The Higgs(-. . .)-Mechanism
I: The Power of Gauge Theories

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NExT PhD Workshop
### Overview of Lectures

#### The Standard Model
- The Uncompromising Gauge Principle

#### The Higgs(-...)-Mechanism
- Mass Generation
- Stability of the Standard Model
- Characteristics of the SM Higgs Boson

#### The Higgs Boson Discovery and Phenomenology
- Discovery of the Resonance, Mass
- Couplings
- \( CP \)-Properties
# The Particle Content of the Standard Model

<table>
<thead>
<tr>
<th>Quarks</th>
<th>Fermions</th>
<th>Bosons</th>
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<tbody>
<tr>
<td>u</td>
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<table>
<thead>
<tr>
<th>Leptons</th>
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<tr>
<td>(\nu_e)</td>
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<td>(\nu_\tau)</td>
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<td>e</td>
<td>g</td>
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<td>μ</td>
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<td>τ</td>
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*Yet to be confirmed*

Source: AAAS
Particle Physics not just “stamp collecting”, hunting new resonances.

The study of the fundamental particles and their interactions is interesting. . .

. . . but what makes it really interesting is that the fundamental interactions are dictated by a few theoretical guidelines and the fundamental dynamics can be directly related to observations.

(fundamental: related to terms in the Lagrangian)
The Standard Model

Standard Model of Particle Physics

Unified theoretical framework describing

1. Electromagnetism
2. Weak Nuclear Force
3. Strong Nuclear Force
4. Matter Particles

Forces

**Electromagnetism**: massless photon. Electric charge

**Weak nuclear force**: massive $W^\pm$, $Z$-boson. Interactions only with left-handed matter and the force carriers themselves. Weak isospin.

**Strong nuclear force**: massless gluons. Interactions with quarks and with gluons. Colour charge.
All the forces can be quantised by the use of **gauge field theory**, based on the groups

\[ SU_c(3) \times SU_L(2) \times U_Y(1) \]

A straightforward introduction of masses - **not only** for the **gauge bosons**, but **also for the matter particles** - would break the gauge invariance of the electro-weak component of the theory.

We will now explore the construction of the SM - in order to expose the **mass generation** through **dynamical electro-weak symmetry breaking** (aka the **Higgs(...)**-mechanism), and the appearance of the **Standard Model Higgs Boson**.
Start from the Lagrangian of a free (Dirac Fermion) field

\[ \mathcal{L} = \bar{\psi}i\partial_\mu \gamma^\mu \psi \]

Introduce **interactions**: Left/right-handed chiral fermion fields:

\[ f_{L,R} = \frac{1}{2} (1 \mp \gamma_5) f. \]

**Left-handed** fermions are in weak **iso-doublets**, **right-handed** fermions in weak **iso-singlets**

\[ L_1 = \left( \begin{array}{l} \nu_e \\ e^- \end{array} \right)_L, \quad e^-_R, \quad Q_1 = \left( \begin{array}{l} u \\ d \end{array} \right)_L, \quad u_R, d_R \]

Similar for the other two generations.
Matter fields in the fundamental representation; acted on by the generators of the adjoint

\[ T^a = \frac{1}{2} \tau^a; \quad \tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]

Commutation relations for weak isospin and hypercharge

\[ [T^a, T^b] = i \epsilon^{abc} T_c \quad \text{and} \quad [Y, Y] = 0 \]
Arrange the Lagrangian as

\[ \mathcal{L} = \bar{L}_i i \partial_\mu \gamma^\mu L_i + \bar{e}_{R_i} i \partial_\mu \gamma^\mu e_{R_i} \]

plus the similar terms for the weak iso-doublets and -singlets of the quark sector.

Already then is invariant under global gauge transformations. **Require** the Lagrangian to be invariant under **local** gauge transformations:

\[
L(x) \rightarrow L'(x) = e^{i\alpha_a(x) T^a + i\beta(x) Y} L(x)
\]

\[
R(x) \rightarrow R'(x) = e^{i\beta(x) Y} R(x)
\]
The Lagrangian of the massless free field can be made invariant under these \textbf{local} gauge transformations by a minimal modification: introduce the \textbf{gauge fields} to counteract the effect of the derivatives on the gauge transform functions. \textbf{Covariant derivative:}

\[
\partial_\mu \rightarrow D_\mu = \partial_\mu - ig_2 T^a W^a_\mu - ig_1 \frac{Y}{2} B_\mu
\]
The gauge fields must then transform as

\[ \tilde{W}_\mu(x) \rightarrow \tilde{W}_\mu'(x) = \tilde{W}_\mu(x) - \frac{1}{g_2} \partial_\mu \tilde{\alpha}(x) - \tilde{\alpha}(x) \times \tilde{W}_\mu(x) \]

\[ B_\mu(x) \rightarrow B_\mu'(x) = B_\mu(x) - \frac{1}{g_1} \partial_\mu \beta(x) \]

This minimal modifications uniquely defines the coupling between the fermion fields and the gauge fields:

\[ -g_i \bar{\psi} V_\mu \gamma^\mu \psi \]
Dynamics of the Gauge Fields

The dynamics of the gauge fields themselves are encoded in the field strength tensors

\[ W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu \]

\[ B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \]

Adding terms of \(-\frac{1}{4} W^a_{\mu\nu} W^a_{\mu\nu}\) to the Lagrangian introduces unique triple and quartic gauge boson couplings

triple : \(ig_2 \text{Tr}(\partial_\nu W_\mu - \partial_\mu W_\nu)[W_\mu, W_\nu]\)

quartic : \(\frac{1}{2} g_2^2 \text{Tr}[W_\mu, W_\nu]^2\)
The (massless) SM Lagrangian

\[ \mathcal{L} = -\frac{1}{4} W_a^{\mu\nu} W_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{L}_i i D_\mu \gamma^\mu L_i + \bar{e}_{R_i} i D_\mu \gamma^\mu e_{R_i} \ldots \]

Unique, starting from free fermion fields, require local gauge invariance of \( SU(2)_L \times U_1(Y) \times SU(3)_c \).

The Problem

Deals so far only with massless fields. But both fermions and the weak gauge fields are heavy. A standard mass term for a fermion breaks gauge invariance, which was the guiding principle in constructing the interactions!
The Problem of Mass-terms and Gauge Invariance

Standard mass terms would break gauge invariance

\[-m_f \bar{\psi} \psi = -m_f \bar{\psi} \left( \frac{1}{2} (1 - \gamma_5) + \frac{1}{2} (q + \gamma_5) \right) \psi = -m_f (\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R)\]

Obviously not gauge invariance, since e.g. only left handed fields transform under $SU(2)$. Also mass terms for the gauge bosons would cause trouble, e.g.

\[\frac{1}{2} M_W^2 W_\mu W^\mu\]

would generate additional terms under a gauge transformation.
In simple words, the Higgs mechanism solves the problem of introducing mass terms by coupling $\bar{\psi}_f \psi_f$ and the relevant $W_\mu W^\mu$ to a new scalar field, which counters the effect of a gauge transformation. This will leave the terms gauge invariant.

Masses are then generated by dynamically (mysteriously?) by requiring this new Higgs field to acquire a vacuum expectation value (vev).
Need to generate masses to three gauge bosons $W^\pm$ and $Z$, but the photon should remain massless for exact QED gauge symmetry. Need at least three degrees of freedom for the scalar field introduced.

Simplest choice: complex $SU(2)$ doublet of scalar fields

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad Y_\phi = +1$$

Add the following terms to the Lagrangian (free scalar field plus simplest $\Phi^4$ potential to generate vev (for $\mu^2 < 0$).

$$\mathcal{L}_S = (D^\mu \Phi)^\dagger (D_\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$$
The vev should not be in the direction of the charged field, for the photon to remain massless ($U(1)_{\text{QED}}$ to survive unbroken)

\[
\langle \Phi \rangle_0 = \langle 0 | \Phi | 0 \rangle = \begin{pmatrix} 0 \\ \frac{\nu}{\sqrt{2}} \end{pmatrix} \quad \text{with} \quad \nu = \sqrt{-\frac{\mu^2}{\lambda}}
\]
To investigate the terms generated, expand the field around the new minimum at $v$:

$$
\Phi(x) = \begin{pmatrix}
\frac{1}{\sqrt{2}}(v + H) + i\theta_1
& \theta_2 + i\theta_1 \\
\frac{1}{\sqrt{2}}(v + H) - i\theta_3
& \frac{1}{\sqrt{2}}(v + H)
\end{pmatrix}
= e^{i\theta_a(x)\tau^a} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + H) \end{pmatrix}
$$

Transform to the Unitary Gauge:

$$
\Phi(x) \rightarrow e^{-i\theta_a(x)\tau^a} \Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}
$$
The Higgs Mechanism of the Standard Model

**Gauge Boson masses:** The covariant derivative of the scalar field generates mass terms for linear combinations of the fields $W, B$:

\[ W^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp iW_\mu^2), \quad Z_\mu = \frac{g_2 W_\mu^3 - g_1 B_\mu}{\sqrt{g_2^2 + g_1^2}}, \quad A_\mu = \frac{g_2 W_\mu^3 + g_1 B_\mu}{\sqrt{g_2^2 + g_1^2}} \]

of

\[ M_W = \frac{1}{2} v g_2, \quad M_Z = \frac{1}{2} v \sqrt{g_2^2 + g_1^2}, \quad M_A = 0 \]

The relationship between the masses are **uniquely** predicted.

\[ M_W = \frac{1}{2} g_2 v = \left( \frac{\sqrt{2} g_2^2}{8 G_\mu} \right)^2 \quad \therefore \quad v = \frac{1}{(\sqrt{2} G_\mu)^{1/2}} \approx 246 \text{ GeV}. \]
**Fermion masses:** Can be generated using the same scalar field $\Phi$, and the iso-doublet $\tilde{\Phi} = i\tau_2 \Phi^*$

\[
\mathcal{L}_F = -\lambda_2 \bar{L} \Phi e_R + \cdots + h.c. = -\frac{1}{\sqrt{2}} \lambda_e (v + H) \bar{e}_L e_R + \cdots
\]

so we identify

\[
m_e = \lambda_e \frac{v}{\sqrt{2}}
\]

One parameter per fermion mass: **fermion masses not predicted, no special relationship between fermion masses.**
Kinetic part (interactions with the other fields etc.) arises from the covariant derivative, whereas the mass of the Higgs boson itself comes from the scalar potential \( V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \).

Using \( v^2 = -\mu^2 / \lambda \) one obtains

\[
V = -\frac{1}{2} \lambda v^2 (v + H)^2 + \frac{1}{4} \lambda (v + H)^4
\]

so the Lagrangian of the Higgs field can be written

\[
\mathcal{L}_H = \frac{1}{2} (\partial_\mu H)(\partial^\mu H) - V
\]

\[
= \frac{1}{2} (\partial H)^2 - \lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4.
\]
From this, we get

\[ M_H^2 = 2\lambda v^2 = -2\mu^2. \]

Higgs mass not predicted, but knowing \( M_H \) fixes \( \lambda \) and \( \mu \) (since \( v \) is fixed by e.g. the mass of the \( W \)).

Furthermore, we find the existence of three and four-Higgs-boson vertices, with couplings

\[ g_{H^3} = (3!)i\lambda v = 3i\frac{M_H^2}{v}, \quad g_{H^4} = (4!)i\frac{\lambda}{4} = 3i\frac{M_H^2}{v^2}. \]
The Higgs Boson of the Standard Model

Leptons
- $e, \mu, \tau$
- $\nu_e, \nu_\mu, \nu_\tau$

Quarks
- $u, c, t$
- $d, s, b$

Photon

$W^+/W^-$

$Z^0$

Higgs Boson

Gluons
Evolution of Higgs Self Coupling

**Triviality Bounds**: The value of the Higgs boson self coupling varies with energy (like all other couplings), determined by RGE.

\[
\frac{d\lambda}{d \log Q^2} = \frac{1}{16\pi^2} \left[ 12\lambda^2 + 6\lambda\lambda_t^2 - 3\lambda_t^4 - \frac{3}{2} \lambda (3g_2^2 + g_1^2) \right. \\
+ \left. \frac{3}{16} \left( 2g_2^4 + (g_1^2 + g_2^2)^2 \right) \right]
\]
Evolution of the Higgs Self Coupling

**Triviality Bounds**: Require that the $\phi^4$-theory remains perturbative, i.e. $\lambda \leq 1$ up to a high scale $\Lambda$: Put bounds on $\lambda$ at the EW scale, and therefore puts an **upper limit** on the Higgs boson mass.

\[
\frac{d\lambda}{d \log Q^2} = \frac{1}{16\pi^2} \left[ 12\lambda^2 + 6\lambda \lambda_t^2 - 3\lambda_t^4 - \frac{3}{2} \lambda (3g_2^2 + g_1^2) 
+ \frac{3}{16} (2g_2^4 + (g_1^2 + g_2^2)^2) \right]
\]
**Stability Bounds**: For small values of lambda, the evolution is determined from the (negative) top quark contribution. \( \lambda \) can go negative! This would mean the minimum of the Higgs potential is moved back to the origin, no spontaneous symmetry break, and no particle masses. This puts a **lower limit** on \( M_H \).

\[
\frac{d\lambda}{d \log Q^2} = \frac{1}{16\pi^2} \left[ 12\lambda^2 + 6\lambda \lambda_t^2 - 3\lambda_t^4 - \frac{3}{2}\lambda(3g_2^2 + g_1^2) \right. \\
\left. + \frac{3}{16}(2g_2^4 + (g_1^2 + g_2^2)^2) \right]
\]
Stability of the Higgs Self Coupling

Conclusion (2005): $130 \, \text{GeV} \leq M_H \leq 180 \, \text{GeV}$

Bounds very dependent on e.g. top-quark mass, $\alpha_s$, matching...
Where did the Higgs potential come from?
How stable is the Higgs mass against radiative corrections?
For a long time, people argued that since, with a cut-off regularisation, the Higgs mass had quadratic divergences, whatever value one found had to be very fine-tuned.
However, cut-off regularisation may be the very source of these problems (violates Lorentz invariance,...)! Certainly, there are no quadratic divergences in dimensional regularisation.

Recent Ideas
Perhaps the Higgs Mass and EWSB is dynamically generated from a condition of $\lambda(Q) = 0$ at some large scale.
The Standard Model Lagrangian was constructed, starting from a free fermion, and by imposing the relevant local gauge invariances. All interactions determined from the gauge principle. The Higgs field is postulated, in order to introduce gauge invariant mass terms.

Gauge Theories Rule!

Nature would rather introduce a new particle, than violate gauge invariance!
Higgs Boson Physics
Characteristics of the Higgs Boson

The masses of all other particles are well established; the characteristics of Higgs boson production and decay can be calculated as a function of the Higgs boson mass only. We can calculate the frequency with which a Higgs boson decays to specific SM particles. The only parameter is the Higgs boson mass.
The masses of all other particles are well established; the characteristics of Higgs boson production and decay can be calculated as a function of the Higgs boson mass only. We can calculate the frequency with which a Higgs boson decays to specific SM particles. The only parameter is the Higgs boson mass.
Gluon-Fusion through top (+bottom+⋯)-loop is the dominant production process. Coupling is small, but gluon flux is large!

Weak Boson Fusion (WBF) second largest production method
The parton density functions are determined using other processes and earlier experiments (non-perturbative low-scale input, perturbative evolution). Hard scattering matrix element calculated in perturbation theory.
Production cross sections

The four main production channels:

- Gluon Fusion
- Weak Boson Fusion
- Associated production/Higgs Strahlung
- $t\bar{t}H$
Part of the challenge

At first it may look hopeless. Higgs production happens much less frequently than e.g. dijet. Calculate the cross section for several simple processes, spanning 12 orders of magnitude. Requires very detailed calculations, and often special refinements for specific processes and channels.
CERN Large Hadron Collider

- 27km in circumference
- proton-proton collider
- “Roughly” 14TeV of centre-off-mass (hadronic) energy
The Higgs Boson Is There!

**ATLAS**

<table>
<thead>
<tr>
<th></th>
<th>ATLAS expected</th>
<th>observed</th>
<th>CMS expected</th>
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<td>7.1</td>
<td>6.7</td>
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<tr>
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<td></td>
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</table>
Current best CMS fit for signal strength compared to SM: $0.80 \pm 0.14$.
Current best ATLAS fit for signal strength compared to SM: $1.30 \pm 0.20$. 
Coupling strength depends on the particle masses as required by the SM Higgs mechanism.
Measurement of New Boson Mass Stable
• In a **combined search** for the SM Higgs boson, a significant excess of events near $m_H = 126$ GeV persists beyond any doubt and now has been **established in individual decay channels**: $ZZ$, $WW$, $\gamma\gamma$

• **New boson’s mass**:
  - CMS: $125.7 \pm 0.4$ GeV
  - ATLAS: $125.5 \pm 0.6$ GeV

• **Is $X_{125}$ the SM Higgs boson?**
  - event yields in all individual channels are consistent with the SM Higgs boson
  - couplings agree with the SM Higgs boson with the current statistical accuracy: 20% ($W$ & $Z$), 25% ($t$), 30% ($\bar{t}$), 60% ($b$)
  - no significant modifications for loop-induced couplings (deviations < 2$\sigma$)
  - $\text{BR}(H \rightarrow \text{BSM}) < 0.5$ (approx.) at 95% CL
  - 100% pure $J^{CP} = 0^-, 1^\pm, 2^+_m$ states are excluded at >99% CL
  - CP-odd fractional contribution: $f(0^-) < 0.58$ at 95% CL

... as presented at Les Houches, June 4.