Polarisation in Sfermions Decays: Determination of $\tan\beta$ and Trilinear Couplings

Gudrid Moortgat-Pick

LHC/LC meeting
CERN, February 14, 2002

1. Summary: parameter determination in the MSSM

2. The crucial case of high tan β

3. Use of τ and t polarisation
   $\rightarrow$ Measuring tan β and A_f

4. Results and outlook

$\rightarrow$ S.Y. Choi, J. Kalinowski, GMP, P.M. Zerwas,'01, '02

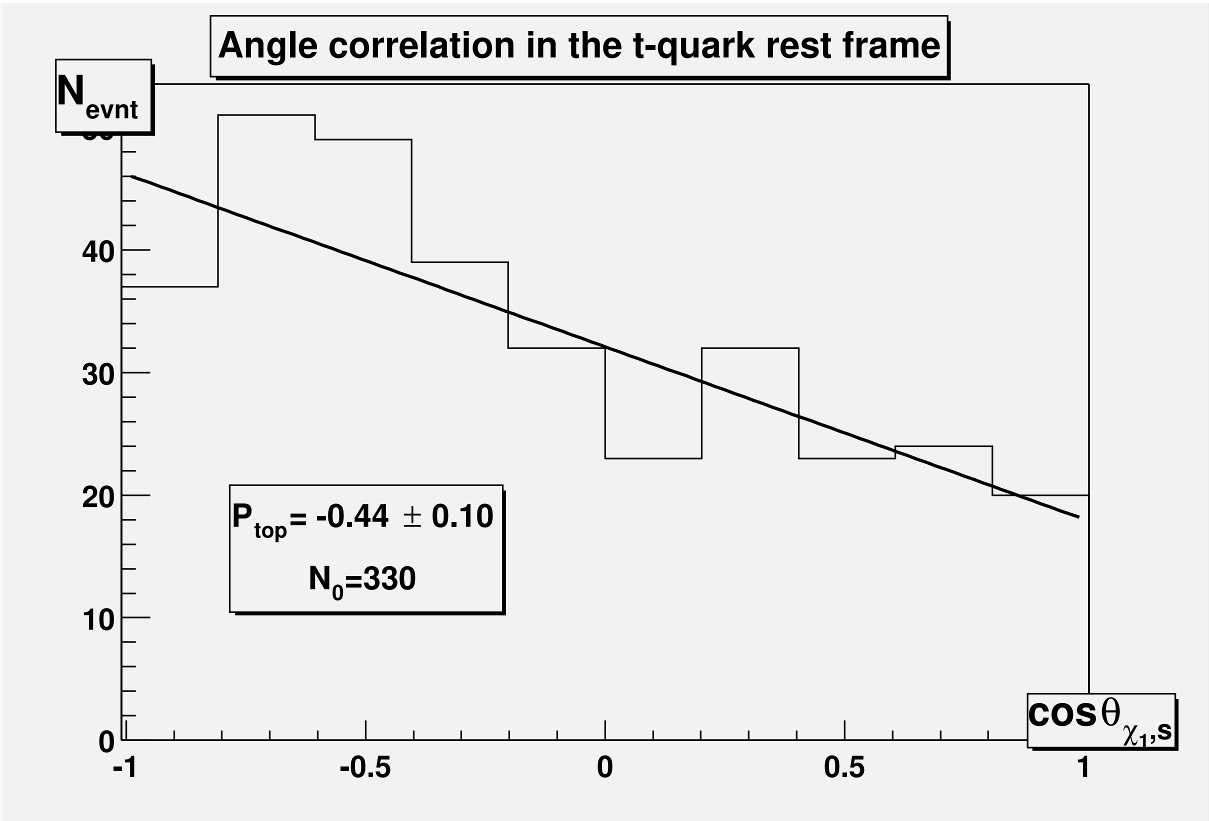
$\rightarrow$ E. Boos, GMP, U. Martyn, M. Sachwitz, A. Vologdin '02

$\rightarrow$ E. Boos, GMP, U. Martyn, M. Sachwitz, A. Sherstnev and P.M. Zerwas '03

Measurement of the $P(\tilde{b}_1 \to t)$

Process: $e^+e^- \to \tilde{b}_i\tilde{b}_1 \to \tilde{b}_i\tilde{\chi}_1^+ t \to \tilde{b}_i\tilde{\chi}_1^+ bc\bar{s}$

$\Rightarrow$ determine t polarisation via angle correlations!

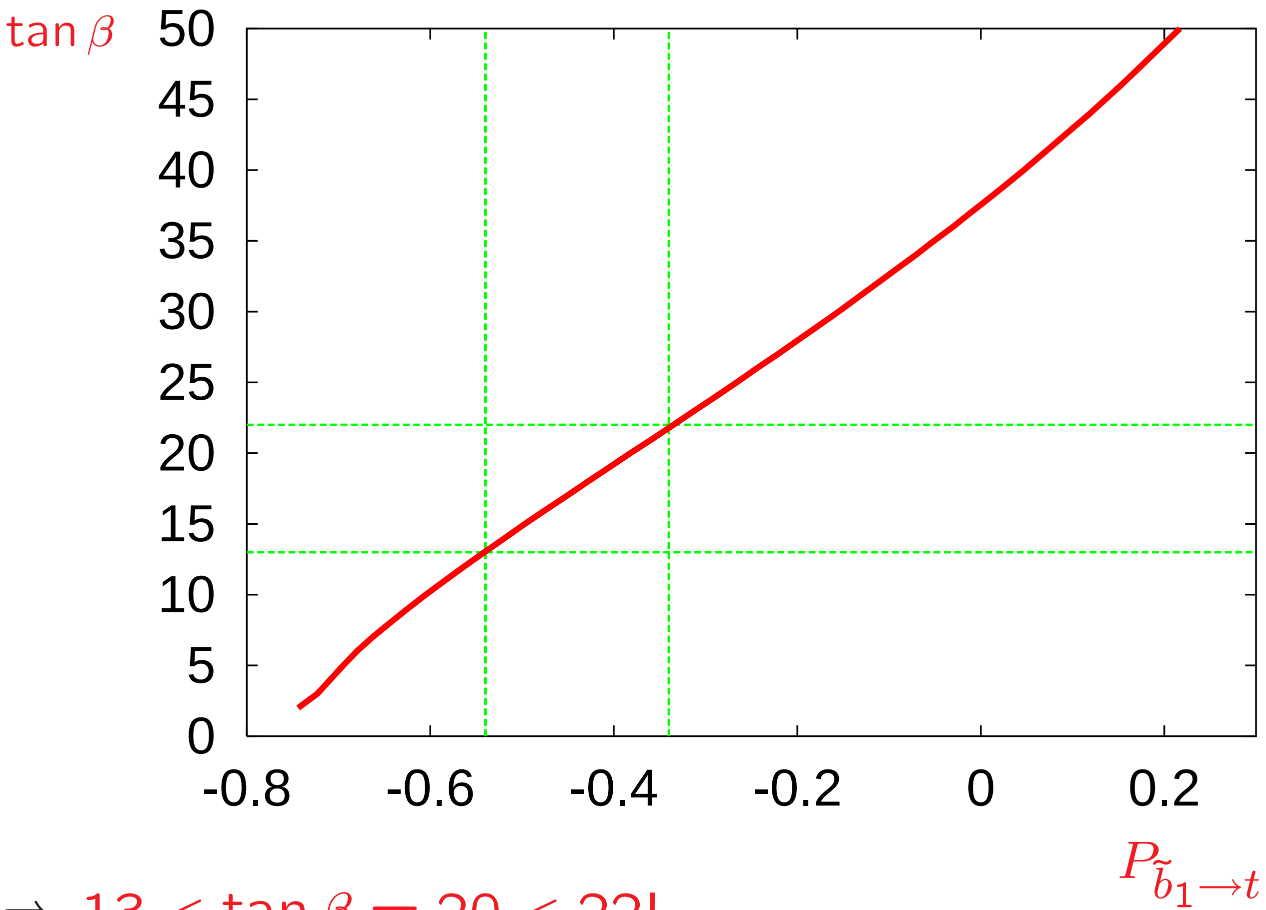


$\Rightarrow$ Determination of the polarization:
$P(\tilde{b}_1 \to t) = -44 \pm 10\%$!

$\Rightarrow$ Inversion of $P(\tilde{b}_1 \to t)$ leads to $\tan\beta$?

Inversion of $P(\tilde{b}_1 \to t)$

Polarization measured $P(\tilde{b}_1 \to t) = -44 \pm 10\%$:



$\Rightarrow$ $13 < \tan\beta = 20 < 22$!

High accuracy again for high $\tan\beta$ possible!

What about A_b?

$\Rightarrow$ Same procedure as in the A_τ case,
but higher precision: $\delta(A_b)/A_b \sim 30\%$

$$A_b = \frac{1}{2m_b}(m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2})\sin(2\theta_b) + \mu\tan\beta$$

Conclusions and Outlook

- In principle: M_2, μ, $\tan\beta$, M_1 can be determined via only the light system! CKMZ'02

- $\tilde{\chi}^0_i$, $\tilde{\chi}^\pm_j$ sector weak dependent on $\tan\beta$:
  $f(1 + const. \times \frac{1}{\tan\beta})$
  $\rightarrow$ not suitable for determining $\tan\beta > 10$

- $\tan\beta$ and A_τ from the $\tilde{\tau}$–$\tilde{\chi}^0_i$ sector
  $\rightarrow$ without assumption on SUSY breaking scheme!
  $\rightarrow$ only input from $\tilde{\chi}^0_{1,2}$, $\tilde{\chi}^\pm_1$ needed

- 'Same' method also applicable for
  A_b and A_t: $\tilde{b}$,$\tilde{t}$ $-\tilde{\chi}^\pm_i$ sector

- What about high $\tilde{q}$ and $\tilde{\tau}$ masses?
  $\rightarrow$ Input from LHC $\rightarrow$ see talk by Lali!

Analysis of the $\tilde{b}$ sector

Mixing matrix of $\tilde{b}_{1,2}$ ... as usual

$$\mathcal{M}_{\tilde{b}} = \begin{pmatrix} M_Q^2 + m_q^2 + m_z^2 \cos(2\beta)(I_{3L}^q - e_q \sin^2\theta_W) & m_b(A_b - \mu \tan\beta) \\ m_b(A_b - \mu \tan\beta) & M_{\tilde{D}}^2 + m_b^2 + e_q m_z^2 \cos(2\beta) \sin^2\theta_W \end{pmatrix}$$

⇒ In principle 'same' $\tan\beta - A_b$ strategy as before

However, since $P(b)$ is not an prospective observable
study the process: $\tilde{b}_i \to \tilde{\chi}_j^{\pm} t$
$\to$ $P(t)$ needed!

$$P^{b,\pm}_{\tilde{b}_1 \to t} = \frac{[(y_h^-)^2 - 1 - (y_h^+)^2 + \cos 2\theta_{\tilde{b}}((y_h^-)^2 - 1 + (y_h^+)^2) + \sin 2\theta_{\tilde{b}} y_h^+] \times f_1}{(y_h^-)^2 + 1 + (y_h^+)^2 + \cos 2\theta_{\tilde{b}}[(y_h^-)^2 + 1 - (y_h^+)^2] - \sin 2\theta_{\tilde{b}} y_h^+ + (1 + \cos 2\theta_{\tilde{q}} - \sin 2\theta_{\tilde{q}} y_h^+) y_h^- \times f_2}$$

$$P^{b,\pm}_{\tilde{b}_2 \to t} = \frac{[(y_h^-)^2 - 1 - (y_h^+)^2 - \cos 2\theta_{\tilde{b}}((y_h^-)^2 - 1 + (y_h^+)^2) - \sin 2\theta_{\tilde{b}} y_h^+] \times f_1}{(y_h^-)^2 + 1 + (y_h^+)^2 - \cos 2\theta_{\tilde{b}}[(y_h^-)^2 + 1 - (y_h^+)^2] + \sin 2\theta_{\tilde{b}} y_h^+ + (1 + \cos 2\theta_{\tilde{q}} + \sin 2\theta_{\tilde{q}} y_h^+) y_h^- \times f_2}$$

with $y_h^- = Y_b U_{j2}/U_{j1}$, $Y_b \sim 1/\cos\beta$ and $y_h^+ = Y_t V_{j2}/U_{j1}$, $Y_t \sim 1/\sin\beta$

⇒ higgsino admixture important!!!

Analysis of the $\tilde{t}$ sector

Mixing matrix of $\tilde{t}_{1,2}$ ... as usual

$$\mathcal{M}_{\tilde{t}} = \begin{pmatrix} M_Q^2 + m_q^2 + m_z^2 \cos(2\beta)(I_{3L}^q - e_q \sin^2\theta_W) & m_t(A_t - \mu \cot\beta) \\ m_b(A_t - \mu \cot\beta) & M_{\tilde{U}}^2 + m_t^2 + e_q m_z^2 \cos(2\beta) \sin^2\theta_W \end{pmatrix}$$

$\Rightarrow$ Again $\tan\beta - A_t$ strategy applicable? $\Rightarrow$ $\cot\beta$ dependence!
not so nice for high $\tan\beta$ ...

Since $P(\tilde{t} \to b\tilde{\chi}^{\pm})$ does not lead to an prospective observable
study the process: $\tilde{t}_i \to \tilde{\chi}_j^0 t$
But $Y_t \sim 1/\sin\beta$ also not very sensitive to high $\tan\beta$ ...

$\Rightarrow$ Use $\tan\beta$ from e.g. $\tilde{b}$ sector and get A_t via mixing angle
$\Rightarrow$ Use $P(t)$ for tests of mixing, Yukawa coupling etc.

Unveiling the MSSM

Problem: $m_p \neq m_{\tilde{p}}$

$\Rightarrow$ SUSY has to be broken:
soft breaking terms lead to $\sim$105 parameters!

$\Rightarrow$ Mixing of eigenstates:

sfermions: e.g. $\tilde{t}_{L,R} \stackrel{mixing}{\longrightarrow} \tilde{t}_{1,2}$

charginos: $\tilde{W}^{\pm}$, $\tilde{H}^{\pm}_{1,2} \stackrel{mixing}{\longrightarrow} \tilde{\chi}^{\pm}_{1,2}$

neutralinos: $\tilde{W}^{3}$, $\tilde{B}^{0}$, $\tilde{H}^{0}_{1,2} \stackrel{mixing}{\longrightarrow} \tilde{\chi}^{0}_{1,2,3,4}$

$\Rightarrow$ Schemes: mSUGRA, AMSB, GMSB, ...

$\uparrow$ $\uparrow$ $\uparrow$

4 parameters

5 parameters 5 parameters

$\Rightarrow$ Search for Step–by–step Strategies together with Hadron Colliders?!

Step–by–step Strategy

General MSSM parameter from $\tilde{\chi}^{\pm}$, $\tilde{\chi}^{0}$, $\tilde{\tau}$

a) charginos: M_2, μ, Φ_μ, $\tan\beta$
b) neutralinos: $+$ M_1, Φ_1

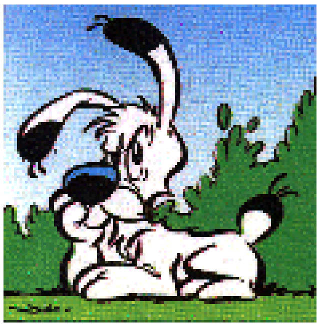
Remember:
only light system, $\tilde{\chi}^{\pm}_1$, $\tilde{\chi}^{0}_1$, $\tilde{\chi}^{0}_2$ would be sufficient!
$\longrightarrow$ and even $m_{\tilde{\chi}^{\pm}_2}$ predictable!...

However, all $m_{\tilde{\chi}_i}$ and couplings $= f(\cos 2\beta, \sin 2\beta)$

High $\tan\beta$: $\sin 2\beta \to 2\frac{1}{\tan\beta}$ and $\cos 2\beta \to -1$

$\Rightarrow$ weak dependend on high $\tan\beta$

No precise determination possible via $\tilde{\chi}_i$ sector!

What to do? 

Possible channels for determining high $\tan\beta$

Higgs sector, e.g.:

- J. Gunion, T. Han, J. Jiang, S. Mrenna, A. Sopczak '01
  $e^+e^- \to HZ \to \bar{b}b\bar{b}b,\ e^+e^- \to b\bar{b}h,\ b\bar{b}A \to \bar{b}b\bar{b}b$
  rates and width $\Rightarrow \Delta\tan\beta > 10\%$ (small m_A)

- V. Barger, T. Han, J. Jiang '00:
  $e^+e^- \to Ht\bar{t},\ Hb\bar{b},\ At\bar{t},\ A, b\bar{b}$
  J.L. Feng, T. Moroi '97:
  $e^+e^- \to Zh,\ AH,\ t\bar{b}H^-,\ \bar{t}bH^+$
  rates and BR

$\tan\beta$	BHJ	FM
3	2.4–3.6	< 5.2
5	4.3–6.3	3–6
10	6.2–12.7	> 6.5
20	14–32	7.5–90
30	18–80	> 8

$\Rightarrow$ high $\tan\beta$ is a problem ...

$\Rightarrow$ Help from $\tilde{\tau}$, $\tilde{t}$, $\tilde{b}$ sector possible?

What has already been done?

- Masses, rates (with pol. beams), BR's, e.g.:
  A. Bartl, H. Eberl, S. Kraml, W. Majerotto, W.Porod '97, '00
  A. Bartl, H. Eberl, S. Kraml, W. Majerotto, W.Porod, A. Sopczak '97
  H. Eberl, S. Kraml, W. Majerotto '99

- Use of τ Polarization, e.g.:
  M. Nojiri '94
  M. Nojiri, K. Fujii, T. Tsukamoto '96
  M. Guchait, D. Roy '01,'02

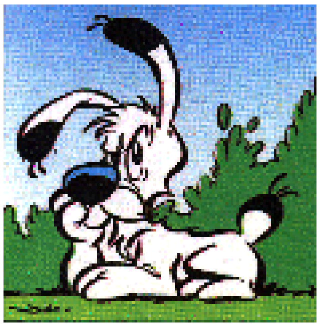
- CP–violation, e.g.:
  S. Choi, M. Drees, B. Gaissmaier, J. Lee (MC) '01
  A. Bartl, K. Hidaka, T. Kernreiter, W. Porod '02

What are we doing?

'Liaison' between $\tilde{\tau}_i \leftrightarrow \tilde{\chi}_j$ sector:

$\Rightarrow$ determination of (high) $\tan\beta$ and A_τ
from $\tilde{\tau}$ sector

But how?



Analysis of the $\tilde{\tau}$ sector

Mixing matrix of $\tilde{\tau}_{1,2}$:

$$\mathcal{M}_{\tilde{\tau}} = \begin{pmatrix} M_L^2 + m_\tau^2 + L\cos(2\beta)m_z^2 & m_\tau(A_\tau - \mu\tan\beta) \\ m_\tau(A_\tau - \mu\tan\beta) & M_E^2 + m_\tau^2 - R\cos(2\beta)m_z^2 \end{pmatrix}$$

$$=: \begin{pmatrix} M_{LL}^2 & M_{LR}^2 \\ M_{LR}^2 & M_{RR}^2 \end{pmatrix}$$

- off–diagonal terms depend strongly on $\tan\beta$ but also on A_τ!
- diagonalization via the mixing angle $\theta_{\tilde{\tau}}$: $\tan(2\theta_{\tilde{\tau}}) = \frac{-2M_{LR}^2}{M_{RR}^2 - M_{LL}^2}$

Masses:

$$m_{\tilde{\tau}_{1,2}}^2 = \frac{1}{2}[m_{LL}^2 + m_{RR}^2 \mp \sqrt{(m_{LL}^2 - m_{RR}^2)^2 + 4(m_{LR}^2)^2}]$$

Two cases:

1. if $\cos(2\theta_{\tilde{\tau}}) = -\frac{1}{\sqrt{1+\xi^2}} < 0$ then $m_{LL}^2 > m_{RR}^2$
2. if $\cos(2\theta_{\tilde{\tau}}) = +\frac{1}{\sqrt{1+\xi^2}} > 0$ then $m_{LL}^2 < m_{RR}^2$

Analysis of the $\tilde{\tau}$ sector

Mixing:

$$\begin{pmatrix}\tilde{\tau}_1\\ \tilde{\tau}_2\end{pmatrix} = \begin{pmatrix}\cos\theta_{\tilde{\tau}} & \sin\theta_{\tilde{\tau}}\\ -\sin\theta_{\tilde{\tau}} & \cos\theta_{\tilde{\tau}}\end{pmatrix}\begin{pmatrix}\tilde{\tau}_L\\ \tilde{\tau}_R\end{pmatrix}$$

$$\theta_{\tilde{\tau}} = [0, \pi]$$

Rates:

$$\sigma(\tilde{\tau}_i\tilde{\tau}_i) = f(\cos^2 2\theta_{\tilde{\tau}}, \cos 2\theta_{\tilde{\tau}})$$
$$\sigma(\tilde{\tau}_1\tilde{\tau}_2) = f(\sin^2 2\theta_{\tilde{\tau}}, \sin 2\theta_{\tilde{\tau}})$$

In principle:

$\Rightarrow$ $\cos 2\theta_{\tilde{\tau}}$ via light system $\sigma(\tilde{\tau}_1\tilde{\tau}_1)$ with 2-fold ambiguity

Resolving ambiguity via 2nd measurement
$\longrightarrow$ use of beam polarization!

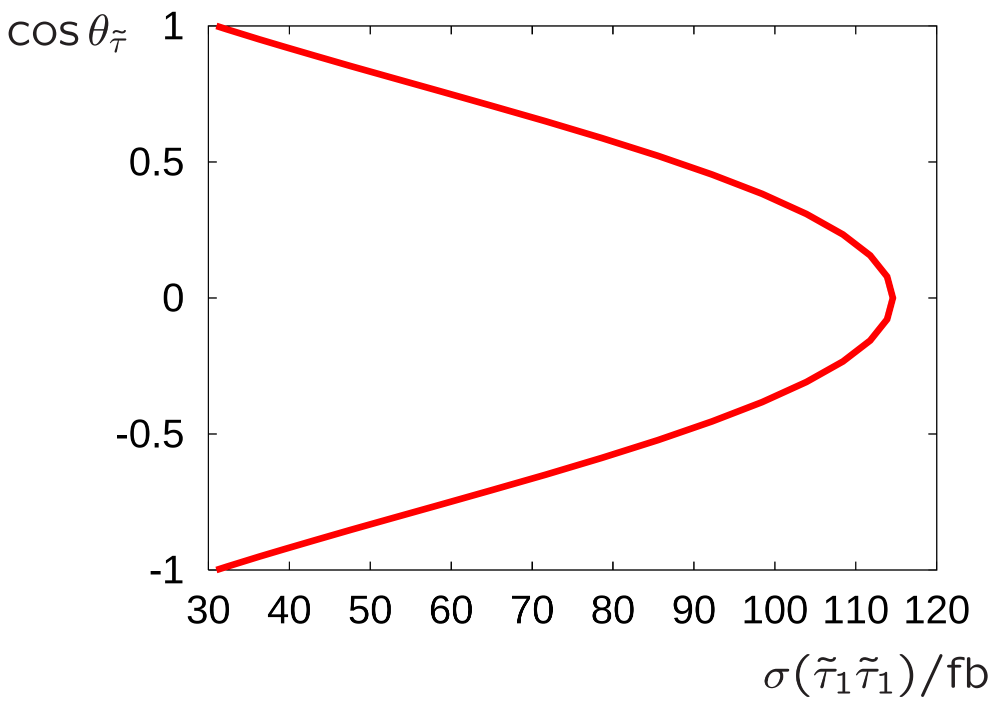
Parameters from the $\tilde{\tau}$ sector

Reference scenario:

$m_{\tilde{\tau}_1}$	$m_{\tilde{\tau}_2}$	M_1	M_2	μ	$\tan\beta$	A_τ
151 GeV	305 GeV	99 GeV	193 GeV	140	20	-254.2

Expected accuracy for masses (see TDR):
$\delta(m_{\tilde{\tau}_{1,2}}) \sim 0(\%)$

Process: $e^+e^- \to \tilde{\tau}_1^+\tilde{\tau}_1^-$
(with $P(e^-) = +80\%, P(e^+) = -60\%$)



$\Rightarrow$ $\sigma(\tilde{\tau}_1\tilde{\tau}_1) = 109$ fb

$\Rightarrow$ $\cos\theta_{\tilde{\tau}} = 0.08$ (with 2-fold ambiguity in $\pm\cos\theta_{\tilde{\tau}}$)

Tau polarization $P(\tilde{\tilde{\tau}}_i \to \tau)$

Process: $\tilde{\tau}_i \to \tau\tilde{\chi}^0_j$

Lagrangian: $\mathcal{L} = \sum \tilde{\tau}_i \bar{\tau}(P_L a^R_{ij} + P_R a^L_{ij})\tilde{\chi}^0_j$

Polarization with complete $\tilde{\chi}^0_i$ mixing:

$$P_{\tilde{\tau}_1\to\tau} = \frac{(4-x_W^2)-(4+x_W^2-2w_h^2)\cos 2\theta_{\tilde{\tau}}+2(2+x_W)w_h \sin 2\theta_{\tilde{\tau}}}{(4+x_W^2+2w_h^2)-(4-x_W^2)\cos 2\theta_{\tilde{\tau}}+2(2-x_W)w_h \sin 2\theta_{\tilde{\tau}}}$$

$$P_{\tilde{\tau}_2\to\tau} = \frac{(4-x_W^2)+(4+x_W^2-2w_h^2)\cos 2\theta_{\tilde{\tau}}-2(2+x_W)w_h \sin 2\theta_{\tilde{\tau}}}{(4+x_W^2+2w_h^2)+(4-x_W^2)\cos 2\theta_{\tilde{\tau}}-2(2-x_W)w_h \sin 2\theta_{\tilde{\tau}}}$$

- x_W: gaugino components from $\tilde{\chi}^0_i$
- $w_h =: \frac{1}{c\beta}\frac{m_\tau}{m_W}\tilde{w}_h$
  $\tilde{w}_h$: higgsino components from $\tilde{\chi}^0_i$
  crucial contribution!

tan β dependence of $P(\tilde{\tau}_i \to \tau)$:

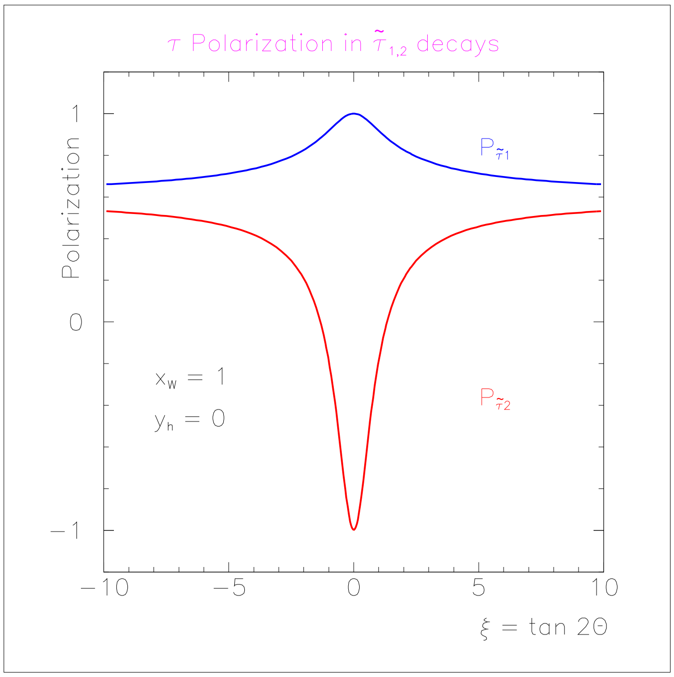
- strong dependence via $\theta_{\tilde{\tau}}$
- weak dependence via neutralino contributions x_W, $\tilde{w}_h$
- strong dependence via $\tau \leftrightarrow \tilde{\chi}^0_i$ liaison: w_h

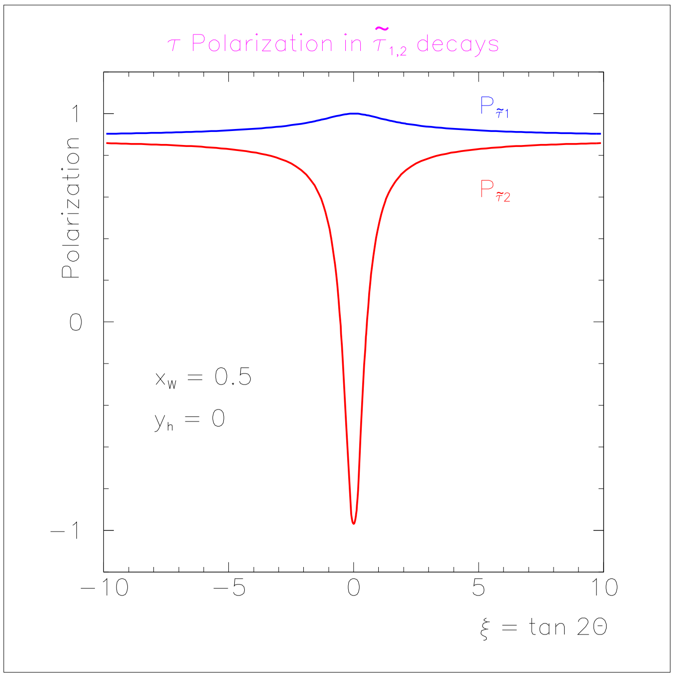
Discussion of τ Polarization

What happens of $\tilde{\chi}_i^0 \sim$ gaugino–like?

$\longrightarrow$ no w_h

$\longrightarrow$ $\tan\beta$ dependence only via x_W and $\cos 2\theta_{\tilde{\tau}}$!

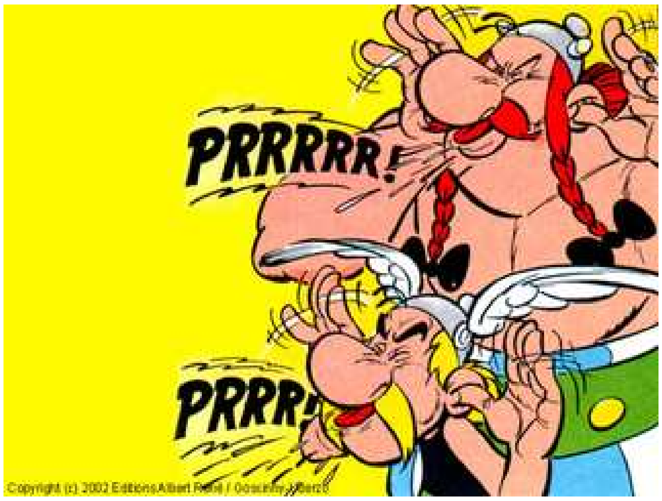




$\Rightarrow$ If $\tilde{w}_h = 0$ and fixed $\theta_{\tilde{\tau}}$ then $P(\tilde{\tau}_i \to \tau)$ only weak dependent on $\tan\beta$!

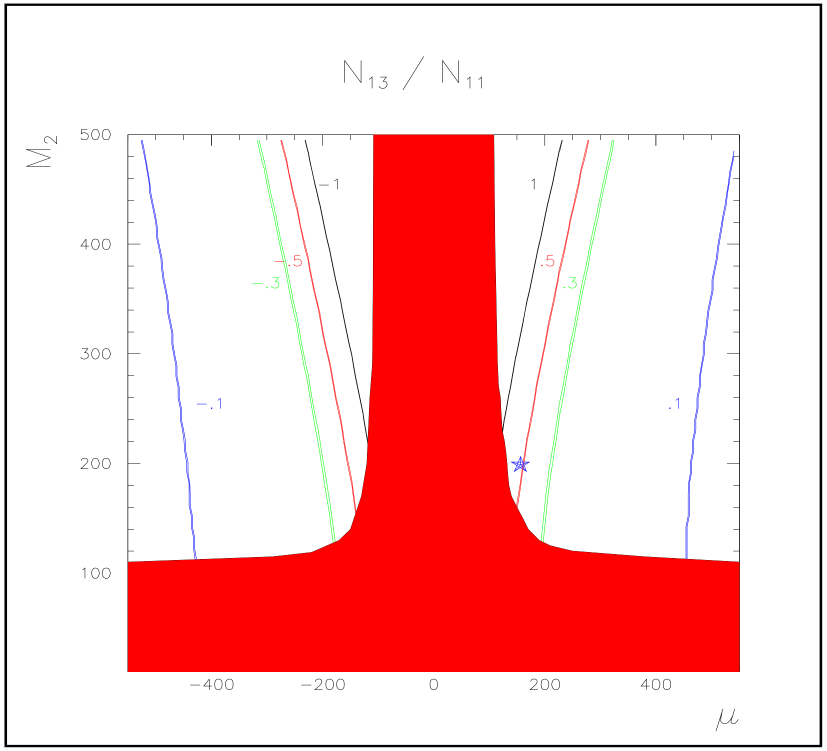
$\Rightarrow$ valid for both $P(\tilde{\tau}_1 \to \tau)$ and $P(\tilde{\tau}_2 \to \tau)$!

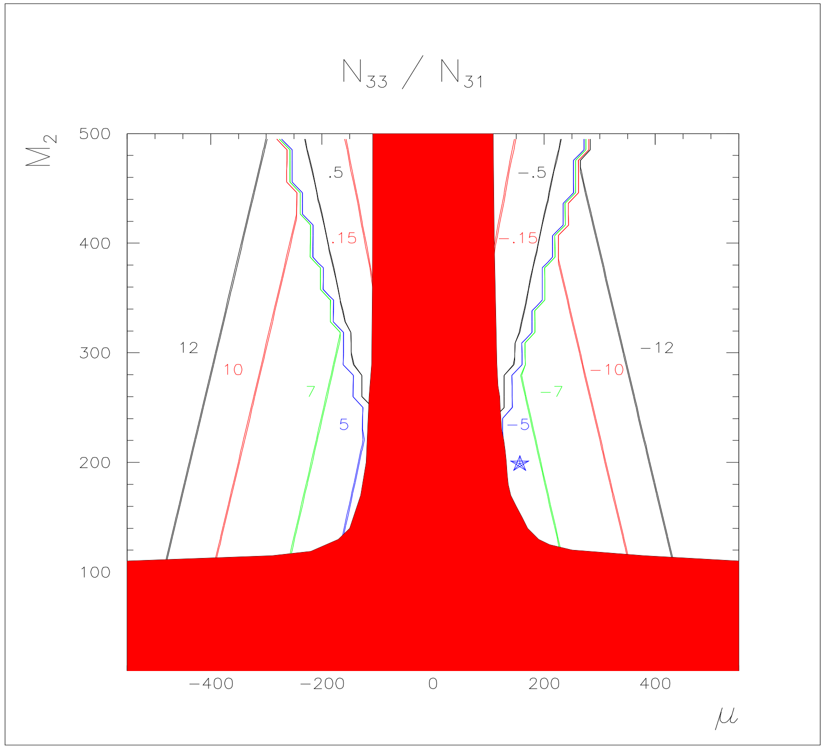
no 'complementarity'



$\Rightarrow$ $\tan\beta$ only via $\cos\theta_{\tilde{\tau}}$: only $(A_\tau - \mu \tan\beta)$ derivable

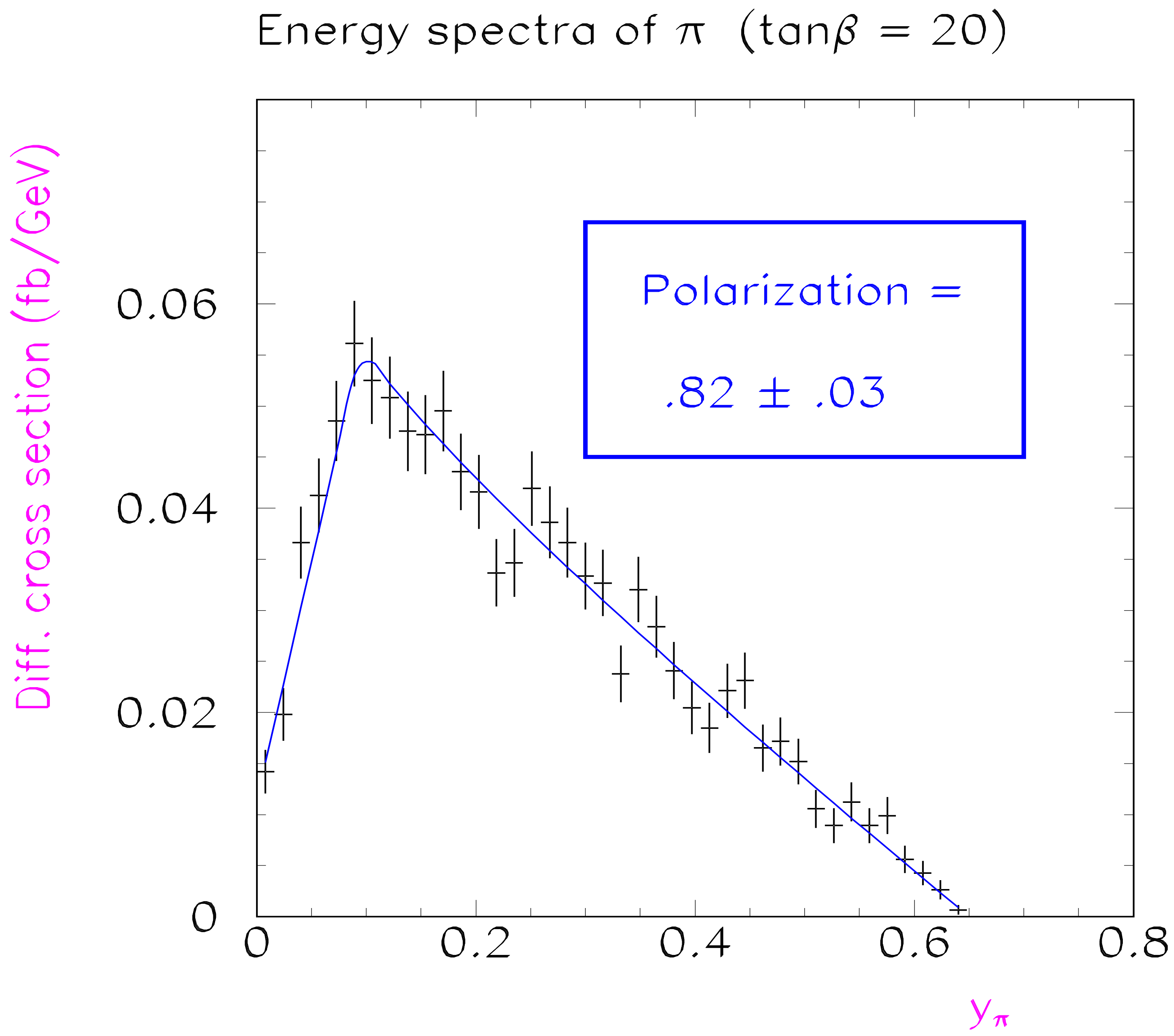
Discussion of $\tilde{\tau}$ Polarization





Measurement of the $P(\tilde{\tilde{\tau}}_1 \to \tau)$

Process: $e^+e^- \to \tilde{\tau}_1^+\tilde{\tau}_1^- \to \tilde{\tau}_1^+\tau\tilde{\chi}_1^0 \to \tilde{\tau}_1^+\nu_\tau\pi^-\tilde{\chi}_1^0$

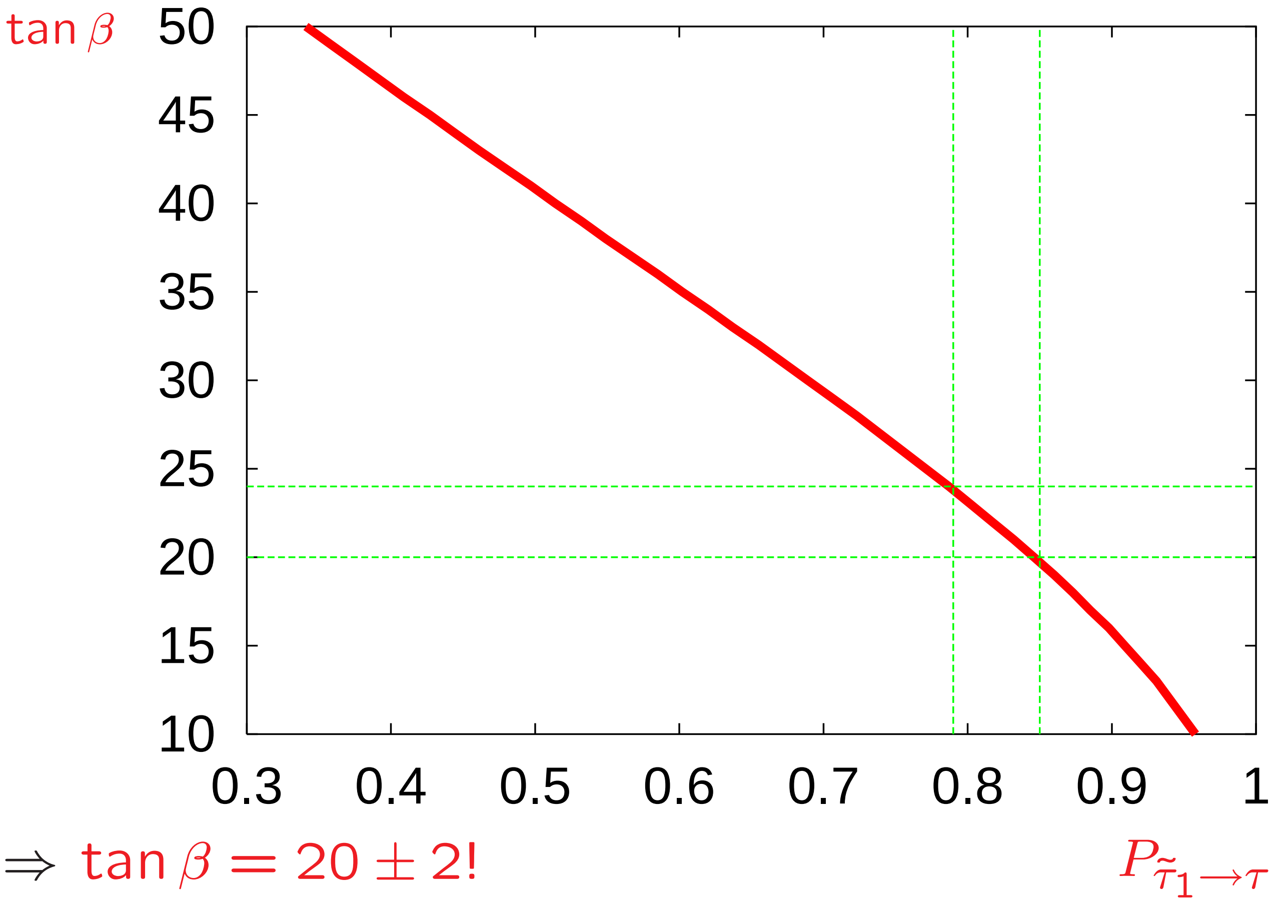


$\Rightarrow$ Precise determination of the polarization:
$P(\tilde{\tau}_1 \to \tau) = 82 \pm 3\%$!

Next step: Inversion of $P(\tilde{\tau}_1 \to \tau)$ leads to tan β!

Inversion of $P(\tilde{\tau}_1 \to \tau)$

Polarization measured $P(\tilde{\tau}_1 \to \tau) = 82 \pm 3\%$:

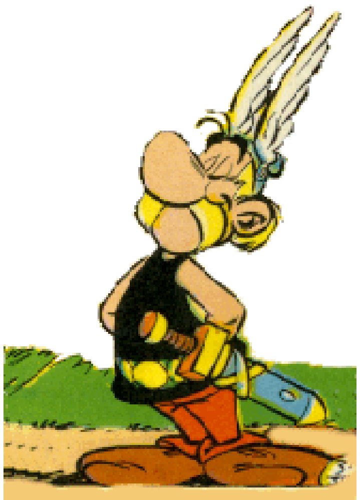


$\Rightarrow$ $\tan\beta = 20 \pm 2$!

High accuracy even for high $\tan\beta$ possible!

What's about the $\tilde{\chi}^0_1$ contributions?

$\longrightarrow$ can be decoupled!



Determination of A_τ

Final step:

Since $\tan\beta = 20 \pm 2$ already known

$\Rightarrow$ use m^2_{LR} to determine A_τ

$A_\tau = \frac{1}{2m_\tau}(m^2_{\tilde{\tau}_1} - m^2_{\tilde{\tau}_2})\sin(2\theta_\tau) + \mu\tan\beta$

$\Rightarrow$ determine $m_{\tilde{\tau}_2} = 305$ GeV with high acc.

$\Rightarrow$ include errors for $m_{\tilde{\tau}_{1,2}}$ and $\tan\beta$

$\Rightarrow$ but $\delta(A_\tau)$ difficult ...