# Testing Leptonic Flavour Models with Future Neutrino Oscillation Experiments

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based on hep-ph/1308.4314 and on-going work in collaboration with S.F. King (Southampton), C. Luhn (Siegen), S. Pascoli (Durham) and M.A. Schmidt (Melbourne).

## Flavour Models with Discrete Symmetries

The flavour structure of the standard model is as yet unexplained. What dictates the pattern of mixing angles that we have observed? One way to explain these features is to promote particle flavour to a symmetry. A popular approach for deriving the large mixing angles of the leptonic sector is to use a discrete symmetry (*e.g.* $A_4$ or $S_4$) which is broken spontaneously, leaving residual symmetries amongst the leptonic mass terms. These reduce the degrees of freedom amongst the mixing parameters and generate a pattern of falsifiable predictions.

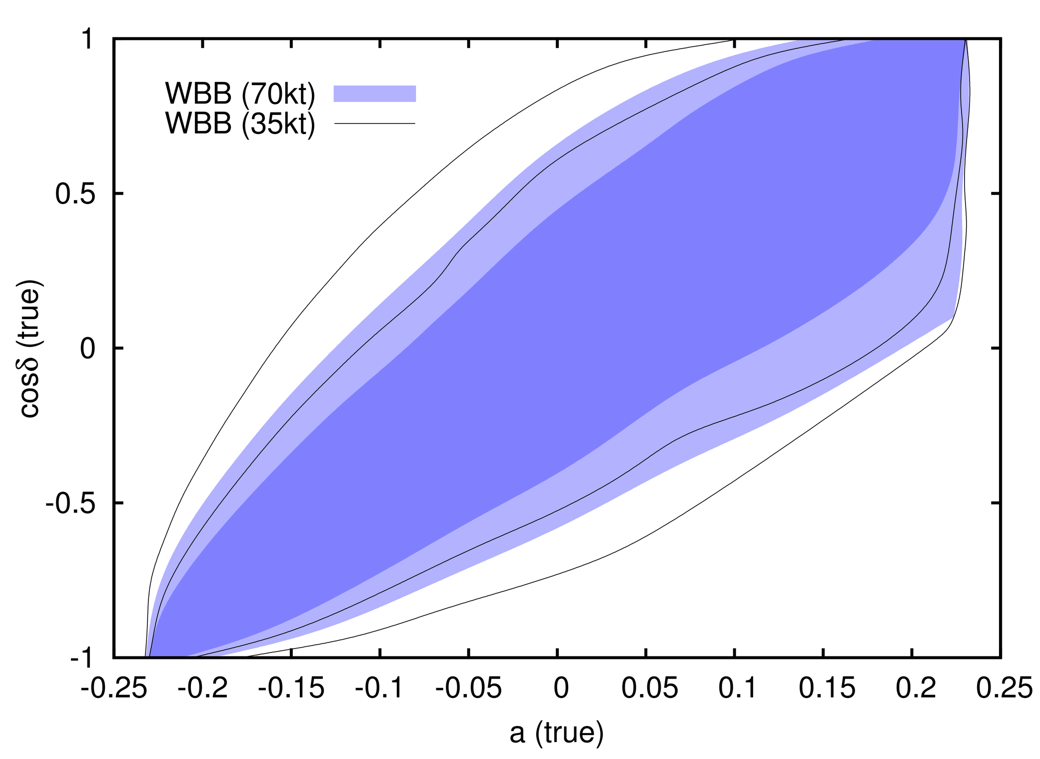
$$D(2,n,m) = G_{\mathrm{F}} \begin{cases} \nearrow \; G_\nu = \mathbb{Z}_2 = \langle S \rangle \\ \searrow \; G_\ell = \mathbb{Z}_n = \langle T \rangle \end{cases}$$

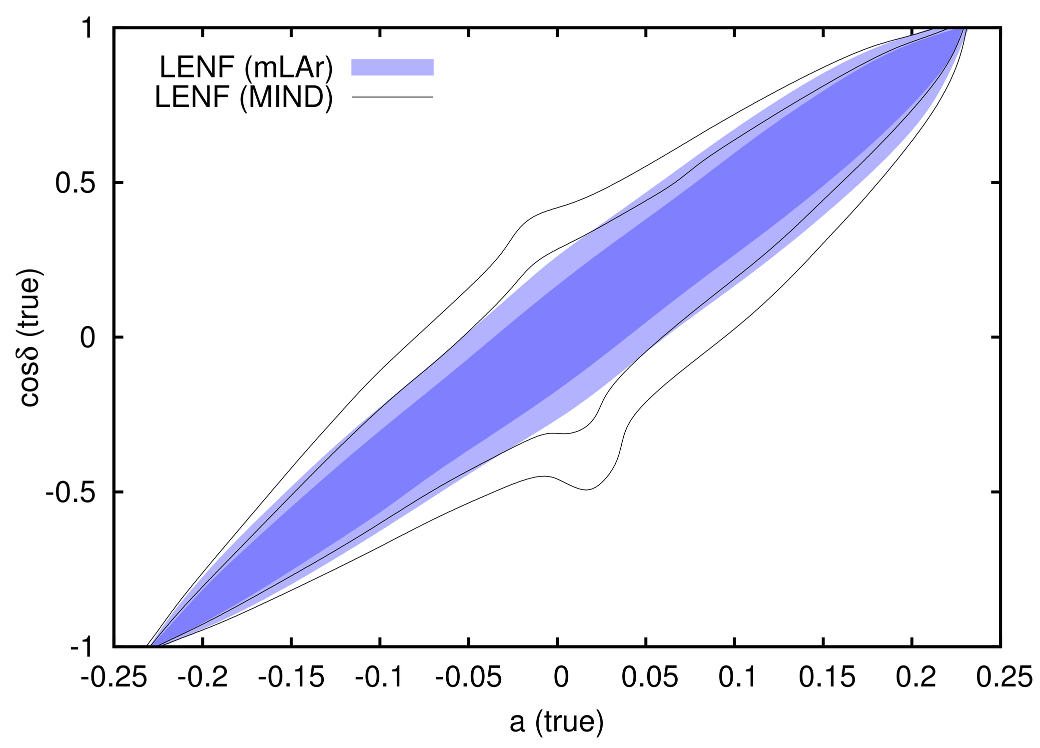
If we start by hypothesizing which symmetries of the leptonic mass terms are residual and which are accidental, we can use this idea to reconstruct the group $G_{\mathrm{F}}$. There is only a finite number of starting points for this process, and our work has focused on cataloguing the viable models in this scheme [based on a formulation of the problem due to Hernández and Smirnov, *Phys.Rev.* **D 86**, 053014 (2012)] and seeing how these predictions can be tested at oscillation facilities.

## Atmospheric Sum Rules at Long-Baseline Facilities

Long-baseline oscillation experiments, such as superbeams (for example, LBNO or LBNE) and neutrino factories, primarily aim to measure appearance probabilities *e.g.* $P(\nu_e \to \nu_\mu)$ over distances of the order $L \approx 1000$ km. These channels are sensitive to the size of the currently unknown parameter $\delta$, and such facilities will allow the first constraints to be placed on flavour symmetric predictions which involve $\delta$.

In our work, we have studied the potential for these facilities to exclude parameter correlations between $a$ and $\cos\delta$ known as atmospheric sum rules (for details, see rightmost panel).




**Upper left:** the regions plotted in the inner (outer) ellipses are the regions of true parameter space for which the sum rule $a = r\cos\delta$ can be excluded at $2\sigma$ ($3\sigma$). The top panel shows the simulation for a Wide Band superBeam (WBB) with large liquid Argon (LAr) detector (mass given by legend), modelled after the LBNO proposal. (Assumptions: baseline distance of 2300 km, $10^{21}$ protons on target per year at 50 GeV, with backgrounds to the appearance channel coming from the intrinsic beam contamination and 0.5% of neutral current events. A 2% uncertainty on the matter density is accounted for, as is a 5% signal normalization error.)

**Lower left:** this plot shows the same regions as the previous plot; however, now we consider a simulation of a Low-Energy Neutrino Factory. This is a more ambitious experiment which can attain a much higher precision measurement on $\delta$, improving its ability to exclude models. (Assumptions: $10^{22}$ total useful muon decays, a baseline of 2000 km and a stored-muon energy of 10 GeV. Two detectors are considered, a 50 kton magnetized LAr (mLAr) and a Magnetized Iron Neutrino Detector (MIND).)

## Predicted Correlations

We have studied a class of models which predict a fixed column of the PMNS matrix

$$e.g.\quad A_4\text{-}T_e\text{-}S_2 \implies \begin{pmatrix} U_{e1} & \frac{1}{\sqrt{3}} & U_{e3} \\ U_{\mu 1} & \frac{1}{\sqrt{3}} & U_{\mu 3} \\ U_{\tau 1} & \frac{1}{\sqrt{3}} & U_{\tau 3} \end{pmatrix}.$$

Constraints of this form generate two correlations amongst the mixing parameters, which can be neatly expressed in terms of new variables [for details, see King, *Phys.Lett.* **B659** (2008)]:

$$\sin\theta_{12} = \frac{1+s}{\sqrt{3}},\quad \sin\theta_{13} = \frac{r}{\sqrt{2}} \quad\text{and}\quad \sin\theta_{23} = \frac{1+a}{\sqrt{2}}.$$

The relations between the $s$, $r$ and $a$ parameters predicted by these models can be divided into two classes:

### A solar prediction

The value of $s$ is predicted in these models as a function of $r$ alone. For example, the model denoted by $A_4$-$T_e$-$S_2$ predicts the correlation

$$s = \frac{1}{\sqrt{1 - r^2/2}}.$$

### An atmospheric sum rule

The value of $a$ is correlated with those of $r$ and $\cos\delta$. This constraint can be expressed in a linearized form by

$$a = a_0 + \lambda r\cos\delta + \mathcal{O}\left(r^2\right)$$

where $a_0$ and $\lambda$ are new model-dependent constants. To test these relations, we require a strong precision on the parameter $\delta$ which motivates the consideration of long-baseline experiments.

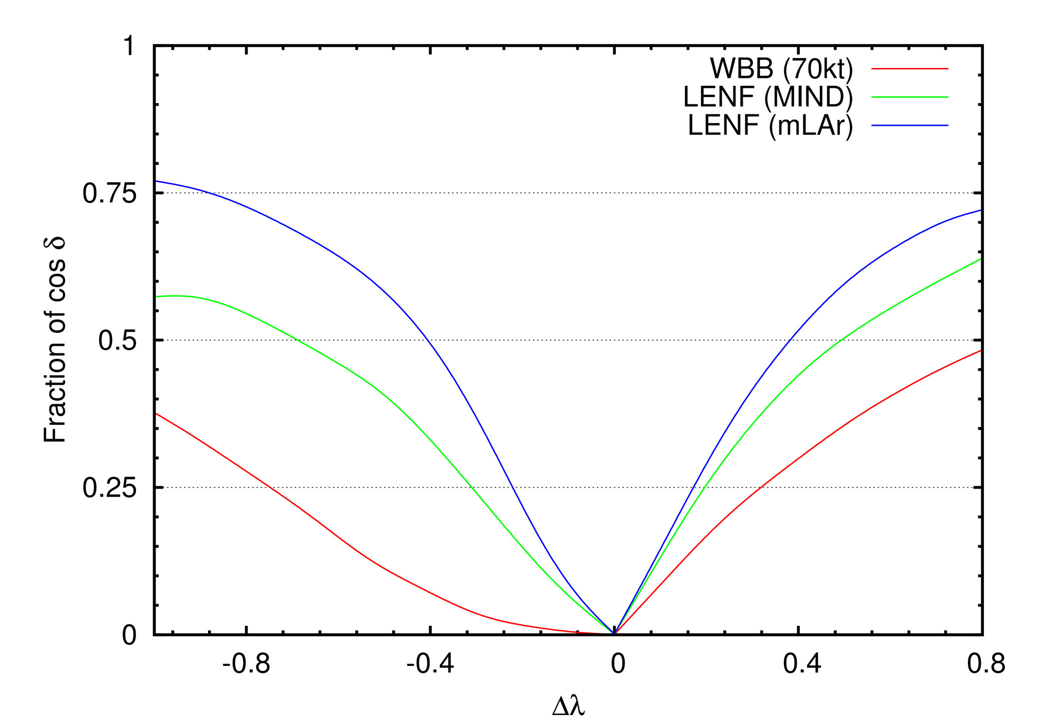
| $G_{\mathrm{F}}$-$T_\alpha$-$S_i$ | $a$-rule |
|---|---|
| $A_4$-$T_l$-$S_2$ | $a = -\frac{1}{2} r\cos\delta$ |
| $S_4$-$T_e$-$S_1$ | $a = r\cos\delta$ |
| $S_4$-$T_\mu$-$S_2$ | $a = \frac{1}{6} - \sqrt{\frac{1}{6}}\, r\cos\delta$ |
| $S_4$-$T_\tau$-$S_2$ | $a = \frac{1}{6} + \sqrt{\frac{1}{6}}\, r\cos\delta$ |
| $A_5$-$T_e$-$S_1$ | $a = \frac{\varphi}{\sqrt{2}} r\cos\delta$ |
| $A_5$-$T_e$-$S_2$ | $a = \frac{1-\varphi}{\sqrt{2}} r\cos\delta$ |
| $A_5$-$T_\mu$-$S_2$ | $a = -\frac{5-4\varphi}{22} - \sqrt{\frac{3+2\varphi}{22}}\, r\cos\delta$ |
| $A_5$-$T_\tau$-$S_2$ | $a = \frac{5-4\varphi}{22} - \sqrt{\frac{3+2\varphi}{22}}\, r\cos\delta$ |

Predictions for atmospheric sum rules derived in Ballett *et al.*, hep-ph/1308.4314 (2013).

## Constraining $\lambda$

In a more general setting, any set of oscillation parameters can be seen as making a prediction for $\lambda$.

In the plot below, we consider how well we can exclude different values of $\lambda$ as a function of $\Delta\lambda \equiv \lambda_{\mathrm{Fit}} - \lambda_{\mathrm{True}}$ for $\lambda_{\mathrm{True}} = 1$.




## Impact of JUNO

To test atmospheric sum rules we need an experiment capable of measuring $\delta$ *e.g.* a superbeam or neutrino factory. However, the models of leptonic flavour that we have studied also predict a correlation between the solar mixing angle $\theta_{12}$ and the reactor mixing angle $\theta_{13}$. If a facility is built with a significant increase in precision on $\theta_{12}$, these relations can also be used to constrain the models. We show here how the JUNO experiment could be used to significantly improve our knowledge of the viability of these models.

**Right:** coloured bands denote the regions of true parameter space where the correlation could not be ruled out by the JUNO experiment. (Assumptions: run time of 6 years, a single reactor of 36 GW power at 52 km, the existence of a 5% uncertainty on the number of events and a 3% linear energy scale uncertainty.)

We see that a clear separation between most models is possible at $5\sigma$. This is a very promising way to test the class of models that we have been considering.