Susy Parameter determination
in combined analyses at LHC/LC
Status Report

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1. Parameter determination in the MSSM without assuming a Susy breaking scheme
   $\rightarrow$ Example SPS1a
   $\rightarrow$ high $\tan\beta = 10$

2. Determination with light $\tilde{\chi}^{\pm}_1$, $\tilde{\chi}^0_{1,2}$ at a LC

3. 'Hand-in-hand' with LHC
   $\rightarrow$ decisive higher accuracy? Yes!

4. Summary

$\rightarrow$ S.Y. Choi, J. Kalinowski, GMP, P.M. Zerwas,'01, '02

$\rightarrow$ J. Kalinowski, GMP, M. Nojiri, G. Polesello '03

Step–by–step Strategy

General MSSM parameter from $\tilde{\chi}^{\pm}$, $\tilde{\chi}^{0}$, $\tilde{\tau}$

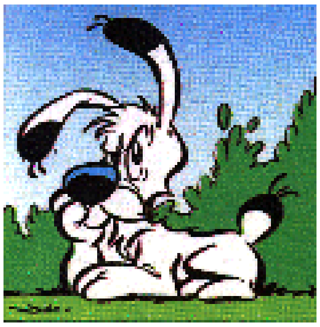
a) charginos: masses, cross sections
$\rightarrow$ mixing angles
$\rightarrow$ M_2, μ, (Φ_μ), $\tan\beta$
b) neutralinos: masses, cross sections
$\rightarrow$ $+$ M_1, (Φ_1)
c) Inversion of equations $\rightarrow$ Susy parameters

($\Phi_\mu \leq \pi/10$, due to EDM$_{e,n,Hg}$ Abel, Khalil, Lebedev'01

Φ_{M_1} experimentally provable via threshold behaviour!
Choi, Kalinowski, GMP, Zerwas,'01, Desch, GMP'02)

Remember:
only light system, $\tilde{\chi}^{\pm}_1$, $\tilde{\chi}^0_1$, $\tilde{\chi}^0_2$ would be sufficient!
$\rightarrow$ and even $m_{\tilde{\chi}^{\pm}_2}$ predictable!...
Choi, Kalinowski, GMP, Zerwas'02

But:
what does this mean in reality? Error bounds?
Help from LHC needed?

What to do? 

Chargino sector

Mass matrix in the basis $(W^{\pm}, H^{\pm})$:

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2} m_W \cos\beta \\ \sqrt{2} m_W \sin\beta & \mu \end{pmatrix}$$

Diagonalization of the mass matrix:

$$U^* \mathcal{M}_C V^{-1} = diag\{m_1^{\pm}, m_2^{\pm}\}$$

Eigenstates: Unitary matrices U, V

$$\chi_i^+ = V_{ij} \binom{W^+}{H^+}_j, \quad \chi_i^- = U_{ij} \binom{W^-}{H^-}_j$$

$\rightarrow$ With m_1, m_2 and $\sigma_{L,R}(\tilde{\chi}_1^+ \tilde{\chi}_1^-) \Rightarrow U$, V

Idea: Use contours of rates to measure U and V

Strategy – chargino

Process: $e^+e^- \to \tilde{\chi}_1^+ \tilde{\chi}_1^-$
with $P_{e^-} = \pm 80\%$, $P_{e^+} = \mp 60\%$ $\to$ $\sigma_{L,R}(11)$
$\to$ occurs via γ, Z and $\tilde{\nu}$ exchange

$\Rightarrow$ U, V with $\sigma_L(11)$, $\sigma_R(11)$
at most 4–fold ambiguity

other observable: transverse rates σ_T? $\to$ too small!

$\Rightarrow$ use $\sigma_L(11)|_{500}$ at $\sqrt{s} = 500$ GeV
and
use $\sigma_R(11)|_{400}$ at $\sqrt{s} = 400$ GeV

$\Rightarrow$ unique U, V in principle ...

Reality

Assumptions: simulation results for SPS1a

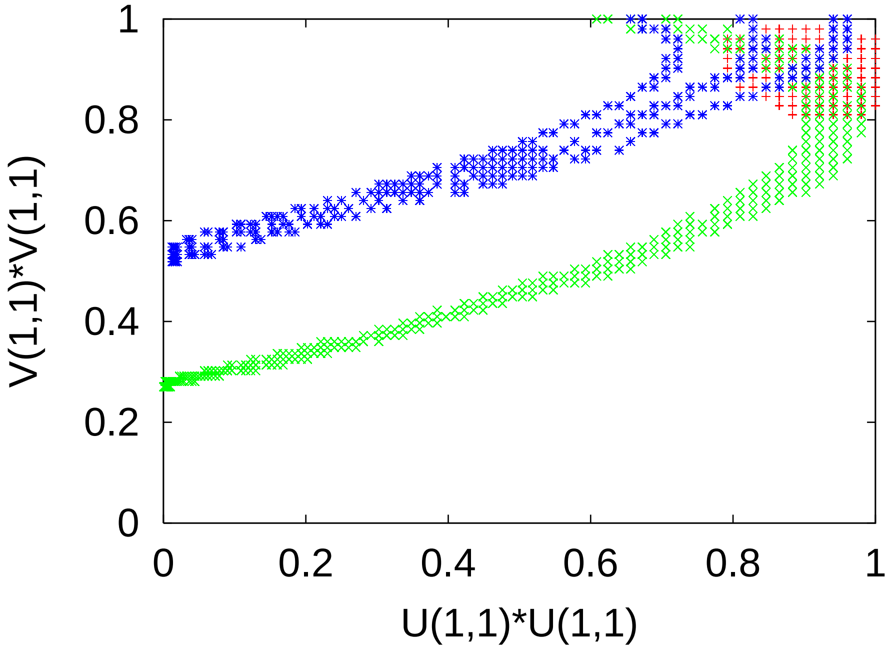
Masses:
$\delta(m_{\tilde{\chi}_1^+}) = 1.5$ GeV (simulation U. Nauenberg)
$\delta(m_{\tilde{\nu}}) = 0.7$ GeV (simulation U. Nauenberg)

Rates: 1 σ stat. error ($\int \mathcal{L} = 500$ fb^{-1})
+ error from $\delta(m_{\tilde{\nu}})$ estimation

$\sigma_L(11)|_{500} = (505 \pm 2)$ fb,
$\sigma_R(11)|_{500} = (11.8 \pm 0.2)$ fb

$\sigma_L(11)|_{400} = (216 \pm 1)$ fb,
$\sigma_R(11)|_{400} = (6.4 \pm 0.2)$ fb

Chargino contour plot



Consistent with:

$m_{\tilde{\chi}_1^+}$, $\sigma_L(11)|_{500}$, $\sigma_R(11)|_{500}$, $\sigma_L(11)|_{400}$

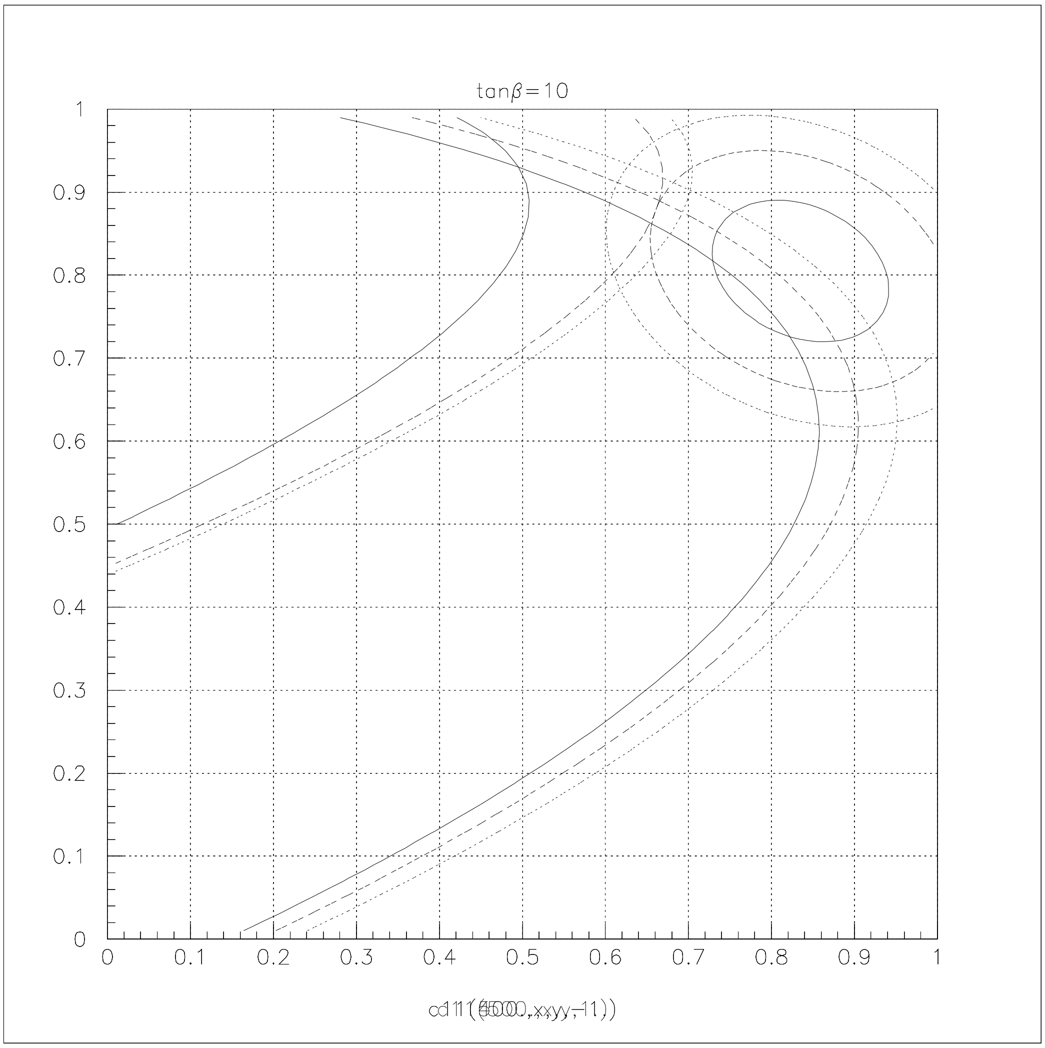
$\Rightarrow$ intervals for mixing angles

$U^2(1,1) = [0.80, 0.88]$ and $V^2(1,1) = [0.90, 0.98]$

In other words ...

Inversion of equations

$$U, V = f(m_1, \sigma_L(11)|_{500}, \sigma_R(11)|_{500}, \sigma_L(11)|_{400})$$



How to get the Susy parameters?

Inversion of $U^2(11)$, $V^2(11)$ leads to:

$$M_2 = [2(m_2^2 - m_W^2) - (m_2^2 - m_1^2)(V_{11}^2 + U_{11}^2)]^{\frac{1}{2}}/\sqrt{2}$$

$$\mu = [2(m_1^2 - m_W^2) + (m_2^2 - m_1^2)(V_{11}^2 + U_{11}^2)]^{\frac{1}{2}}/\sqrt{2}$$

$$\tan\beta = \sqrt{\frac{4m_W^2 + 2(m_2^2 - m_1^2)(V_{11}^2 - U_{11}^2)}{4m_W^2 - 2(m_2^2 - m_1^2)(V_{11}^2 - U_{11}^2)}}$$

$$\cos\Phi_\mu = \frac{(m_2^2-m_1^2)^2(U_{11}^2+V_{11}^2-(U_{11}^2)^2-(V_{11}^2)^2)-2m_W^2(m_2^2+m_1^2-2m_W^2)}{[4m_W^4-(m_2^2-m_1^2)^2(V_{11}^2-U_{11}^2)^2]^{\frac{1}{2}}[(m_2^2+m_1^2-2m_W^2)^2-(m_2^2-m_1^2)^2(V_{11}^2+U_{11}^2-1)^2]^{\frac{1}{2}}}$$

$\Rightarrow m_{\tilde{\chi}_2^\pm}$ is missing, however range is given:

$$\sqrt{s}/2 - m_{\tilde{\chi}_1^\pm} \leq m_{\tilde{\chi}_2^\pm} \leq [m^2_{\tilde{\chi}_1^\pm} + 2m_W^2/|V^2(11) - U^2(11)|]^{\frac{1}{2}}$$

Idea: Help from $e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0$?

Chargino + Neutralino Sector

$\Rightarrow$ Parameter M_1 comes in addition

Input: M_2, μ, $\tan\beta$ as $f(m_{\tilde{\chi}_2^+})$
and M_1

$\Rightarrow$ Contours of $\sigma_{L,R}(\tilde{\chi}_1^0\tilde{\chi}_2^0)$
or/and $\sigma_{L,R}(\tilde{\chi}_2^0\tilde{\chi}_2^0)$ at $\sqrt{s} = 500$ GeV

$\Rightarrow$ fix simultanously M_1 and $m_{\tilde{\chi}_2^\pm}$!

In principle . . .

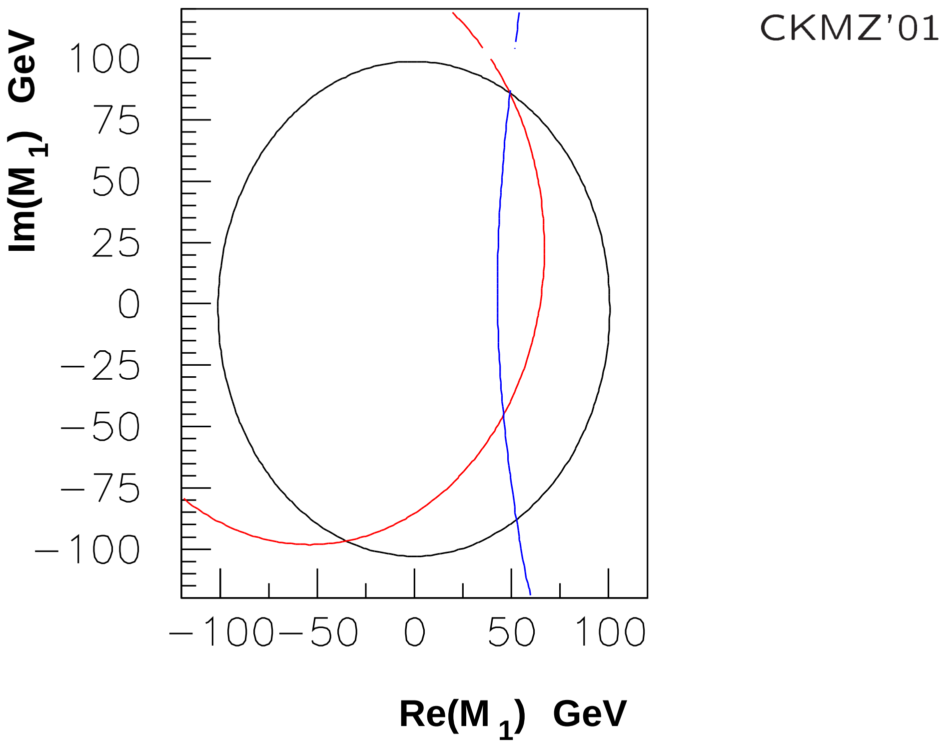
Short reminder

Eigenvalue equation of neutralino mass matrix:

$$(ReM_1)^2 + (ImM_1)^2 + u_i ReM_1 + v_i ImM_1 = w_i,$$
$$u_i, v_i, w_i = f(M_2, \mu, \tan\beta, m_{\tilde{\chi}^0_i}), \quad (i = 1, \ldots, 4)$$

$\rightarrow$ ambiguity with only two neutralino masses:

$\rightarrow$ take $m_{\tilde{\chi}^0_1}$, $m_{\tilde{\chi}^0_2}$, $\sigma_L(12)$



Processes: $e^+e^- \rightarrow \tilde{\chi}^0_1\tilde{\chi}^0_2$ (and $\tilde{\chi}^0_2\tilde{\chi}^0_2$ at $\sqrt{s} = 500$ GeV) with $P_{e^-} = \pm 80\%$, $P_{e^-} = \mp 60\%$

$\rightarrow$ occurs via Z, $\tilde{e}_L$ and $\tilde{e}_R$ exchange

In reality

Assumptions: simulation results for SPS1a

Masses:

$\delta(m_{\tilde{\chi}^0_1}) = 0.2$ GeV (simulation U. Martyn)
$\delta(m_{\tilde{\chi}^0_2}) = 1.4$ GeV (simulation M. Ball/K. Desch)
$\delta(m_{\tilde{e}_R}) = 0.1$ GeV (simulation U. Martyn)
$\delta(m_{\tilde{e}_L}) = 0.5$ GeV (est. corresp. U. Nauenberg)

Rates: simulation study of Ball/Desch
$\to \delta(\sigma(12)) \sim 2.2\%$
$\to$ larger than 1 σ stat. error ($\int \mathcal{L} = 500$ fb^{-1})
therefore: 2.2% (for 12 and 22 assumed)
$+$ error from $\delta(m_{\tilde{e}_{L,R}})$

$e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2$:

$\sigma_L(12) = (168\pm4)$ fb and $\sigma_R(12) = (20.8\pm0.5)$ fb

$e^+e^- \to \tilde{\chi}^0_2\tilde{\chi}^0_2$:

$\sigma_L(22) = (217\pm5)$ fb and $\sigma_R(22) = (6.1\pm0.2)$ fb

Results (still without LHC)

Procedure:

a) vary mixing angles U, V (@$\tilde{\chi}_1^{\pm}$!)
vary M_1 and $m_{\tilde{\chi}_2^{\pm}}$ in interval (@$\tilde{\chi}^0_{12,22}$)

b) take data samples which are consistent with
$m_{\tilde{\chi}_1^{\pm}}$, $\sigma_{L,R}(11)|_{400,500}$
$m_{\tilde{\chi}_1^0}$, $m_{\tilde{\chi}_2^0}$, $\sigma_{L,R}(12)$, $\sigma_{L,R}(22)$

1. $U^2(1,1)$ from sample $\{m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, \sigma_L(12)\}$:
   $\Rightarrow \bar{U}^2(11) = 0.847 \pm 0.003$
2. $V^2(1,1)$ from sample $\{m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, \sigma_L(12)\}$:
   $\Rightarrow \bar{V}^2(11) = 0.953 \pm 0.002$
3. Interval for $m_{\tilde{\chi}_2^{\pm}}$ from $\{m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, \sigma_L(22), \sigma_R(12)\}$:
   $\Rightarrow m_{\tilde{\chi}_2^{\pm}} = [376, 417]$ GeV
   $\Rightarrow m_{\tilde{\chi}_2^{\pm}} = 389.5 \pm 15$ GeV
4. M_1 from sample $\{m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, \sigma_L(12)\}$:
   $\Rightarrow M_1 = 99.28 \pm 0.06$ GeV $\quad (\chi^2 \sim 0.24)$
5. M_2, μ, $\tan\beta$ from equations
   $+$ pure error propagation:
   $\Rightarrow M_2 = 191.27 \pm 3.22$ GeV
   $\Rightarrow \mu = 364.93 \pm 14.41$ GeV
   $\Rightarrow \tan\beta = 13.87 (\pm 68.49)$

Parameter determination at the LC

M_1, M_2, μ rather good:
$\rightarrow$ in particular input for $\tilde{b}$,$\tilde{t}$@LHC! (see M. Nojiri)

Why is the determination of $\tan\beta$ so bad?

$\rightarrow$ all $m_{\tilde{\chi}}$ and couplings $f(\cos 2\beta, \sin 2\beta)$ ...

(Remark:
high $\tan\beta$ from Higgs sector to some extent ...
high $\tan\beta$ from $\tilde{\tau}/\tilde{b}$ sector Boos,Martyn,GMP,Zerwas'03

Two cases: 1. no assumption on A_τ
$\rightarrow$ sensitive if decay $\tilde{\chi}$ higgsino–like
$\rightarrow$ not the case in SPS1a!
2. take $A_\tau = 0$ in $m_\tau(A_\tau - \mu\tan\beta) = \frac{m^2_{t\tilde{a}u_1} - m^2_{\tilde{\tau}_2}}{2} \sin\theta_{\tilde{\tau}}$
with $\delta(m_{\tilde{\tau}_1}) = 0.3$ GeV, $\delta(m_{\tilde{\tau}_2}) = 2$ GeV, $\delta(\cos\theta_{\tilde{\tau}}) = 0.02$
(simulation U. Martyn)
$\rightarrow$ $\mu\tan\beta = 3778.2 \pm 274.8$ GeV)

Hmm, but, is help from LHC for
$\tan\beta$@$\tilde{\chi}$ sector possible?

Any further improvement possible via LHC input?

Now: Results with input from LHC

Procedure: In principle same procedure,
but in addition $m_{\tilde{\chi}^0_4}$ from LHC

$\Rightarrow m_{\tilde{\chi}^0_4} = 387 \pm 5.5$ GeV ('Giacomo's endpoints')

(Remark: heavier $\tilde{\chi}^0_1\tilde{\chi}^0_{3,4}$ at LC possible,
but very small rates ~ 1 fb in SPS1a ...)

1. $U^2(1,1)$ from sample for $\{m_{\tilde{\chi}^0_1}, m_{\tilde{\chi}^0_2}, m_{\tilde{\chi}^0_4}, \sigma_L(12)\}$:
   $\Rightarrow \bar{U}^2(1,1) = 0.835 \pm 0.003$, $\quad (\chi^2 \sim 0.27)$
2. $V^2(1,1)$ from sample for $\{m_{\tilde{\chi}^0_1}, m_{\tilde{\chi}^0_2}, m_{\tilde{\chi}^0_4}, \sigma_R(12)\}$:
   $\Rightarrow \bar{V}^2(11) = 0.948 \pm 0.004$, $\quad (\chi^2 \sim 0.20)$
3. Intervall for $m_{\tilde{\chi}^\pm_2}$ from sample for $\{m_{\tilde{\chi}^0_1}, m_{\tilde{\chi}^0_2}, m_{\tilde{\chi}^0_4}, \sigma_R(12)\}$:
   $\Rightarrow m_{\tilde{\chi}^\pm_2} = [376, 382]$GeV
   $\Rightarrow m_{\tilde{\chi}^\pm_2} = 378.8 \pm 1.4$ GeV

   Data from $\tilde{\chi}^0_2\tilde{\chi}^0_2$ not needed any more but could give consistency checks!

Parameter determination at LHC+LC

4. M_1 from the sample $\{m_{\tilde{\chi}^0_1}, m_{\tilde{\chi}^0_2}, m_{\tilde{\chi}^0_4}, \sigma_L(12)\}$:

   $\Rightarrow M_1 = 99.22 \pm 0.15$ GeV $\qquad (\chi^2 \sim 0.16)$

5. M_2, μ, $\tan\beta$ from equations
   $+$ pure error propagation:

   $\Rightarrow M_2 = 191.61 \pm 1.14$ GeV
   $\Rightarrow \mu = 353.26 \pm 1.42$ GeV
   $\Rightarrow \tan\beta = 10.79 (\pm 14.27)$

All central values better, even for high $\tan\beta$!

Original:

M_1	M_2	μ	$\tan\beta$
99.13	192.7	352.4	10

And now? $\mu \tan\beta$ for stop/sbottom studies...

- Parameter determination at a LC precisely worked out without assuming a breaking scheme
  $\rightarrow$ however, high $\tan\beta$ might be tricky

- Hand–in–hand procedure with LHC$\leftrightarrow$ LC

  LC $\left(\overset{\text{light masses}}{\longrightarrow}\right)$ LHC

  LHC $\left(\overset{\text{heavier}\tilde{\chi}^0_j}{\longrightarrow}\right)$ LC

  LC$\left(\overset{M_2, M_1, \mu, \tan\beta}{\longrightarrow}\right)$ LHC ...

$\rightarrow$ detailed analysis seems to be very promising

$\rightarrow$ precise Susy parameter determination at LHC+LC
without assuming a Susy breaking scheme