The Approach to the Decoupling Limit

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This talk is based on work that appears in:

1. J.F. Gunion and H.E. Haber, "The CP-conserving two-Higgs-doublet model: the approach to the decoupling limit," hep-ph/0207010, Phys. Rev. D in press.

2. S. Davidson, J.F. Gunion and H.E. Haber, "Basis-independent methods for analyzing the two-Higgs-doublet model," in preparation.

3. G. Branco, L. Lavoura and J.P. Silva, *CP Violation* (Oxford University Press, Oxford, England, 1999), chapters 22 and 23.

Outline

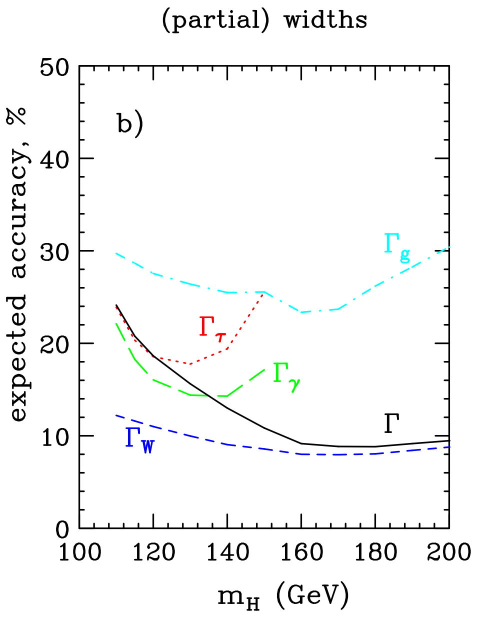
- Motivation
  - Theoretical underpinning of precision Higgs studies
  - The Decoupling Limit defined
- The general Two-Higgs-Doublet Model (2HDM)
  - Basis-independent techniques
  - Approaching the decoupling limit
  - A survey of Higgs couplings
- The Higgs-fermion interaction
  - Where did $\tan\beta$ go?
  - Many $\tan\beta$-like parameters?
  - The canonical example: the MSSM Higgs sector
- Lessons for future Higgs studies

Motivation

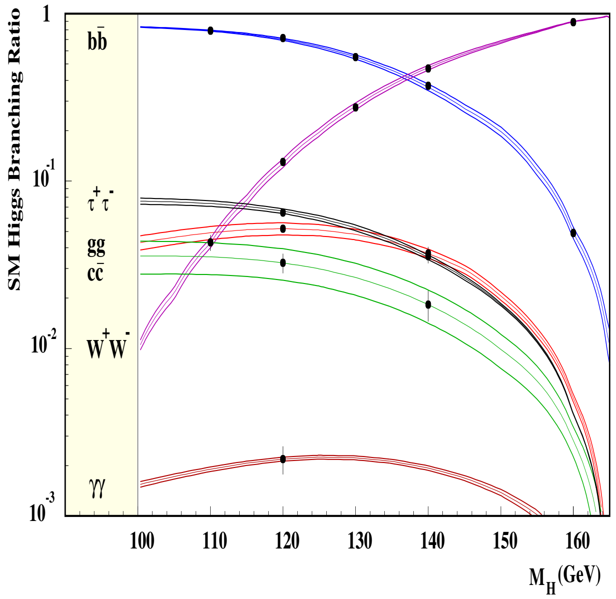
Assume that the Higgs boson is discovered at the LHC and is observed to be consistent with the SM Higgs boson.

LHC expectations: some Higgs couplings can be measured with an accuracy in the range 5–15%

Precision Higgs measurements at the LC in some cases can achieve precisions of a few percent.



from Zeppenfeld *et al.*



from Battaglia and Desch

Precision Higgs physics at the LC

- Results from the American Linear Collider Collaboration

	$m_{h_{\rm SM}} = 120$ GeV		$m_{h_{\rm SM}} = 140$ GeV	
	BR	δBR/BR	BR	δBR/BR
$h_{\rm SM} \to b\bar{b}$	$(69 \pm 2.0)\%$	2.9%	$(34 \pm 1.3)\%$	3.8%
$h_{\rm SM} \to WW^*$	$(14 \pm 1.3)\%$	10%	$(51 \pm 1.5)\%$	3.0%
$h_{\rm SM} \to c\bar{c}$	$(2.8 \pm 1.1)\%$	39%	$(1.4 \pm 0.64)\%$	44%
$h_{\rm SM} \to gg$	$(5.2 \pm 0.93)\%$	18%	$(3.5 \pm 0.79)\%$	23%
$h_{\rm SM} \to \tau^+\tau^-$	$(7.1 \pm 0.56)\%$	8.0%	$(3.6 \pm 0.38)\%$	10%

Predicted branching ratio precisions in the LCD Large detector and typical vertex detector configuration for 500 fb^{-1} and $\sqrt{s} = 500$ GeV. [Brau, Iwasaki and Potter]

- Results from the European LC studies

Higgs coupling	δBR/BR	$\delta g/g$
hWW	5.1%	1.2%
hZZ	—	1.2%
$ht\bar{t}$	—	2.2%
$hb\bar{b}$	2.4%	2.1%
$hc\bar{c}$	8.3%	3.1%
$h\tau\tau$	5.0%	3.2%
$h\mu\mu$	$\sim 30\%$	$\sim 15\%$
hgg	5.5%	
$h\gamma\gamma$	16%	
hhh	—	$\sim 20\%$

Expected fractional uncertainties for LC measurements of Higgs branching ratios [BR($h \to XX$)] and couplings [g_{hXX}], for various choices of final state XX, assuming $m_h = 120$ GeV. In all but four cases, the analysis is based on 500 fb^{-1} of data at $\sqrt{s} = 500$ GeV. Results for $h\gamma\gamma$, $h\mu\mu$ and hhh are based on 1 ab^{-1} of data at $\sqrt{s} = 500$ GeV (for $\gamma\gamma$ and hh) and $\sqrt{s} = 800$ GeV (for tt and $\mu\mu$), respectively. [Battaglia, Boos, DeRoeck, Desch, and others]

Question: If a Higgs boson consistent with the SM is observed, what will we have learned?

Answer: Let Λ be the scale of EWSB dynamics. (Assume $\Lambda \gg v = 246$ GeV.) Then below Λ, the effective theory is the SM with one Higgs doublet. This is the decoupling limit. Above Λ, new physics associated with EWSB dynamics will be revealed.

But, the low-energy effective theory exhibits corrections of $\mathcal{O}(v^2/\Lambda^2)$ that can be probed with precision measurements.

Thus, if the LHC has discovered a SM-like Higgs boson, use the LHC to search for EWSB dynamics and the LC to probe the $\mathcal{O}(v^2/\Lambda^2)$ effects of the effective low-energy theory.

examples from theory:

- Little Higgs Models
- The MSSM Higgs sector, with $m_A \gg m_Z$
- The NMSSM Higgs sector
- ......

The General Two-Higgs-Doublet Model

Why study the most general case? For example, the MSSM is a constrained 2HDM. True, but only at tree-level. At one-loop, due to SUSY-breaking, all possible 2HDM interactions allowed by gauge invariance are generated.

Aside: at one-loop, one must also include the effects of, *e.g.*, $d = 6$ effective operators, whose effects are suppressed by $\mathcal{O}(v^2/\Lambda^2)$ as well. However, in many cases, the radiatively-generated terms of the Higgs potential and Higgs-fermion Yukawa couplings (some of which are absent in the SUSY limit) are the leading effects contributing to the deviation from the decoupling limit.

The general 2HDM consists of two identical (hypercharge-one) scalar doublets Φ_1 and Φ_2. One can always redefine the basis, so the parameter $\tan\beta \equiv v_2/v_1$ is not meaningful!

To determine the physical quantities, one must develop basis-independent techniques.

Consider the 2HDM potential:

$$\mathcal{V} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \tfrac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2$$

$$+ \tfrac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)$$

$$+ \left\{ \tfrac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \left[\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)\right] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\}$$

Assume CP conservation (for simplicity). The vevs are $\langle \Phi_i^0 \rangle = v_i/\sqrt{2}$, with $\tan\beta \equiv v_2/v_1$ and $v^2 \equiv v_1^2 + v_2^2 = (246 \text{ GeV})^2$. By assumption, $\lambda_i \lesssim \mathcal{O}(1)$.

The parameters m_{11}^2, m_{22}^2, m_{12}^2, $\lambda_1, \ldots, \lambda_7$ and v_1, v_2 would change under a U(2) transformation $\Phi_a \to U_{ab}\Phi_b$. To identify invariants, write $\widehat{v}_a = (v_1/v, v_2/v) \equiv (c_\beta, s_\beta)$ and, $\widehat{w}_a = -\epsilon_{ab}\widehat{v}_b^* = (-s_\beta, c_\beta)$.

$$\mathcal{V} = Y_{ab}\Phi_a^\dagger \Phi_b + \tfrac{1}{2} Z_{abcd} (\Phi_a^\dagger \Phi_b)(\Phi_c^\dagger \Phi_d)$$

Here is one possible invariant:

$$Z_1 \equiv Z_{abcd}\, \widehat{v}_a^*\, \widehat{v}_b\, \widehat{v}_c^*\, \widehat{v}_d\,.$$

Explicit evaluation yields

$$Z_1 = \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 + 2 s_{2\beta}(c_\beta^2 \lambda_6 + s_\beta^2 \lambda_7)\,,$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$.

The complete list:

$$Y_1 \equiv Y_{ab}\,\widehat{v}_a^*\,\widehat{v}_b\,,$$

$$Y_2 \equiv Y_{ab}\,\widehat{w}_a^*\,\widehat{w}_b\,,$$

$$Y_3 \equiv Y_{ab}\,\widehat{v}_a^*\,\widehat{w}_b\,,$$

and

$$Z_1 \equiv Z_{abcd}\,\widehat{v}_a^*\,\widehat{v}_b\,\widehat{v}_c^*\,\widehat{v}_d\,,$$

$$Z_2 \equiv Z_{abcd}\,\widehat{w}_a^*\,\widehat{w}_b\,\widehat{w}_c^*\,\widehat{w}_d\,,$$

$$Z_3 \equiv Z_{abcd}\,\widehat{v}_a^*\,\widehat{v}_b\,\widehat{w}_c^*\,\widehat{w}_d\,,$$

$$Z_4 \equiv Z_{abcd}\,\widehat{w}_a^*\,\widehat{v}_b\,\widehat{v}_c^*\,\widehat{w}_d\,,$$

$$Z_5 \equiv Z_{abcd}\,\widehat{v}_a^*\,\widehat{w}_b\,\widehat{v}_c^*\,\widehat{w}_d\,,$$

$$Z_6 \equiv Z_{abcd}\,\widehat{v}_a^*\,\widehat{v}_b\,\widehat{v}_c^*\,\widehat{w}_d\,,$$

$$Z_7 \equiv Z_{abcd}\,\widehat{v}_a^*\,\widehat{w}_b\,\widehat{w}_c^*\,\widehat{w}_d\,.$$

For convenience, we also introduce $Z_{345} \equiv Z_3 + Z_4 + Z_5$. When the above invariants are evaluated in the generic basis, one finds:

$$
\begin{aligned}
Y_1 &= m_{11}^2 c_\beta^2 + m_{22}^2 s_\beta^2 - m_{12}^2 s_{2\beta}\,, \\
Y_2 &= m_{11}^2 s_\beta^2 + m_{22}^2 c_\beta^2 + m_{12}^2 s_{2\beta}\,, \\
Y_3 &= \tfrac{1}{2}(m_{22}^2 - m_{11}^2) s_{2\beta} - m_{12}^2 c_{2\beta}\,,
\end{aligned}
$$

$$
\begin{aligned}
Z_1 =&\lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 + 2 s_{2\beta}(c_\beta^2 \lambda_6 + s_\beta^2 \lambda_7)\,, \\
Z_2 =&\lambda_1 s_\beta^4 + \lambda_2 c_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 - 2 s_{2\beta}(s_\beta^2 \lambda_6 + c_\beta^2 \lambda_7)\,, \\
Z_3 =&\tfrac{1}{4} s_{2\beta}^2(\lambda_1 + \lambda_2 - 2\lambda_{345}) + \lambda_3 - s_{2\beta} c_{2\beta}(\lambda_6 - \lambda_7)\,, \\
Z_4 =&\tfrac{1}{4} s_{2\beta}^2(\lambda_1 + \lambda_2 - 2\lambda_{345}) + \lambda_4 - s_{2\beta} c_{2\beta}(\lambda_6 - \lambda_7)\,, \\
Z_5 =&\tfrac{1}{4} s_{2\beta}^2(\lambda_1 + \lambda_2 - 2\lambda_{345}) + \lambda_5 - s_{2\beta} c_{2\beta}(\lambda_6 - \lambda_7)\,, \\
Z_6 =& -\tfrac{1}{2} s_{2\beta}(\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345} c_{2\beta}) + c_\beta c_{3\beta} \lambda_6 + s_\beta s_{3\beta} \lambda_7\,, \\
Z_7 =& -\tfrac{1}{2} s_{2\beta}(\lambda_1 s_\beta^2 - \lambda_2 c_\beta^2 + \lambda_{345} c_{2\beta}) + s_\beta s_{3\beta} \lambda_6 + c_\beta c_{3\beta} \lambda_7\,.
\end{aligned}
$$

Interpretation: Put $c_\beta = 1$ and $s_\beta = 0$. This corresponds to the "Higgs basis" [Branco, *et al.*], where only one of the two Higgs fields has a non-zero vacuum expectation value. Then, the parameters above are simply the coefficients of $\mathcal{V}$ in the Higgs basis.

The Higgs spectrum in the CP-conserving model contains a CP-odd state, A, a charged scalar pair, $H^\pm$, and two CP-even states obtained by diagonalizing a 2×2 squared-mass matrix:

$$H = (\sqrt{2}\mathrm{Re}\,\Phi_1^0 - v_1)\cos\alpha + (\sqrt{2}\mathrm{Re}\,\Phi_2^0 - v_2)\sin\alpha\,,$$

$$h = -(\sqrt{2}\mathrm{Re}\,\Phi_1^0 - v_1)\sin\alpha + (\sqrt{2}\mathrm{Re}\,\Phi_2^0 - v_2)\cos\alpha\,.$$

The angle α (like β) is basis-dependent. Define $\widehat{n} = (c_\alpha, s_\alpha)$. Then the basis independent quantities:

$$\widehat{n}_a \widehat{v}_a = \cos(\beta-\alpha) \equiv c_{\beta-\alpha}\,, \qquad \epsilon_{ab}\widehat{n}_a\widehat{v}_b = \sin(\beta-\alpha) \equiv s_{\beta-\alpha}\,.$$

are meaningful. The end result of the diagonalization is:

$$m^2_{H,h} = \tfrac{1}{2}\left[m_A^2 + v^2(Z_5+Z_1) \pm \sqrt{[m_A^2 + (Z_5-Z_1)v^2]^2 + 4Z_6^2v^4}\right]\,,$$

$$\sin\left[2(\beta-\alpha)\right] = 2s_{\beta-\alpha}c_{\beta-\alpha} = \frac{-2Z_6v^2}{m_H^2 - m_h^2},$$

$$\cos\left[2(\beta-\alpha)\right] = c^2_{\beta-\alpha} - s^2_{\beta-\alpha} = \frac{(Z_1-Z_5)v^2 - m_A^2}{m_H^2 - m_h^2}\,.$$

In addition, $m^2_{H^\pm} = m_A^2 + \frac{1}{2}v^2(Z_5 - Z_4)$.

Implications: $m_h \lesssim \mathcal{O}(v)$, and if $m_A \gg v$, then $m_H \simeq m_A \simeq m_{H^\pm}$ [up to corrections of $\mathcal{O}(v^2/m_A)$] and $c_{\beta-\alpha} \sim \mathcal{O}(v^2/m_A^2)$.

2HDM Tree-Level Higgs Couplings

Higgs couplings to gauge bosons: suppression factors

$\cos(\beta-\alpha)$	$\sin(\beta-\alpha)$	no angle factor
HW^+W^-	hW^+W^-	
HZZ	hZZ	
ZAh	ZAH	$ZH^+H^-, \gamma H^+H^-$
$W^\pm H^\mp h$	$W^\pm H^\mp H$	$W^\pm H^\mp A$
$ZW^\pm H^\mp h$	$ZW^\pm H^\mp H$	$ZW^\pm H^\mp A$
$\gamma W^\pm H^\mp h$	$\gamma W^\pm H^\mp H$	$\gamma W^\pm H^\mp A$

Higgs self-couplings

Here are two examples:

$$g_{hhh} = -3v\Big[Z_1 s^3_{\beta-\alpha} + Z_{345} s_{\beta-\alpha} c^2_{\beta-\alpha} + 3Z_6 c_{\beta-\alpha} s^2_{\beta-\alpha} + Z_7 c^3_{\beta-\alpha}\Big]$$

$$g_{hhhh} = -3\Big[Z_1 s^4_{\beta-\alpha} + Z_2 c^4_{\beta-\alpha} + 2Z_{345} c^2_{\beta-\alpha} s^2_{\beta-\alpha} + 4Z_6 c_{\beta-\alpha} s^3_{\beta-\alpha} + 4Z_7 c^3_{\beta-\alpha} s_{\beta-\alpha}\Big]$$

Indeed, all couplings above depend only on basis-independent invariants.

We can work out the leading corrections in the approach to the decoupling limit. First, we note the following exact result:

$$m_h^2 = v^2 \left[Z_1 + \frac{Z_6\, c_{\beta-\alpha}}{s_{\beta-\alpha}} \right] .$$

Thus, in the decoupling limit [$c_{\beta-\alpha} = 0$], $g_{hhh} = -3m_h^2/v$ and $g_{hhhh} = -3m_h^2/v^2$, which reproduces the SM result. Keeping only the leading terms at $\mathcal{O}(c_{\beta-\alpha})$,

ABC	X_{ABC}	Y_{ABC}
hhh	$3Z_1$	$9Z_6$
hhH	$-3Z_6$	$3Z_1 - 2Z_{345}$
hHH	Z_{345}	$3(Z_7 - 2Z_6)$
hAA	$Z_3 + Z_4 - Z_5$	Z_7
hH^+H^-	Z_3	Z_7
HHH	$-3Z_7$	$3Z_{345}$
HAA	$-Z_7$	$Z_3 + Z_4 - Z_5$
HH^+H^-	$-Z_7$	Z_3

Three-Higgs vertex Feynman rules in the approach to the decoupling limit are given by $ig_{ABC} = -iv(X_{ABC} + Y_{ABC}\, c_{\beta-\alpha})$, where the coefficients X and Y are listed above.

$ABCD$	X_{ABCD}	Y_{ABCD}
$hhhh$	$3Z_1$	$12Z_6$
$hhhH$	$-3Z_6$	$3(Z_1 - Z_{345})$
$hhHH$	Z_{345}	$6(Z_7 - Z_6)$
$hhAA$	$Z_3 + Z_4 - Z_5$	$2Z_7$
hhH^+H^-	Z_3	$2Z_7$
$hHHH$	$-3Z_7$	$3(Z_{345} - Z_2)$
$hHAA$	$-Z_7$	$-Z_2 + Z_3 + Z_4 - Z_5$
hHH^+H^-	$-Z_7$	$Z_3 - Z_2$
$HHHH$	$3Z_2$	$-12Z_7$
$HHAA$	Z_2	$-2Z_7$
HHH^+H^-	Z_2	$-2Z_7$
$AAAA$	$3Z_2$	0
AAH^+H^-	Z_2	0
$H^+H^-H^+H^-$	$2Z_2$	0

Four-Higgs vertex Feynman rules in the approach to the decoupling limit are given by $ig_{ABCD} = -i(X_{ABCD} + Y_{ABCD}\, c_{\beta-\alpha})$, where the coefficients X and Y are listed above.

Higgs couplings to fermion pairs

$$-\mathcal{L}_Y = \overline{Q}^0_L \widetilde{\Phi}_1 \eta_1^{U,0} U^0_R + \overline{Q}^0_L \Phi_1 \eta_1^{D,0} D^0_R$$
$$+ \overline{Q}^0_L \widetilde{\Phi}_2 \eta_2^{U,0} U^0_R + \overline{Q}^0_L \Phi_2 \eta_2^{D,0} D^0_R + \text{h.c.}\,,$$

where $\widetilde{\Phi}_i \equiv i\sigma_2\Phi_i^*$, Q^0_L is the weak isospin quark doublet, and U^0_R, D^0_R are weak isospin quark singlets in an interaction eigenstate basis, and $\eta_1^{U,0}$, $\eta_2^{U,0}$, $\eta_1^{D,0}$, $\eta_2^{D,0}$ are matrices in flavor space. Identify the fermion mass eigenstates by employing the appropriate biunitary transformation of the quark mass matrices involving unitary matrices V_L^U, V_L^D, V_R^U, V_R^D, where $K \equiv V_L^U V_L^{D\,\dagger}$ is the CKM matrix. Then, define $\eta^Q \equiv (\eta_1^Q, \eta_2^Q)$, where $\eta_i^Q \equiv V_L^Q \eta_i^{Q,0} V_R^{Q\,\dagger}$, for $Q = U,\ D$. One can then introduce the invariant quantities:

$$\kappa^Q \equiv \widehat{v}\cdot\eta^Q = c_\beta\eta_1^Q + s_\beta\eta_2^Q\,,$$

$$\rho^Q \equiv \widehat{w}\cdot\eta^Q = -s_\beta\eta_1^Q + c_\beta\eta_2^Q\,.$$

The quark mass terms are identified by replacing the scalar fields with their vevs. V_L^U, V_L^D, V_R^U and V_R^D are then chosen so that κ^D and κ^U are diagonal with real non-negative entries. The resulting quark mass matrices are then diagonal:

$$M_D = \frac{v}{\sqrt{2}}\kappa^D\,, \qquad M_U = \frac{v}{\sqrt{2}}\kappa^U\,.$$

The Higgs-fermion Yukawa couplings are then given by:

$$
\begin{aligned}
-\mathcal{L}_Y =& \frac{1}{v}\overline{D}\left[M_D s_{\beta-\alpha} + \frac{v}{\sqrt{2}}(\rho^D P_R + \rho^{D\dagger} P_L) c_{\beta-\alpha}\right] Dh \\
& + \frac{1}{v}\overline{D}\left[M_D c_{\beta-\alpha} - \frac{v}{\sqrt{2}}(\rho^D P_R + \rho^{D\dagger} P_L) s_{\beta-\alpha}\right] DH \\
& + \frac{i}{\sqrt{2}}\overline{D}(\rho^D P_R - \rho^{D\dagger} P_L) DA \\
& + \frac{1}{v}\overline{U}\left[M_U s_{\beta-\alpha} + \frac{v}{\sqrt{2}}(\rho^U P_R + \rho^{U\dagger} P_L) c_{\beta-\alpha}\right] Uh \\
& + \frac{1}{v}\overline{U}\left[M_U c_{\beta-\alpha} - \frac{v}{\sqrt{2}}(\rho^U P_R + \rho^{U\dagger} P_L) s_{\beta-\alpha}\right] UH \\
& - \frac{i}{\sqrt{2}}\overline{U}(\rho^U P_R - \rho^{U\dagger} P_L) UA \\
& + \left\{\overline{U}\left[K\rho^D P_R - \rho^{U\dagger} K P_L\right] DH^+ + \text{h.c.}\right\} .
\end{aligned}
$$

where $P_{R,L} \equiv \frac{1}{2}(1 \pm \gamma_5)$. The Higgs-fermion Yukawa couplings depend only on invariant quantities: the quark masses, the ρ^Q and $\beta - \alpha$. In general, ρ^U and ρ^D are complex non-diagonal matrices. Hence, the Yukawa couplings exhibit tree-level Higgs-mediated flavor-changing neutral currents and CP-violating Higgs-fermion couplings. But, near the decoupling limit, the couplings of h approach their SM values.

In a one generation model, the Higgs-quark interaction produces the following Feynman rules of the form $-ig_{\phi f_1 \bar{f}_2}$:

$$
\begin{aligned}
g_{hq\bar{q}} &= \frac{m_q}{v} s_{\beta-\alpha} + \tfrac{1}{\sqrt{2}}(S_q + i\gamma_5 P_q) c_{\beta-\alpha} \,, \\
g_{Hq\bar{q}} &= \frac{m_q}{v} c_{\beta-\alpha} - \tfrac{1}{\sqrt{2}}(S_q + i\gamma_5 P_q) s_{\beta-\alpha} \,, \\
g_{Au\bar{u}} &= -\tfrac{1}{\sqrt{2}}(iS_u\gamma_5 - P_u) \,, \\
g_{Ad\bar{d}} &= \tfrac{1}{\sqrt{2}}(iS_d\gamma_5 - P_d) \,, \\
g_{H^+ d\bar{u}} &= \tfrac{1}{2}[\rho^D(1+\gamma_5) - \rho^{U*}(1-\gamma_5)] \,,
\end{aligned}
$$

where $S_q \equiv \mathrm{Re}\ \rho^Q$ and $P_q \equiv \mathrm{Im}\ \rho^Q$.

The MSSM Higgs sector is a type-II 2HDM, *i.e.*, $\eta_1^U = \eta_2^D = 0$. A basis-independent condition for type-II is: $\eta^U \cdot \eta^D = 0$. But, this means that $\tan\beta$ has been promoted to a physical parameter since $\eta_1^U = \eta_2^D = 0$ occurs only for one choice of β.* It follows that:

$$
\tan\beta = -\frac{\rho^D}{\kappa^D} = \frac{\kappa^U}{\rho^U} \,.
$$

For these two definitions to be consistent, $\kappa^U\kappa^D + \rho^U\rho^D = 0$ must be satisfied. But this is equivalent to the type-II condition, $\eta^U \cdot \eta^D = 0$.

* Actually two choices: β and $\beta - \pi/2$, corresponding to whether the Higgs doublet that couples to, say, the up-type quark is Φ_1 or Φ_2.

Moreover, using $\kappa^Q = \sqrt{2}M_Q/v = \sqrt{2}v^{-1}\mathrm{diag}(m_u, m_d)$,

$$\rho^D = \frac{-\sqrt{2}m_d \tan\beta}{v}\,, \qquad \rho^U = \frac{\sqrt{2}m_u \cot\beta}{v}\,.$$

This then yields the well-known type-II Higgs-quark interactions:

$$hb\bar{b}: \quad -\frac{\sin\alpha}{\cos\beta} = 1\,\sin(\beta-\alpha) - \tan\beta\,\cos(\beta-\alpha)\,,$$

$$ht\bar{t}: \quad \frac{\cos\alpha}{\sin\beta} = 1\,\sin(\beta-\alpha) + \cot\beta\,\cos(\beta-\alpha)\,,$$

$$Hb\bar{b}: \quad \frac{\cos\alpha}{\cos\beta} = \cos(\beta-\alpha) + \tan\beta\,\sin(\beta-\alpha)\,,$$

$$Ht\bar{t}: \quad \frac{\sin\alpha}{\sin\beta} = \cos(\beta-\alpha) - \cot\beta\,\sin(\beta-\alpha)\,.$$

In the more general (type-III) 2HDM, $\tan\beta$ is not a meaningful parameter. Nevertheless, one can introduce three $\tan\beta$-like parameters:†

$$\tan\beta_d \equiv \frac{-\rho^D}{\kappa^D}\,, \qquad \tan\beta_u \equiv \frac{\kappa^U}{\rho^U}\,, \qquad \tan\beta_e \equiv \frac{-\rho^E}{\kappa^E}\,,$$

the last one corresponding to the Higgs-lepton interaction. In a type-III 2HDM, there is no reason for the three parameters above to coincide.

† Interpretation: In the Higgs basis, up and down-type quarks interact with both Higgs doublets. But, clearly there exists some basis (*i.e.*, a rotation by angle β_u from the Higgs basis) for which only one of the two up-type quark Yukawa couplings is non-vanishing. This defines the physical angle β_u.

The MSSM Higgs sector is a type-III 2HDM

The tree-level Higgs potential of the MSSM satisfies:

$$\lambda_1 = \lambda_2 = -\lambda_{345} = \tfrac{1}{4}(g^2 + g'^2)\,, \lambda_4 = -\tfrac{1}{2}g^2\,, \lambda_5 = \lambda_6 = \lambda_7 = 0\,.$$

But, one-loop radiative corrections generate corrections to these relations, due to SUSY-breaking. *E.g.*, at one-loop, λ_5, λ_6, $\lambda_7 \neq 0$.

For MSSM Higgs couplings to fermions, Yukawa vertex corrections modify the effective Lagrangian that describes the coupling of the Higgs bosons to the third generation quarks:

$$-\mathcal{L}_{\rm eff} = \epsilon_{ij}\left[(h_b + \delta h_b)\bar{b}_R H_d^i Q_L^j + (h_t + \delta h_t)\bar{t}_R Q_L^i H_u^j\right] + \Delta h_b \bar{b}_R Q_L^k H_u^{k*} + \Delta h_t \bar{t}_R Q_L^k H_d^{k*} + {\rm h.c.}$$

Indeed, this is a general type-III model. For example, in some MSSM parameter regimes (corresponding to large $\tan\beta$),[‡]

$$\Delta h_b \simeq h_b\left[\frac{2\alpha_s}{3\pi}\mu M_{\tilde{g}}\, I(M^2_{\tilde{b}_1}, M^2_{\tilde{b}_2}, M^2_{\tilde{g}}) + \frac{h_t^2}{16\pi^2}\mu A_t\, I(M^2_{\tilde{t}_1}, M^2_{\tilde{t}_2}, \mu^2)\right]$$

Strictly speaking, one should define three $\tan\beta$-like quantities corresponding to the Yukawa interactions of the third generation fermions. For example, large $\tan\beta$ above really means large $\tan\beta_d$.

[‡] $I(a,b,c) = [ab\ln(a/b) + bc\ln(b/c) + ca\ln(c/a)]/(a-b)(b-c)(a-c)$.

Lessons for future Higgs studies

• To probe the physics of EWSB dynamics, one must directly detect the phenomena associated with EWSB dynamics (at the scale Λ). Without such data, the only window of opportunity is to detect deviations from the decoupling limit.

• If the 2HDM provides a good description of the low-energy scalar sector (as it does in the MSSM), then the pattern of h couplings and their dependence on $c_{\beta-\alpha}$ is precisely known. These will provide important checks to the interpretation of the Higgs data.

• To successfully test the interpretation of the theory via precision Higgs studies, inclusion of the radiative corrections is imperative. In this case, there is no longer one unique $\tan\beta$. One must be very careful in deciding which physical parameters of the theory are being measured.

• Precision Higgs measurements can provide critical tests of the supersymmetric interpretation of new physics beyond the SM.

• It is possible to be fooled by a SM-like Higgs boson that isn't. This can happen if $s_{\beta-\alpha} \simeq 0$, in which case H has SM-like properties. Nevertheless, in such a scenario, the entire 2HDM spectrum must be light and accessible to the LHC and/or LC.