Instanton Test of Non-Supersymmetric Deformations of $\mathrm{AdS}_5 \times \mathrm{S}^5$

Callum Durnford hep-th/0606111
With G. Georgiou and V. V. Khoze
Durham University

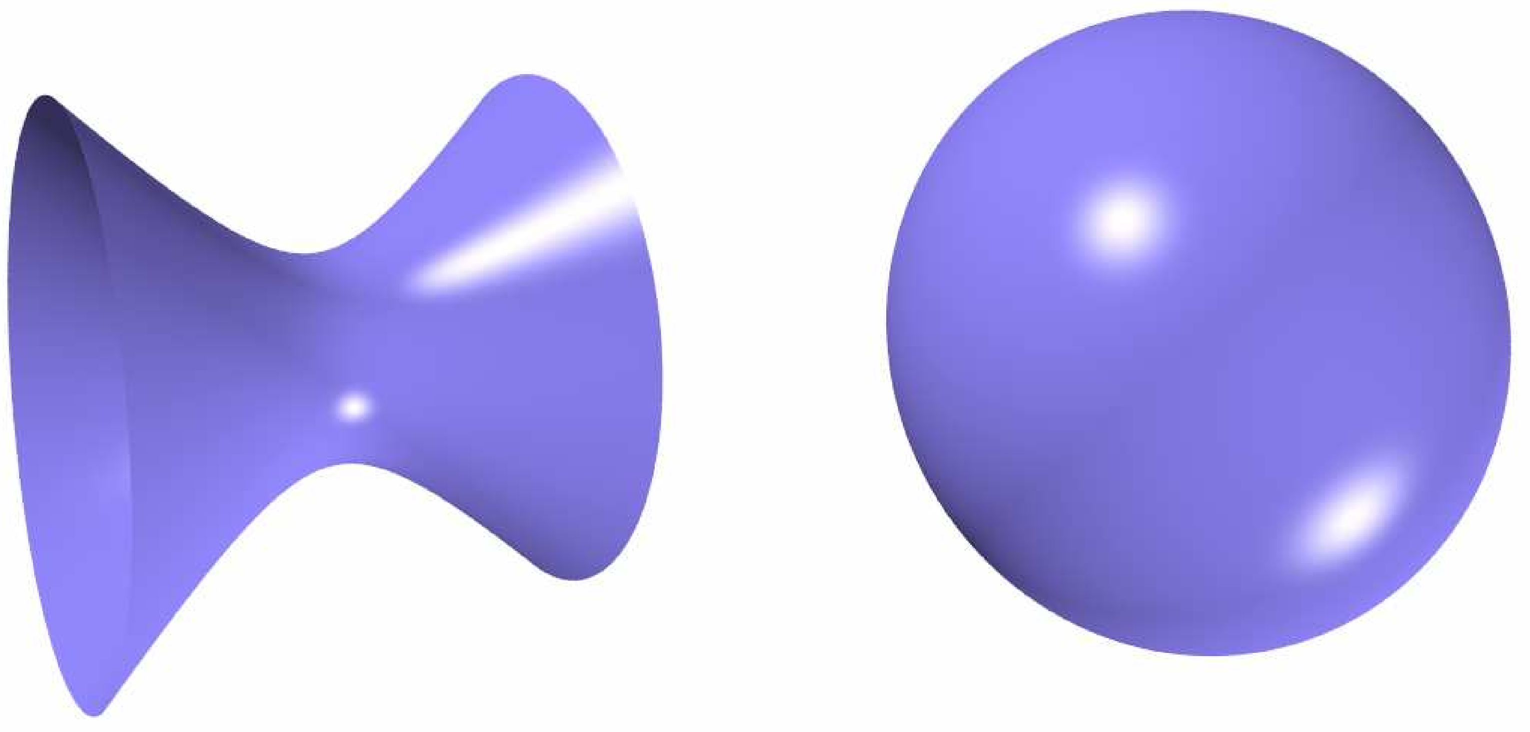
The most well known example of the gravity/gauge theory correspondence is between type IIb supergravity on the background $\mathrm{AdS}_5 \times S^5$ (see right) and $\mathcal{N} = 4$ $SU(N)$ Super Yang-Mills theory in the large N, strong coupling limit.

Under the duality, the radius R of the AdS_5 and S^5 is related to the 't Hooft coupling $\lambda = g^2 N$ of the gauge theory by $R^2 = \alpha\sqrt{\lambda}$ and the dilaton/axion field $\tau = ie^{-\phi} + C$ is identified with the complexified gauge coupling $\tau_0 = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$.

The correspondence is conjectured to hold between the full superstring theory and quantum conformal field theory, so D-instanton corrections to supergravity should match (at least up to numerical factors) with instanton contributions on the Yang-Mills side. S-duality of the IIb superstring requires the correction received by various correlation functions to be modular forms in τ. For example the 16 dilatino vertex contains a modular form with weak string coupling expansion:

$$f_{16} = a_0\, e^{-3\phi/2} + a_1\, e^{\phi/2} + \sum_{k=1}^{\infty} \Big(\sum_{n|k} \frac{1}{n^2} \Big) \left(ke^{-\phi}\right)^{25/2} \exp\left(-2\pi i\tau k\right) e^{\phi/2}$$

The SYM multi-instanton calculation was performed by Dorey et al hep-th/9808157 and exactly matches the expansion of these modular forms. This result tests the conjecture beyond the supergravity approximation and also indicates the existence of a non-renormalisation theorem for Yang-Mills instantons: the Yang-Mills calculation should only be valid for weak coupling $\lambda = g^2 N \ll 1$, but matches with string theory in the supergravity limit, which has $g^2 N \gg 1$.



$\mathrm{AdS}_5 \times S^5$: The near horizon geometry of N stacked D3-branes

An interesting way to explore the AdS/CFT correspondence is to deform the $\mathcal{N} = 4$ SYM theory with exactly marginal operators. This way the theory remains conformal and so is expected to be dual to $\mathrm{AdS} \times$ Something. One such deformation is the β-deformation, where the superpotential is modified:

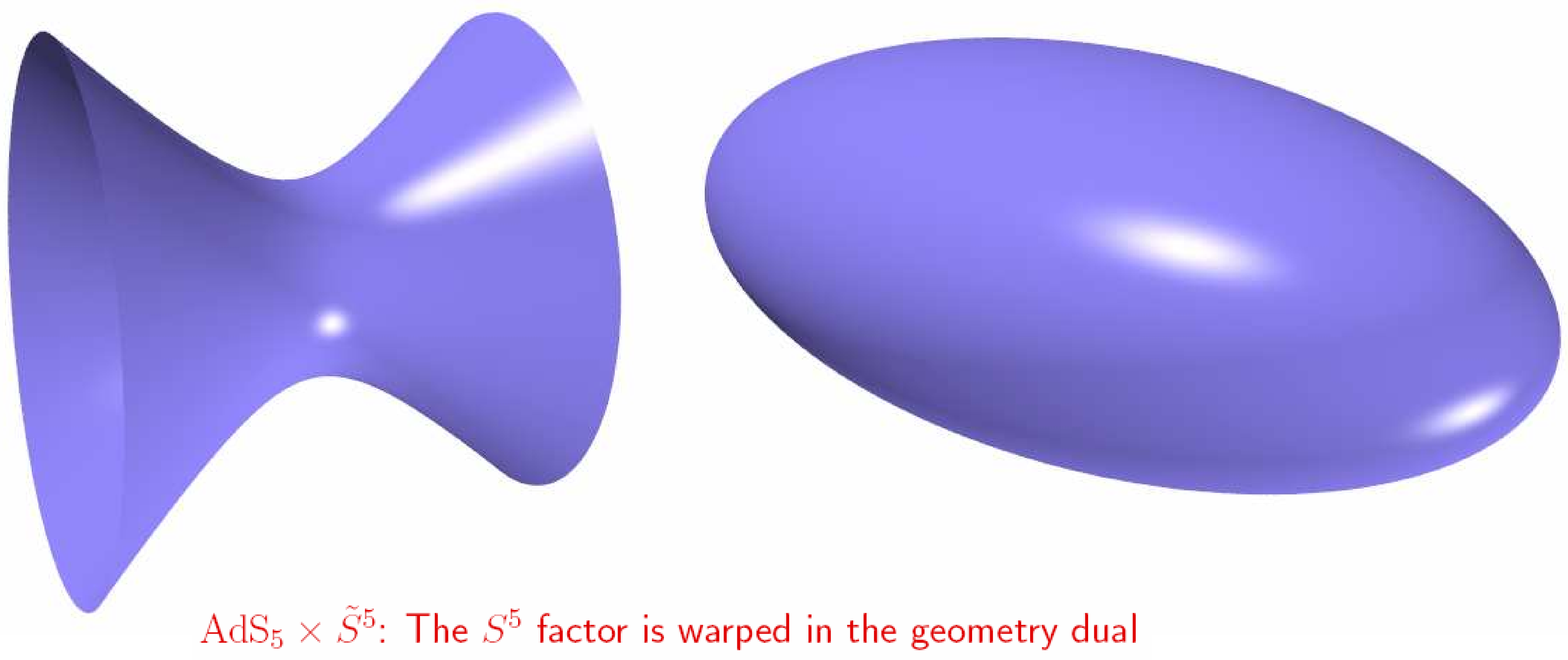
$$ig\,\mathrm{Tr}(\Phi_1\Phi_2\Phi_3 - \Phi_1\Phi_3\Phi_2) \;\rightarrow\; ig\,\mathrm{Tr}(e^{i\pi\beta}\Phi_1\Phi_2\Phi_3 - e^{-i\pi\beta}\Phi_1\Phi_3\Phi_2)$$

The resulting theory has $\mathcal{N} = 1$ supersymmetry, and a global $U(1) \times U(1)$ symmetry (distinct from the $U(1)_R$ automorphism of the supersymmetry algebra) which can be taken to act on the three chiral superfields as:

$$U(1)_1: \quad (\Phi_1, \Phi_2, \Phi_3) \rightarrow (\Phi_1, e^{i\varphi_1}\Phi_2, e^{-i\varphi_1}\Phi_3)$$
$$U(1)_2: \quad (\Phi_1, \Phi_2, \Phi_3) \rightarrow (e^{-i\varphi_2}\Phi_1, e^{i\varphi_2}\Phi_2, \Phi_3)$$

This symmetry was used by Lunin and Maldacena hep-th/0502086 to find the dual supergravity theory for small β. The deformation parameter β enters by deforming the geometry of the S^5 (see left). The above matching of D-instantons and SYM instantons was shown by Georgiou and Khoze hep-th/0602141 to work for this one-parameter family of deformations by modifying the way the dilaton/axion field is associated under duality with the complexified gauge coupling:

$$\tau = \tau_0 + 2\pi i N \beta^2 (\mu_1^2\mu_2^2 + \mu_2^2\mu_3^2 + \mu_3^2\mu_1^2)$$

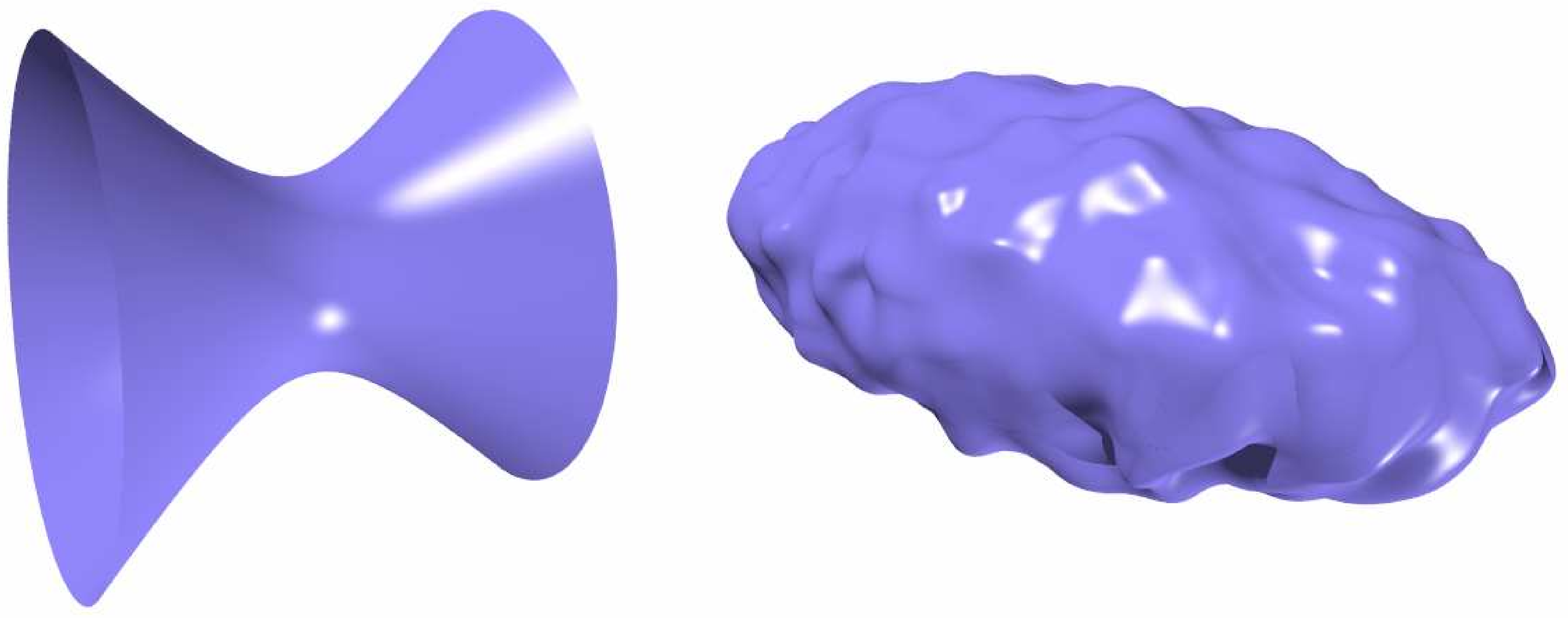


$\mathrm{AdS}_5 \times \tilde{S}^5$: The S^5 factor is warped in the geometry dual to β-deformed SYM

The SUGRA solution generating technique of Lunin and Maldacena can be viewed as a TsT transformation acting on the torus in SUGRA geometry which is dual to $U(1) \times U(1)$. Such a transformation, with parameter $\hat{\gamma}$ goes as follows:

> If the torus is parameterised by (ϕ_1, ϕ_2), we first T-dualise in the ϕ_1 direction, then shift $\phi_2 \rightarrow \phi_2 + \hat{\gamma}\,\phi_1$, then T-dualise again along ϕ_1.

Frolov hep-th/0503201 generalised the SUGRA background by performing 3 TsT's on different tori within the S^5. This results in even more warping of the 5-sphere (see right). TsT transformations on the AdS_5 factor can also be considered, but one encounters the issue of T-duality in a time-like direction.



$\mathrm{AdS}_5 \times \hat{S}^5$: Non-supersymmetric deformations of $\mathcal{N} = 4$ SYM lead to even more deformation of the dual S^5

↓↑↓↓↓↑↓↓ Integrability ↓↑↓↓↓↑↑↓

$\mathcal{N} = 4$ $SU(N)$ SuperYang-Mills is of special interest as it has an integrable sub-sector: the Dilatation operator can be modelled as the Hamiltonian of an integrable spin-chain. This new correspondence allows us to use integrable field theoretic tools such as the Bethe Ansatz to find the anomalous dimensions of various operators in the original gauge theory.

Frolov constructed a Lax pair for the string theory dual to β-deformed SYM, thereby showing that the integrable structure of the original theory persists under the deformation.

People have recently been investigating the direct correspondence between string theory and the spin chain model. Interest has centred on the fundamental spin chain excitation, known as a *magnon*, which maps over to a spinning string configuration living on an $\mathbb{R} \times S^2$ subspace of $\mathrm{AdS}_5 \times S^5$ — a *giant magnon* hep-th/0604135. Comparison of the scattering and bound states of these magnons in both theories has lead to useful checks of the correspondence.

Magnon solutions can also be constructed for the β-deformed correspondence hep-th/0606220.

The gauge theories dual to Frolov's SUGRA solution form a multi-parameter family of conformal deformations of planar $\mathcal{N} = 4$ SYM which, in general, will have no supersymmetry. Fortunately they still have the supersymmetric field content of the original theory so determinants arising from bosonic and fermionic excitations in any path integral will still cancel. Calculating the multi-instanton contribution to the partition function then reduces to understanding the phases introduced at fermionic interactions by the deformation. By integrating out the fermionic collective coordinates we showed Yang-Mills instantons again give the same correction to deformed gauge theory as D-instantons do to Frolov's supergravity background.

As above, the exact matching of results indicates a non-renormalisation theorem for the Yang-Mills instantons, but now we learn that this protection from renormalisation cannot be due to supersymmetry in the underlying gauge theory. We expect that the origin of the agreement stems from the fact the instantons and D-instantons that are being identified are 'extended objects' or defects in the dual theories.