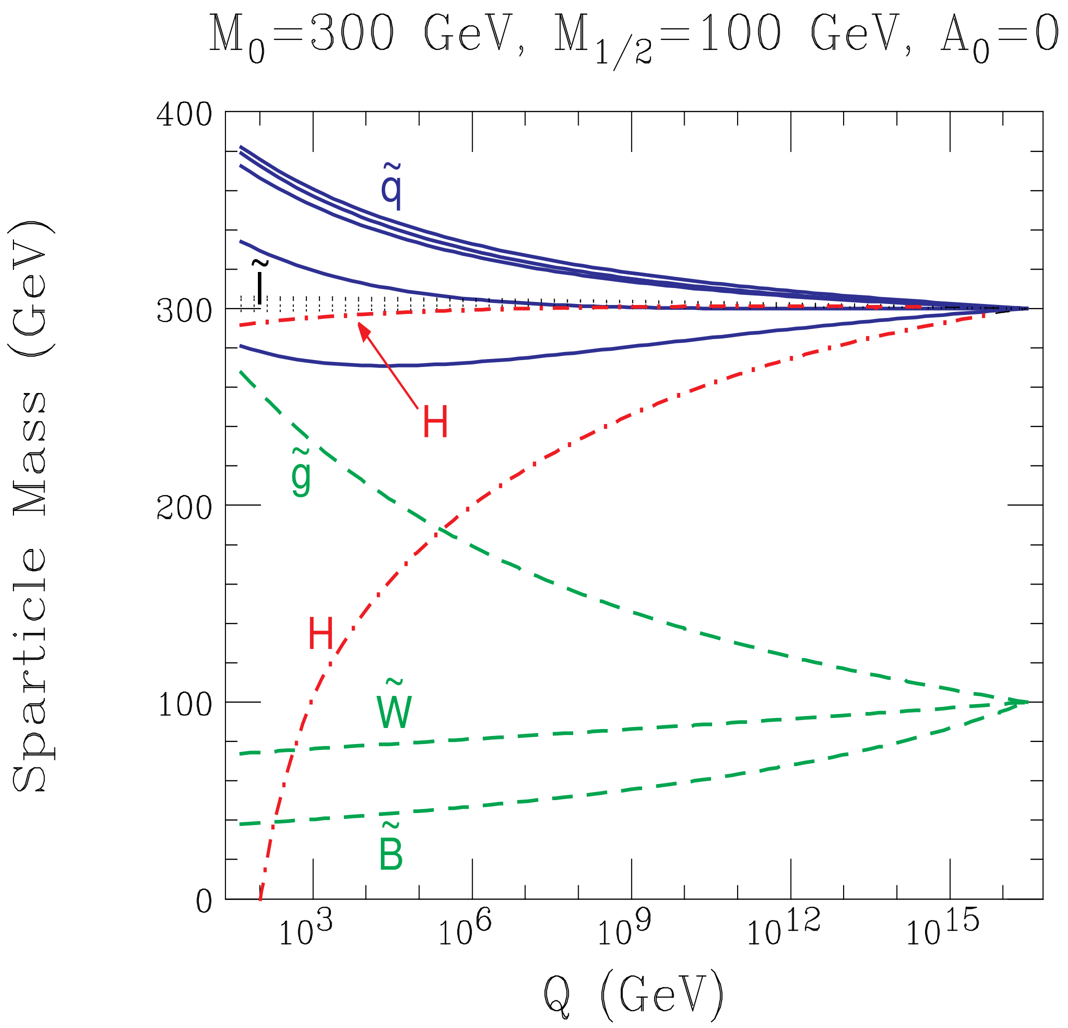


[*U. Amaldi, W. de Boer, H. Fürstenau '91*]

⇒ coupling constant unification in MSSM for $M_{\text{SUSY}} \lesssim 1$ TeV

Radiative electroweak symmetry breaking:

Universal boundary conditions at GUT scale,
renormalization group running down to weak scale



large corrections from
top-quark Yukawa coupling

$\Rightarrow$ $m^2_{H_u}$ driven to negative values

$\Rightarrow$ ew symmetry breaking

emerges naturally at scale $\sim 10^2$ GeV
for 100 GeV $\lesssim m_t \lesssim$ 200 GeV

R parity

Most general gauge-invariant and renormalizable superpotential with chiral superfields of the MSSM:

$$\mathcal{V} = \mathcal{V}_{\mathsf{MSSM}} + \underbrace{\frac{1}{2}\lambda^{ijk} L_i L_j E_k + \lambda'^{ijk} L_i Q_j D_k + \mu'^{i} L_i H_u}_{\text{violate lepton number}} + \underbrace{\frac{1}{2}\lambda''^{ijk} U_i D_j D_k}_{\text{violates baryon number}}$$

If both lepton and baryon number are violated

⇒ rapid proton decay

Minimal choice (MSSM) contains only terms in the Lagrangian with even number of SUSY particles

⇒ additional symmetry: "R parity"

⇒ all SM particles have even R parity, all SUSY particles have odd R parity

Phenomenological consequences of R parity conservation:

- SUSY particles can only be produced in pairs, e.g. $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$

- Decay of SUSY particles into SM particles
  $+$ odd number of SUSY particles
  $\Rightarrow$ Lightest SUSY particle (LSP) is stable

- All SUSY particles will eventually decay into LSP
  $\Rightarrow$ Decay cascades of heavy SUSY particles

- LSP stable
  $\Rightarrow$ some LSP must have survived from Big Bang
  $\Rightarrow$ Weakly interacting massive particle ("WIMP")
  (otherwise ruled out from bounds on exotic isotopes etc.)
  $\Rightarrow$ Candidate for cold dark matter in the Universe (if $m_{\text{LSP}} \lesssim 1$ TeV)

- LSP neutral, uncolored $\Rightarrow$ leaves no traces in collider detectors
  $\Rightarrow$ Typical SUSY signatures: "missing energy"

Relations between SUSY parameters

Symmetry properties of MSSM Lagrangian (SUSY, gauge invariance) give rise to coupling and mass relations

Soft SUSY breaking does not affect SUSY relations between dimensionless couplings

E.g.:

gauge boson–fermion coupling

=

gaugino–fermion–sfermion coupling

for U(1), SU(2), SU(3) gauge groups

In SM: all masses are free input parameters
(except M_W–M_Z interdependence)

MSSM:

- Upper bound on mass of lightest $\mathcal{CP}$-even Higgs boson

- Relations between neutralino and chargino masses

- Sfermion mass relations, e.g.

$$m^2_{\tilde{e}_L} = m^2_{\tilde{\nu}_L} - M_W^2 \cos(2\beta)$$

All relations receive corrections from loop effects
$\Leftrightarrow$ effects of soft SUSY breaking, electroweak symmetry breaking

$\Rightarrow$ Experimental verification of parameter relations is a crucial test of SUSY!

The Higgs sector of the MSSM

Two Higgs doublets:

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1^0 + i\chi_1^0)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2^0 + i\chi_2^0)/\sqrt{2} \end{pmatrix}$$

Higgs potential:

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$
$$+ \underbrace{\frac{g'^2 + g^2}{8}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM

Mixing of $\mathcal{CP}$-even, $\mathcal{CP}$-odd, charged fields:

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \end{pmatrix}, \quad \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}$$

$$\tan(2\alpha) = \tan(2\beta)\frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}$$

Three Goldstone bosons (as in SM): G^0, $G^\pm$

$\longrightarrow$ longitudinal components of $W^\pm$, Z

$\Rightarrow$ Five physical states: $h^0, H^0, A^0, H^\pm$

h, H: neutral, $\mathcal{CP}$-even, A^0: neutral, $\mathcal{CP}$-odd, $H^\pm$: charged

Gauge-boson masses:

$$M_W^2 = \frac{1}{2}g'^2(v_1^2 + v_2^2), \quad M_Z^2 = \frac{1}{2}(g^2 + g'^2)(v_1^2 + v_2^2), \quad M_\gamma = 0$$

Problem:

MSSM contains term $\mu H_1 H_2$ in superpotential

μ: dimensionful parameter

For ew symmetry breaking required: $\mu \sim$ electroweak scale

But: no a priori reason for $\mu \neq 0$, $\mu \ll M_{\mathrm{P}}$

(problem mainly in GMSB svenario, easier to overcome in mSUGRA)

Possible solution:

μ related to v.e.v. of additional field

$\Rightarrow$ Introduction of extra singlet field S, v.e.v. s $\Rightarrow$ "NMSSM"

Superpotential: $\mathcal{V} = \lambda H_1 H_2 S + \frac{1}{3} \kappa S^3 + \ldots$

Physical states in NMSSM Higgs-sector:

S_1, S_2, S_3 (CP-even), P_1, P_2 (CP-odd), $H^{\pm}$

Parameters in MSSM Higgs potential V (besides g, g'):

$$v_1,\ v_2,\ m_1,\ m_2,\ m_{12}$$

relation for M_W^2, M_Z^2 $\Rightarrow$ 1 condition

minimization of V w.r.t. neutral Higgs fields H_1^1, H_2^2 $\Rightarrow$ 2 conditions

$\Rightarrow$ only two free parameters remain in V, conventionally chosen as

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2(\tan\beta + \cot\beta)$$

$\Rightarrow$ m_h, m_H, mixing angle α, $m_{H^\pm}$: no free parameters, can be predicted

In lowest order:

$$M_{H^\pm}^2 = M_A^2 + M_W^2$$

Predictions for m_{h}, m_{H} from diagonalization of tree-level mass matrix:

$\phi_1 - \phi_2$ basis:

$$M_{\mathrm{Higgs}}^{2,\mathrm{tree}} = \begin{pmatrix} m_{\phi_1}^2 & m_{\phi_1\phi_2}^2 \\ m_{\phi_1\phi_2}^2 & m_{\phi_2}^2 \end{pmatrix} =$$

$$\begin{pmatrix} M_{\mathrm{A}}^2 \sin^2\beta + M_{\mathrm{Z}}^2 \cos^2\beta & -(M_{\mathrm{A}}^2 + M_{\mathrm{Z}}^2) \sin\beta \cos\beta \\ -(M_{\mathrm{A}}^2 + M_{\mathrm{Z}}^2) \sin\beta \cos\beta & M_{\mathrm{A}}^2 \cos^2\beta + M_{\mathrm{Z}}^2 \sin^2\beta \end{pmatrix}$$

$$\Big\Downarrow \quad \leftarrow \quad \text{Diagonalization}, \alpha$$

$$\begin{pmatrix} m_{\mathrm{H}}^{2,\mathrm{tree}} & 0 \\ 0 & m_{\mathrm{h}}^{2,\mathrm{tree}} \end{pmatrix}$$

Tree-level result for m_{h}, m_{H}:

$$m^2_{\mathrm{H,h}} = \frac{1}{2}\left[M_{\mathrm{A}}^2 + M_{\mathrm{Z}}^2 \pm \sqrt{(M_{\mathrm{A}}^2 + M_{\mathrm{Z}}^2)^2 - 4M_{\mathrm{Z}}^2 M_{\mathrm{A}}^2 \cos^2 2\beta}\right]$$

$\Rightarrow m_{\mathrm{h}} \leq M_{\mathrm{Z}}$ at tree level

$\Rightarrow$ Light Higgs boson h required in SUSY

Measurement of m_{h}, Higgs couplings

$\Rightarrow$ test of the theory (more directly than in SM)

Higgs couplings, tree level:

$$g_{\mathrm{hVV}} = \sin(\beta-\alpha)\, g_{\mathrm{HVV}}^{\mathrm{SM}}, \qquad V = W^{\pm}, Z$$

$$g_{\mathrm{HVV}} = \cos(\beta-\alpha)\, g_{\mathrm{HVV}}^{\mathrm{SM}}$$

$$g_{\mathrm{hAZ}} = \cos(\beta-\alpha)\, \frac{g'}{2\cos\theta_{\mathrm{W}}}$$

$$g_{\mathrm{hb\bar{b}}}, g_{\mathrm{h}\tau^+\tau^-} = -\frac{\sin\alpha}{\cos\beta}\, g_{\mathrm{Hb\bar{b}},\mathrm{H}\tau^+\tau^-}^{\mathrm{SM}}$$

$$g_{\mathrm{ht\bar{t}}} = \frac{\cos\alpha}{\sin\beta}\, g_{\mathrm{Ht\bar{t}}}^{\mathrm{SM}}$$

$$g_{\mathrm{Ab\bar{b}}}, g_{\mathrm{A}\tau^+\tau^-} = \gamma_5 \tan\beta\, g_{\mathrm{Ab\bar{b}}}^{\mathrm{SM}}$$

$\Rightarrow$ $g_{\mathrm{hVV}} \leq g_{\mathrm{HVV}}^{\mathrm{SM}}$, $\quad g_{\mathrm{hVV}}$, g_{HVV}, g_{hAZ} cannot all be small

$g_{\mathrm{hb\bar{b}}}, g_{\mathrm{h}\tau^+\tau^-}$: significant suppression or enhancement w.r.t. SM coupling possible

Higgs mass bounds in SUSY theories

MSSM predicts upper bound on m_{h}:

tree-level bound: $m_{\mathrm{h}} < M_{\mathrm{Z}}$, excluded by LEP Higgs searches!

Large radiative corrections:

Yukawa couplings: $\frac{e\, m_{\mathrm{t}}}{2 M_{\mathrm{W}} s_{\mathrm{W}}}, \frac{e\, m_{\mathrm{t}}^2}{M_{\mathrm{W}} s_{\mathrm{W}}}, \ldots$

$\Rightarrow$ Dominant one-loop corrections: $G_\mu m_{\mathrm{t}}^4 \ln\left(\frac{m_{\tilde{\mathrm{t}}_1} m_{\tilde{\mathrm{t}}_2}}{m_{\mathrm{t}}^2}\right)$

Present status of m_{h} prediction in the MSSM:

Complete one-loop and ‘almost complete’ two-loop result available

Upper bound on m_{h} in the MSSM:

"Unconstrained MSSM":

M_{A}, $\tan\beta$, 5 parameters in $\tilde{\mathrm{t}}$–$\tilde{\mathrm{b}}$ sector, μ, $m_{\tilde{\mathrm{g}}}$, M_2

Diagrammatic result: *FeynHiggs*

[*S. Heinemeyer, W. Hollik, G. W. '98, '00, '02*]

`http://www.feynhiggs.de`

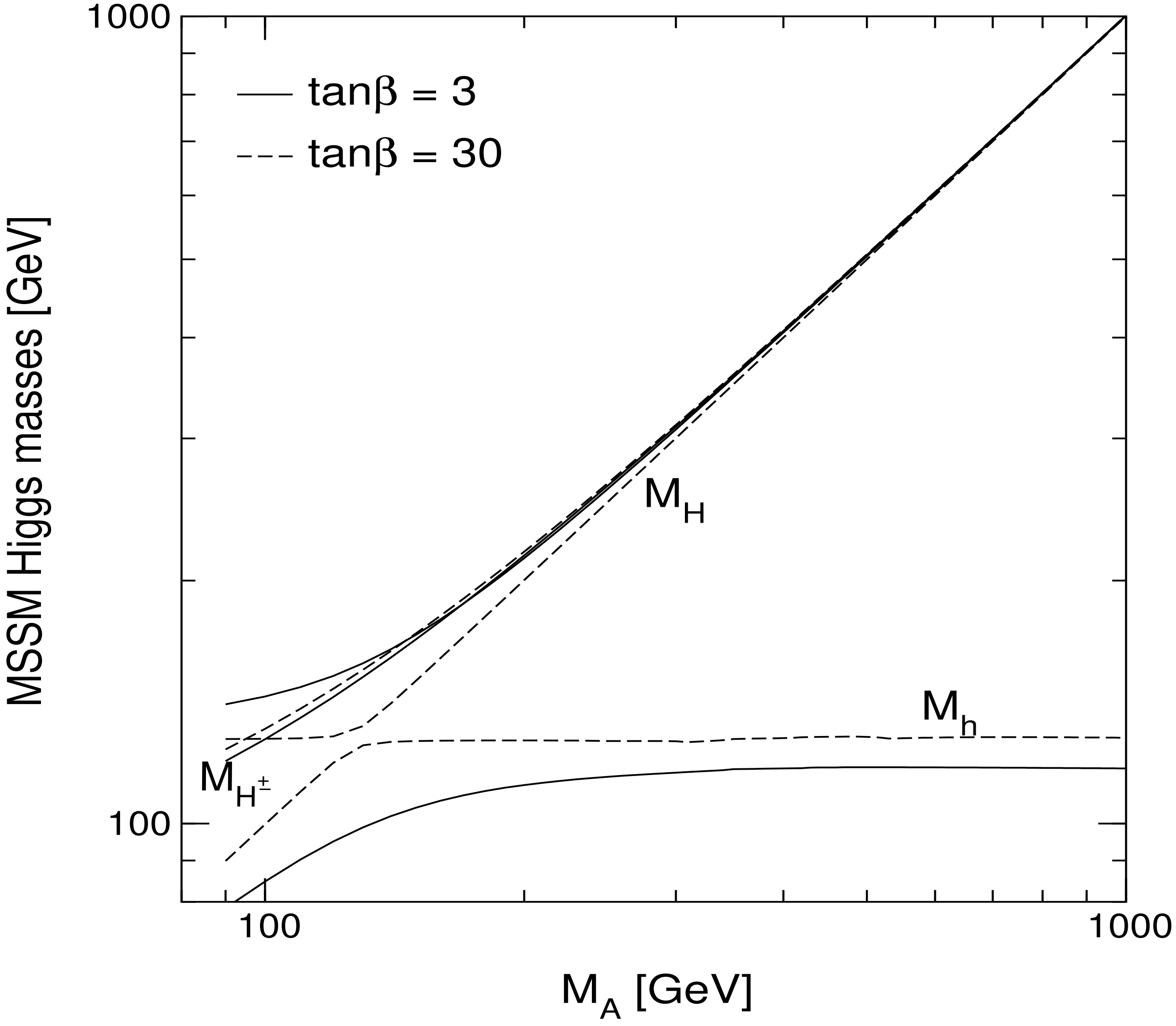
$$m_{\mathrm{h}} \lesssim 135\,\mathrm{GeV}$$

for $m_{\mathrm{t}} = 175\,\mathrm{GeV}$

no theoretical uncertainties from unknown higher orders included

Upper bound on m_{h} saturated for large M_{A}, large $\tan\beta$, significant mixing in $\tilde{\mathrm{t}}$ sector

[S. Heinemeyer, W. Hollik, G. W. '01]



Upper bound $m_\mathsf{h} \lesssim 135$ GeV reduced by

$\approx 6, 11, 8$ GeV in mSUGRA, GMSB, AMSB scenarios

[*S. Ambrosanio, A. Dedes, S. Heinemeyer, S. Su, G. W. '01*]

Upper bound on m_h in extensions of MSSM: $m_\mathsf{h} \lesssim 200$ GeV

[*G. Kane, C. Kolda, J. Wells '93*] [*J. Espinosa, M. Quirós '93, '98*]

$\Rightarrow$ **SUSY requires light Higgs boson:**

definite and robust prediction of SUSY models

testable at next generation of colliders

Remaining theoretical uncertainties in prediction for m_{h} in the MSSM:

[*G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich, G. W. '02*]

– From unknown higher-order corrections:
$\Rightarrow \Delta m_{\mathrm{h}} \approx 3$ GeV

– From uncertainties in input parameters

$m_{\mathrm{t}}, \ldots, M_{\mathrm{A}}, \tan\beta, m_{\tilde{\mathrm{t}}_1}, m_{\tilde{\mathrm{t}}_2}, \theta_{\tilde{\mathrm{t}}}, m_{\tilde{\mathrm{g}}}, \ldots$

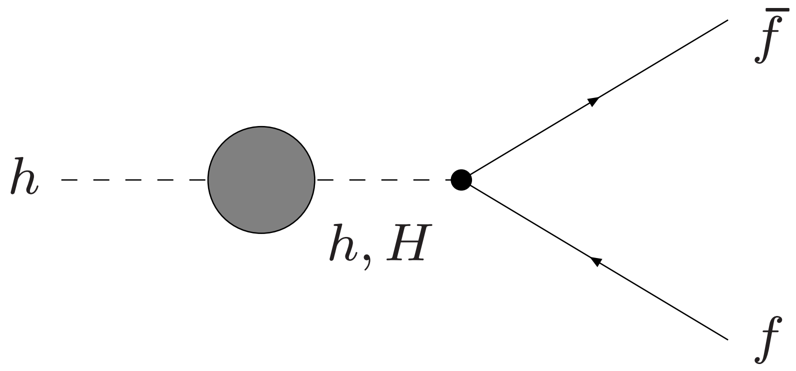
$\Delta m_{\mathrm{t}} \approx 5$ GeV $\Rightarrow \Delta m_{\mathrm{h}} \approx 5$ GeV

Higgs couplings, production cross sections

Also affected by large SUSY loop corrections

FD two-loop corrections implemented in results for $h \to f\bar{f}$, $e^+e^- \to hZ, hA$
[*S. Heinemeyer, W. Hollik, G. W. '00*]
[*S. Heinemeyer, W. Hollik, J. Rosiek, G. W. '01*]

$hf\bar{f}$ coupling:



$$A(h \to f\bar{f}) = \sqrt{Z_h}\left(\Gamma_h - \frac{\hat{\Sigma}_{\mathrm{hH}}(m_{\mathrm{h}}^2)}{m_{\mathrm{h}}^2 - m_{\mathrm{H}}^2 + \hat{\Sigma}_{\mathrm{HH}}(m_{\mathrm{h}}^2)}\Gamma_H\right)$$

$\Rightarrow$ Effective $hf\bar{f}$ coupling can vanish for large $\hat{\Sigma}_{\mathrm{hH}}$

Gluino vertex corrections to $h \to q\bar{q}$:

$\Rightarrow$ ratio $\Gamma(h \to \tau^+\tau^-)/\Gamma(h \to b\bar{b})$ can significantly differ from SM value for large $\tan\beta$

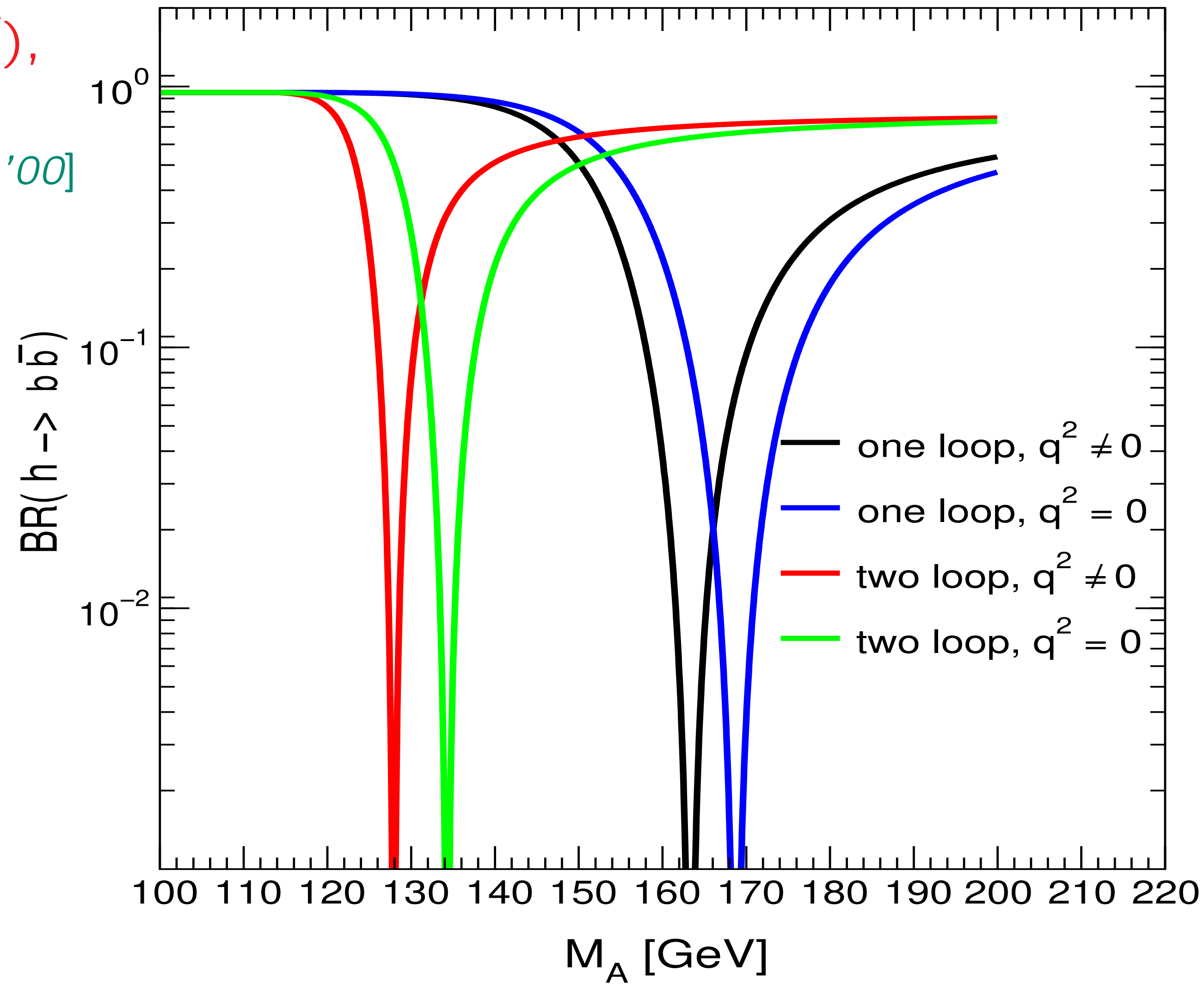
Effective $hf\bar{f}$ coupling can go to zero for large $\hat{\Sigma}_{hH}$

$\Rightarrow$ "Pathological regions"

[*W. Loinaz, J. Wells '98*] [*M. Carena, S. Mrenna, C. Wagner '99*]

$\Rightarrow$ Suppression of BR($h \to b\bar{b}$), BR($h \to \tau\tau$), . . .

[*S. Heinemeyer, W. Hollik, G. W. '00*]



$\mathcal{CP}$ violation in the Higgs sector:

MSSM Higgs sector is $\mathcal{CP}$-conserving at tree level

Complex parameters enter via loop corrections:

– μ: Higgsino mass parameter

– $A_{t,b,\tau}$: trilinear couplings $\Rightarrow$ $X_{t,b,\tau} = A_{t,b} - \mu^* \{\cot\beta, \tan\beta\}$ complex

– $M_{1,2}$: gaugino mass parameter (one phase can be eliminated)

– $m_{\tilde{g}}$: gluino mass

$\Rightarrow$ can induce $\mathcal{CP}$-violating effects

$\Rightarrow$ Mixing between neutral Higgs bosons h_1, h_2, h_3

Inclusion of higher-order corrections:

Propagator / mass matrix with higher-order corrections:

$$\begin{pmatrix} q^2 - M_\mathrm{A}^2 + \hat{\Sigma}_{AA}(q^2) & \hat{\Sigma}_{AH}(q^2) & \hat{\Sigma}_{Ah}(q^2) \\ \hat{\Sigma}_{HA}(q^2) & q^2 - m_\mathrm{H}^2 + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{Hh}(q^2) \\ \hat{\Sigma}_{hA}(q^2) & \hat{\Sigma}_{hH}(q^2) & q^2 - m_\mathrm{h}^2 + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

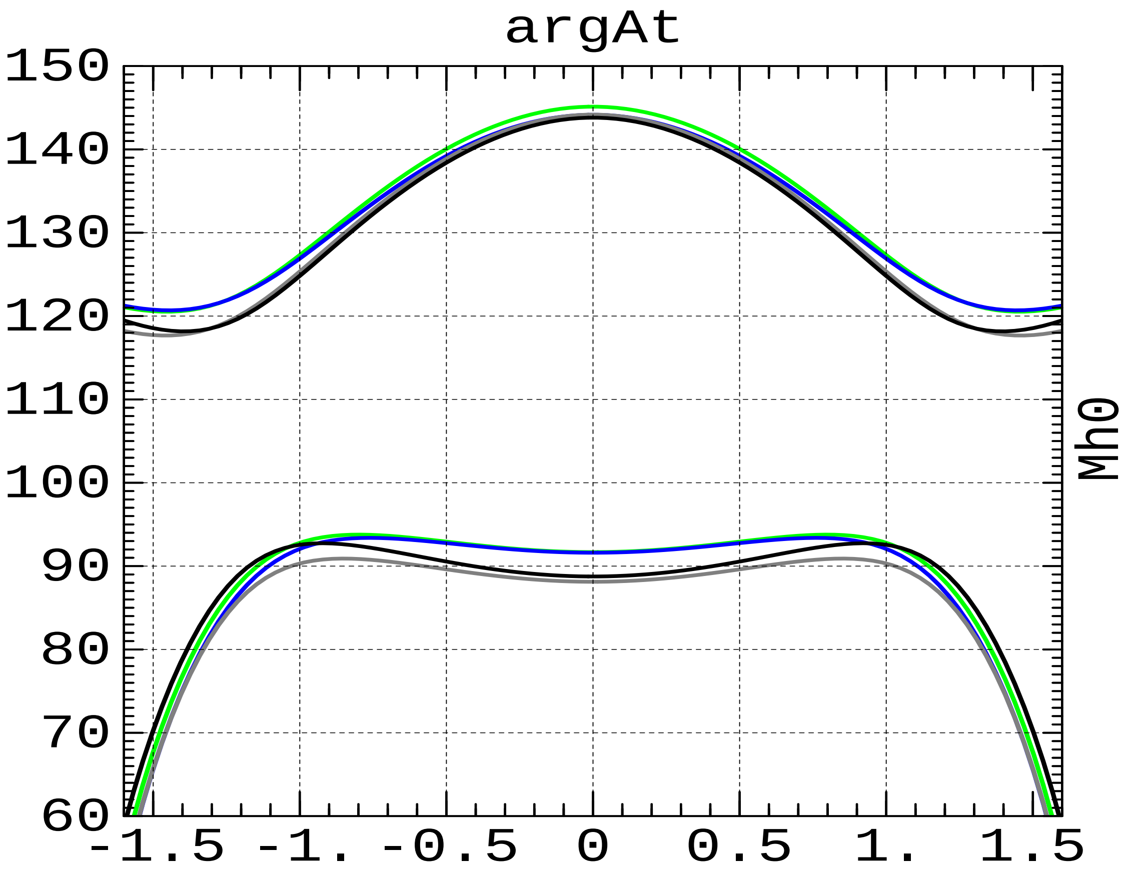
$\hat{\Sigma}_{ij}(q^2)$ $(i,j = h, H, A)$: renormalized Higgs self-energies

$\hat{\Sigma}_{Ah}, \hat{\Sigma}_{AH} \neq 0 \;\Rightarrow\; \mathcal{CP}$V, $\mathcal{CP}$-even and $\mathcal{CP}$-odd fields can mix

Result: $(A, H, h) \to (h_3, h_2, h_1)$ with $m_{\mathrm{h}_3} > m_{\mathrm{h}_2} > m_{\mathrm{h}_1}$

Example: ϕ_{A_t} dependence of m_{h_1}, m_{h_2} ($\phi_\mu = 0$, $|A_\mathrm{t}| = 2\, M_\mathrm{SUSY}$):

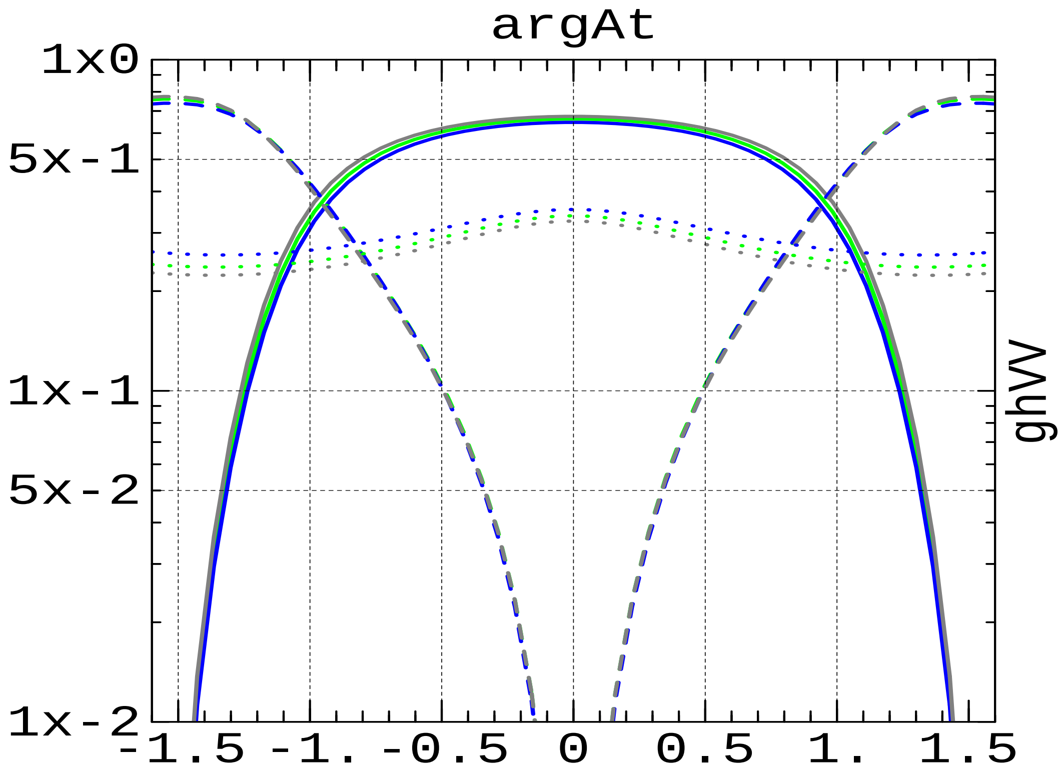
[*M. Frank, S. Heinemeyer, W. Hollik, G. W. '03*]



$\mathrm{t}/\tilde{\mathrm{t}}, \mathrm{b}/\tilde{\mathrm{b}}$, $q^2 = 0$, (s)fermion, $q^2 = 0$, full MSSM, $q^2 = 0$, full MSSM, $q^2 \neq 0$

Complex phases can have large effects on Higgs couplings

Example: g_{hVV} for h_1, h_2, h_3:
[*M. Frank, S. Heinemeyer, W. Hollik, G. W. '03*]

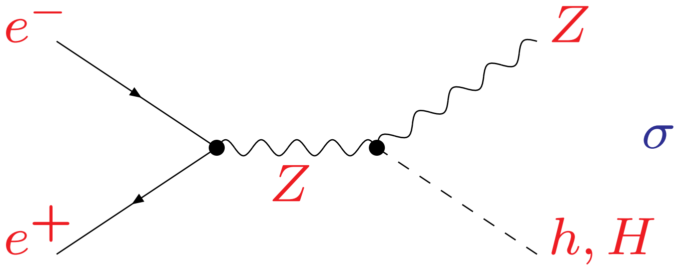


full: h_1, dashed: h_2, dotted: h_3

Parameters: $M_{\text{SUSY}} = 500$ GeV, $M_2 = 500$ GeV, $\mu = 2000$ GeV, $|A_t| = 1000$ GeV,
$M_{H^\pm} = 150$ GeV, $\tan\beta = 5$

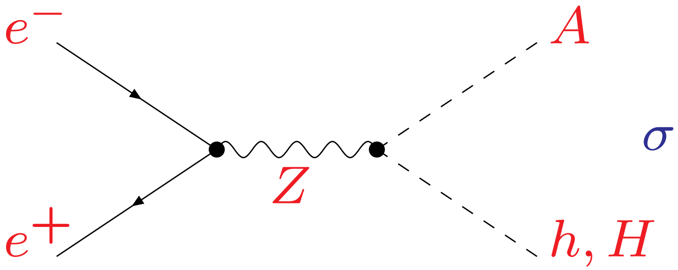
Search for neutral MSSM Higgs bosons at LEP:

$e^+e^- \longrightarrow Zh, ZH$



$$\sigma_{\mathrm{hZ}} \approx \sin^2(\beta - \alpha_{\mathrm{eff}})\sigma_{\mathrm{hZ}}^{\mathrm{SM}}$$

$e^+e^- \longrightarrow Ah, AH$

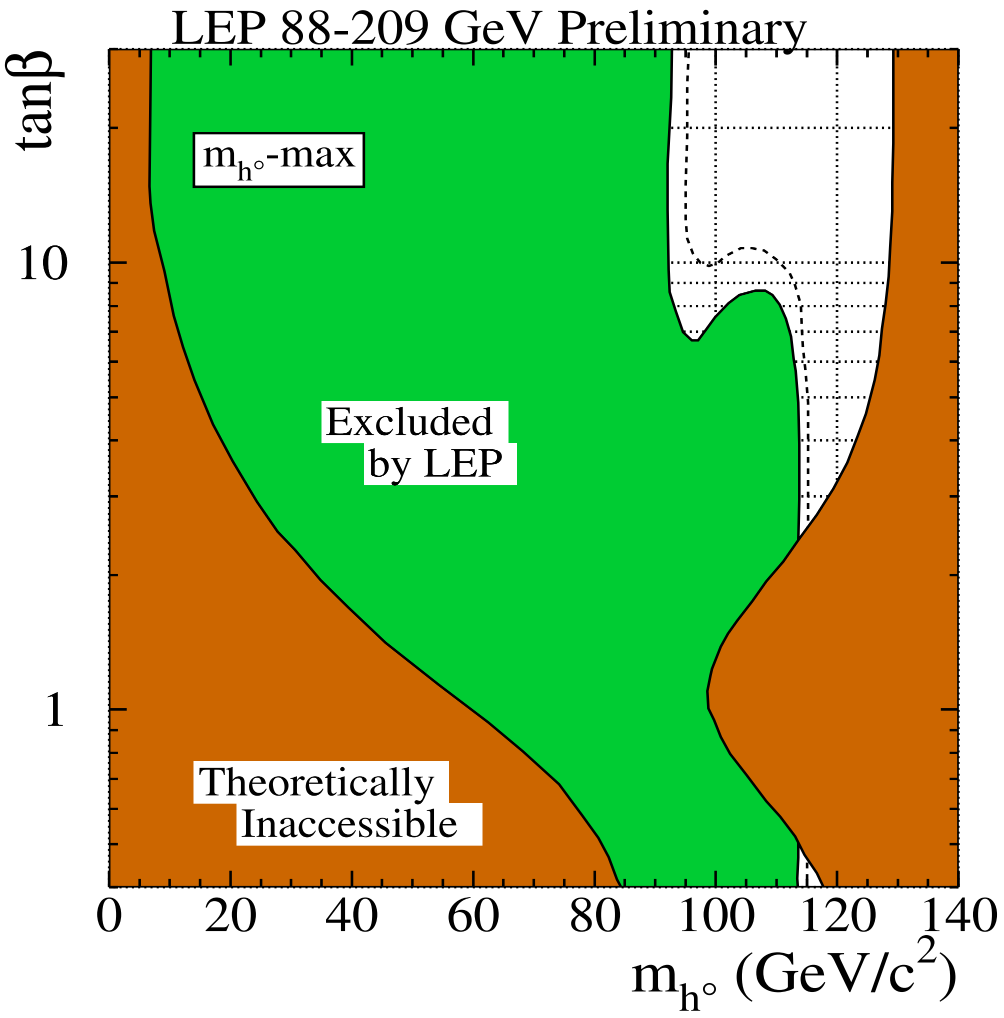


$$\sigma_{\mathrm{hA}} \propto \cos^2(\beta - \alpha_{\mathrm{eff}})\sigma_{\mathrm{hZ}}^{\mathrm{SM}}$$

Constraints from the Higgs search at LEP [*LEP Higgs Working Group '01*]

Experimental search vs. upper m_h-bound (*FeynHiggs* 1.0)

m_h^{max}-scenario ($m_t = 174.3$ GeV, $M_{SUSY} = 1$ TeV):



$m_h > 91.0$ GeV
(expected: 94.6 GeV), 95% C.L.

$M_A > 91.9$ GeV
(expected: 95.0 GeV)

Parameter region where experimental lower bound on m_{h} is significantly lower than SM bound, $M_{\text{H}} > 114.4$ GeV, corresponds to $\sin^2(\beta - \alpha_{\text{eff}}) \ll 1$

"Excluded" $\tan\beta$ region:

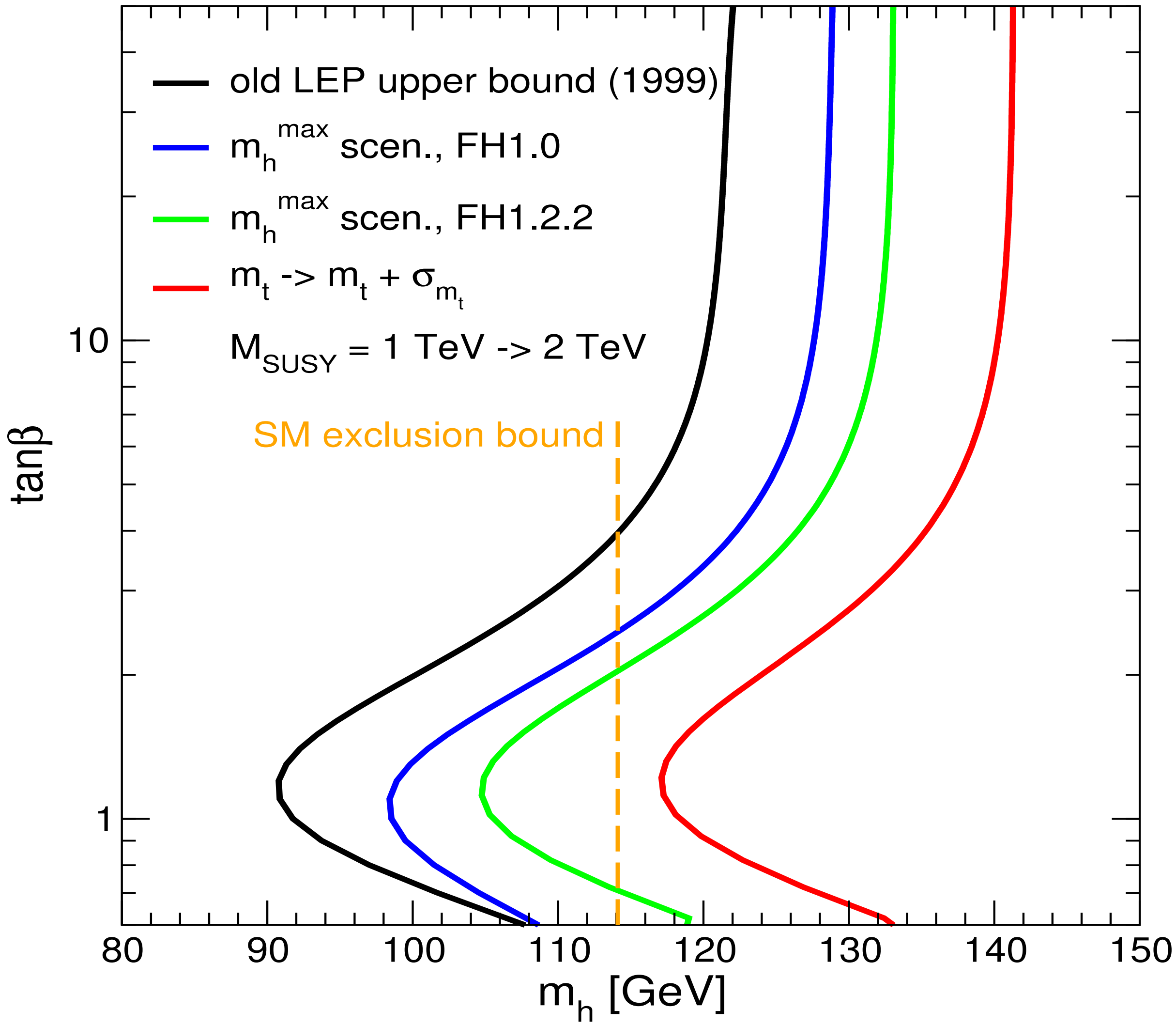
$$0.5 < \tan\beta < 2.4$$

Note: this exclusion bound assumes

m_{t}, M_{SUSY} fixed, $m_{\text{t}} = 174.3$ GeV, $M_{\text{SUSY}} = 1$ TeV

no theoretical uncertainties included

Effect of new corrections and $m_t \to m_t + \sigma_{m_t}$, $M_{SUSY} = 1$ TeV $\to$ 2 TeV
[*G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich, G. W. '02*]



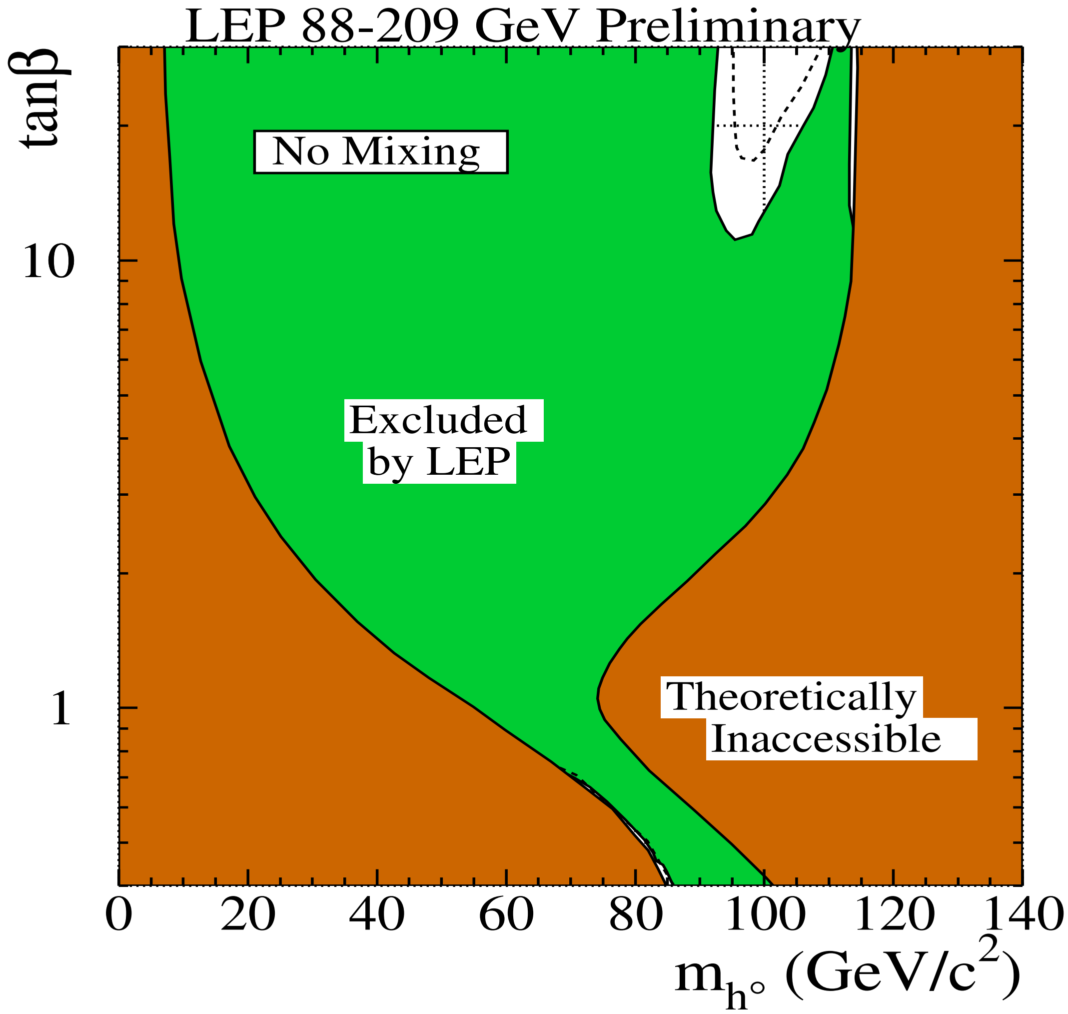
$\Rightarrow$ no $\tan\beta$ exclusion for $m_t \to m_t + \sigma_{m_t}$, $M_{SUSY} = 1$ TeV $\to$ 2 TeV

$\Rightarrow$ Low $\tan\beta$ region not fully excluded by LEP!

Experimental search vs. upper m_h-bound (*FeynHiggs*)

[*LEP Higgs Working Group '01*]

no-mixing scenario:



The LSP as a dark matter candidate

Astrophysical data (cosmic microwave background, . . .) $\Rightarrow$ existence of non-baryonic cold dark matter in the Universe

SUSY with R parity conservation $\Rightarrow$ LSP relic density falls naturally in favoured range if $m_{\mathrm{LSP}} \lesssim 1$ TeV

Most recent result (WMAP)
[*C. Bennet et al. '03*], [*D. Spergel et al. '03*]

$\Rightarrow$ cold dark matter density: $\Omega_{\mathrm{CDM}} h^2 = 0.1126^{+0.0161}_{-0.0181}$ at 95% C.L.
(h: expansion rate)

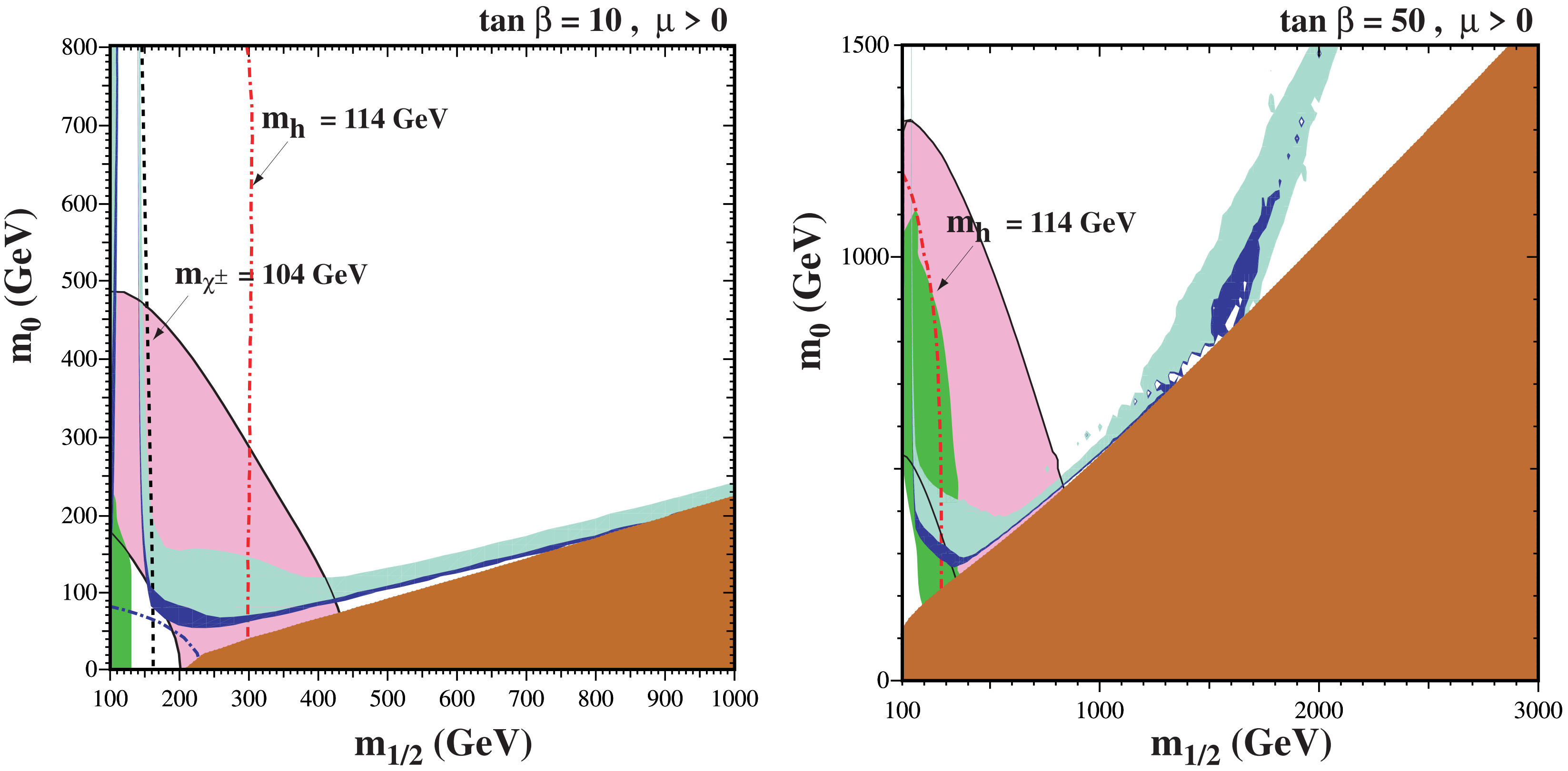
$\Rightarrow$ Constraints on the SUSY parameter space

Example: mSUGRA scenario with $\mu > 0$, $A_0 = 0$, $\tan\beta = 10, 50$

Parameter region allowed by the CDM constraint in the $m_{1/2}$–m_0 plane with and without the new WMAP data

(also shown: favoured region from $(g_\mu - 2)$, region excluded by $b \to s\gamma$)

[J. Ellis, K. Olive, Y. Santoso, V. Spanos '03]

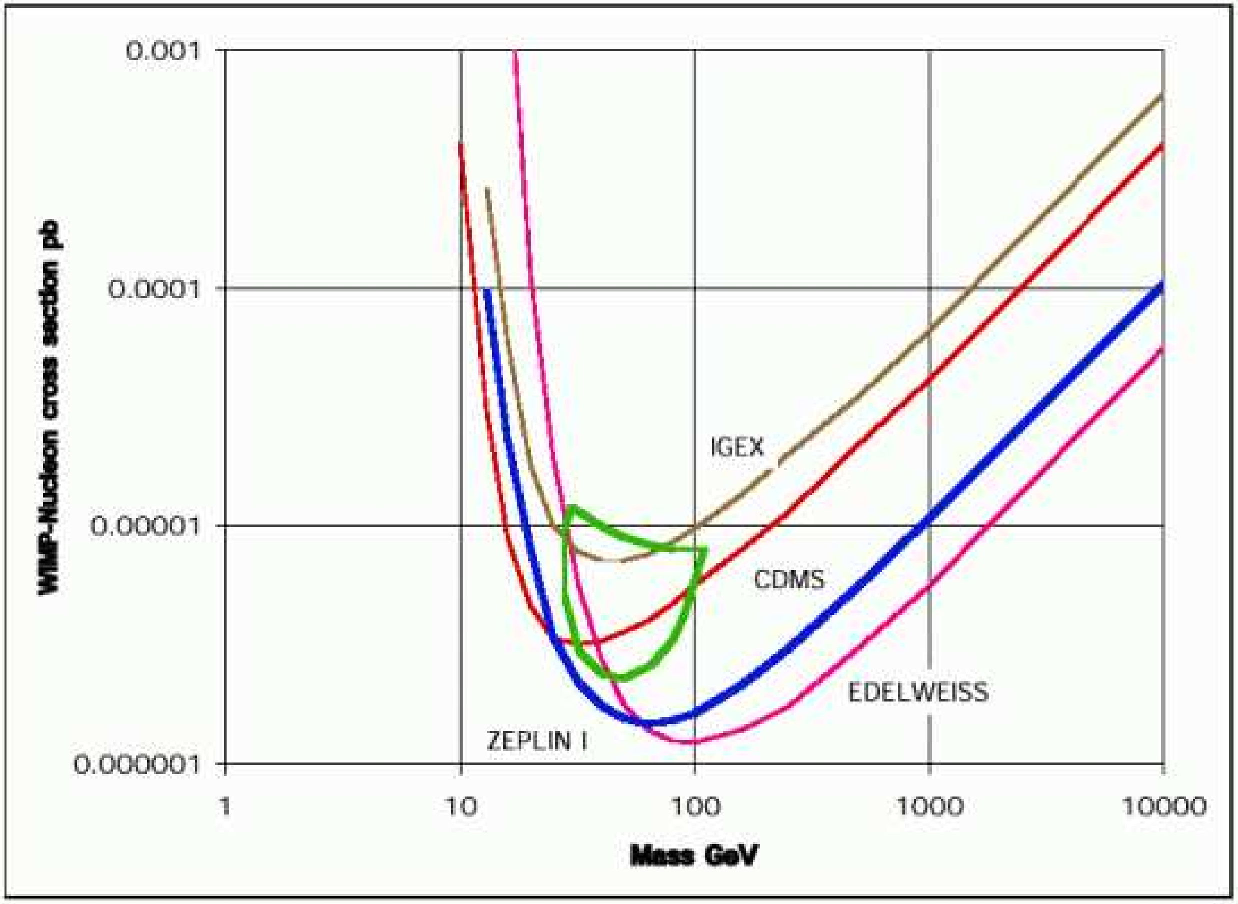


⇒ mSUGRA scenario consistent with all experimental constraints

<u>Direct search for dark matter:</u>

E.g.: scattering off nuclei $\Rightarrow$ measurement of nuclear recoil

Present limits in the plane of m_{WIMP} and the WIMP–nucleon cross section σ_{SI} from different experiments

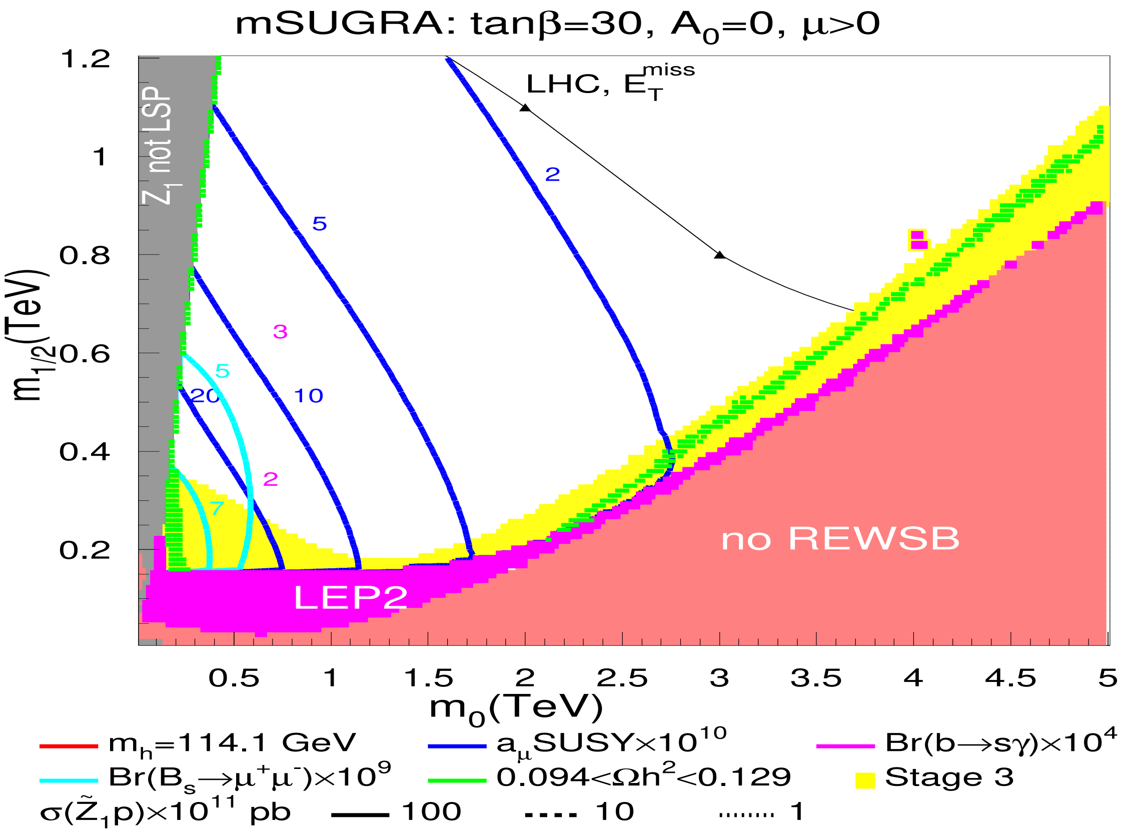


⇒ Present limits on σ_{SI} ("Stage 1" dark matter experiments): $\approx 10^{-5}$–10^{-6} pb

Anticipated sensitivity of possible future dark matter experiments ("Stage 3"): $\sigma_{SI} \approx 10^{-9}$–$10^{-10}$ pb

Accessible parameter region (yellow) in mSUGRA scenario ($\tan\beta = 30$, $\mu > 0$, $A_0 = 0$) with Stage 3 sensitivity vs. CDM allowed region and LHC sensitivity

[*H. Baer, C. Bal'azs, A. Belyaev, J. O'Farrill '03*]



Summary of Lecture 4:

- Properties of SUSY theories: allow gauge coupling unification, radiative electroweak symmetry breaking

- SUSY parameter relations $\Rightarrow$ crucial test of SUSY

- SUSY requires light Higgs boson:

  definite and robust prediction of SUSY theories, testable at next generation of colliders

- LSP: attractive candidate for cold dark matter in the Universe

- mSUGRA in agreement with exp. constraints from Higgs search, CDM, $(g_\mu - 2)$, $B \to X_s\gamma$

Lectures on Supersymmetry

Georg Weiglein

IPPP Durham

Torino, 07/2003

Lecture 5:

- Electroweak precision tests: SM vs. MSSM

- Prospects for SUSY searches at the next generation of colliders

Electroweak precision tests: SM vs. MSSM

Electroweak precision measurements:

$$
\begin{array}{llll}
M_Z[\,\mathrm{GeV}] & = & 91.1875 \pm 0.0021 & 0.002\% \\
G_\mu[\,\mathrm{GeV}^{-2}] & = & 1.16637(1)\,10^{-5} & 0.0009\% \\
m_t[\,\mathrm{GeV}] & = & 174.3 \pm 5.1 & 2.9\% \\
 & & & \\
M_W[\,\mathrm{GeV}] & = & 80.426 \pm 0.034 & 0.04\% \\
\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} & = & 0.23148 \pm 0.00017 & 0.07\% \\
\Gamma_Z[\,\mathrm{GeV}] & = & 2.4952 \pm 0.0023 & 0.09\% \\
\ldots & & &
\end{array}
$$

Quantum effects of the theory: loop corrections: $\sim \mathcal{O}(1\%)$

SM: M_H is free parameter

precise measurement of M_W, $\sin^2\theta_{\mathrm{eff}}$, $\ldots \Rightarrow$ constraints on M_H

MSSM: m_h is predicted

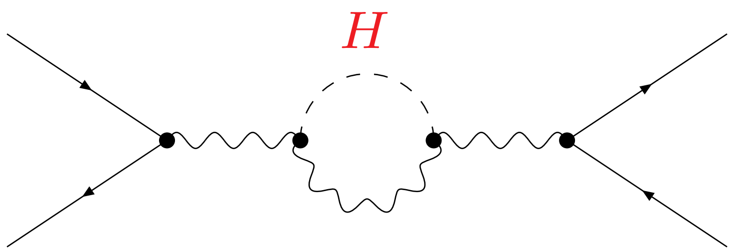
precise meas. of M_W, $\sin^2\theta_{\mathrm{eff}}$, m_h, $\ldots \Rightarrow$ constr. on $m_{\tilde{t}}$, $\theta_{\tilde{t}}$, $m_{\tilde{b}}$, $\theta_{\tilde{b}}$, $\ldots$

Comparison of ew precision data with theory:

EW precision data:	Theory:
$M_Z, M_W, \sin^2\theta_{eff}^{lept}, \ldots$	SM, MSSM, . . .

$$\Downarrow$$

Test of theory at quantum level:



$$\Downarrow$$

Improve indirect constraints on unknown parameters: M_H, $m_{\tilde{t}}$, . . .

effects of "new physics"?

Indirect determination of m_t from precision data:

$$m_t = 180.3^{+11.7}_{-9.2} \text{ GeV}$$

Direct measurement:

$$m_t = 174.3 \pm 5.1 \text{ GeV}$$

Leading corrections to precision observables:

$$\sim m_t^2$$
$$\sim \ln M_H$$

$\Rightarrow$ Very high accuracy of measurements and theoretical predictions needed

Theoretical uncertainties:

– unknown higher-order corrections

– experimental error of input parameters: m_t, $\Delta\alpha_{had}$, . . .

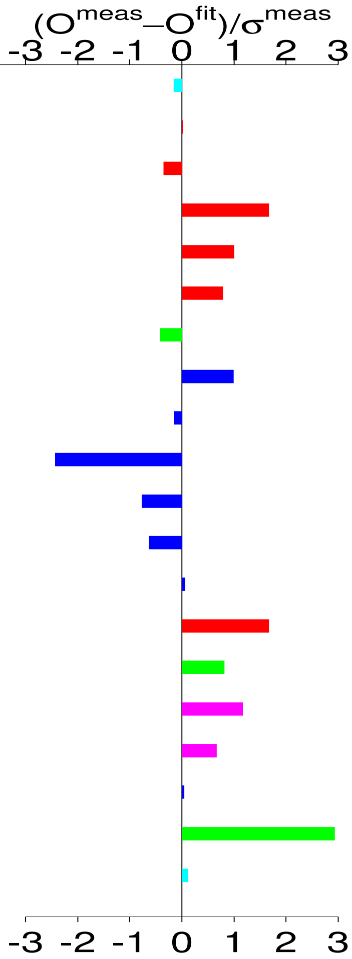
Global fit of the SM to all data:

Basic assumption: SM provides correct description of experimental data

Comparison of SM prediction with the data: [*LEPEWWG '03*]

Winter 2003

	Measurement	Pull
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02761 ± 0.00036	-0.16
m_Z [GeV]	91.1875 ± 0.0021	0.02
Γ_Z [GeV]	2.4952 ± 0.0023	-0.36
σ^0_{had} [nb]	41.540 ± 0.037	1.67
R_l	20.767 ± 0.025	1.01
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.79
$A_l(P_\tau)$	0.1465 ± 0.0032	-0.42
R_b	0.21644 ± 0.00065	0.99
R_c	0.1718 ± 0.0031	-0.15
$A^{0,b}_{fb}$	0.0995 ± 0.0017	-2.43
$A^{0,c}_{fb}$	0.0713 ± 0.0036	-0.78
A_b	0.922 ± 0.020	-0.64
A_c	0.670 ± 0.026	0.07
A_l(SLD)	0.1513 ± 0.0021	1.67
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.82
m_W [GeV]	80.426 ± 0.034	1.17
Γ_W [GeV]	2.139 ± 0.069	0.67
m_t [GeV]	174.3 ± 5.1	0.05
$\sin^2\theta_W(\nu N)$	0.2277 ± 0.0016	2.94
Q_W(Cs)	-72.83 ± 0.49	0.12



Overall fit probability (quality of the fit): 4.4%

Comparison of current experimental errors with anticipated precision at

Run II of the Tevatron ($p\bar{p}$ collider, $E_{\rm CM} \approx 2$ TeV; ≥ 2001),

LHC (pp collider, $E_{\rm CM} \approx 14$ TeV; $\gtrsim 2007$,

LC (e^+e^- collider, $E_{\rm CM} \approx 500$–1000 GeV, $\gtrsim 201x$?)
with and without low-energy running mode (GigaZ)

	now	Tev. Run IIA	Run IIB	LHC	LC	GigaZ
$\delta \sin^2\theta_{\rm eff} (\times 10^5)$	17	78	29	14–20	—	1.3
$\delta M_{\rm W}$ [MeV]	34	27	16	15	10	7
$\delta m_{\rm t}$ [GeV]	5.1	2.7	1.4	1.0	0.1	0.1
$\delta m_{\rm h}$ [MeV]	—	—	$\mathcal{O}(2000)$	100	50	50

Additional sources for sizable radiative corrections in the MSSM:

- Mass and couplings of light $\mathcal{CP}$-even Higgs:

  Large Yukawa corrections: $\sim G_\mu m_{\mathrm{t}}^4 \ln\left(\frac{m_{\tilde{\mathrm{t}}_1} m_{\tilde{\mathrm{t}}_2}}{m_{\mathrm{t}}^2}\right)$, ...

- t / $\tilde{\mathrm{t}}$ loops, b / $\tilde{\mathrm{b}}$ loops (for large $\tan\beta$)

- Corr. to relation between bottom mass and bottom Yukawa coupling:

$$y_b = \frac{\sqrt{2}}{v\cos\beta}\frac{m_{\mathrm{b}}}{1+\Delta_b},$$

  $\Delta_b = \mu\tan\beta\,(\alpha_{\mathrm{s}} I(\ldots) + \alpha_{\mathrm{t}} I(\ldots))$

  $\Rightarrow$ Coupling non-perturbative for $\Delta_b \to -1$

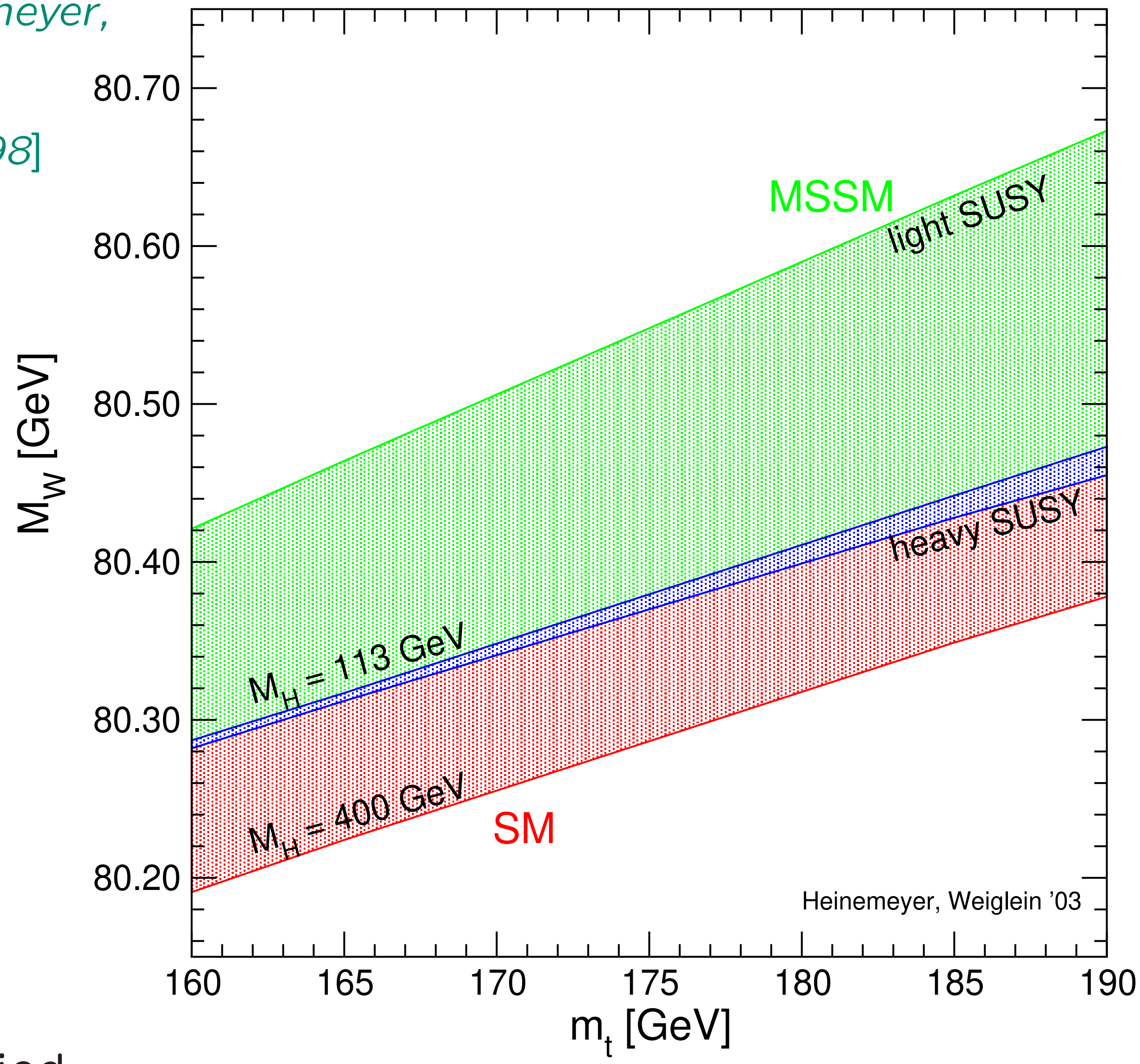
- Loop contributions from light SUSY particles

- ...

Prediction for M_W in the SM and the MSSM:

[*A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger, G. W. '97*]

[*S. Heinemeyer, W. Hollik, G. W. '98*]



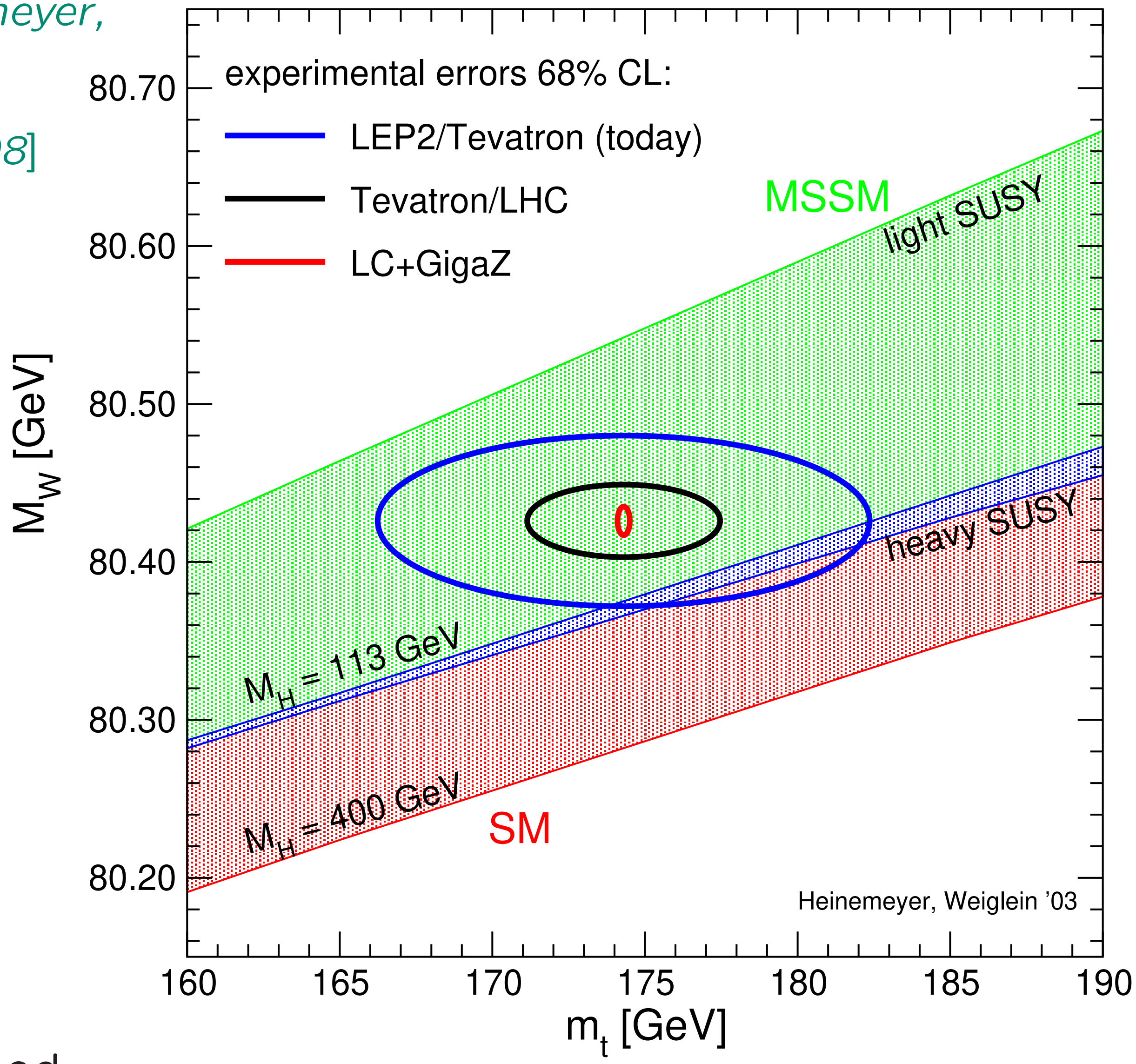
SM: M_{H} varied

MSSM: SUSY parameters varied

Prediction for M_W in the SM and the MSSM:

[A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger, G. W. '97]

[S. Heinemeyer, W. Hollik, G. W. '98]

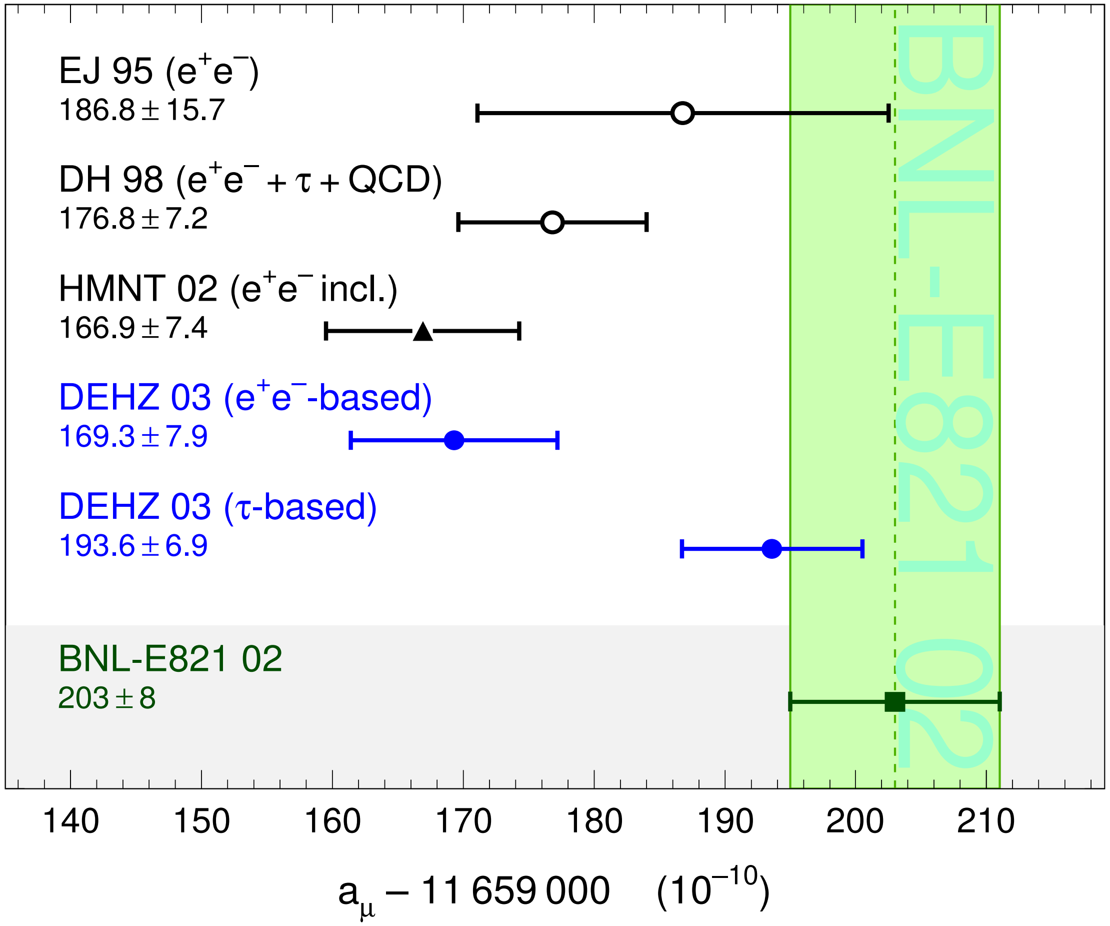


SM: M_H varied

MSSM: SUSY parameters varied

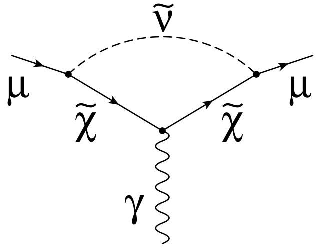
Experimental result for anomalous magnetic moment of the muon vs. SM prediction:

[*Muon $(g-2)$ Collaboration '02*]



$\Rightarrow$ a_μ(exp) $-$ a_μ(SM): 1–3 σ deviation (sizable hadronic uncertainties)

Compatible with effect of SUSY contributions to a_μ:

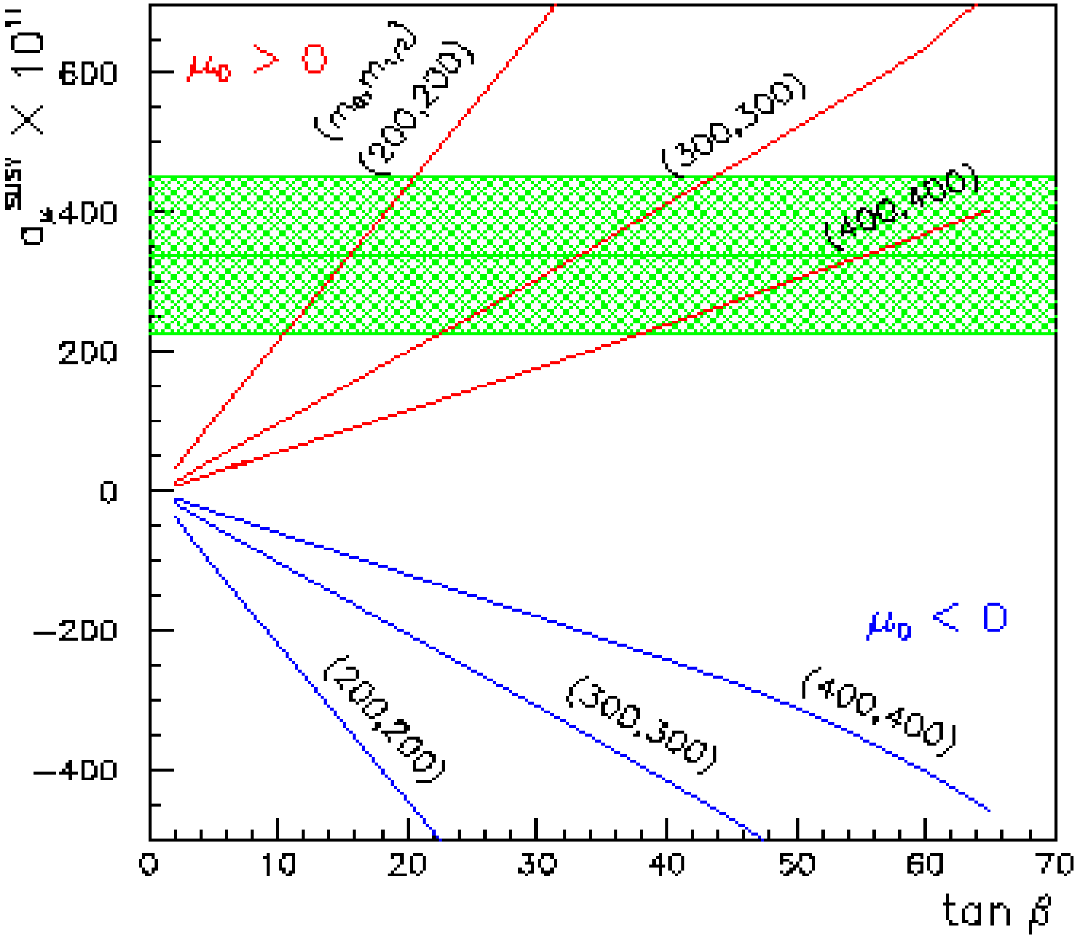


mSUGRA predictions for $(g_\mu - 2)$:

[*W. de Boer, C. Sander '02*]

$$a_\mu = \frac{1}{2}(g_\mu - 2)$$

$$a_\mu - a_\mu^{\mathrm{SM}} = (338 \pm 112) \times 10^{-11}$$



$\Rightarrow$ Preference for $\mu > 0$

note: influenced by hadronic uncertainties of SM result

No convincing new physics model for "explaining" deviations from SM in $\sin^2\theta_{\mathrm{eff}}$ and ν–nucleon scattering from NuTeV

Note, however:
if NuTeV result interpreted as deviation in $Z\nu\bar{\nu}$ coupling:
(ν neutral current rate)/prediction:

0.995 ± 0.003	LEP1 lineshape (invisible Z width)
0.988 ± 0.004	NuTeV

Need some patience until experimental clarification:

A_{LR}, A_{FB}:	LC with GigaZ option?
ν–nucleon scatt.:	ν factory?

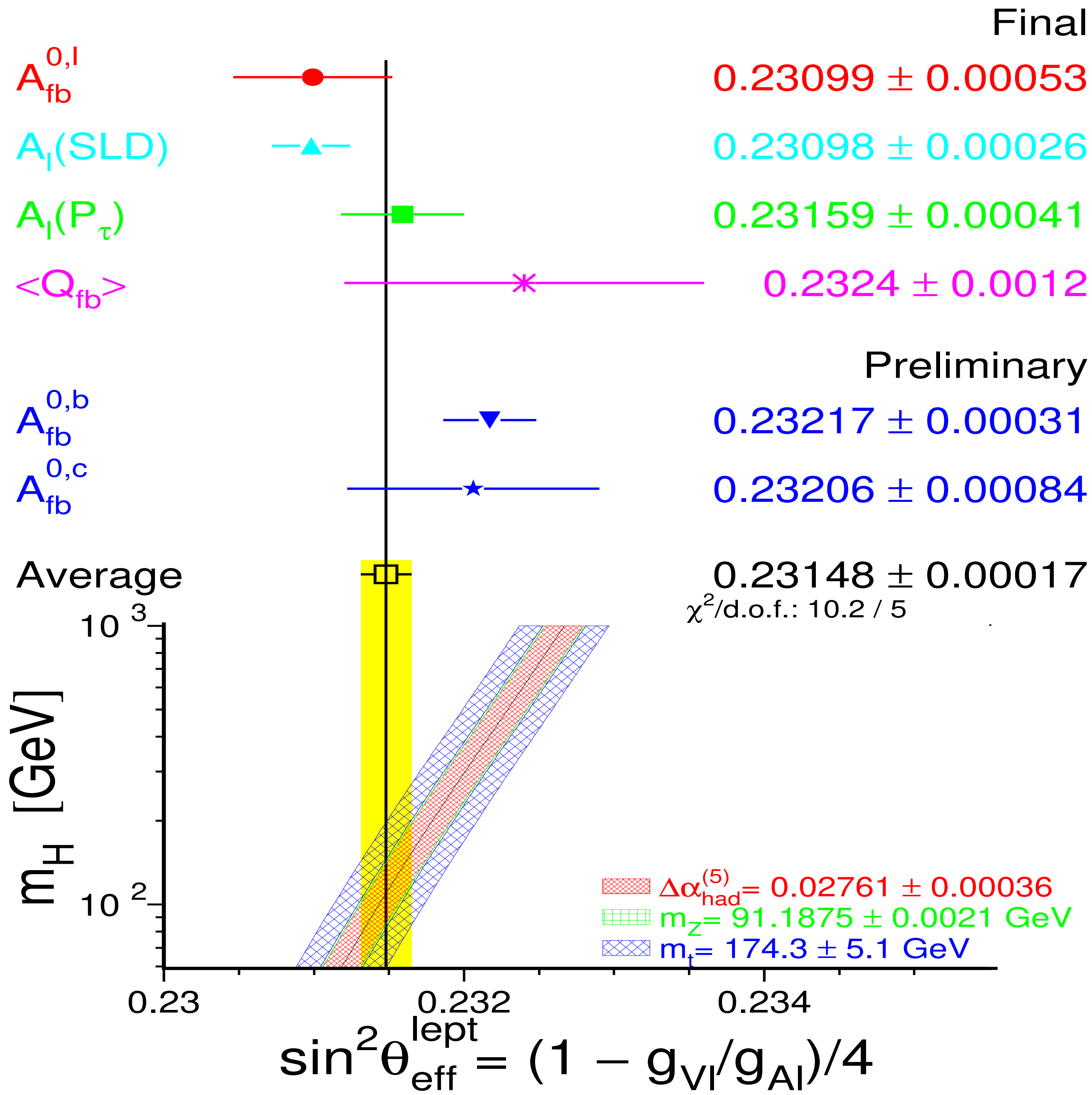
If hadronic data on A_{FB} and NuTeV data on ν–nucleon scattering were removed from fit:

much better fit quality, probability $\approx 70\%$

stronger preference for light Higgs: $M_{\mathrm{H}} \lesssim 150$ GeV, 95% C.L.

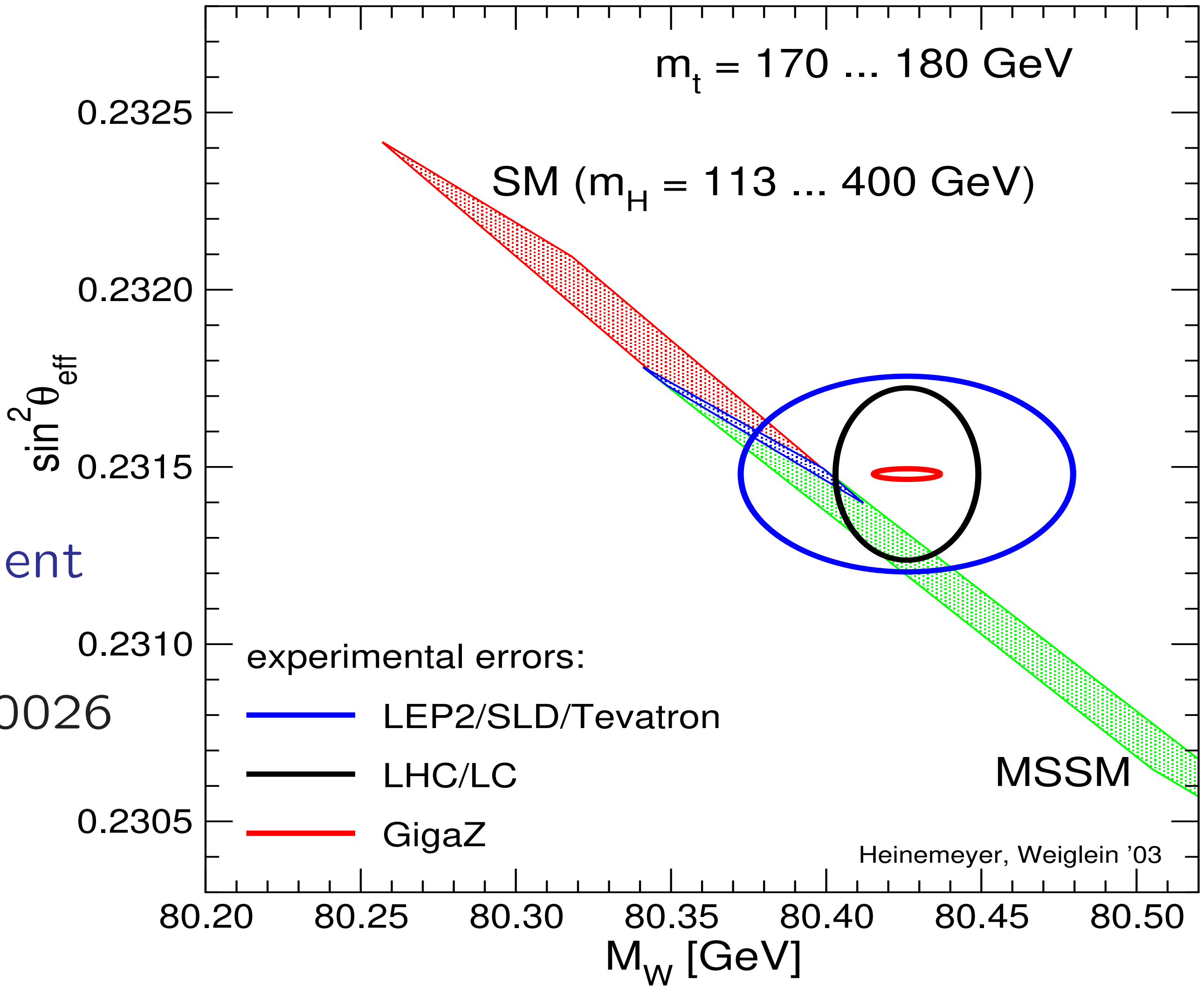
The effective weak mixing angle $\sin^2\theta_{\rm eff}$:

[*LEPEWWG '03*]



Prediction for M_W, $\sin^2\theta_{\mathrm{eff}}$ in SM and MSSM:

[*S. Heinemeyer, G. W. '01*]



Scenario where measurement of $A_{\mathrm{FB}}^{\mathrm{b}}$ is discarded:

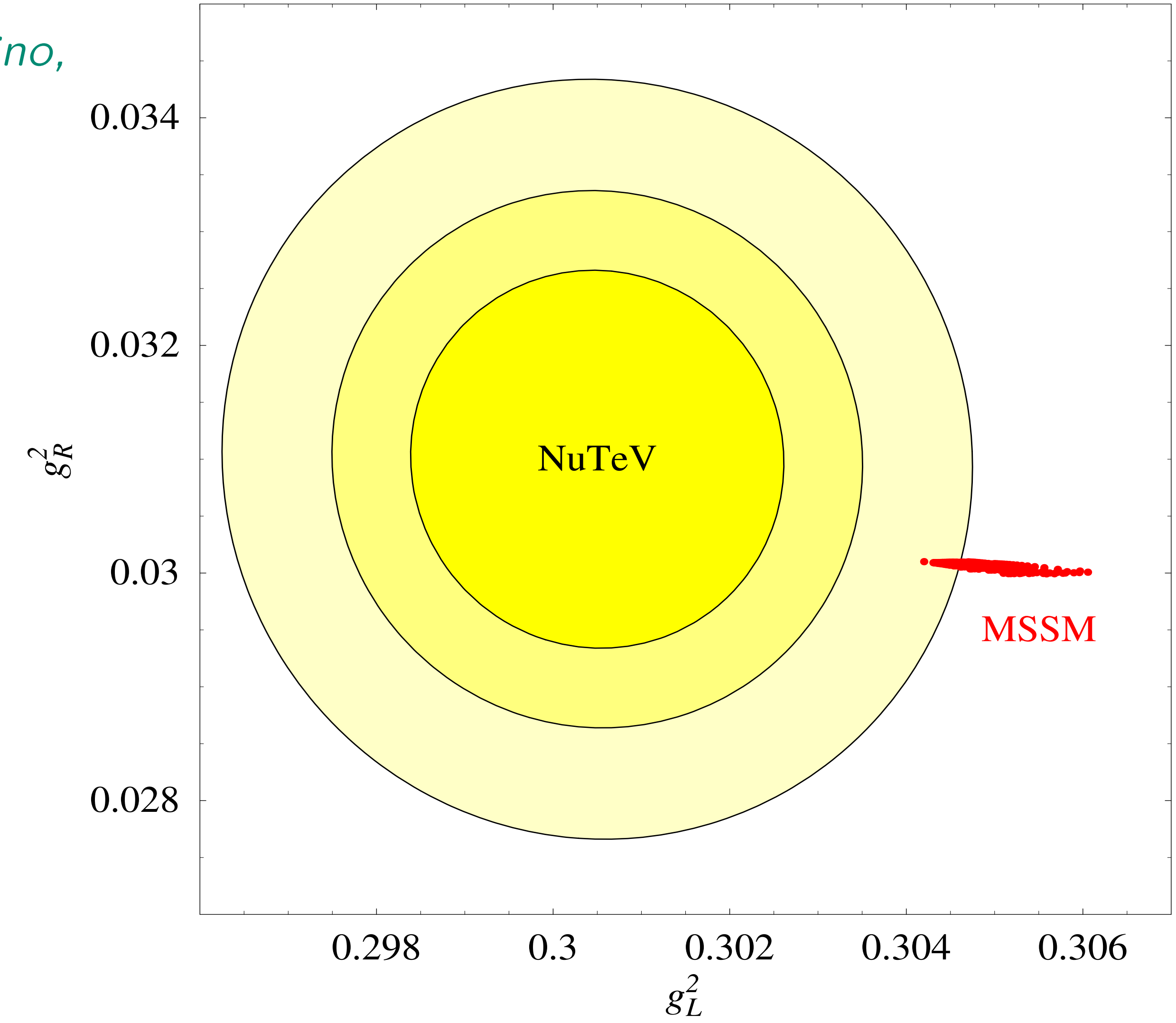
$\sin^2\theta_{\mathrm{eff}}^{\mathrm{SLD}} = 0.23098 \pm 0.00026$

$\Rightarrow$ good agreement with MSSM prediction

$\Rightarrow$ High sensitivity to deviations both from the SM and the MSSM

MSSM prediction for neutrino–nucleon scattering (measured at NuTeV):

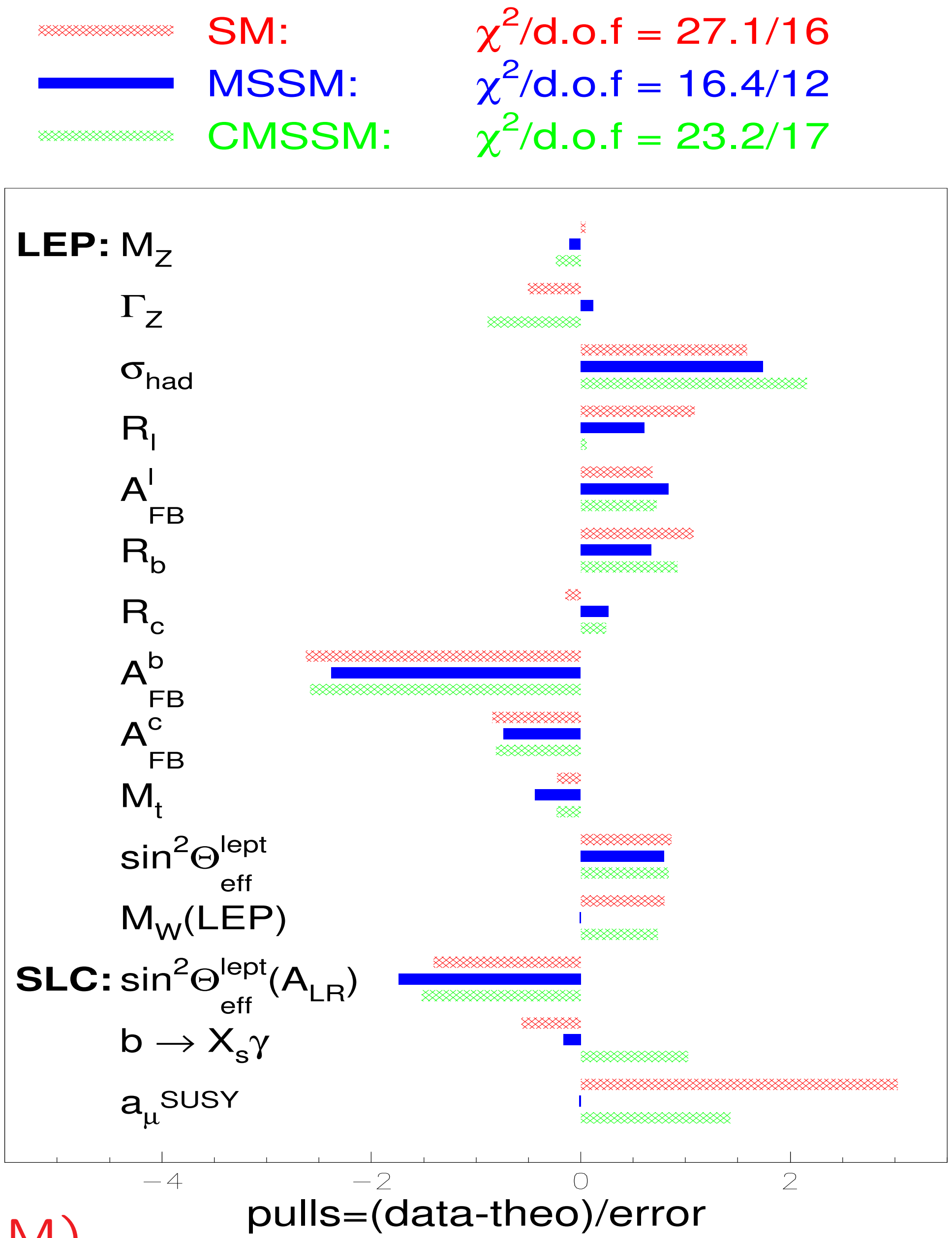
[S. Davidson, S. Forte, P. Gambino, N. Rius, A. Strumia '02]



$\Rightarrow$ Discrepancy cannot be explained in MSSM (light slepton corrections)

Global fit: SM / unconstrained MSSM / mSUGRA

[*W. de Boer, C. Sander '02*]



⇒ MSSM contributions can improve agreement with data for $g_\mu - 2$ (for large $\tan\beta$) and M_W

no significant improvement for A^b_{FB} and neutrino–nucleon cross section

Similar quality of global fit (slightly better in MSSM than in SM)

Prospects for SUSY searches at the next generation of colliders

Limits on SUSY particles from LEP, Tevatron Run I: $\mathcal{O}(100\,\text{GeV})$

But: some SUSY particles can be much lighter if they have small coupling to Z boson

E.g.:

– no strict lower bound on lightest neutralino if "GUT relation" between M_1 and M_2 is relaxed

– A light sbottom with $m_{\tilde{b}_1} \approx 5$ GeV is in agreement with electroweak precision tests, Higgs searches, . . .
[*M. Carena, S. Heinemeyer, C. Wagner, G. W. '01*]

– . . .

SUSY searches at the Tevatron, Run II:

compared to Run I: $\approx 100\times$ higher luminosity, slightly increased energy ($1.8 \to 2$ TeV)

Limited mass window in which discovery of SUSY particles above Run I is possible

Best prospects for:

– 'Trilepton signal': $\tilde{\chi}_2^0 \tilde{\chi}_1^+ \to \ell^+ \ell^- \tilde{\chi}_1^0 \ell^+ \nu \chi_1^0$

– $\tilde{t}$, $\tilde{b}$ searches

– light SUSY Higgs h in region of large $\tan\beta$

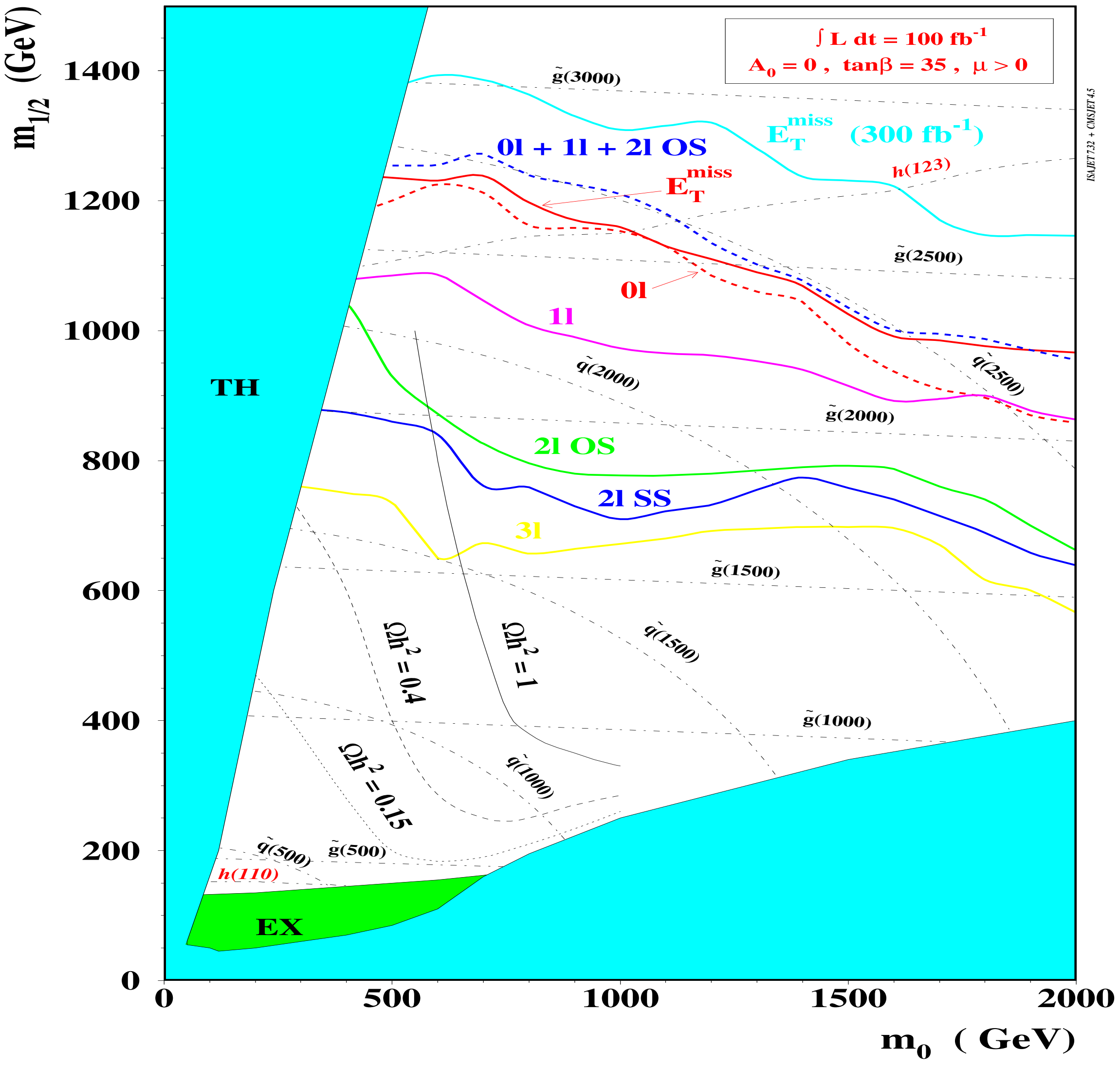
SUSY searches at the LHC:

Dominated by production of colored particles: gluino, squarks

Very large mass range in the searches for jets + missing energy

$\Rightarrow$ gluino, squarks accessible up to 2–3 TeV

Discovery reach contours in m_0–$m_{1/2}$ plane (mSUGRA scenario) for various final states with 100 fb^{-1}: [*CMS '99*]



⇒ discovery of SUSY particles expected if low-energy SUSY is realized

Production of SUSY particles at the LHC will in general result in complicated final states, e.g.

$$\tilde{g} \to \bar{q}\tilde{q} \to \bar{q}q\tilde{\chi}_2^0 \to \bar{q}q\tilde{\tau}\tau \to \bar{q}q\tau\tau\tilde{\chi}_1^0$$

Production of uncolored particles via cascade decays often dominates over direct production

Many states are produced at once

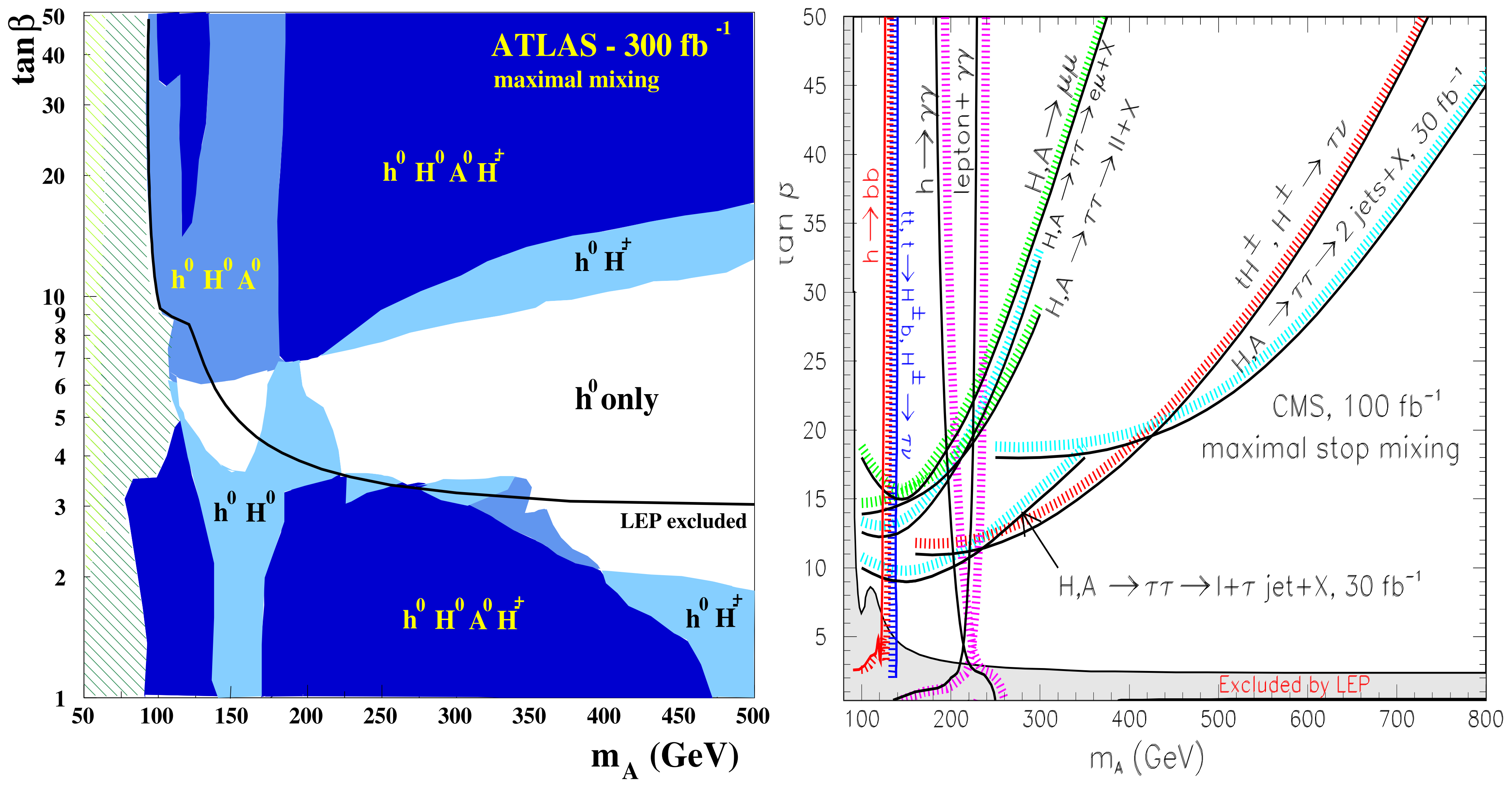
$\Rightarrow$ **Main background for SUSY is SUSY itself!**

Searches for MSSM Higgs bosons:

good prospects for detecting light Higgs h

H/A discovery possible in significant part of parameter space

MSSM Higgs discovery contours in M_A–$\tan\beta$ plane (m_h^{max} benchmark scenario): [ATLAS '99] [CMS '03]



In order to establish SUSY experimentally:

Need to demonstrate that:

– every particle has superpartner

– their spins differ by $1/2$

– their gauge quantum numbers are the same

– their couplings are identical

– mass relations hold

...

$\Rightarrow$ Precise measurements of masses, branching ratios, cross sections, angular distributions, ... mandatory for

- establishing SUSY experimentally
- disentangling patterns of SUSY breaking

Requires clean experimental environment, high luminosity, beam polarization, . . .

$\Rightarrow$ High luminosity LC necessary, complementary to hadron machines

SUSY searches at the LC:

Clean signatures, small backgrounds

Thresholds for pair production of SUSY particles

$\Rightarrow$ precise determination of mass and spin of SUSY particles, mixing angles, complex phases, . . .

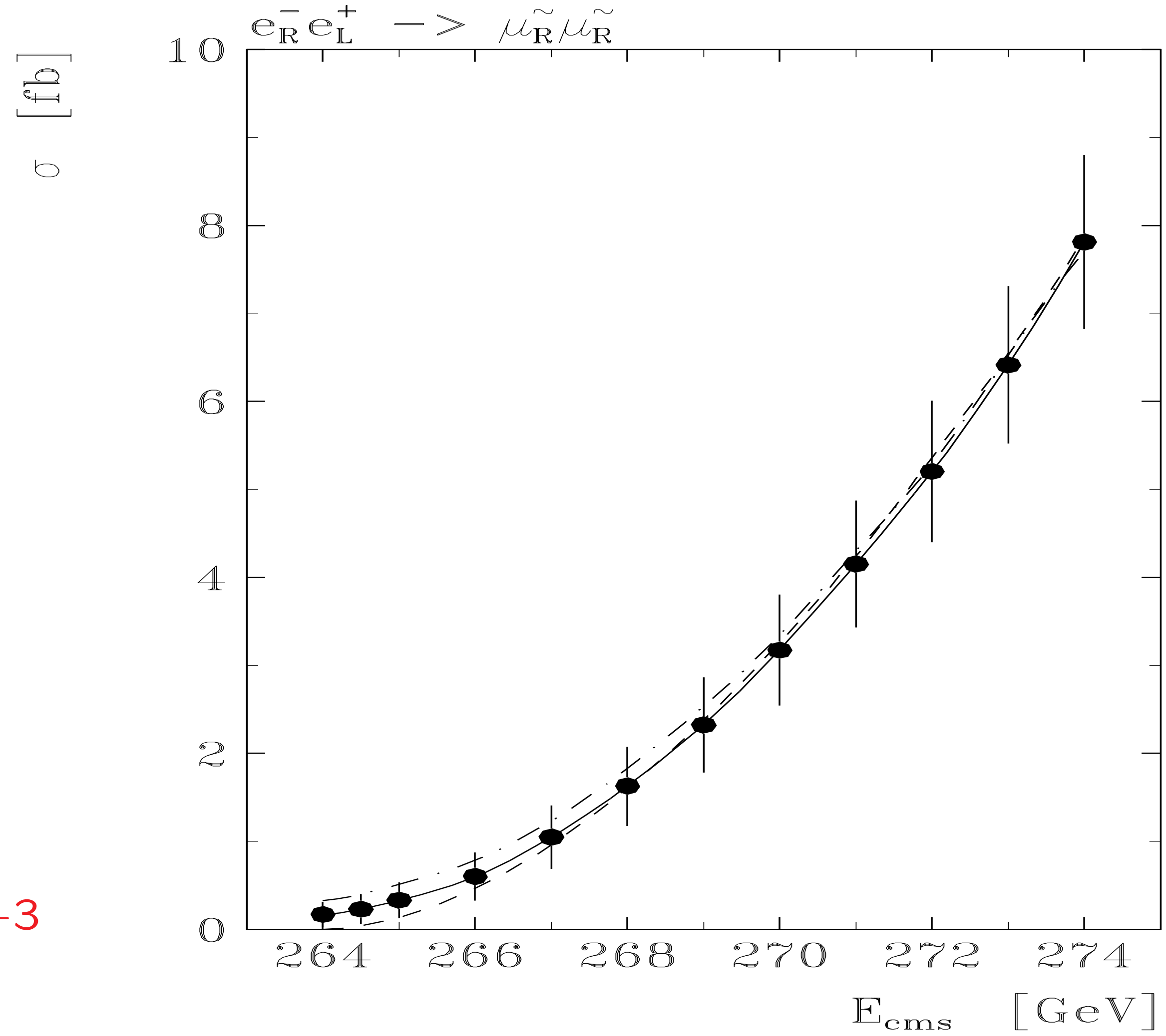
Limited by kinematic reach

Good prospects for production of uncolored particles

$\Rightarrow$ LHC / LC complementarity

Examples for SUSY physics at the LC:

Determination of mass and spin of $\tilde{\mu}_R$ from production at threshold:
[*TESLA TDR '01*]

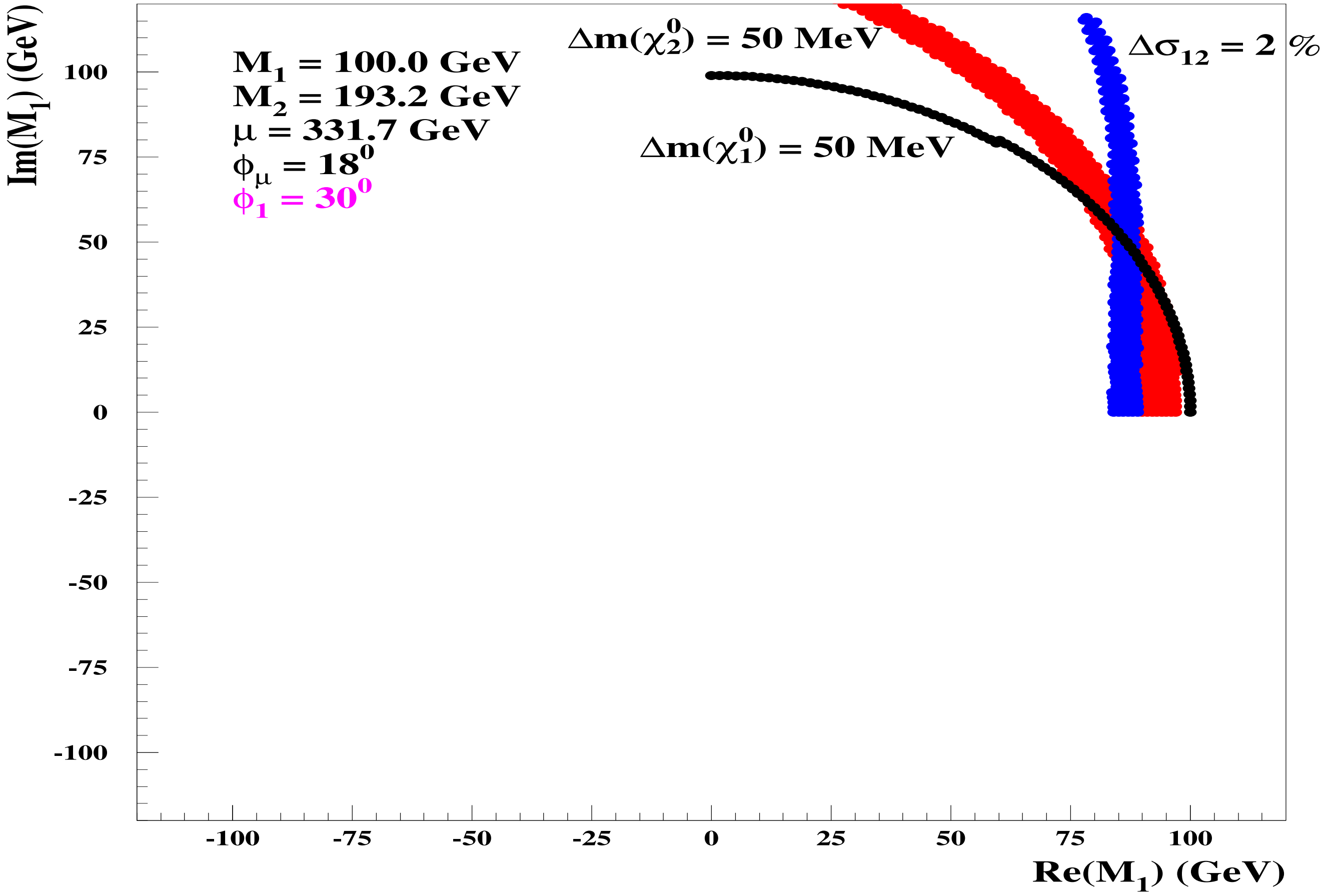


$$\Rightarrow \quad \frac{\Delta m_{\tilde{\mu}_R}}{m_{\tilde{\mu}_R}} < 1 \times 10^{-3}$$

$\Rightarrow$ test of $J = 0$ hypothesis

Determination of phase ϕ_1 in neutralino sector from measurement of $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2$ and $e^+e^- \to \tilde{\chi}^+_1\tilde{\chi}^-_1$, $\mathcal{L} = 500$ fb^{-1}:

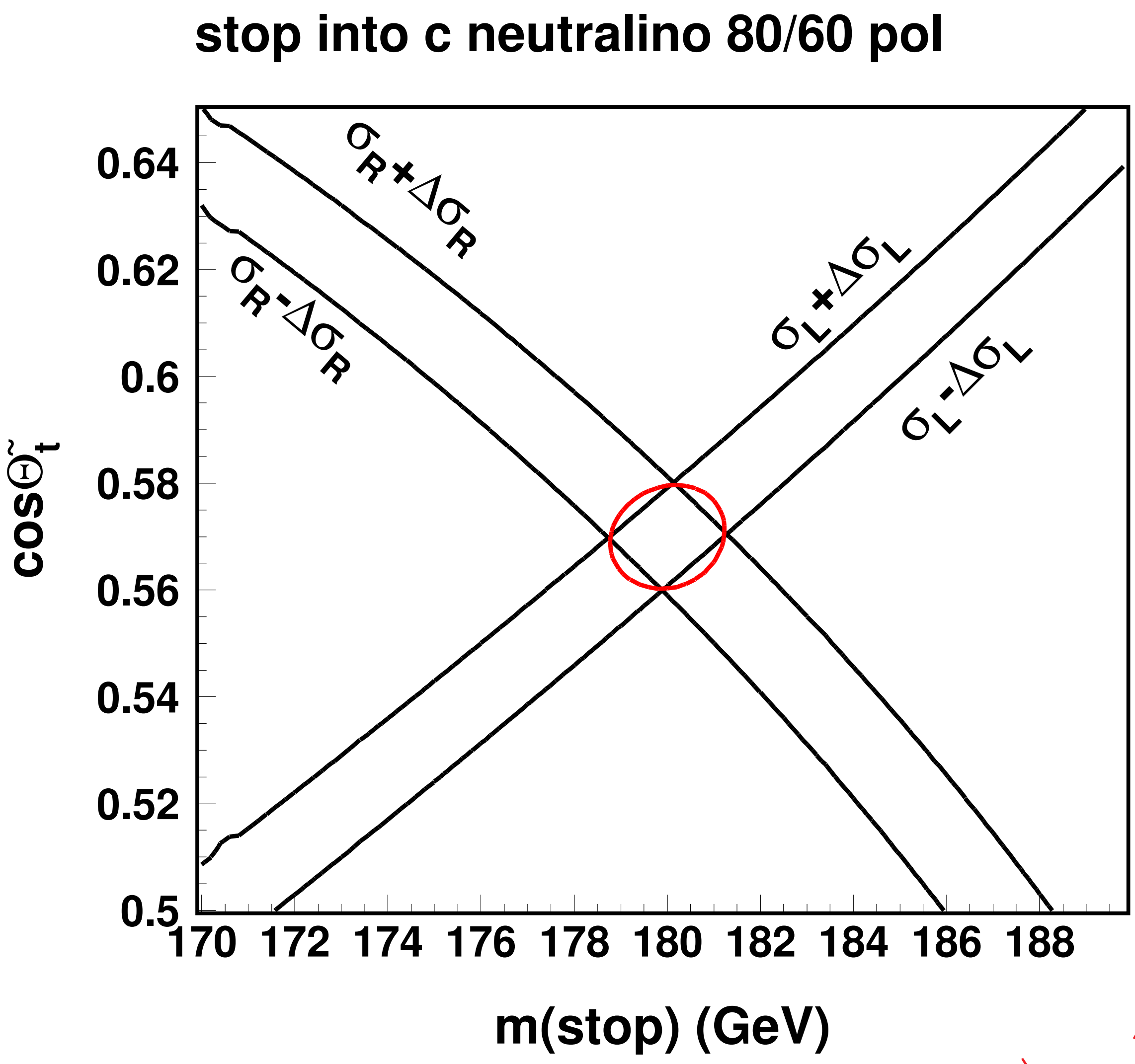
$m_{\tilde{\chi}^0_1} = 96.12 \pm 0.05$ GeV, $m_{\tilde{\chi}^0_2} = 177.13 \pm 0.05$ GeV, $m_{\tilde{\chi}^\pm_1} = 174.90 \pm 0.05$ GeV, $\Delta\sigma(\tilde{\chi}^0_1\tilde{\chi}^0_2) = \pm 2\%$ [K. Desch, G. Moortgat-Pick '02]



$\Rightarrow \Phi_1 = 30^0 \pm 3^0$

Determination of $m_{\tilde{t}_1}$, $\theta_{\tilde{t}}$ from $\sigma(e^+e^- \to \tilde{t}_1\tilde{t}_1)$ with polarized beams:

[*R. Keränen, H. Nowak, A. Sopczak '00*]



$$\Rightarrow \quad \frac{\Delta m_{\tilde{t}_1}}{m_{\tilde{t}_1}} \approx 0.5\%, \qquad \frac{\Delta \cos\theta_{\tilde{t}}}{\cos\theta_{\tilde{t}}} \approx 1.5\%$$

Complementarity of LHC and LC:

$\Rightarrow$ Results obtained at one collider can be used for improving experimental analyses at the other

$\Rightarrow$ investigated in "LHC / LC Study Group"

`www.ippp.dur.ac.uk/~georg/lhclc`

Collaborative effort of Hadron Collider (HC) and Linear Collider (LC) community

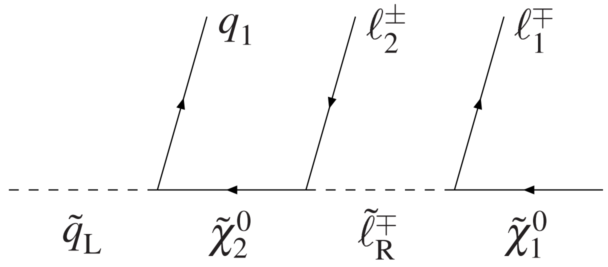
Started in spring 2002, currently about 190 working group members from ATLAS, CMS, LC working groups, theory + Tevatron contact person

Example: SUSY parameters at LHC and LC

Reconstruction of sparticle masses at the LHC

[B. Gjelsten, E. Lytken, D. Miller, P. Osland, G. Polesello, M. Chiorboli, A. Tricomi]

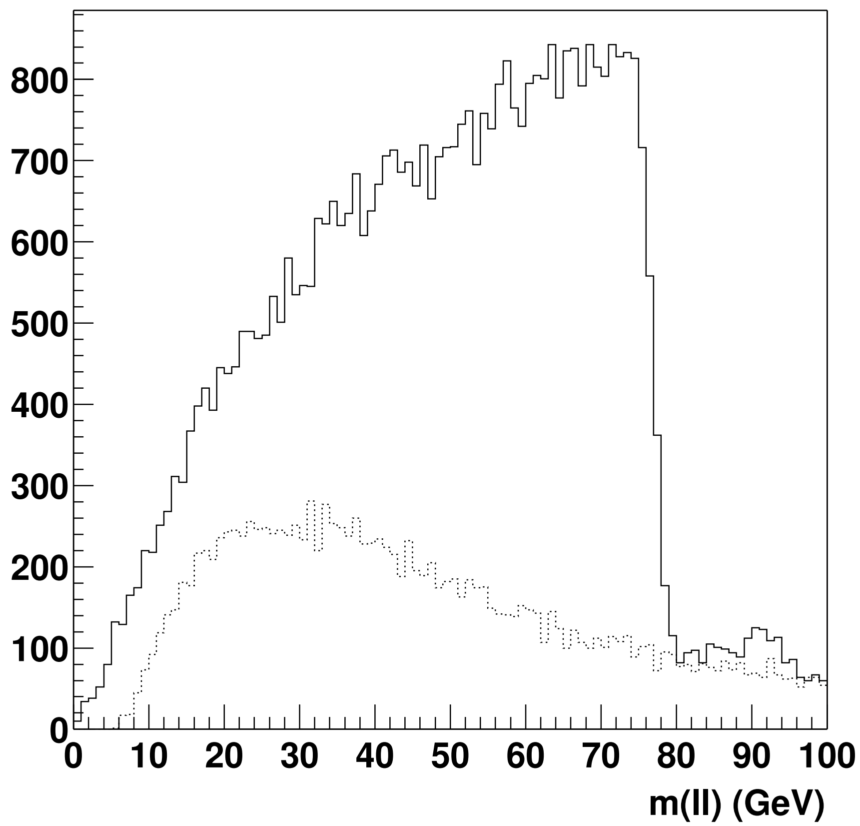
Complicated decay chains for squarks and gluinos

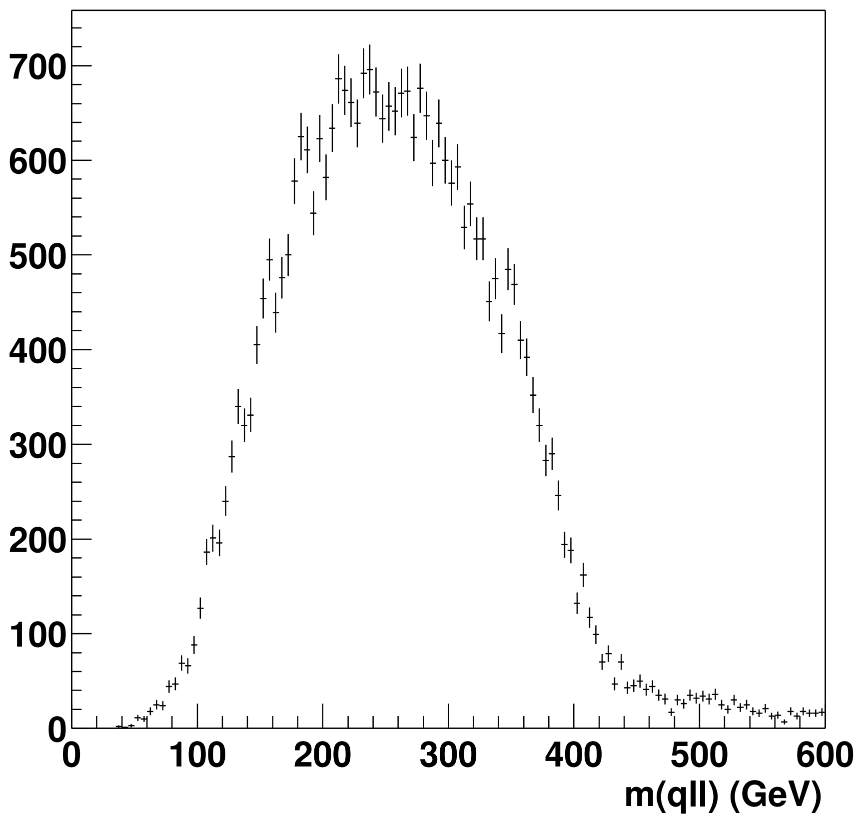


Examples worked out for SPS1a from ATLAS and CMS
main tool: dilepton "edge" from $\tilde{\chi}_2^0 \to \ell^+\ell^-\tilde{\chi}_1^0$

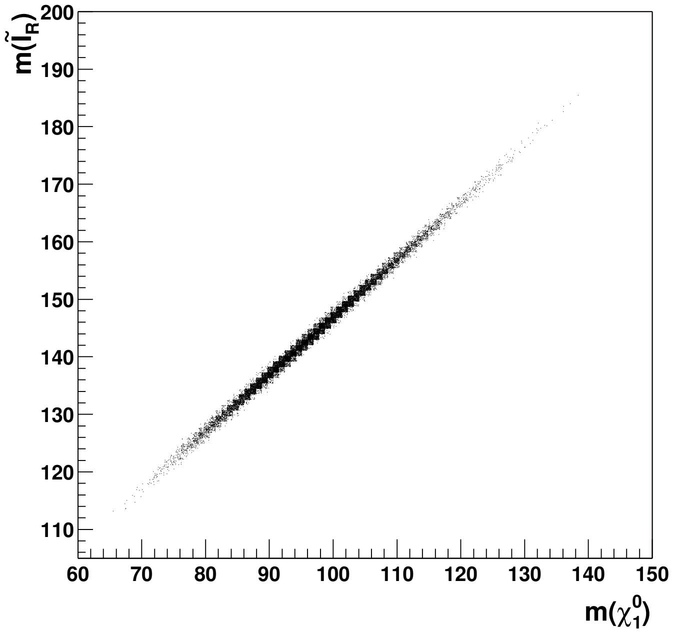
Sbottom/squark and gluino reconstruction:

Edge in same flavour-opposite sign lepton distribution (left), invariant mass distributions with kinematical endpoints (right)





Strong correlation between slepton mass and LSP mass, LSP mass can be constrained at LHC at the 10% level only:



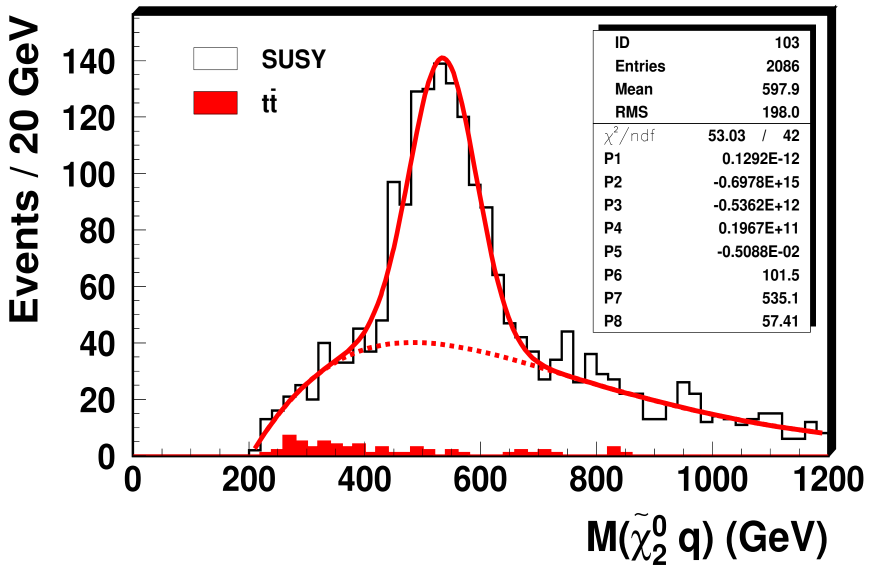
$\Rightarrow$ Take LSP mass as input from LC

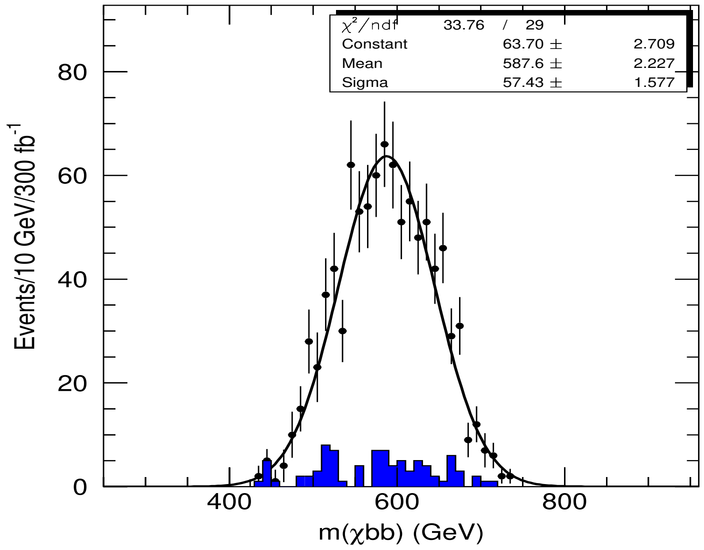
Select events close to the edge and combine with b-jet / q-jet

$$\vec{p}(\tilde{\chi}_2^0) = \left(1 - \frac{m(\tilde{\chi}_1^0)}{m(\ell\ell)}\right) \vec{p}_{\ell\ell}$$

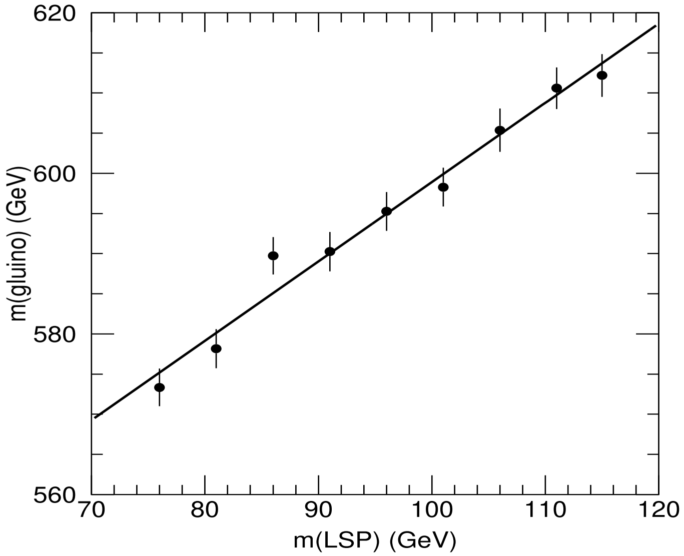
$\Rightarrow$ Get sbottom/squark mass if LSP mass is known

Squark peak (left) and gluino reconstruction from $(\chi_2^0 bb)$ invariant mass distribution (right):





$m_{\tilde{g}}$ as function of the LSP mass:



$\Rightarrow \Delta m_{\tilde{g}} \approx \Delta m_{\mathrm{LSP}}$

Accuracies for the case of the LHC alone (left) and with the LC measurement of the LSP mass with 0.2% accuracy (middle) and 1.0% accuracy (right column) in GeV:

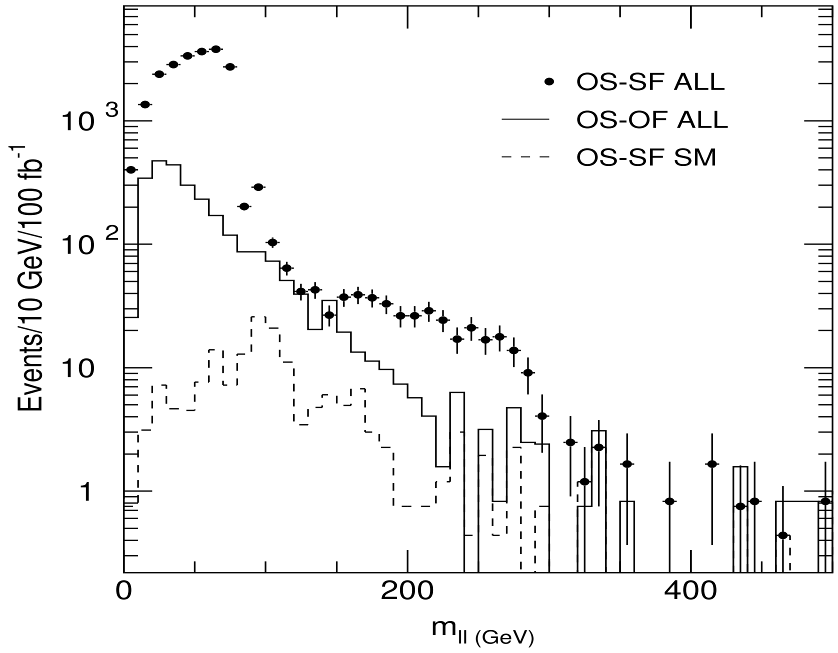
	LHC	LHC+LC (0.2%)	LHC+LC (1.0%)
$\Delta m_{\tilde{\chi}_1^0}$	9.2	0.2	1.0
$\Delta m_{\tilde{l}_R}$	9.2	0.5	1.0
$\Delta m_{\tilde{\chi}_2^0}$	9.0	0.3	1.0
$\Delta m_{\tilde{b}_1}$	23.1	16.9	17.0
$\Delta m_{\tilde{q}_L}$	15.0	5.1	5.3

$\Rightarrow$ LC input improves accuracy significantly

One step further:

Determination of the mass of the heaviest neutralino at the LHC using LC input from the neutralino/chargino sector:

[*J. Kalinowski, G. Moortgat-Pick, M. Nojiri, G. Polesello*]



⇒ Need besides LSP mass also masses of sleptons and charginos from LC in order to correctly identify $\tilde{\chi}_4^0$

⇒ Feeding $m(\tilde{\chi}_4^0)$ back into LC analysis improves accuracy of parameter determination at the LC

Even further:

Full reconstruction of stop/sbottom parameters with LHC $\otimes$ LC

[*J. Hisano, K. Kawagoe, M. Nojiri*]

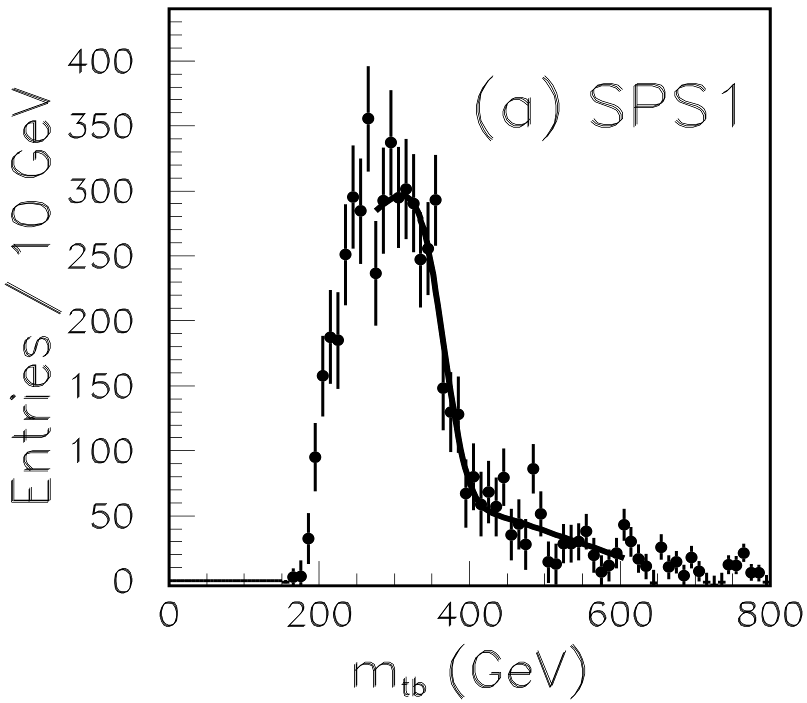
Complete set of electroweak SUSY parameters (from LC) and branching ratios used to exploit LHC rate measurements

Stop/sbottom sector determined by 5 parameters, e.g. $m_{\tilde{\mathrm{b}}_1}$, $m_{\tilde{\mathrm{b}}_2}$, $m_{\tilde{\mathrm{t}}_1}$, $\theta_{\tilde{\mathrm{b}}}$, $\theta_{\tilde{\mathrm{t}}}$

- Take $m_{\tilde{\mathrm{b}}_1}$, $m_{\tilde{\mathrm{b}}_2}$ from previous study

- $\Rightarrow$ need three more observables:
  - tb invariant mass distribution
  - rate of “edge-events” in m_{tb} distribution (chargino chain)
  - rate of events in $\ell\ell b$ distribution ($\tilde{\chi}_2^0$ chain)

m_{tb} distribution for SPS1a:

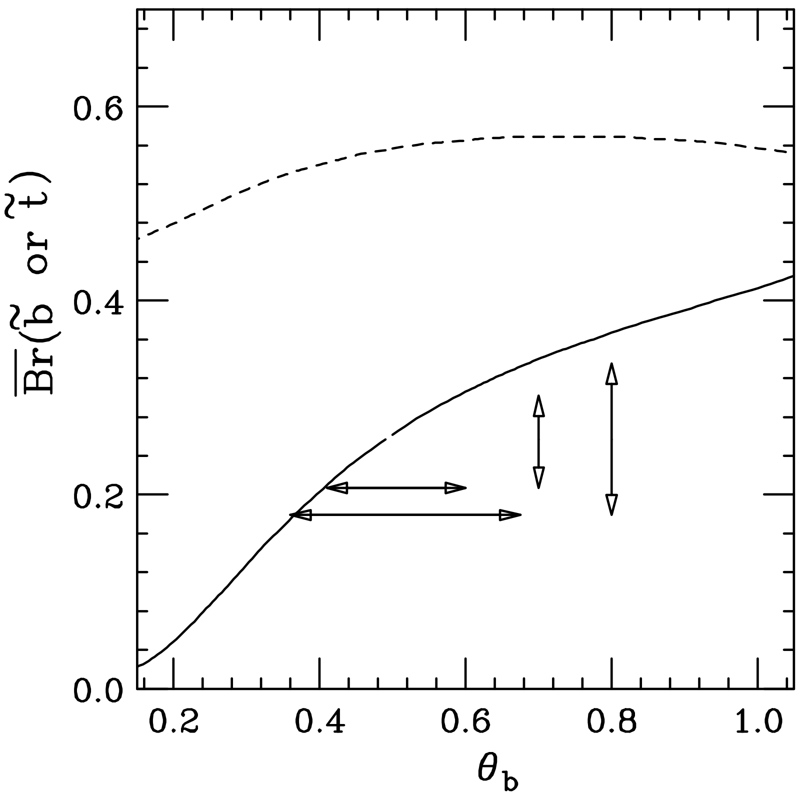


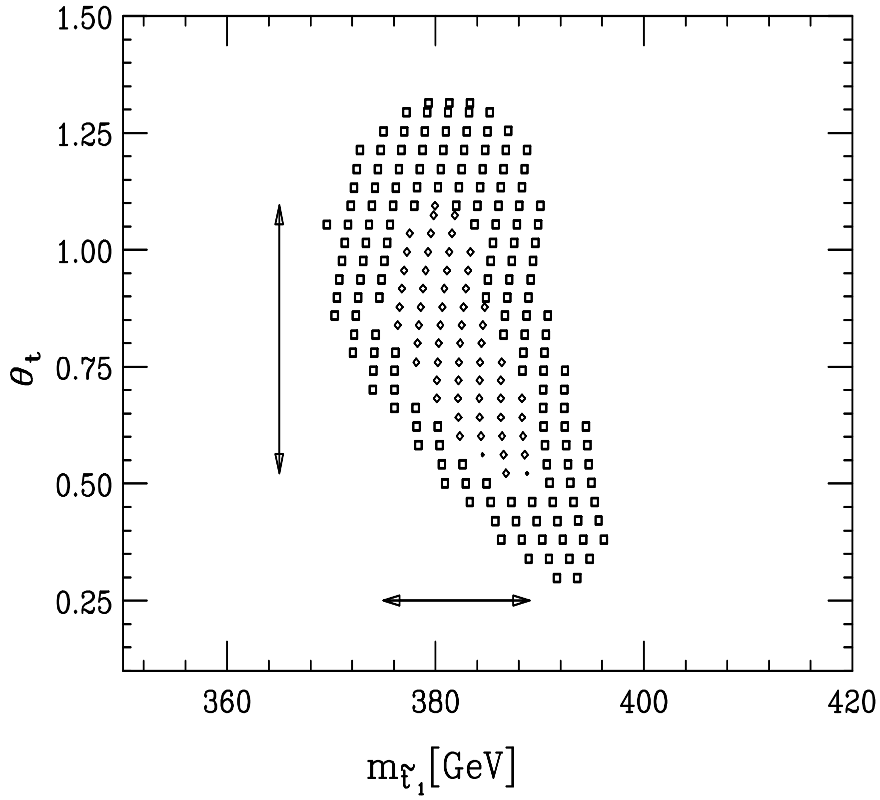
Involved couplings are stop–sbottom–W, top–sbottom–chargino and stop–bottom–chargino

$\Rightarrow$ If chargino couplings + BR's are known then observed rates are sensitive to sbottom/stop mixing parameters

Determination of $\theta_{\tilde{b}}$ from $\mathrm{BR}(\tilde{g} \to b\tilde{b}_2 \to bb\tilde{\chi}^0_2)/\mathrm{BR}(\tilde{g} \to b\tilde{b}_1 \to bb\tilde{\chi}^0_2)$ in SPS1a, sbottom masses and parameters of chargino/neutralino sector are assumed to be known (left)

Determination of $m_{\tilde{t}_1}$, $\theta_{\tilde{t}}$ assuming also $\theta_{\tilde{b}}$ is known (right)





$\Rightarrow$ $\mathcal{O}(50\%)$ determination of mixing angles, $\Delta m_{\tilde{t}_1}/m_{\tilde{t}_1} < 5\%$

Results used for:

mSUGRA fit to LHC $\oplus$ LC data in SPS1a

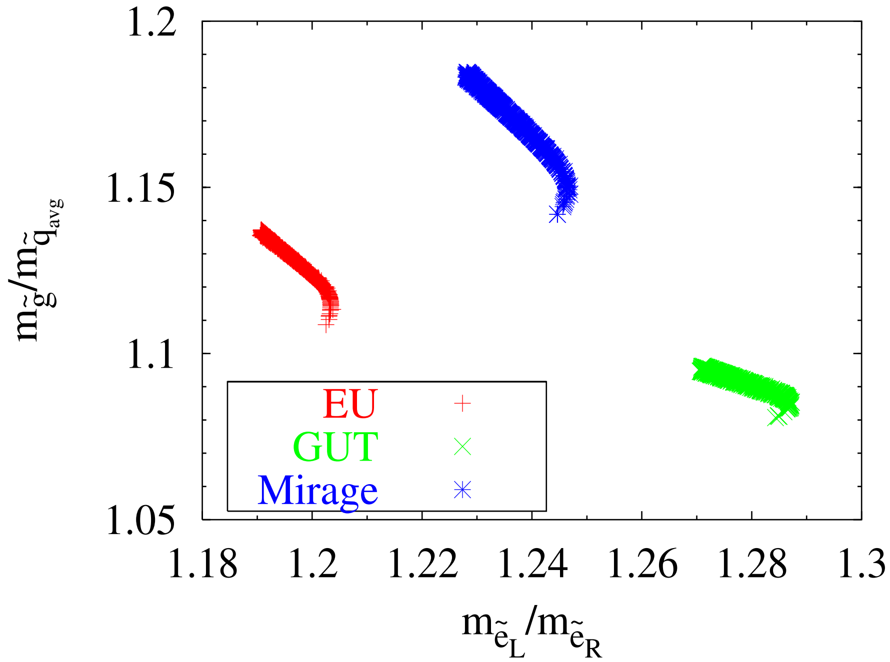
[*B. Allanach, S. Kraml, W. Porod*]

LHC / LC complementarity in mSUGRA fits

[*D. Tovey*]

Discrimination between different SUSY-breaking scenarios

[*B. Allanach, D. Grellscheid, F. Quevedo*]



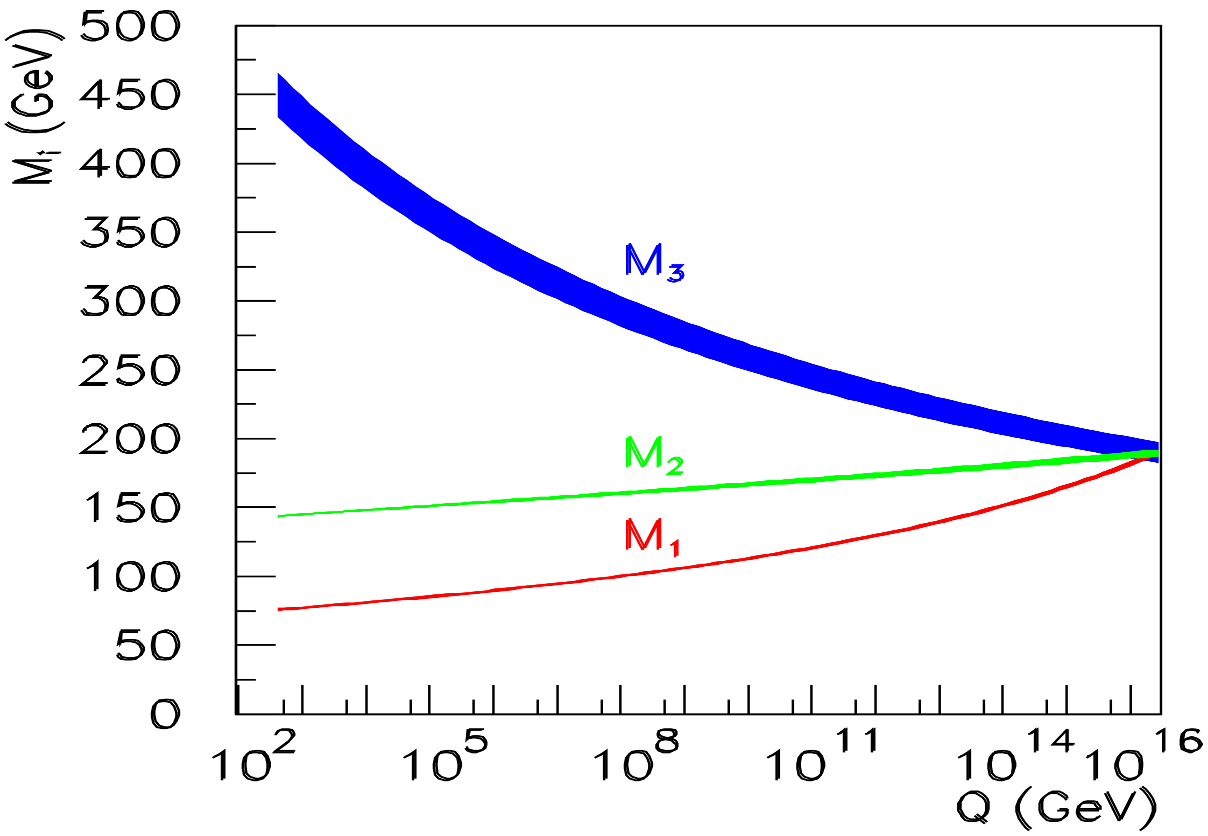
$\Rightarrow$ Need %-level accuracy to distinguish between different models

Extrapolation to physics at high scales

from combination of LHC and LC results, precise measurement of masses of SUSY particles, couplings

E.g.: Test of gaugino mass unification

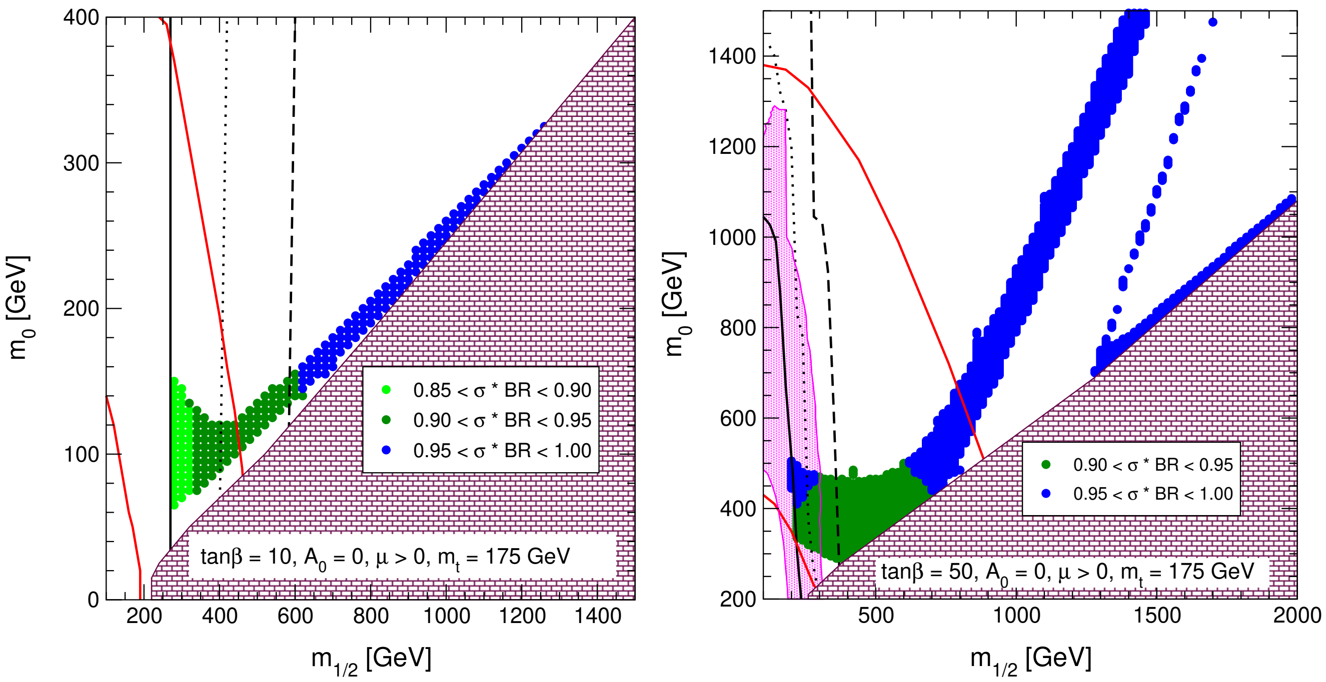
[G. Blair, W. Porod, P. Zerwas '01]



Higgs searches at the Tevatron and the LHC

mSUGRA scenario with CDM constraints

$\mu > 0$, $\tan\beta = 10, 50$: $\quad \Big[\sigma(gg \to h) \times \mathrm{BR}(h \to \gamma\gamma)\Big]_{\mathrm{CMSSM}} \Big/ \Big[\sigma(gg \to h) \times \mathrm{BR}(h \to \gamma\gamma)\Big]_{\mathrm{SM}}$:



$\Rightarrow$ no significant suppression of $\sigma(gg \to h) \times \mathrm{BR}(h \to \gamma\gamma)$ compared to SM

$\Rightarrow$ Discovery of lightest Higgs boson within about one year at LHC possible
[*J. Ellis, S. Heinemeyer, K. Olive, G. W. '01*]

Similar results in GMSB and AMSB scenarios [*A. Dedes, S. Heinemeyer, S. Su, G. W. '03*]

Suggested benchmarks for Higgs searches at the Tevatron and the LHC

[*M. Carena, S. Heinemeyer, C. Wagner, G. W. '02*]

Scenarios for general MSSM, no specific SUSY-breaking scenario assumed, no indirect constraints, M_A, $\tan\beta$ varied

- m_h^{max}-scenario: $X_t = 2\,M_{SUSY}$ (FD), $M_{SUSY} = 1$ TeV

  $\Rightarrow$ maximal $m_h(\tan\beta)$ for fixed m_t, M_{SUSY}

- no-mixing scenario: $X_t = 0$, $M_{SUSY} = 2$ TeV

- gluophobic Higgs scenario: $M_{SUSY} = 350$ GeV, $X_t = -750$ GeV (FD)

  $\Rightarrow$ suppression of $gg \to h$

- small α_{eff} scenario:

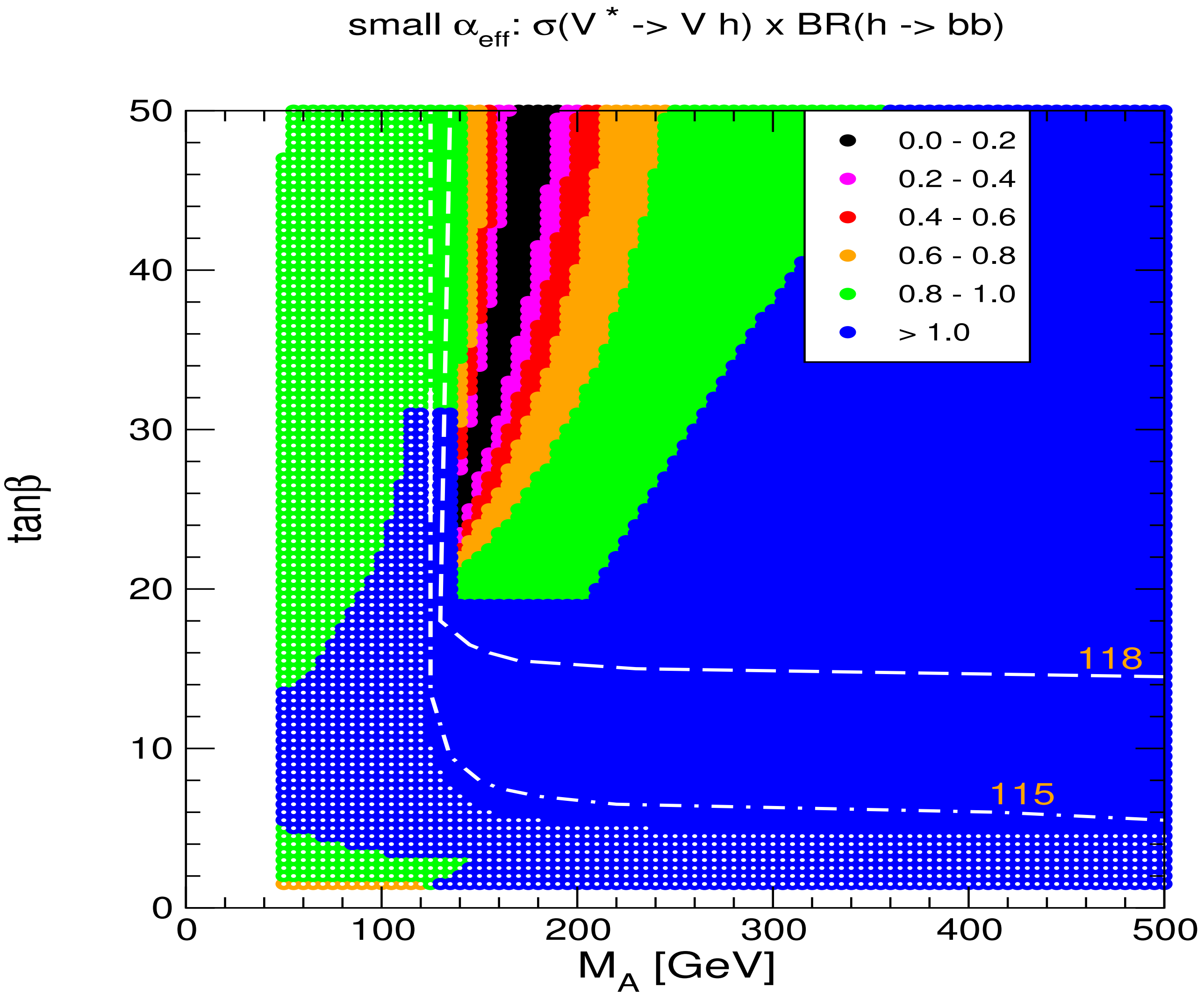
  $M_{SUSY} = 800\,\text{GeV}$, $\mu = 2.5\,M_{SUSY}$, $X_t = -1100$ GeV (FD)

  $\Rightarrow$ suppression of $h \to b\bar{b}$, $h \to \tau\tau$

Small α_{eff} scenario:

$\sigma(q\bar{q} \to Vh) \times \text{BR}(h \to b\bar{b})$ at the Tevatron:

[*M. Carena, S. Heinemeyer, C. Wagner, G. W. '02*]

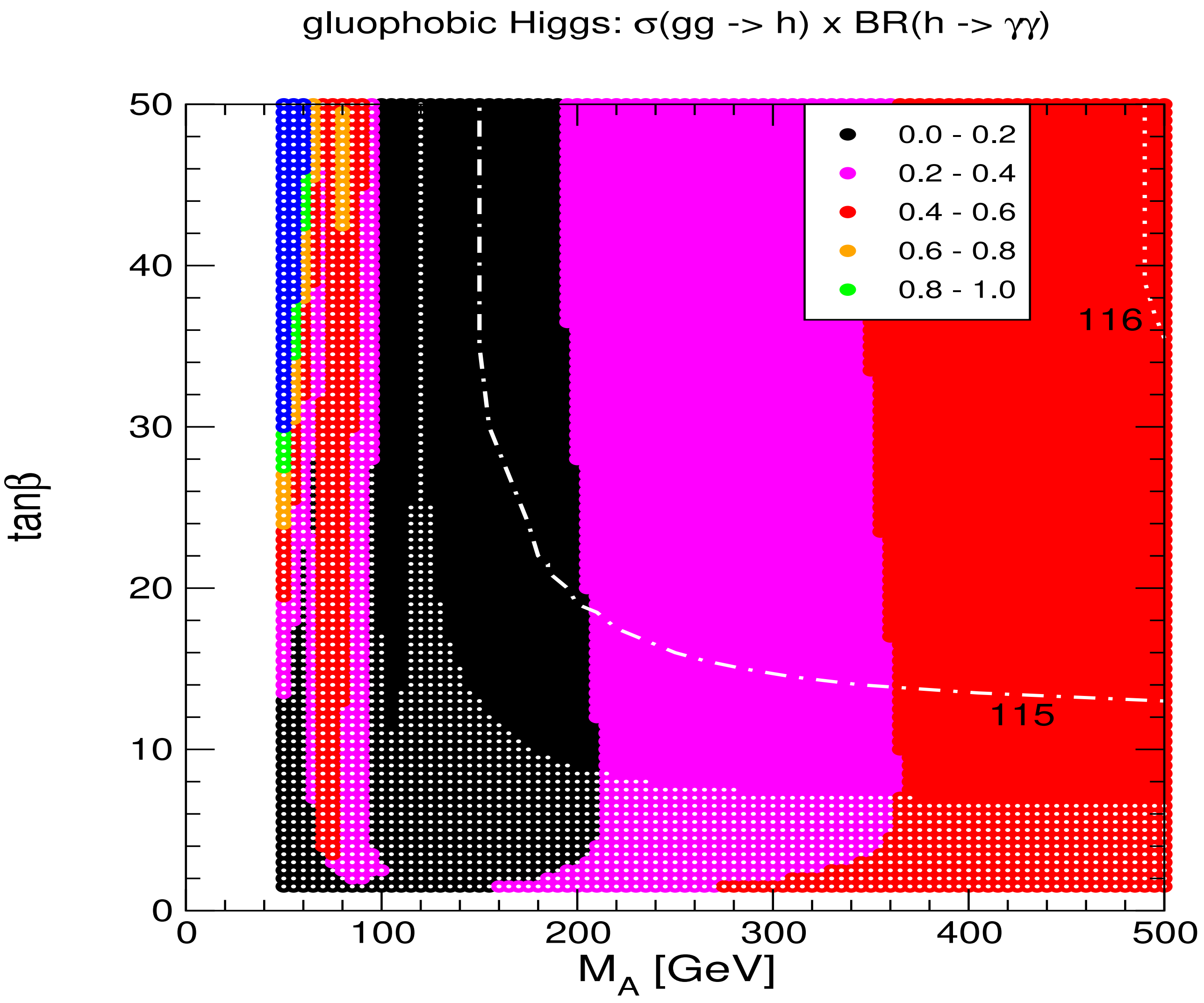


$\Rightarrow$ Significant suppression for $M_A \lesssim 250$ GeV

Gluophobic Higgs scenario:

$\sigma(gg \to h) \times \text{BR}(h \to \gamma\gamma)$ at the LHC:

[*M. Carena, S. Heinemeyer, C. Wagner, G. W. '02*]



⇒ Large suppression in whole M_A–$\tan\beta$ plane

Precision physics in the MSSM Higgs sector

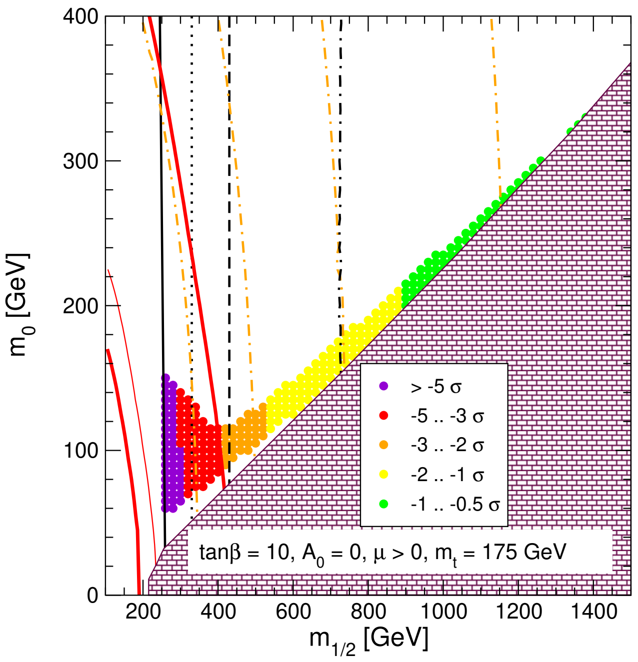
Precise measurement of Higgs branching ratios

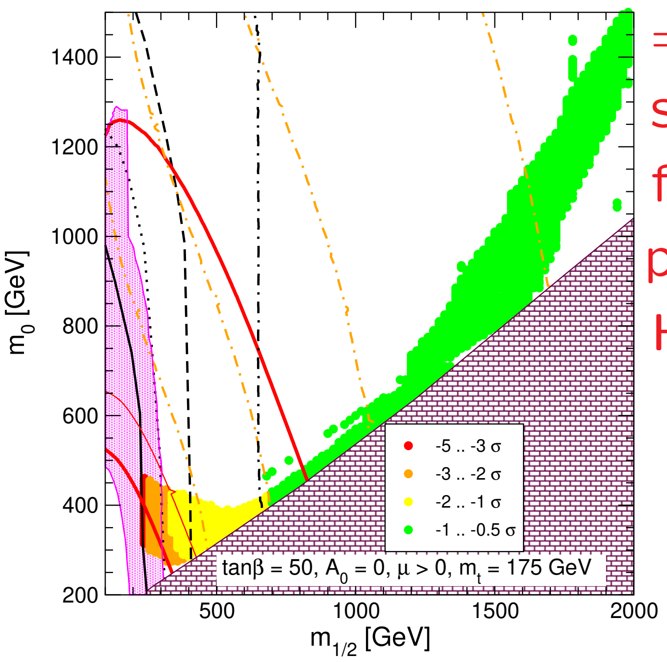
$\Rightarrow$ Sensitivity to deviations SM / MSSM

E.g.: Prediction for $\sigma(e^+e^- \to Zh) \times \mathrm{BR}(h \to WW^*)$ in parameter region allowed by cosmology: comparison mSUGRA – SM:

[*J. Ellis, S. Heinemeyer, K. Olive, G. W. '02*]

$\mu > 0$, $\tan\beta = 10, 50$:





$\Rightarrow$ In allowed parameter space: sizable deviations from SM predictions for precision observables in the Higgs sector possible

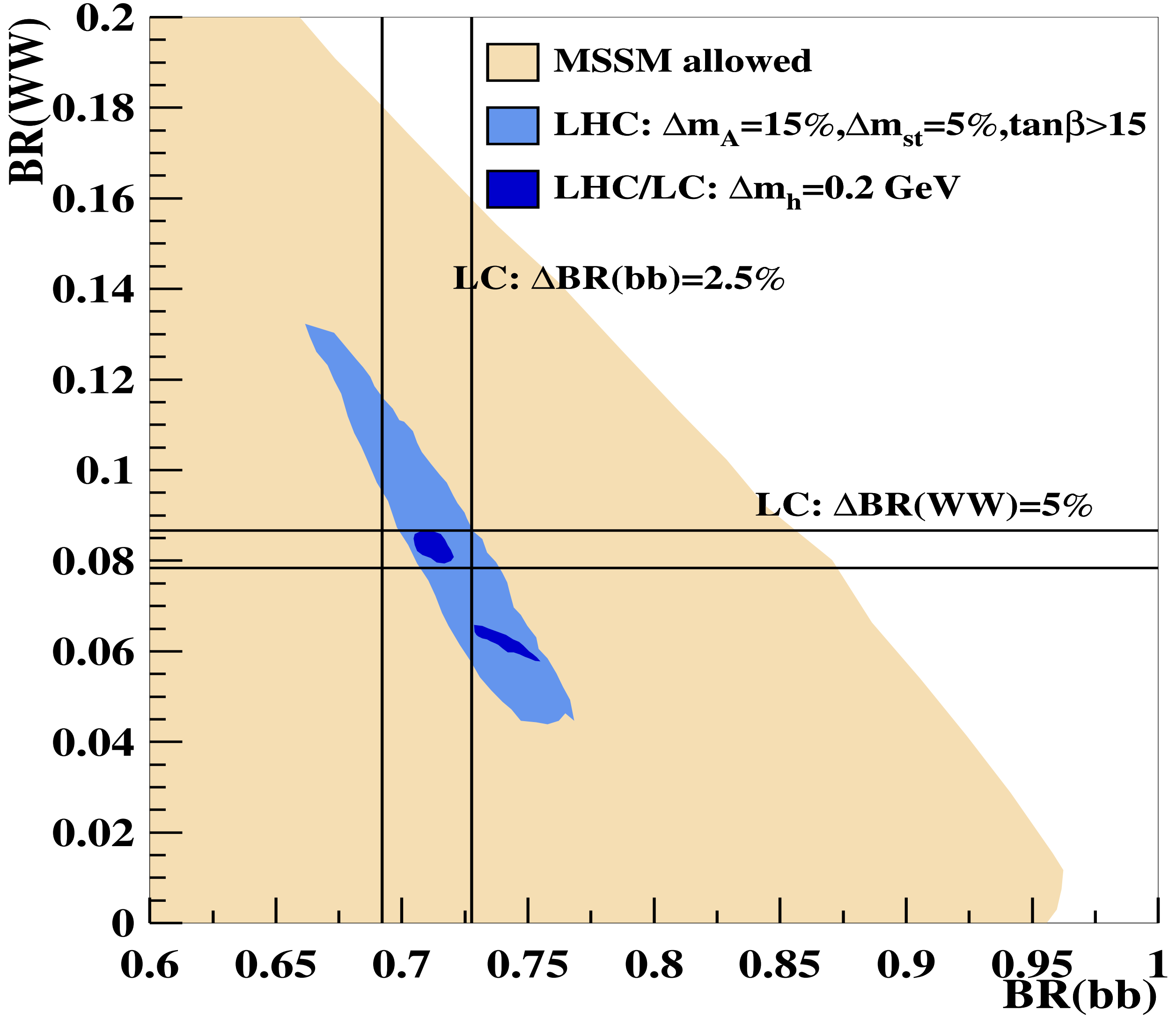
Combination of LHC data on heavy Higgs states with LC data on the light $\mathcal{CP}$-even Higgs

[K. Desch, S. Heinemeyer, G.W.]

Assume: LHC information on M_A, $\tan\beta \oplus$ (LHC $\otimes$ LC) information on stop/sbottom masses $\oplus$ LHC / LC measurement of m_h:

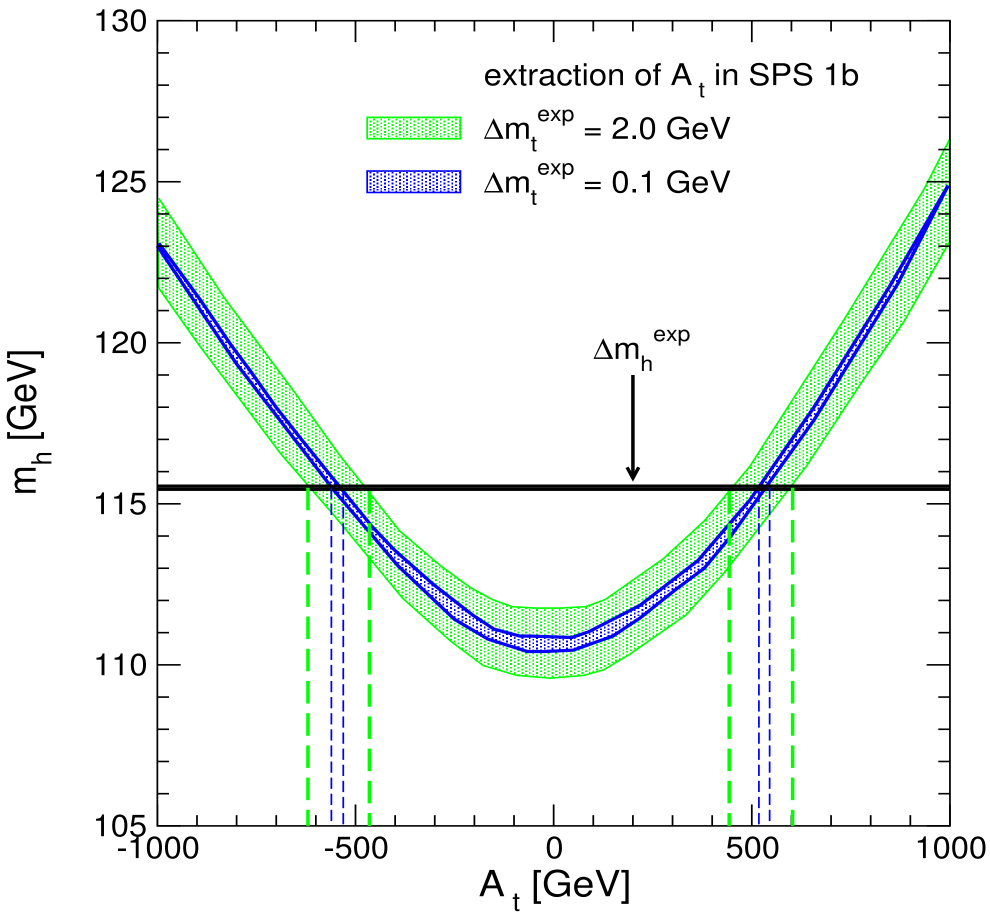
M_A: 15% accuracy, $m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}$: 5% accuracy, $\tan\beta > 15$

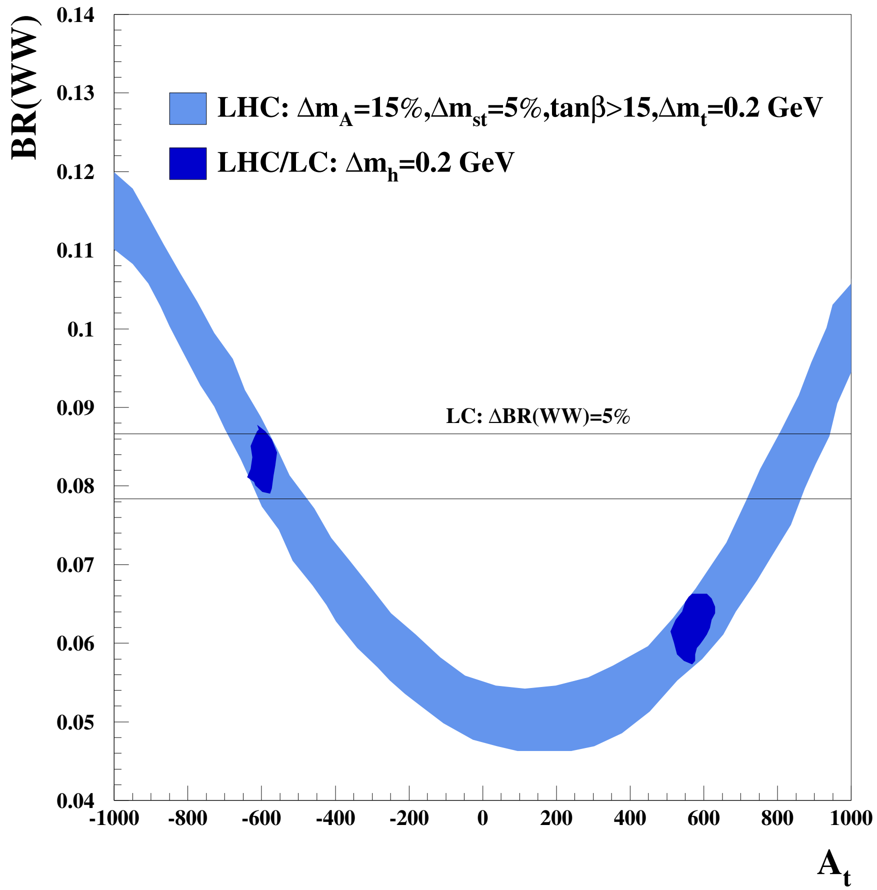
BR($h \to b\bar{b}$): 2.5% accuracy, BR($h \to WW^*$): 5% accuracy



⇒ Comparison of MSSM prediction based on assumed inputs with BR's measured at the LC yields very sensitive test of the model

⇒ Indirect determination of trilinear coupling A_t:





Precise measurement of m_t at the LC crucial, $\delta m_t \lesssim 100$ MeV

Δm_t^{LC} vs. Δm_t^{LHC} ⇒ accuracy of A_t determination improved by factor 3

Necessary improvements in accuracy of theoretical prediction in order to match experimental precision at LHC, $\delta m_{\mathrm{h}}^{\mathrm{exp}} \approx 0.2$ GeV:

- Uncertainty from experimental errors of input parameters:

  $\Rightarrow$ Complementarity example:

  In order to match

  experimental precision at LHC, $\delta m_{\mathrm{h}}^{\mathrm{exp}} \approx 0.2$ GeV

  need

  LC precision on m_{t}, $\delta m_{\mathrm{t}}^{\mathrm{exp}} \lesssim 0.2$ GeV

- Uncertainty from unknown higher-order corrections:

  $\Rightarrow$ Need improvement by more than a factor 10!

Summary of Lecture 5:

- EW precision tests: MSSM yields equally complete description as SM

  Global fit: better agreement of MSSM with data for $g_\mu - 2$, M_W than SM, no improvement for A_FB^b, NuTeV, similar fit quality

- SUSY searches (same for other kinds of new physics) at next generation of colliders:

  Tevatron and LHC: big discovery potential

  LC: high-precision physics

  $\Rightarrow$ need both to

  - establish SUSY experimentally
  - disentangle patterns of SUSY breaking

- Complementarity LHC / LC can be exploited for improving analyses at both machines

- Precision physics in Higgs sector: $\Rightarrow$ Very sensitive test of ew theory

Outlook:

- Low-energy Supersymmetry continues to be our best bet for physics beyond the Standard Model

- Data rules:

  We need experimental information from Tevatron, LHC, LC,
  ν experiments, dark matter searches, low-energy experiments, ...
  to verify / falsify models of new physics understanding

- The experiments in the next years will bring a decisive test of our ideas about low-energy SUSY

$\Rightarrow$ Very exciting prospects for the coming years

Expect the unexpected!