Theories with TeV-Scale Extra Dimensions

Many interesting scenarios!
with distinct phenomenology!

- General pedagogical introduction
- Examine phenomenology of various scenarios
  - ⇒ Collider signatures

Why are we interested in Extra Dimensions?

- They address many outstanding questions:
  - hierarchy problem
  - electroweak symmetry breaking (w/o Higgs!)
  - fermion masses / CKM / $\not{CP}$
  - neutrino masses
  - unification (w/o SUSY!)
  - new Dark Matter candidate
    + new cosmology perspective

    ⋮

- They lead to testable predictions in current + future experiments
- They would dramatically alter our view of the universe
- They could lead to string theory

A Few Concepts from Kindergarten String Theory…

Physics of Branes: Spatial Dimensional Subspace

Our 3+1-dim subspace = 3-brane

Embedded in D = 3+6+1 space = bulk

Bulk

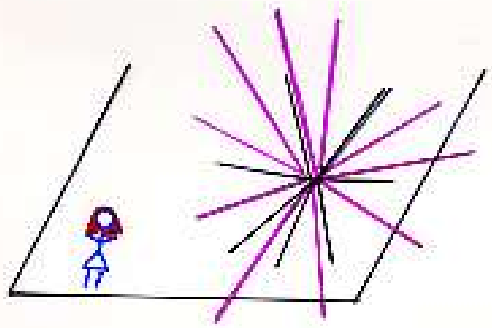
Size + geometry of bulk can vary

String Theory provides mechanism to `localize'
fields on brane

- Gauge theories live on brane
- Gauge particles live at end of strings
- Closed strings are neutral

  ⇒ can pop off brane = bulk gravitons

Compactification



Standard Model forces stuck on 3-brane

Gravitational fields spread out over all spacetime

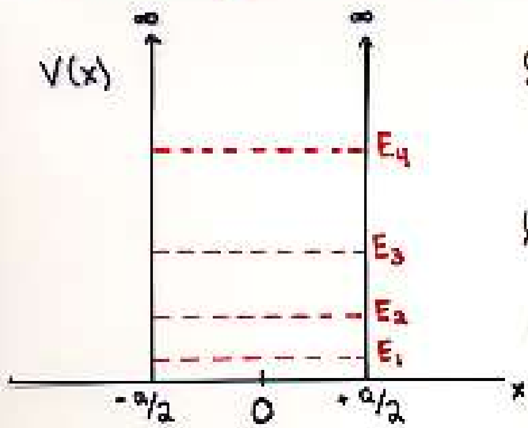
Are gravitational fields diluting too quickly?

⇒ Extra dimensions must be compactified!

$F_{Gr} \sim \frac{1}{r^2}$ recovered on 3-brane

Particle in a Box

Infinite Square-Well potential



Sol'n to Schroedinger Eqn:

$$\psi_n(x) = \begin{cases} A_n \cos K_n x, & n = 1,3,5,\ldots \\ B_n \sin K_n x, & n = 2,4,6,\ldots \end{cases}$$

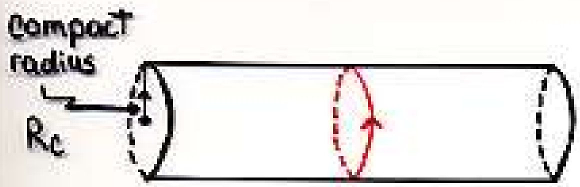
where $K_n = n\pi/a$

Momentum of the particle is <u>Quantized!</u>

$E_n \sim n^2/a^2$ (non-relativistic)

Fields in Compact Dimensions:

Expand into Kaluza-Klein towers



δ-d Kinetic motion is quantized!

$$p_\delta^2 = \frac{\vec{n}\cdot\vec{n}}{R_c^2}$$

Appears as tower of massive particles in 4-d

$$m_{\vec{n}}^2 = \frac{\vec{n}\cdot\vec{n}}{R_c^2}$$

with identical spin + quantum numbers

mode numbers $\vec{n} = (n_1, n_2, \ldots n_\delta)$ label KK excitation

Space-like vs Time-like

- Consider particle of mass M in 5D co-ords
- Assume Lorentz invariance holds in 5D

$$\Rightarrow \quad p^2 = M^2$$

$$p^2 = p_0^2 - \vec{p}^{\,2} \pm p_5^2 \qquad \{\, O(4,1) \text{ or } O(3,2)?\}$$

energy

momentum

momentum in 5^{th} dim

SIGN?

$+$ time-like 5^{th} dim
$-$ space-like

Is there a preference?

$$p^2 = M^2 \longrightarrow p_0^2 - \vec{p}^{\,2} = p_\mu p^\mu = M^2 \mp p_5^2$$

if $p_5^2 > M^2$ (why not?) and 5^{th} dim is time-like, then $\underline{p_\mu p^\mu < 0}$

$\Rightarrow$ Tachyon with possible causality problems

To avoid tachyons we generally choose extra dims to be space-like!

$\therefore$ only 1 time dimension

Consider one extra dimension:

a simple extension of 4D

$$ds^2 = \eta_{\mu\nu}\, dx^\mu dx^\nu - dy^2 \quad [dx_5^2]$$

flat, space-like 5^{th} d

5D Klein Gordon Eqn: (real scalar)

$$\underbrace{\partial_A \partial^A}_{\partial_\mu \partial^\mu - \partial_y^2} \Phi(x_\mu, y) = 0$$

separate the variables: $\Phi \equiv \sum_n \chi_n(y)\, \phi_n(x_\mu)$

→ Kaluza-Klein (KK) decomposition

Klein Gordon says: $\sum_n \left(\chi_n \partial_\mu \partial^\mu \phi_n - \phi_n \partial_y^2 \chi_n\right) = 0$

if $\partial_y^2 \chi_n = -m_n^2 \chi_n$ (quantized)

$$\sum_n \chi_n \underbrace{(\partial_\mu \partial^\mu + m_n^2)\, \phi_n = 0}_{0}$$

a set of n independent eqns

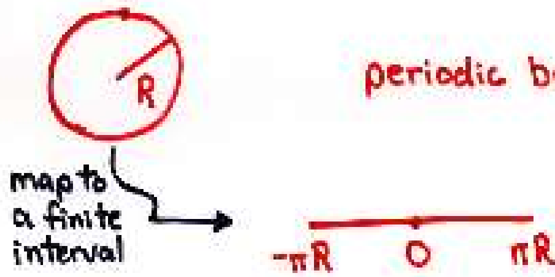
∞ set of massive scalar states

→ a KK tower

$A \to (0, 1\text{-}5)$
$\mu \to (0, 1\text{-}3)$

Motion in a circle (orbital angular momentum)

$S^1 \equiv$



periodic boundary conditions

$$\chi_n(y + 2\pi R) = \chi(y)$$

$$\Rightarrow \chi_n = A_n \cos{}^{ny}/_R + B_n \sin{}^{ny}/_R \quad ; \quad n = 0, 1, 2, \cdots$$

$$\boxed{m_n = {}^{n}/_{R}}$$

Recall
$\partial_y^2 \chi_n = -m_n^2 \chi_n$

KK masses $\sim \frac{1}{\text{size}}$ of extra dimension

(for flat space)

Action Approach:

$$S = \int d^4x \int_{y_1}^{y_2} dy \ \underbrace{\tfrac{1}{2}\partial_A \Phi \partial^A \Phi}_{\tfrac{1}{2}\partial_\mu \Phi \partial^\mu \Phi - \tfrac{1}{2}\partial_y \Phi \partial^y \Phi}$$

let $\Phi \equiv \sum_n \chi_n(y)\, \phi_n(x_\mu)$

$$S = \int d^4x \int_{y_1}^{y_2} dy \left\{ \tfrac{1}{2} \sum_{nm} \chi_n \chi_m \overset{①}{\partial_\mu} \phi_n \partial^\mu \phi_m - \tfrac{1}{2} \sum_{nm} \phi_n \overset{②}{\phi_m} \partial_y \chi_n \partial_y \chi_m \right\}$$

To Diagonalize:

① $\int_{y_1}^{y_2} dy \ \chi_n \chi_m = \delta_{nm}$ orthonormal wavefunctions

② integrate-by-parts $\int_{y_1}^{y_2} dy \ \partial_y \chi_n \partial_y \chi_m$

$$= \underbrace{\chi_m \partial_y \chi_n \Big|_{y_1}^{y_2}}_{0} - \int_{y_1}^{y_2} dy \ \chi_m \underbrace{\partial_y^2 \chi_n}_{-m_n^2 \chi_n}$$

Boundary conditions! quantized

$$S = \int d^4x \sum_n \left(\tfrac{1}{2}\partial_\mu \phi_n \partial^\mu \phi_n - \tfrac{1}{2} m_n^2 \phi_n^2 \right)$$

n massive scalars

$$(\partial_y^2 + m_n^2)\chi_n = 0 \Rightarrow \chi_n = A_n e^{i m_n y} + B_n e^{-i m_n y}$$

plane waves!

Higher Dimensional Field Decomposition

We saw a 5D scalar → a tower of 4D scalars

What about a 5-vector A^m
or symmetric tensor h^{mN} ???

Recall:

	Lorentz (4D)	⟷	Rotations (3D)
	scalar		scalar
4-vector	A^μ		$\vec{A}, \phi$
tensor	$F^{\mu\nu}$		$\vec{E}, \vec{B}$

Now

	5D	⟷	4D
	scalar		scalar $(s)_n$
vector	A^m		$(A^\mu, A^5)_n$
tensor	h^{mN}		$\underbrace{(h^{\mu\nu}, h^{\mu 5}, h^{55})_n}_{\text{KK towers}}$

For δ extra dimensions:

	δ D	⟷	4D $(i = 1 \ldots \delta)$
	scalar		δ scalars
vector	A^m		$(A^\mu, A^i)_{ni}$
tensor	h^{mN}		$(h^{\mu\nu}, h^{\mu i}, h^{ij})_{ni}$

1 tensor, δ 4-vectors, $\frac{1}{2}\delta(\delta+1)$ scalars

- **Experimental observation of KK states:**

  Signals existence of extra dimensions

- **Properties of KK states:**

  Determined by geometry of extra dimensions

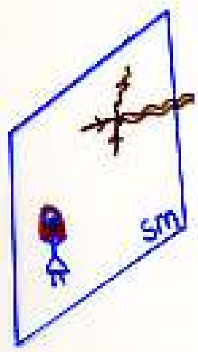
  $\Rightarrow$ measured by experiment!

The physics of extra dimensions is
the physics of the KK excitations

Large Extra Dimensions

Arkani-Hamed,
Dimopoulos, Dvali
SLAC-PUB-7801

Motivation: Solve the hierarchy problem by removing it!



SM fields confined to 3-brane

Gravity becomes strong in the bulk

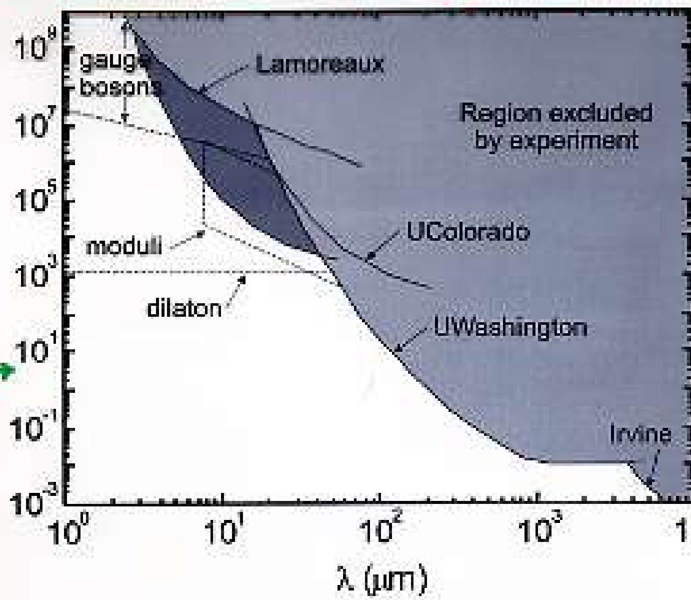
Gauss' Law: $M_{Pl}^2 = V_\delta M_D^{2+\delta}$; $V_\delta \sim R_c^\delta$

M_D = Fundamental scale in the bulk
$\simeq$ TeV

$\delta = 1$	$R_c \sim 10^{11}$ m	Excluded!
2	0.4 mm	$m_c = 1/R_c \sim 5 \times 10^{-4}$ eV
4	10^{-5} mm	20 KeV
6	30 fm	7 MeV

Constraints from Cavendish-type exp'ts

Parameterized as $\Delta V = -G_N m m \left\{ \alpha \, e^{-\lambda r}/r \right\}$



2 extra dims $\rightarrow$ 10^1

Chiaverini etal hep-ph/020932

$$V_{gravity} \sim \frac{m_1 m_2}{M_D^{2+\delta}} \frac{1}{r^{\delta+1}} \qquad (r < R_c)$$

$$\sim \frac{m_1 m_2}{M_{Pl}^2} \frac{1}{r} \qquad (r > R_c)$$

For $\delta = 2$: $\lambda \leq 190\,\mu$ $[M_D \gtrsim 1.8\text{ TeV}]$

Constraints from Astrophysics/Cosmology

(*) Supernova Cooling

Cullen + Perelstein
Barger et al
Savage et al

$NN \rightarrow NN + G_n$ can cool supernova too rapidly

(*) Cosmic Diffuse γ Rays

$NN \rightarrow NN + G_n$, $G_n \hookrightarrow \gamma\gamma$ — Hannestad + Raffelt

$\nu\bar{\nu} \rightarrow G_n \rightarrow \gamma\gamma$ — Hall + Smith

(*) Matter Dominated Early Universe — Fairbairn

too many KK states

(*) Neutron Star Heat Excess — Hannestad + Raffelt

$NN \rightarrow NN + G_n$

$\hookrightarrow$ becomes trapped in neutron star halo + heats it

Constraints from Astrophysics/Cosmology

M_D (TeV)	$\delta = 2$	3	4	5
Supernova Cooling	30	2.5		
Cosmic Diffuse γ-Rays				
Sne	80	7		
$\upsilon\bar{\upsilon}$ Annihilation	110	5		
Reheating	170	20	5	1.5
Neutron Star	450	30		
Matter Dominated Universe	85	7	1.5	
Neutron Star Heating	1700	60	4	1

Low M_D disfavored for $\delta \leq 3$

Bulk Metric: Linearized Quantum Gravity

$$G_{AB} = \eta_{AB} + \frac{h_{AB}(x^\mu, y^a)}{M_D^{\delta/2+1}}$$

$A = 0, \ldots, 3+\delta$
$\mu = 0,1,2,3$
$a = 4, \ldots, 3+\delta$

Interactions:

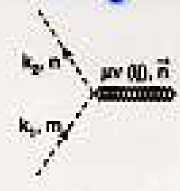
$$S_{int} = \frac{-1}{M_D^{\delta/2+1}} \int d^4x\, d^\delta y\, h_{AB}(x^\mu, y^a)\, T_{AB}(x^\mu, y^a)$$

- Perform Graviton KK reduction
- Expand h_{AB} into KK tower
- SM on 3-brane
  - $\Rightarrow$ Set $T_{AB} = \eta^\mu{}_A \eta^\nu{}_B T_{\mu\nu} \delta(y^a)$
- Pick a gauge
- Integrate over $d^\delta y$

$\Rightarrow$ Interactions of Graviton KK states with SM fields on 3-brane

Feynman Rules - Graviton KK tower

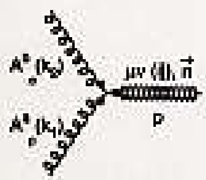
Massless 0-mode + KK states have identical coupling to matter



$$\tilde{h}^{\vec{n}}_{\mu\nu}\phi\phi: \quad -i\kappa/2\,\delta_{nm}\,(m_\phi^2\,\eta_{\mu\nu} + C_{\mu\nu,\rho\sigma}\,k_1^\rho k_2^\sigma)$$

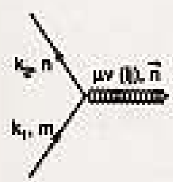
$$\tilde{\phi}^{\vec{n}}_{ij}\phi\phi: \quad i\omega\kappa\,\delta_{ij}\,\delta_{nm}\,(k_1\cdot k_2 - 2\,m_\phi^2)$$

$\kappa \sim \dfrac{1}{M_{Pl}}$



$$\tilde{h}^{\vec{n}}_{\mu\nu}AA: \quad -i\kappa/2\,\delta^{ab}\,(\,(m_A^2 + k_1\cdot k_2)\,C_{\mu\nu,\rho\sigma} + D_{\mu\nu,\rho\sigma}(k_1,k_2) + \xi^{-1}\,E_{\mu\nu,\rho\sigma}(k_1,k_2)\,)$$

$$\tilde{\phi}^{\vec{n}}_{ij}AA: \quad i\omega\kappa\,\delta_{ij}\,\delta^{ab}\,(\eta_{\rho\sigma}\,m_A^2 + \xi^{-1}\,(k_{1\sigma}\,p_\rho + k_{2\rho}\,p_\sigma))$$



$$\tilde{h}^{\vec{n}}_{\mu\nu}\psi\psi: \quad -i\kappa/8\,\delta_{nm}\,(\gamma_\mu\,(k_{1\nu} + k_{2\nu}) + \gamma_\nu\,(k_{1\mu} + k_{2\mu}) - 2\,\eta_{\mu\nu}\,(\not{k}_1 + \not{k}_2 - 2\,m_\psi))$$

$$\tilde{\phi}^{\vec{n}}_{ij}\psi\psi: \quad i\omega\kappa\,\delta_{ij}\,\delta_{nm}\,(3/4\,\not{k}_1 + 3/4\,\not{k}_2 - 2\,m_\psi)$$

Figure 4: Three-point vertex Feynman rules. The KK states are plot in double-sinusoidal curves. The symbols $C_{\mu\nu,\rho\sigma}$, $D_{\mu\nu,\rho\sigma}(k_1,k_2)$ and $E_{\mu\nu,\rho\sigma}(k_1,k_2)$ are defined in Eqs. (A.10), (A.11) and (A.12) respectively. m_ϕ, m_A and m_ψ are masses of the scalar, vector and fermion. $\omega = \sqrt{\frac{2}{3(n+2)}}$, $\kappa = \sqrt{16\pi G_N}$ and ξ is the gauge-fixing parameter.

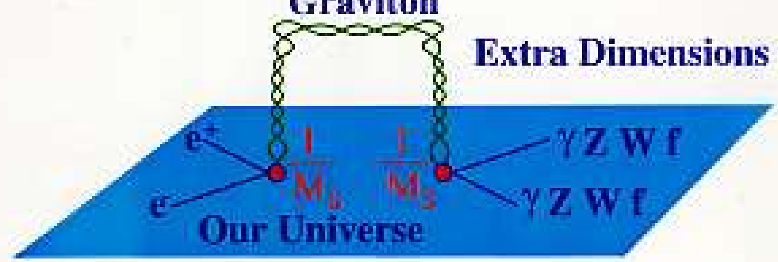
Han, Lykken, Zhang
Giudice, Rattazzi, Wells

Collider Tests

Search Strategy

Direct Search: 1 photon or 1 Z boson + missing energy.

Indirect Search: Look for deviations from $(d\sigma/d\Omega)_{SM}$.



Graviton Tower Exchange

$XX \to G_n \to YY$

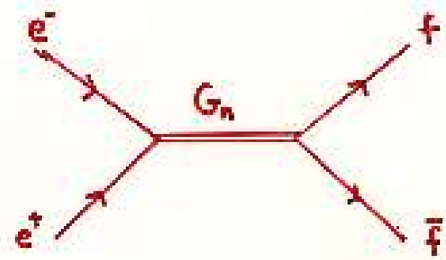
Search for 1) Deviations in SM processes

2) New processes! $(gg \to \ell^+\ell^-)$

Angular distributions reveal spin-2 exchange

Consider $e^+e^- \to f\bar{f}$

$$\mathcal{M} = \frac{1}{16\, m_{Pl}^2} \sum_{n} \frac{T^{e}_{\mu\nu}\, P^{\mu\nu\lambda\sigma}\, T^{f}_{\lambda\sigma}}{s - m_n^2 + i\epsilon}$$



G_n are densely packed!

$(M_D R_c)^\delta$ states are exchanged! $(\sim 10^{32}$ for $\delta = 2)$

$$\Rightarrow \sum_{n} \to \int dm^2 \rho(m^2)$$

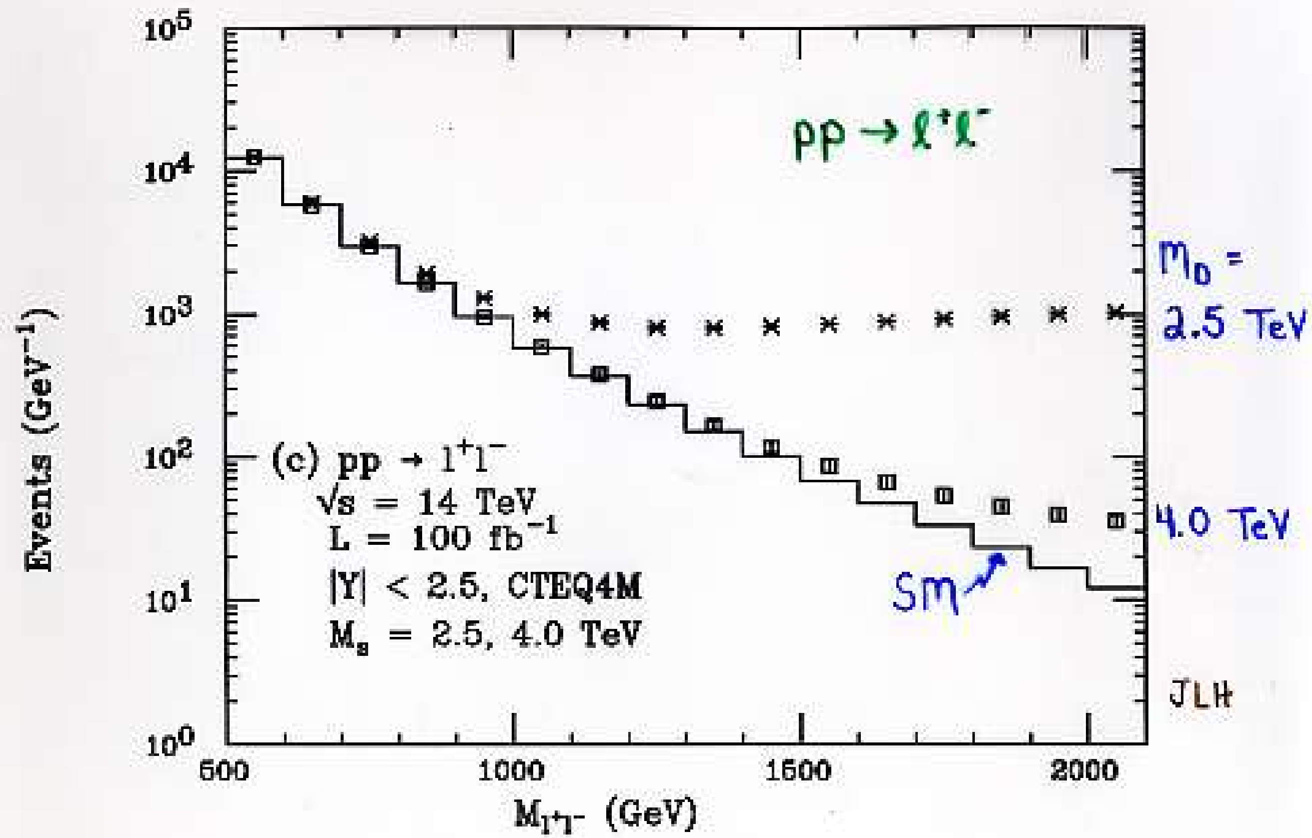
$$\frac{1}{m_{Pl}^2} \sum_{n} \frac{1}{s - m_n^2} \to \frac{1}{M_D^4} \to \frac{1}{\Lambda^4}$$

JLH, PRL 99

Giudice, Rattazzi, Wells

$$\mathcal{O} = \frac{4}{\Lambda^4}\, T_{\mu\nu} T^{\mu\nu}$$

Drell-Yan Production at LHC



Search Reach at Future Colliders

		$\sqrt{s}$	M_D (TeV)
LC:	$e^+e^- \to f\bar{f}$	500 GeV	5.0
		1 TeV	8.4
	$\gamma\gamma \to \gamma\gamma$	1 TeV	3.5
	$\gamma\gamma \to WW$		13.0
	$e\gamma \to e\gamma$		8.0
LHC:	$pp \to l^+l^-$	14 TeV	7.5
	$pp \to \gamma\gamma$		7.1

(@ design luminosity)

LHC/LC Explore the parameter space which is relevant to the hierarchy!

Limits from Virtual G_{KK} Effects

H. Zheng

- Different notations used in different processes:
  - M_D is the fundamental mass scale – real graviton
  - M_S is the ultraviolet cutoff of the divergent sum over the KK excitations – virtual effects
- No exact relation between M_D and M_S is available
- M_D and M_S are expected to be of the same order

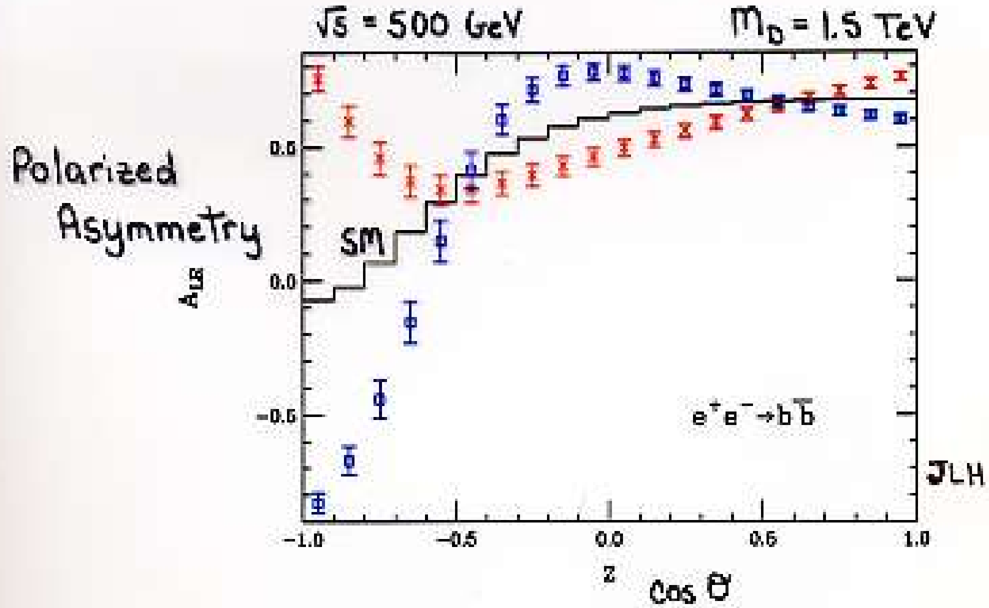
Hewett convention

DØ [PRL 86 (2001) 1156]: Ms (λ = +1) > 1.1 TeV; Ms (λ = -1) > 1.0 TeV

LEP Combined Results [hep-ex/0111063 v2]: Ms (λ = +1) > 1.0 TeV; Ms (λ = -1) > 1.1 TeV

CDF Preliminary [hep-ex/0111063 v2]: Ms (λ = +1) > 0.8 TeV; Ms (λ = -1) > 0.9 TeV

Angular Distributions in $e^+e^- \to f\bar{f}$



- Governed by spin of exchanged particle

Expand $\frac{d\sigma}{d\cos\theta}$ in moments of $P_n(\cos\theta)$

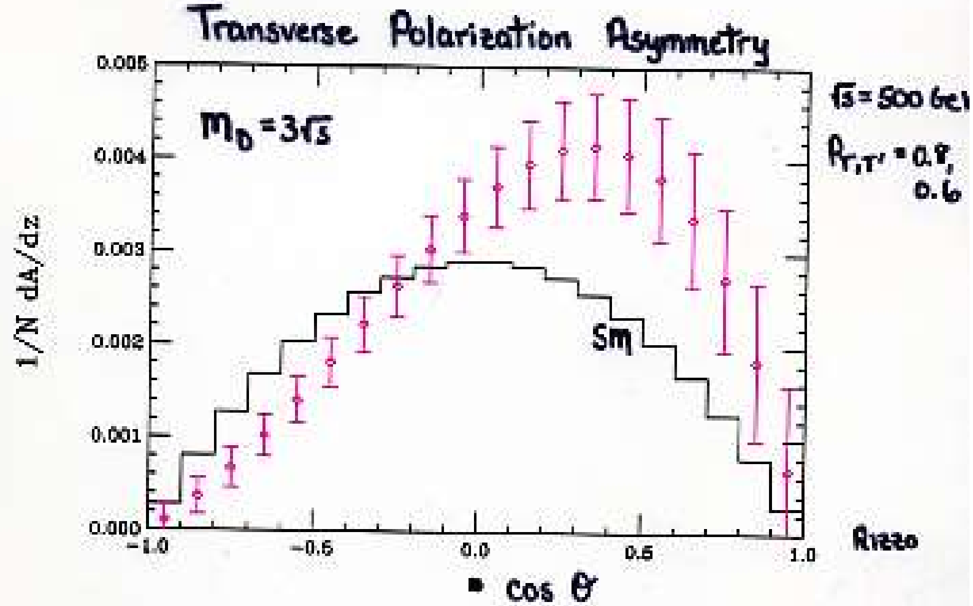
Spin-2 exchange: $\langle P_{3,4}(\cos\theta)\rangle \neq 0$
$\langle P_{n>4}(\cos\theta)\rangle = 0$

Fit to simulated $e^+e^- \to f\bar{f}$ data: Rizzo

5σ ID of spin-2 for $M_D \lesssim (5\text{-}6)\sqrt{s}$

Graviton Exchange with Transverse Polarization

$$|\mathcal{M}|^2 = \frac{1}{4}(1-P_L P_L')(|T_+|^2+|T_-|^2) + (P_L - P_L')(|T_+|^2 - |T_-|^2)$$
$$+ (2P_T P_T')[\cos 2\phi \, Re(T_+T_-^*) - \sin 2\phi \, Im(T_+T_-^*)]$$



$\sqrt{s}$ (TeV)	Search Reach 95% CL (TeV)	Spin-2 ID Reach 95% CL (TeV)
0.5	10.2	5.4
1.0	21.5	11.1
1.5	32.7	16.7
	$\simeq 21\sqrt{s}$	$\simeq 11\sqrt{s}$!

Graviton Tower Emission

Giudice, Rattazzi, Wells
Mirabelli, Perelstein, Peskin

- $e^+e^- \rightarrow \gamma/Z + G_n$
- $q\bar{q} \rightarrow g + G_n$
- $Z \rightarrow f\bar{f} + G_n$

G_n appears as missing energy
Model independent - Probes M_D directly
Sensitive to δ

Parameterized by density of states

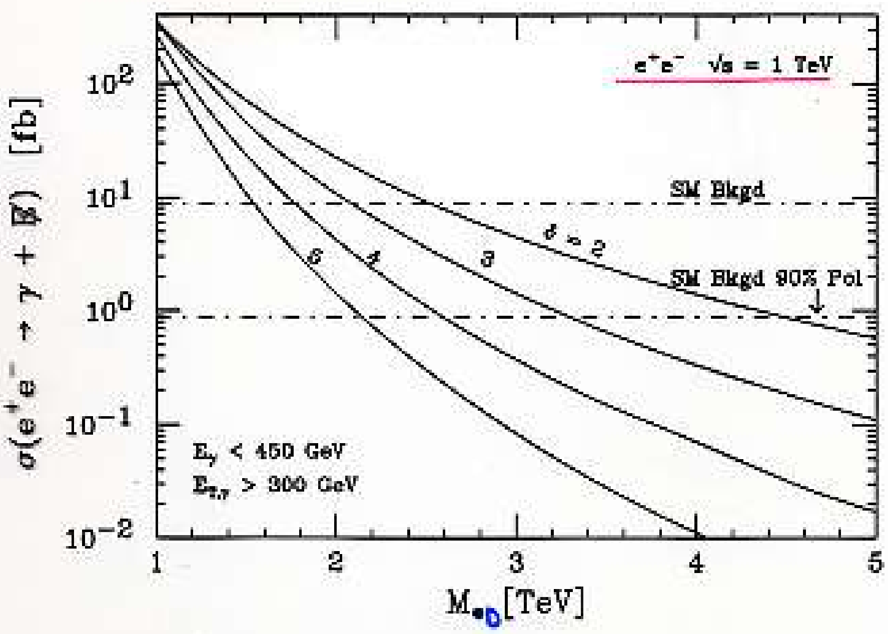
$$\sigma \sim \frac{1}{M_{Pl}^2} (E R_c)^\delta \rightarrow \frac{1}{M_D^2} \left(\frac{E}{M_D}\right)^\delta$$

Discovery Reach for M_D (TeV)

$e^+e^- \rightarrow \gamma + G_n$	$\sqrt{s} = 800$ GeV	$\delta = 2$	4	6
	$P_+ = 0$	5.9	3.5	2.5
Tesla TDR	$P_- = 0.8$	8.3	4.4	2.9
	$P_- = 0.8$, $P_+ = 0.6$	10.4	5.1	3.3

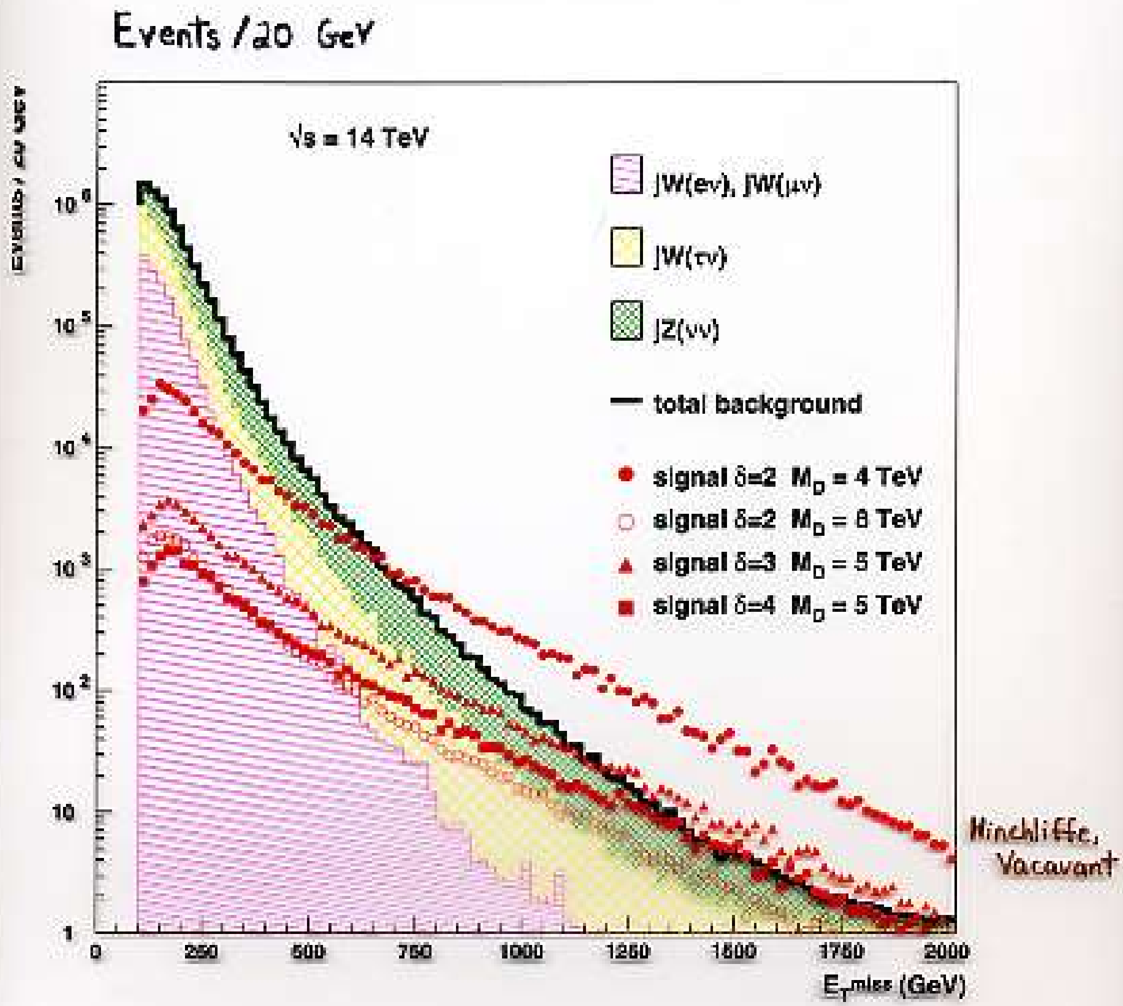
$pp \rightarrow g + G_n$	LHC	$\delta = 2$	3	4
Hinchliffe + Vacavant		4 - 8.9	4.5 - 6.8	5.0 - 5.8

Graviton Emission : $e^+e^- \to \gamma + G^{(n)}$

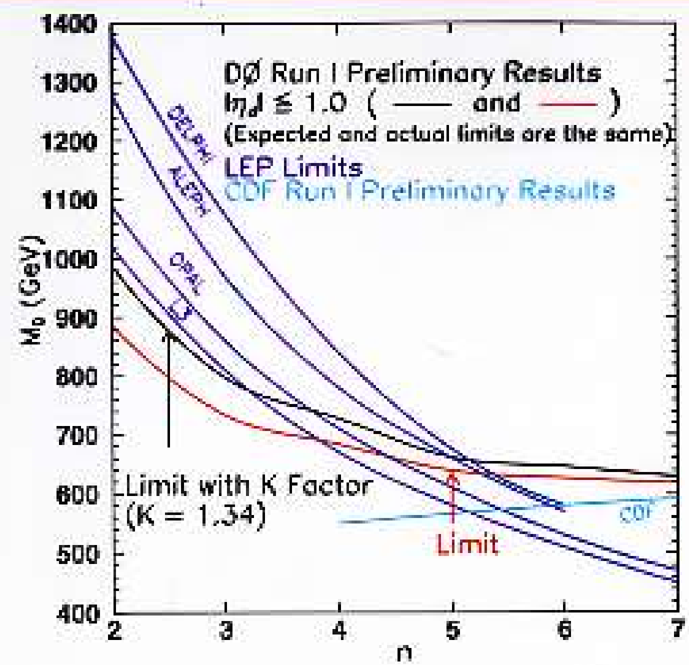


Giudice, Rattazzi, Wells

Simulated Graviton Emission at LHC



Limits


DØ Run I Preliminary Results
$|\eta_d| \leq 1.0$ (—— and ——)
(Expected and actual limits are the same)
LEP Limits
CDF Run I Preliminary Results
DELPHI
ALEPH
OPAL
L3
CDF
Limit with K Factor
(K = 1.34)
Limit
M_0 (GeV)
n
1400
1300
1200
1100
1000
900
800
700
600
500
400
2
3
4
5
6
7

Limits from $G_{KK}\gamma$ Emission

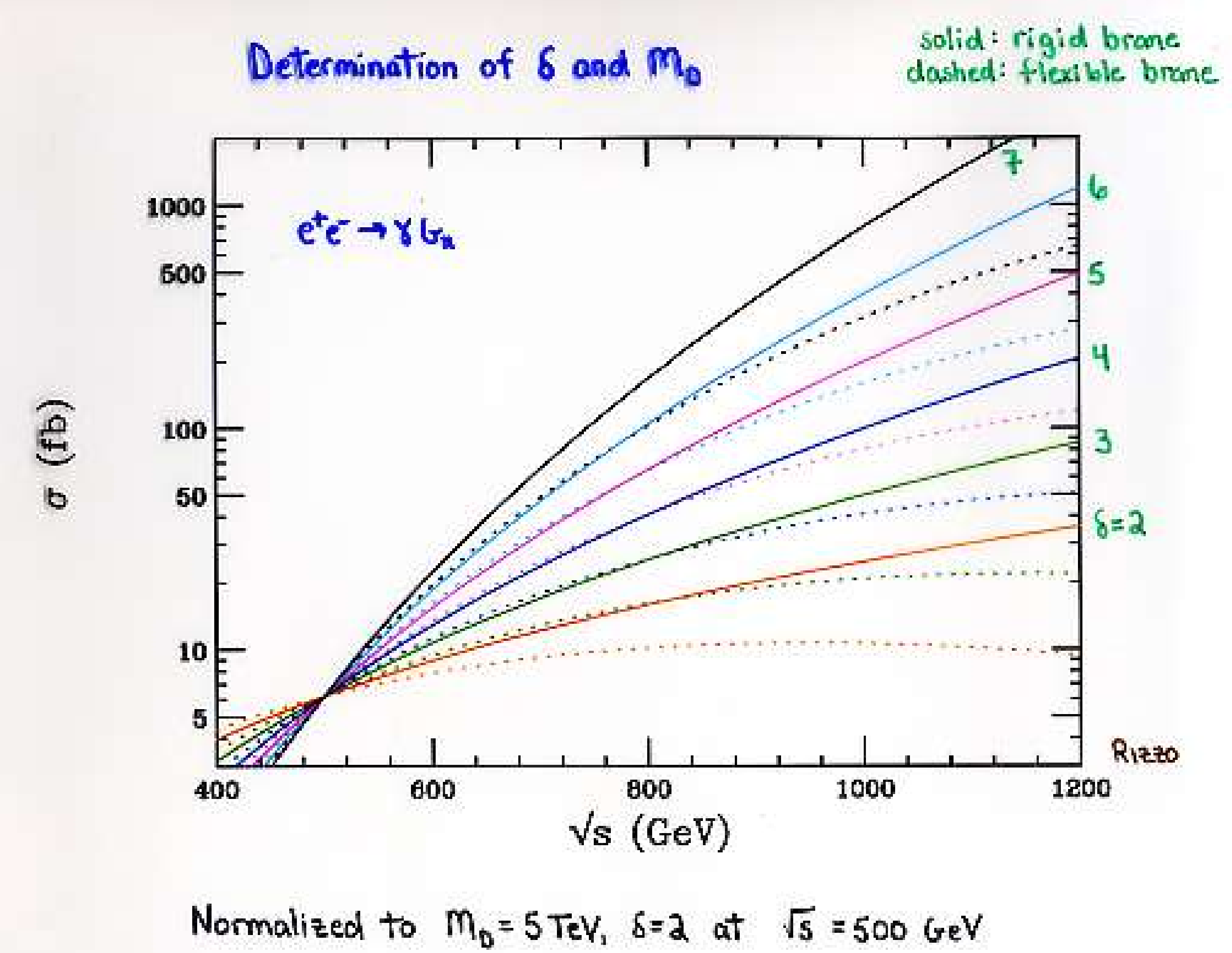
H. Zheng

LEP: B. Vachon; hep-ex/0201029 v2

n	2	3	4	5	6	7
ALEPH (189-209 GeV) M_D Limit (TeV)	1.28	0.97	0.78	0.66	0.57	—
DELPHI (181-209 GeV) M_D Limit (TeV)	1.38	—	0.84	—	0.58	—
L3 (189 GeV) M_D Limit (TeV)	1.02	0.81	0.67	0.58	0.51	0.45
OPAL (189 GeV) M_D Limit (TeV)	1.09	0.86	0.71	0.61	0.53	0.47

CDF: hep-ex/0205057

n	4	6	8
M_D Limit (TeV)	0.55	0.58	0.60



Determination of δ and M_D
solid: rigid brane
dashed: flexible brane
$e^+e^- \to \gamma G_n$
7
6
5
4
3
δ=2
1000
500
100
50
10
5
σ (fb)
400
600
800
1000
1200
√s (GeV)
Rizzo
Normalized to M_D = 5 TeV, δ=2 at √s = 500 GeV

Branes gone soft: Graviton Emission $e^+e^- \to \gamma G_n$

soft brane
can recoil

Suppresses KK tower
Couplings

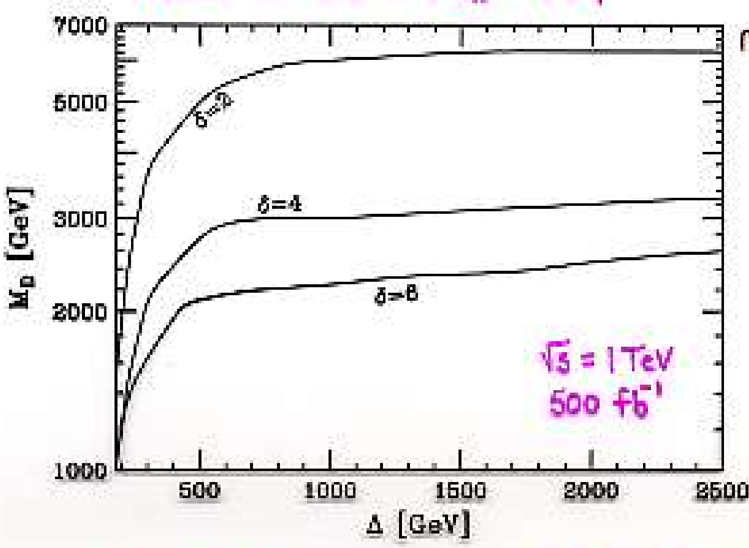
$$g_n^2 \to g_n^2\, e^{-m_n^2/\Delta^2}$$

$\Delta \sim \sqrt{T}$ wall tension
$\lesssim m_D$

Search Reach is reduced

$$\left.\frac{d^2\sigma}{dx_\gamma\, d\cos\theta}\right|_{soft} \to \left.\frac{d^2\sigma}{dx_\gamma\, d\cos\theta}\right|_{stiff} e^{-s(1-x_\gamma)/\Delta^2}$$

Reach in $e^+e^- \to \gamma G_n \to \gamma \not{E}_T$



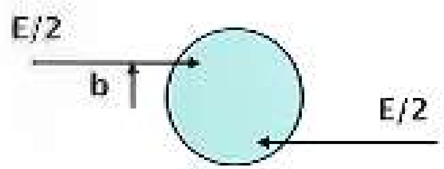
⇐ soft brane stiff brane ⇒

Black Hole Production @ LHC:

Dimopoulos, Landsberg
Giddings, Thomas

Black Holes produced when $\sqrt{s} > M_*$

Classical Approximation: [space curvature << E]



$b < R_s(E) \Rightarrow$ BH forms

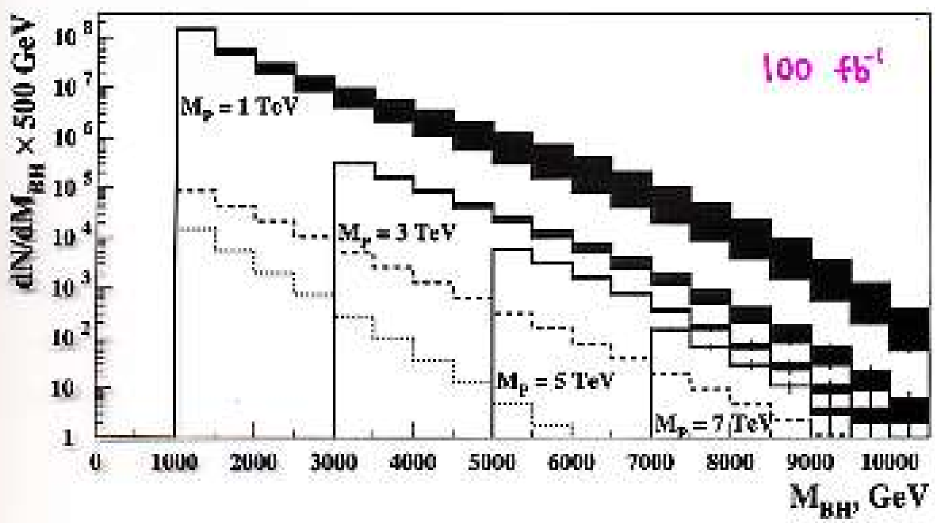
$$M_* R_s = \left[\frac{\Gamma(\frac{n+3}{2})}{(n+2)\pi^{(n+3)/2}} \frac{M_{BH}}{M_*} \right]^{1/(n+1)}$$

$M_{BH} \sim \sqrt{\hat{s}}$

Geometric Considerations:

$\sigma_{\text{Naïve}} = \pi R_s^{\,2}(E)$, details show this holds up to a factor of a few

Number of BHs produced at LHC



Dimopoulos
Landsberg

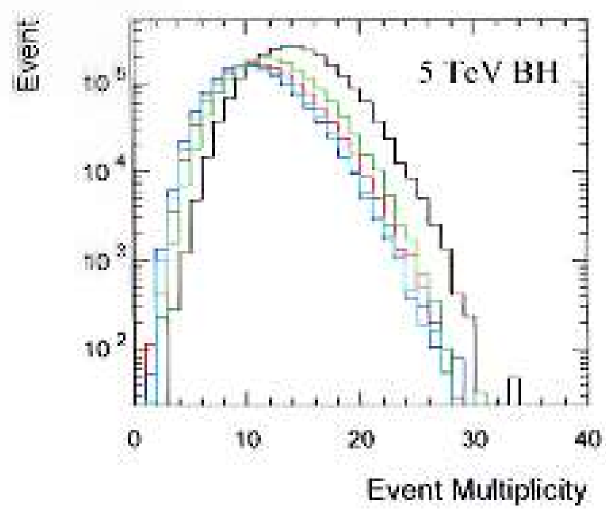
Decay via Hawking radiation into ~25 partons

Decay Properties of Black Holes:

Decay proceeds by thermal emission of Hawking radiation

$$T_H = \frac{(n+1)M_*}{4\pi}\left[\frac{\Gamma(\frac{n+3}{2})}{(n+2)\pi^{(n+3)/2}}\frac{M_{BH}}{M_*}\right]^{-1/(n+1)}$$

At fixed M_{BH}, higher dimensional BH's are hotter



$\langle N \rangle \sim 1/\langle T \rangle$

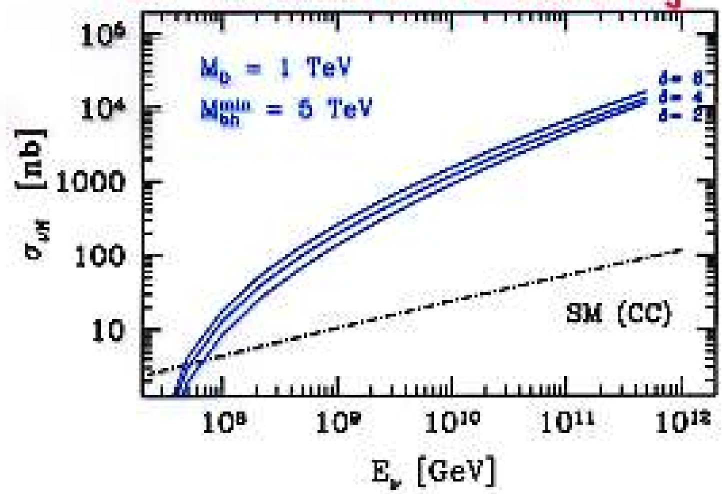
$\Rightarrow$ higher dimensional BH's emit fewer quanta, with each quanta having higher energy

Multiplicity for n = 2 to n = 6

Harris etal hep-ph/0411022

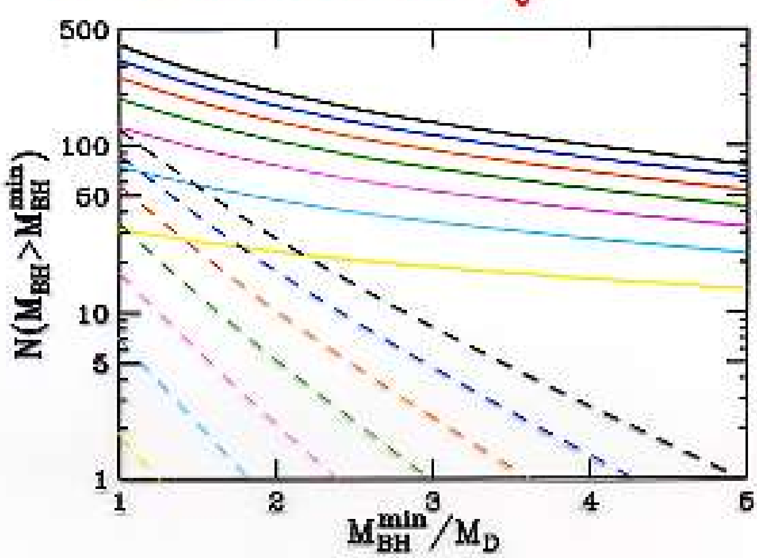
Cosmic Ray Sensitivity to Black Hole Production

BH Production in νN Scattering



Ringwald, Tu

BH Event rates at Auger

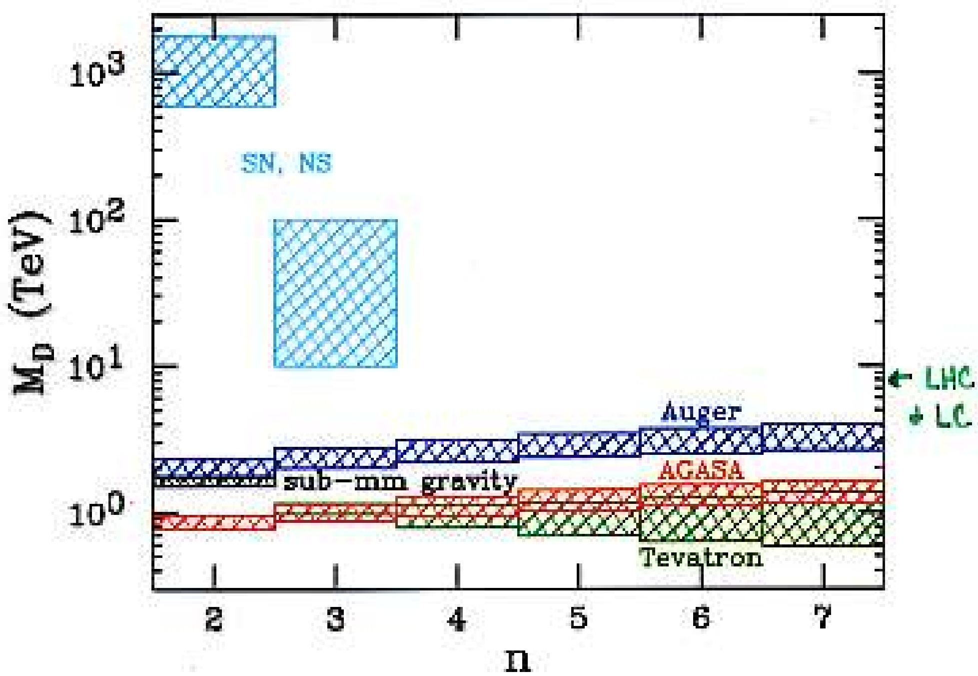


Anchordoqui et al

without

with Voloshin suppression

Summary of Exp't Constraints on M_D



Anchordoqui, Feng, Goldberg, Shapere

Questions you might ask:

1) Doesn't string / M-theory fix $n=6,7$?

2) Aren't there models where the SM gauge fields have TeV scale KK excitations ?

3) Do all n dimensions have to be the same size?

$$m_{Pl}^2 = V_n m_*^{n+2} \Rightarrow \text{in principle } V_n \sim R_1 R_2 \ldots R_n$$

Let $R^n = R_1^p R_2^{n-p}$ with $R_1 \sim$ large
$R_2 \sim$ small $\sim 1/\text{TeV} \sim 1/m_*$

$$\Rightarrow m_{Pl}^2 = R_1^p m_*^{p-n} m_*^{n+2}$$
$$= R_1^p m_*^{p+2} \quad \text{with } 2 \leq p \leq 6$$

SM fields can propagate in small R_2^{n-p} dimensions

TeV^{-1} - size Extra Dimensions

Can arise naturally in string theory   Antoniadis

The SM goes into the bulk!

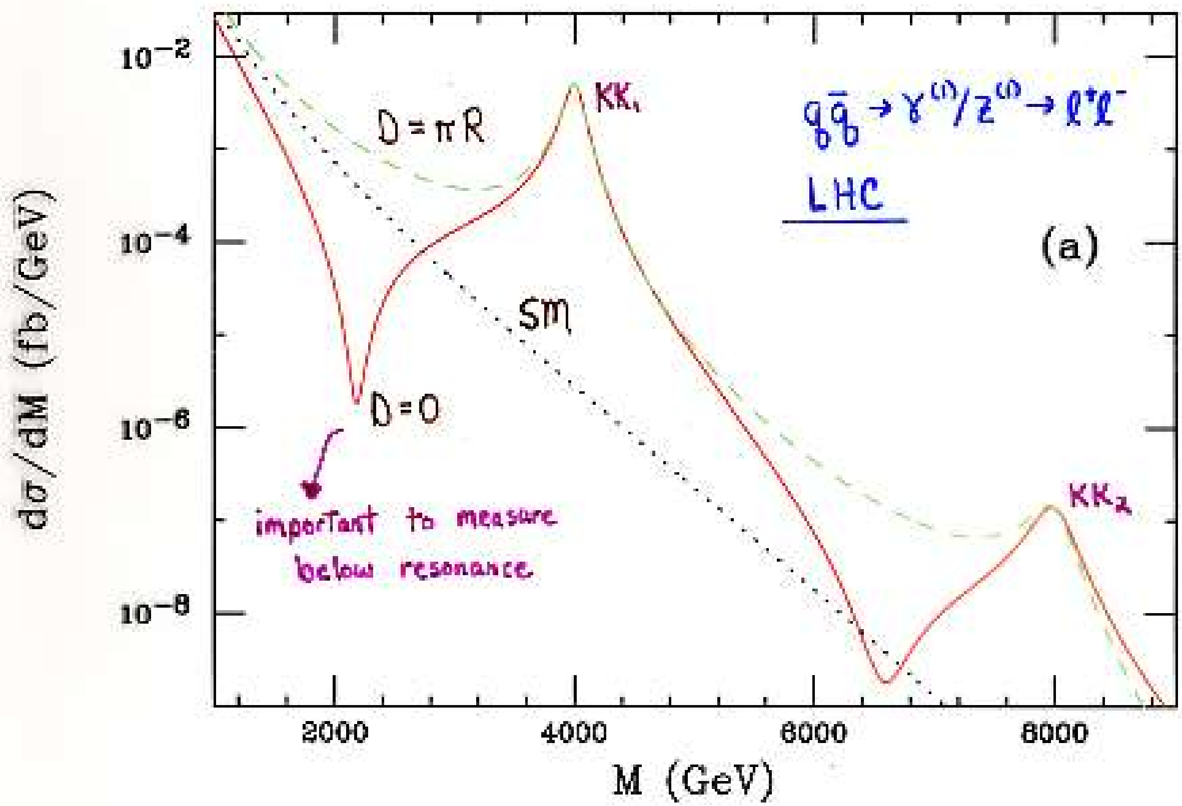
Model building choices:

- Gauge fields in the bulk
- Higgs in bulk or on brane?
- Fermions
  - fixed points
  - bulk
  - localized

Discovery Reach for γ/Z KK state (TeV)

LHC:	100 fb^{-1}		6.3	direct production
LC:	500 fb^{-1}	$\sqrt{s}$ = 500 GeV	13.0	indirect exchange
		1.0 TeV	23.0	
		1.5 TeV	31.0	

D = separation of fermions in 5th dimension



Precision EW Data:

Exchange of gauge KK excitations alter precision EW observables

- tree-level KK interactions
- KK - zero mode mixing
- zero mode loop corrections

KK Tower exchanges induce new dim-6 operators with coefficients

$$V = \sum_n \frac{g_n^2}{g_0^2} \frac{m_W^2}{m_n^2}$$

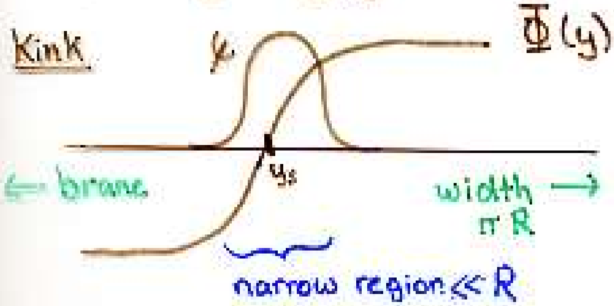
Rizzo, Wells

Perform full fit to global precision EW data set via Z-fitter

Localized Fermions on a Thick Brane

Arkani-Hamed, Schmaltz

Spatially varying domain wall scalar



• Consider a 5-d fermion ψ_5 coupled to a scalar

$m_5 \to m_5 + \lambda \Phi(y)$

$$(m_5 + \lambda \Phi \mp \partial_y)\, \chi^{(n)}_{R,L} = -m_n\, \chi^{(n)}_{L,R}$$

$\Rightarrow$ zero-mode $\quad \chi^{(0)}_{L,R} = N_{L,R} \exp\left[\pm \int dy \left[m_5 + \lambda \Phi(y)\right]\right]$

$\Phi(y) \sim \sigma^{-2} y \qquad = N_{L,R} \exp\left\{\pm \lambda (y - y_s)^2 / 2\sigma^2\right\}$

$$m_5 = \lambda/\sigma^2\, y_s$$

$\Rightarrow$ A gaussian of width $\sigma/\sqrt{\lambda}$

centered on $y_s = m_5 \sigma^2 / |\lambda|$

Sign of λ determines if normalizable sol'n exists for L or R

Proton Decay

Induced by short distance physics above M_*

Local QQQL interaction:

$$S \sim \int d^4x\, dy \, \frac{(Q^T C_5 L)^\dagger (U^{c\dagger} C_5 D^c)}{M_*^3}$$

$C_5 = \gamma^0 \gamma^2 \gamma^5$

Corresponding 4-d operator:

5d fields → 0-mode fields

Calculate wavefunction overlap in y

$$S \sim \int d^4x \; \delta \, \frac{(q\ell)^\dagger (u^c d^c)}{M_*^2}$$

$$\delta \sim \int dy \left[e^{-\lambda y^2 / 2\sigma^2} \right]^3 e^{-\lambda (y-r)^2 / 2\sigma^2}$$

$$\sim e^{-3\sigma^2/4 \, \lambda r^2} \quad \sim 10^{-33} \quad \text{if } \frac{\sqrt{\lambda}}{2\sigma} r = 10$$

½-width of Gaussian

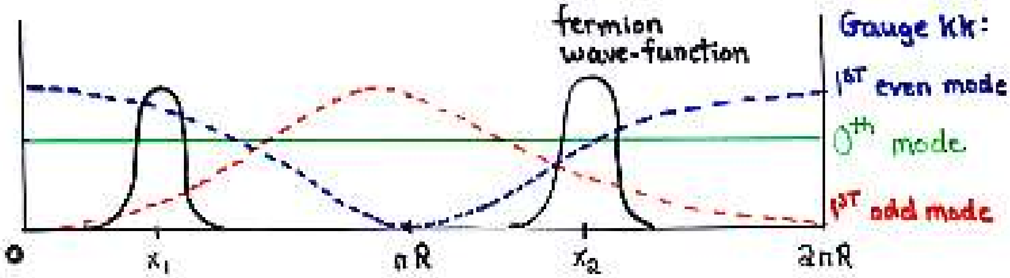
separation distance between $q + \ell$

→ 4-d coupling is exponentially supressed!

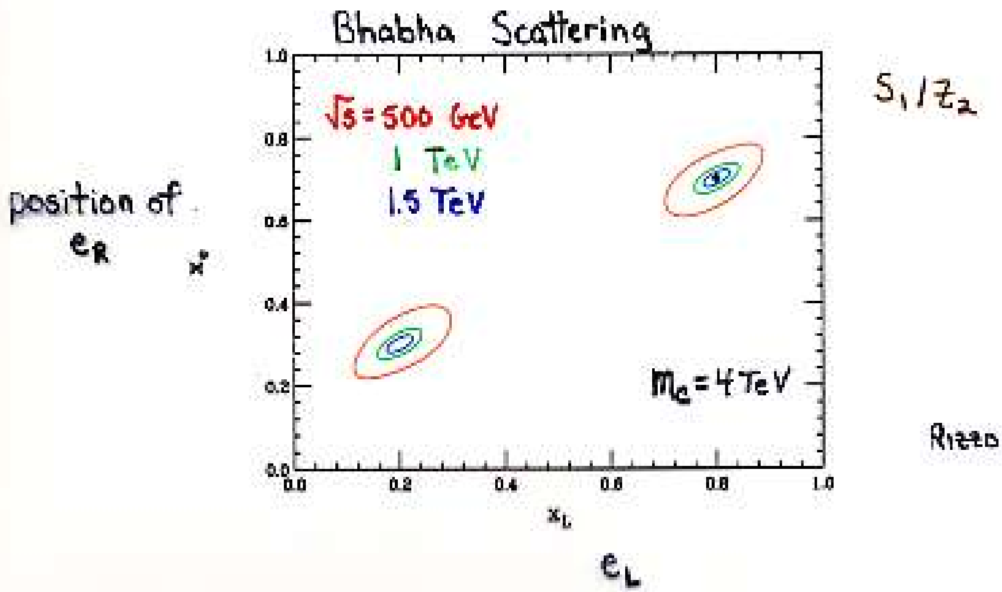
Separated Fermions

Arkani-Hamed, Schmaltz

Fermions can be localized at different points in a thick brane



Gauge KK couplings probe relative fermion locations!



Universal Extra Dimensions: Bosonic SUSY

All SM fields in TeV^{-1} 5-d Bulk

KK-parity is conserved, $(-1)^n$, due to 5-d momentum conservation

$\Rightarrow$ Lightest KK Particle (LKP) is stable: Dark Matter Candidate!

Present data constrains

$R^{-1} \leq 300$ GeV

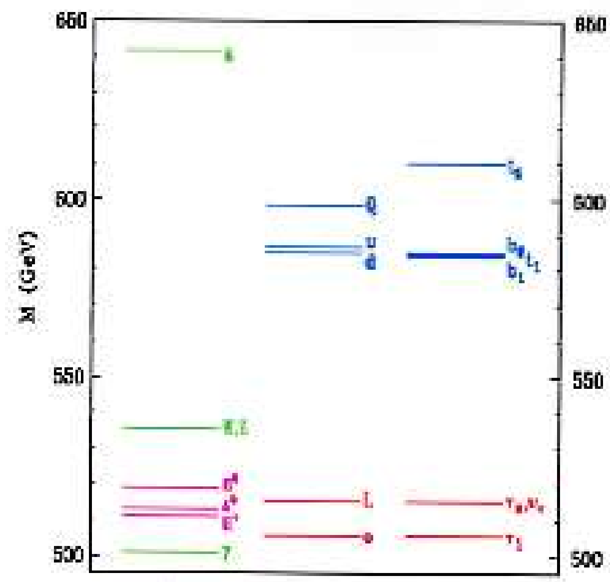
LKP: Photon KK state

appears as missing Energy

SUSY-like Spectroscopy

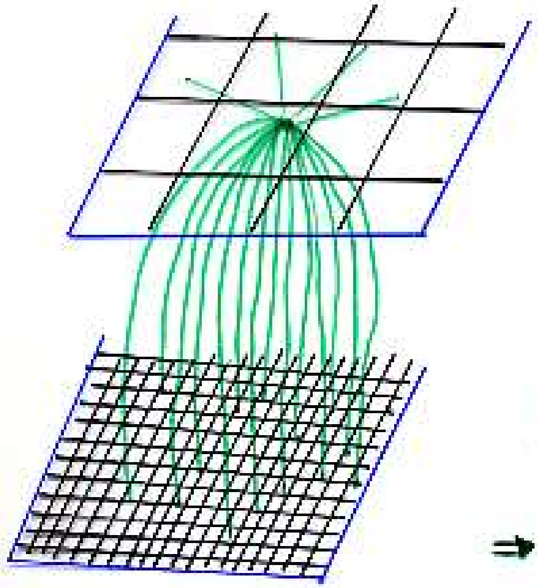
Confusion with SUSY if discovered @ LHC !

Spectrum looks like SUSY !



Chang, Matchev,Schmaltz

Non-Factorizable Curved Geometry - 'Warped' Space



Area of each grid is equal

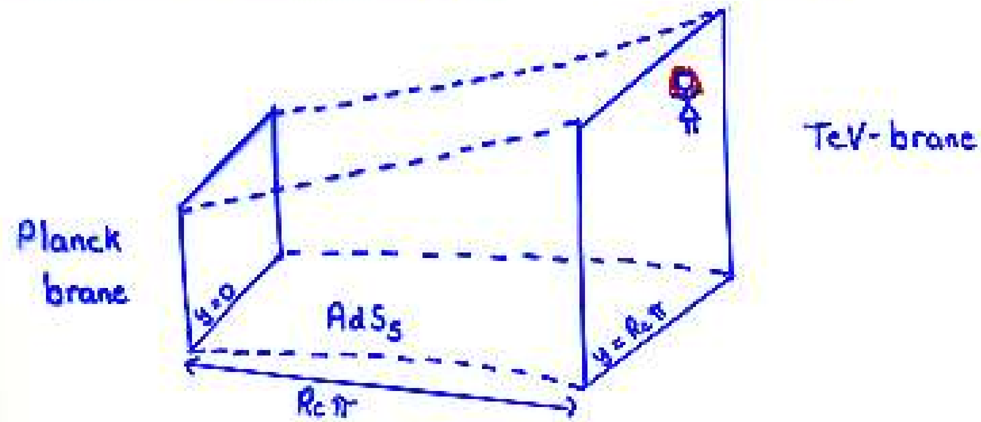
Field lines spread out faster with more volume

⇒ Drop to bottom brane

Gravity appears weak on top brane!

Localized Gravity

Randall, Sundrum



Bulk = Slice of AdS_5 $\qquad \Lambda_5 = -24\, M_5^3\, k^2$ → curvature scale

5-D non-factorizable geometry:

$$ds^2 = e^{-2ky}\eta_{\mu\nu}\,dx^\mu dx^\nu - dy^2$$

Warp factor

$M_5 \sim M_{Pl} \sim k$ ⇒ no additional hierarchies!

Physical scales on SM 3-brane:

$$\Lambda_\pi = e^{-kR_c\pi} M_{Pl}$$
$$\simeq \text{TeV if } kR_c \sim 11$$

Naturally stabilized via Goldberger-Wise

Hierarchy is generated by an exponential!

4-d Effective Theory

Davoudiasl, JLH, Rizzo

Linear expansion of flat metric

$$G_{AB} = e^{-2ky}\left(\eta_{AB} + \frac{h_{AB}(x^\mu, y)}{M_5^{3/2}}\right)$$

Expand into KK tower

$$h_{AB}(x^\mu, y) = \sum_{n=0}^{\infty} h_{AB}^{(n)}(x^\mu)\,\frac{\chi_h^{(n)}(y)}{\sqrt{R_c}}$$

Employ Boundary Conditions + find

$$\chi_h^{(n)}(y) = \frac{e^{2ky}}{N_n}\left[J_2\left(\frac{m_n}{k}e^{ky}\right) + \alpha_n Y_2\left(\frac{m_n}{k}e^{ky}\right)\right]$$

$$m_n = x_n k\, e^{-kR_c\pi} \qquad \text{with } J_1(x_n) = 0$$
$$= x_n \Lambda_\pi \frac{k}{M_{Pl}}$$

→ KK excitations are not evenly spaced!

Phenomenology governed by 2 free parameters

Λ_π / m_1 and k/M_{Pl}

$\sim$ TeV $\quad\quad \lesssim 0.1$

$\left[\text{5-d curvature: } |R_5| = 20k^2 < M_5^2\right]$

Interactions

$$\mathcal{L} \sim \frac{-1}{M_5^{3/2}} T^{AB}(x^\mu, y)\, h_{AB}(x^\mu, y), \qquad y = R_c\pi$$

$$= \frac{-1}{M_{Pl}} T^{\alpha\beta}(x)\, h^{(0)}_{\alpha\beta}(x) - \frac{1}{\Lambda_\pi} T^{\alpha\beta}(x) \sum_{n=1}^{\infty} h^{(n)}_{\alpha\beta}(x)$$

zero-mode decouples

TeV-strength couplings

Can be produced directly!

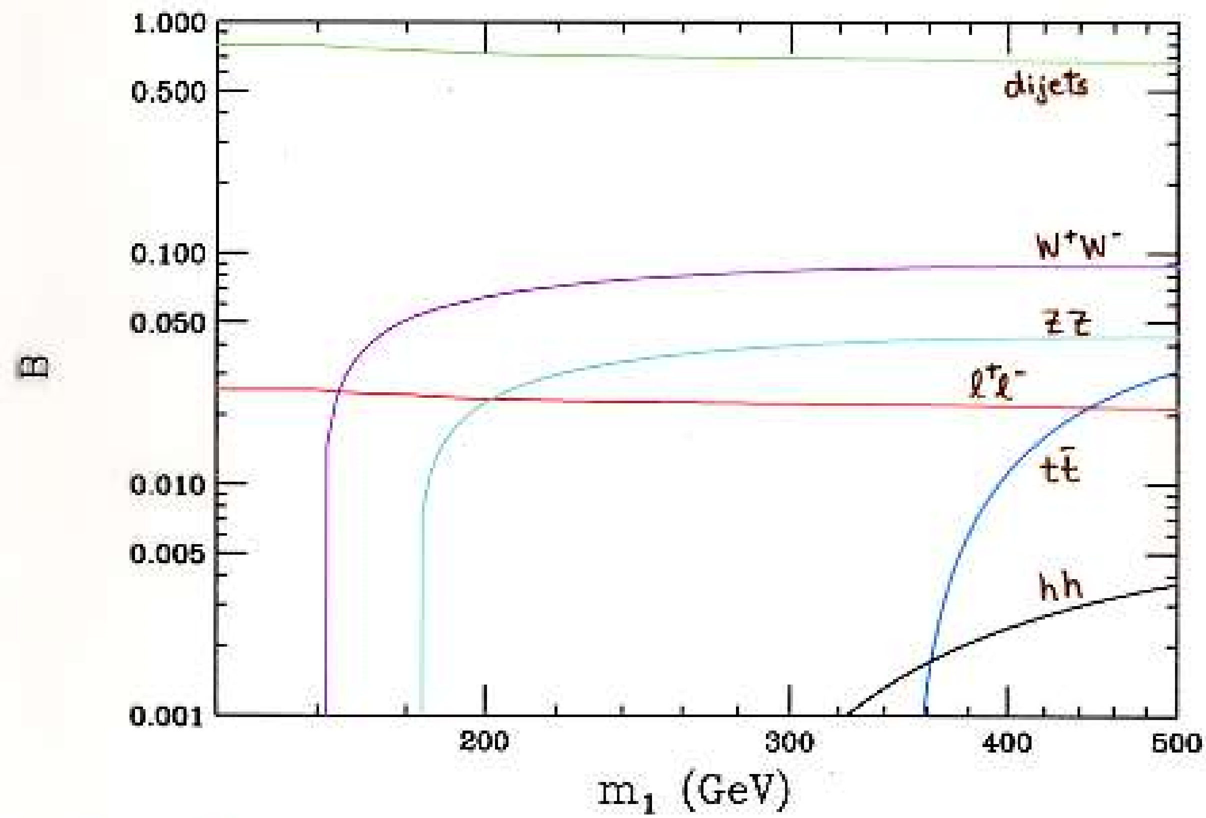
Phenomenology

- Graviton resonance production
- 'Light, skinny' Gravitons [$k/M_{Pl} \lesssim 0.01$]
- Below resonance virtual exchange [contact ints]
- Graviton induced cosmic rays
- SM fields in the bulk
- KK contributions to EW oblique parameters
- KK contributions to $g-2|_\mu$

Davoudiasl, JLH, Rizzo
PRL 00
PLB 00
PRD 01
PLB 00
PRD 02
+ Fall '02
PRD 03
JHEP 03

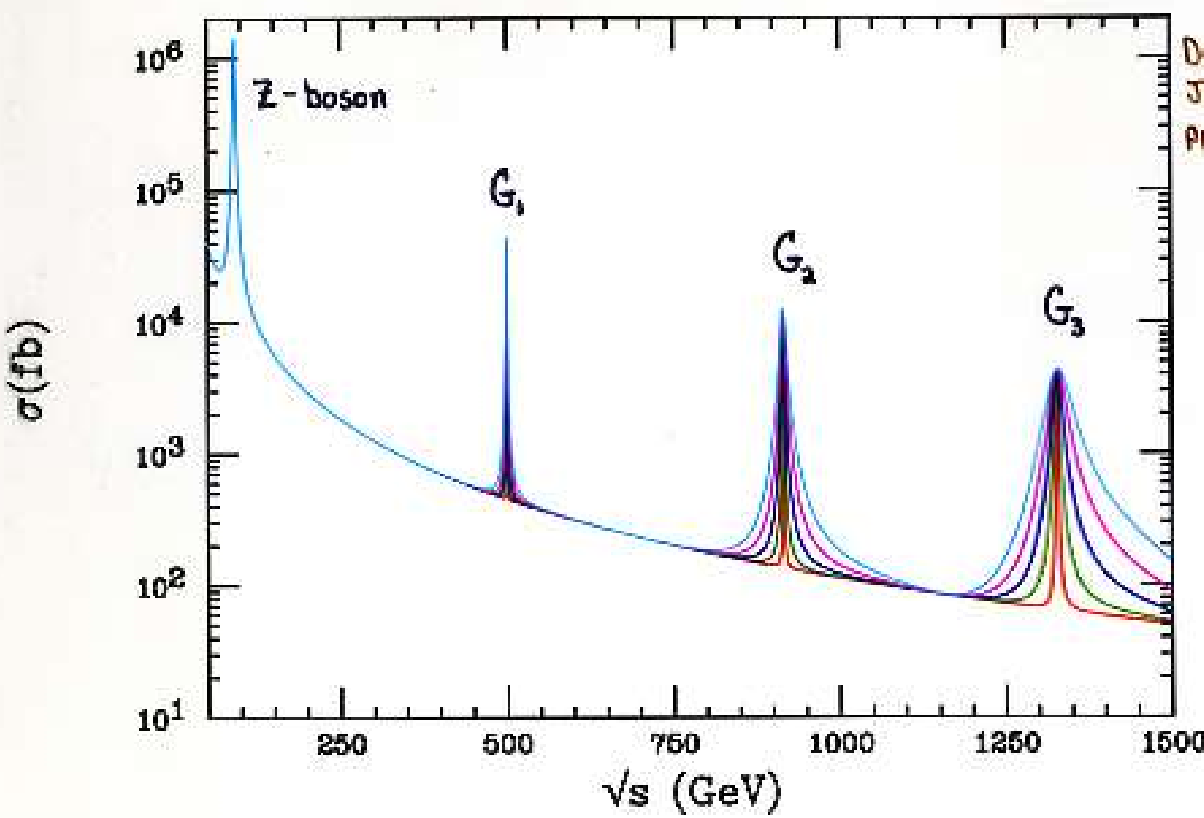
JLH, Petriello, Rizzo
hep-ph/0203091

Graviton Branching Fractions



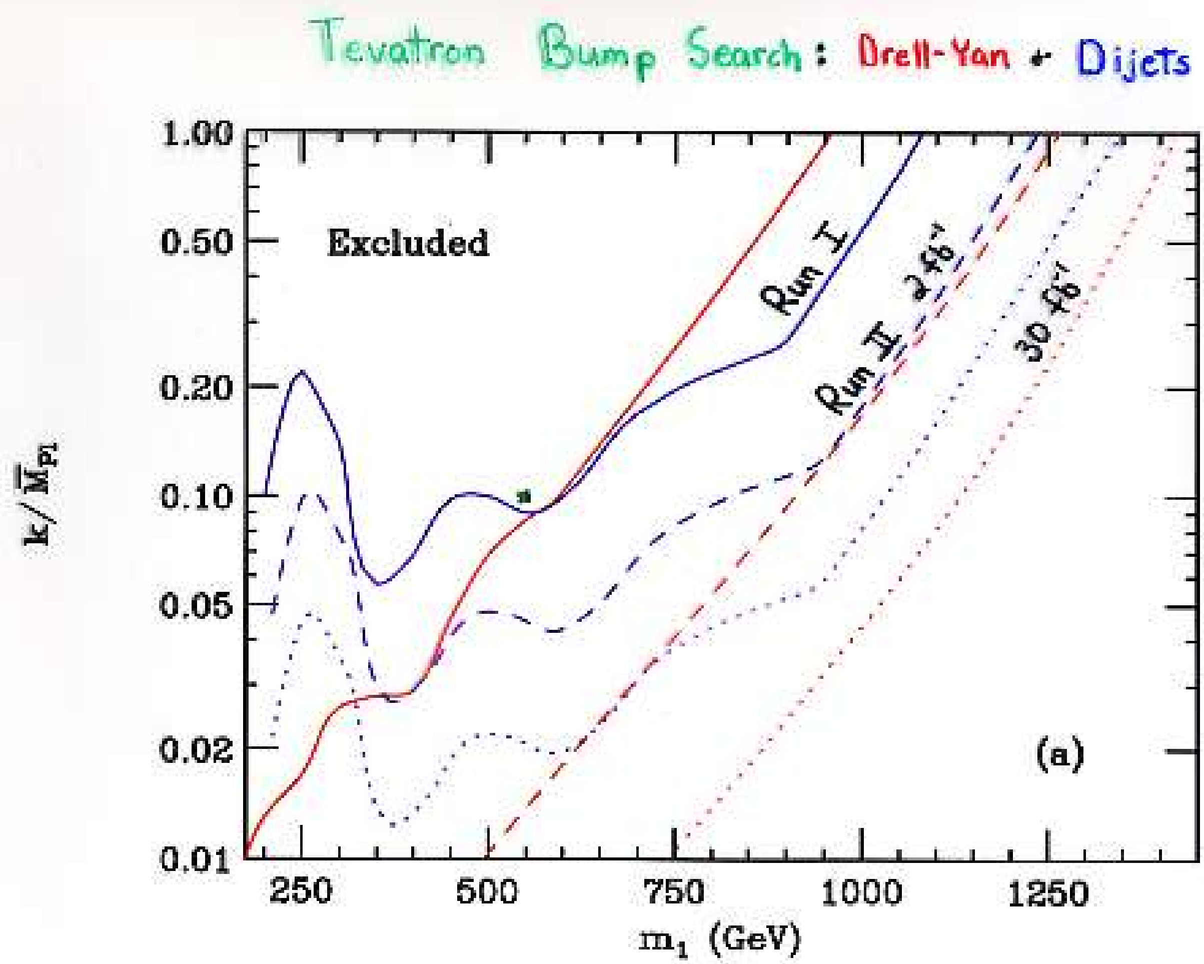
$B_{\gamma\gamma} = 2 B_{\ell^+\ell^-}$

$e^+e^- \to \mu^+\mu^-$ Line Shape

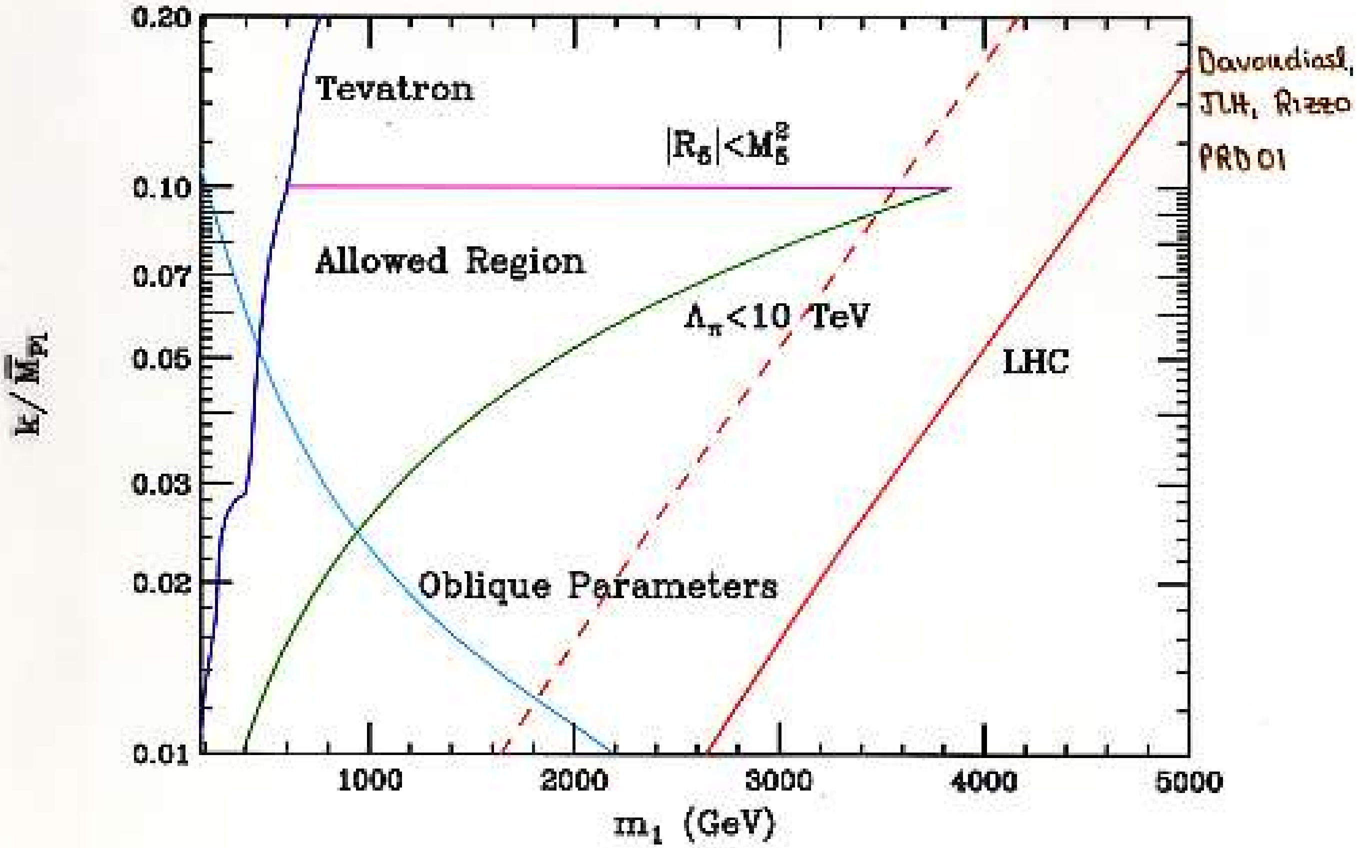


Davoudiasl, JLH, Rizzo PRL00

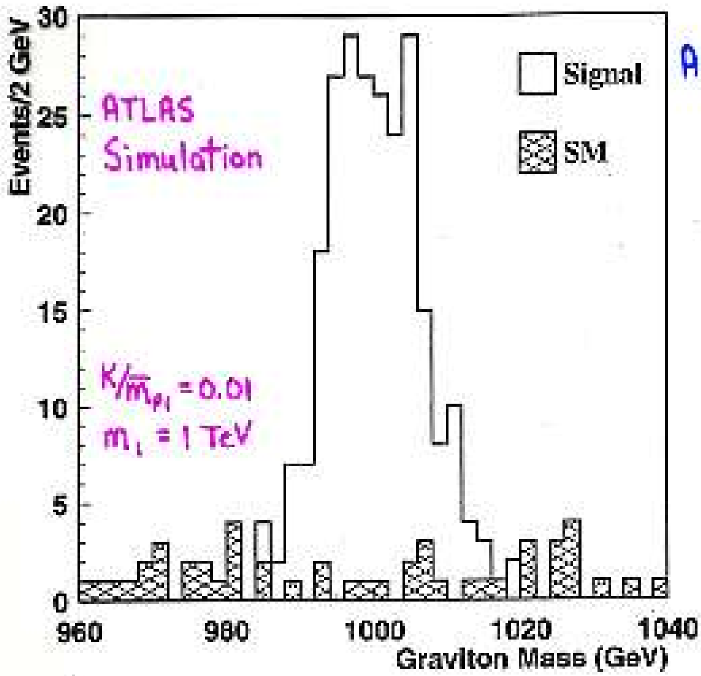
LC becomes a Graviton Factory!


Tevatron Bump Search: Drell-Yan + Dijets
Excluded
Run I
Run II
2 fb⁻¹
30 fb⁻¹
(a)
1.00
0.50
0.20
0.10
0.05
0.02
0.01
250
500
750
1000
1250
k/M̄Pl
m1 (GeV)

Summary of Theory + Experimental Constraints



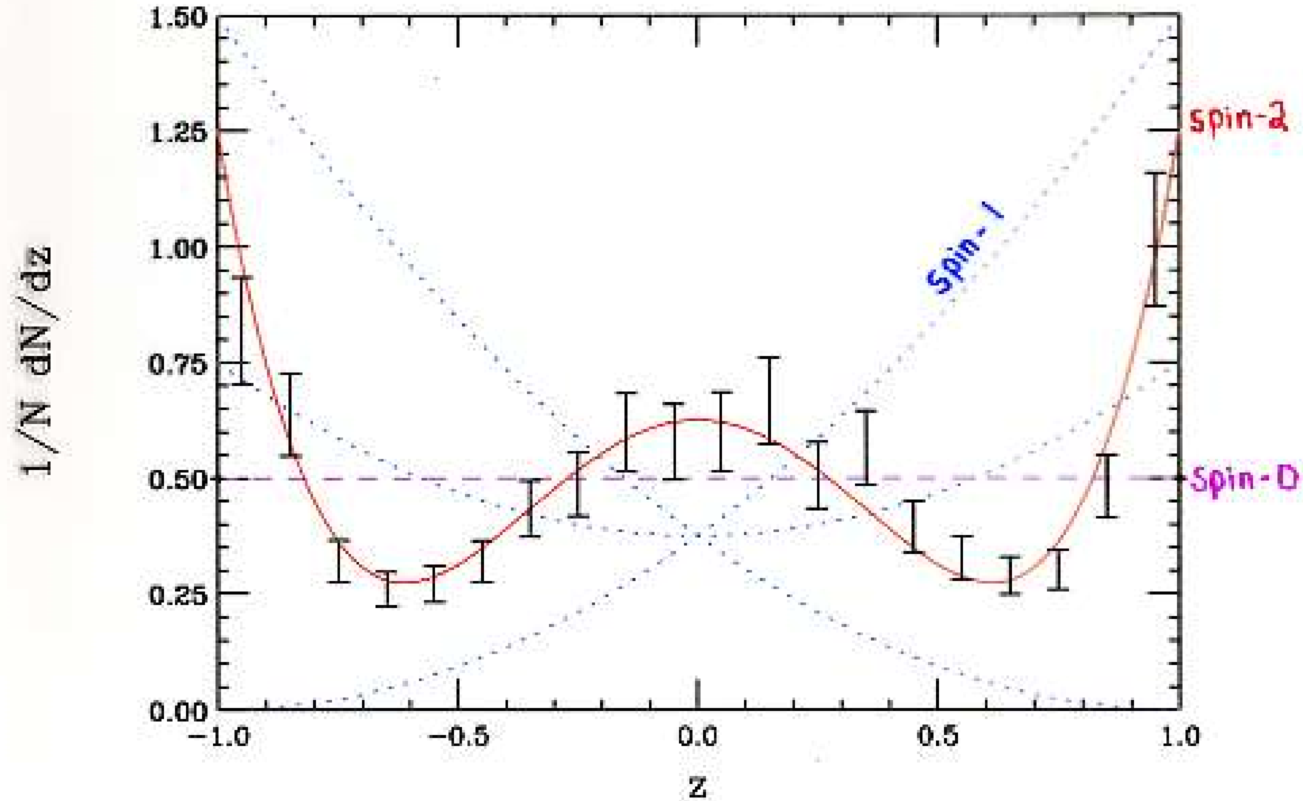
Narrow-width Graviton Resonance



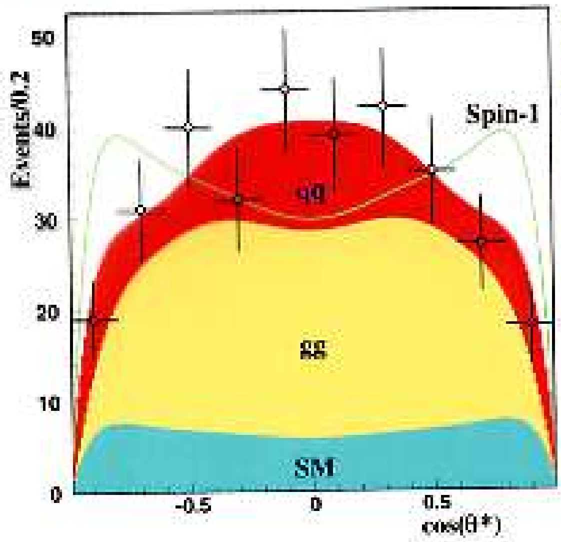
Allanach et al

ATLAS search reach: $m_1 \sim 1830$ GeV for $K/\overline{m}_{Pl} = 0.01$

On-Resonance Spin Determination



Spin-2 Determination from Drell-Yan



Allanach etal
ATLAS Simulation

m_1 = 1 TeV
100 fb^{-1}

Figure 4: The angular distribution of data (points with errors) in the test model for $m_G = 1000$ GeV and 100 fb^{-1} of integrated luminosity. The stacked histograms show the contributions from the Standard Model (SM), gg production (gg) and $q\bar{q}$ production ($q\bar{q}$). The curve shows the distribution expected from a spin-1 resonance.

distributions, defined as

$$\begin{aligned} L = {} & x_q \cdot f_q(\theta^*) \cdot A_q(M,\theta^*)/I_q(M) + x_g \cdot f_g(\theta^*) \cdot A_g(M,\theta^*)/I_g(M) \\ & + x_{DY} \cdot f_{DY}(\theta^*) \cdot A_{DY}(M,\theta^*)/I_{DY}(M) \end{aligned} \tag{4.1}$$

where x_i is the fraction of the events from each contributing process, $f_i(\theta^*)$ is the angular distribution of the process, $A_i(M,\theta^*)$ is the acceptance of the detector as a function of the mass of the electron pair and θ^*, and

$$I_i(M) = \int_{-1}^{1} f_i(\theta^*) \cdot A_i(M,\theta^*)\, d\cos\theta^* \tag{4.2}$$

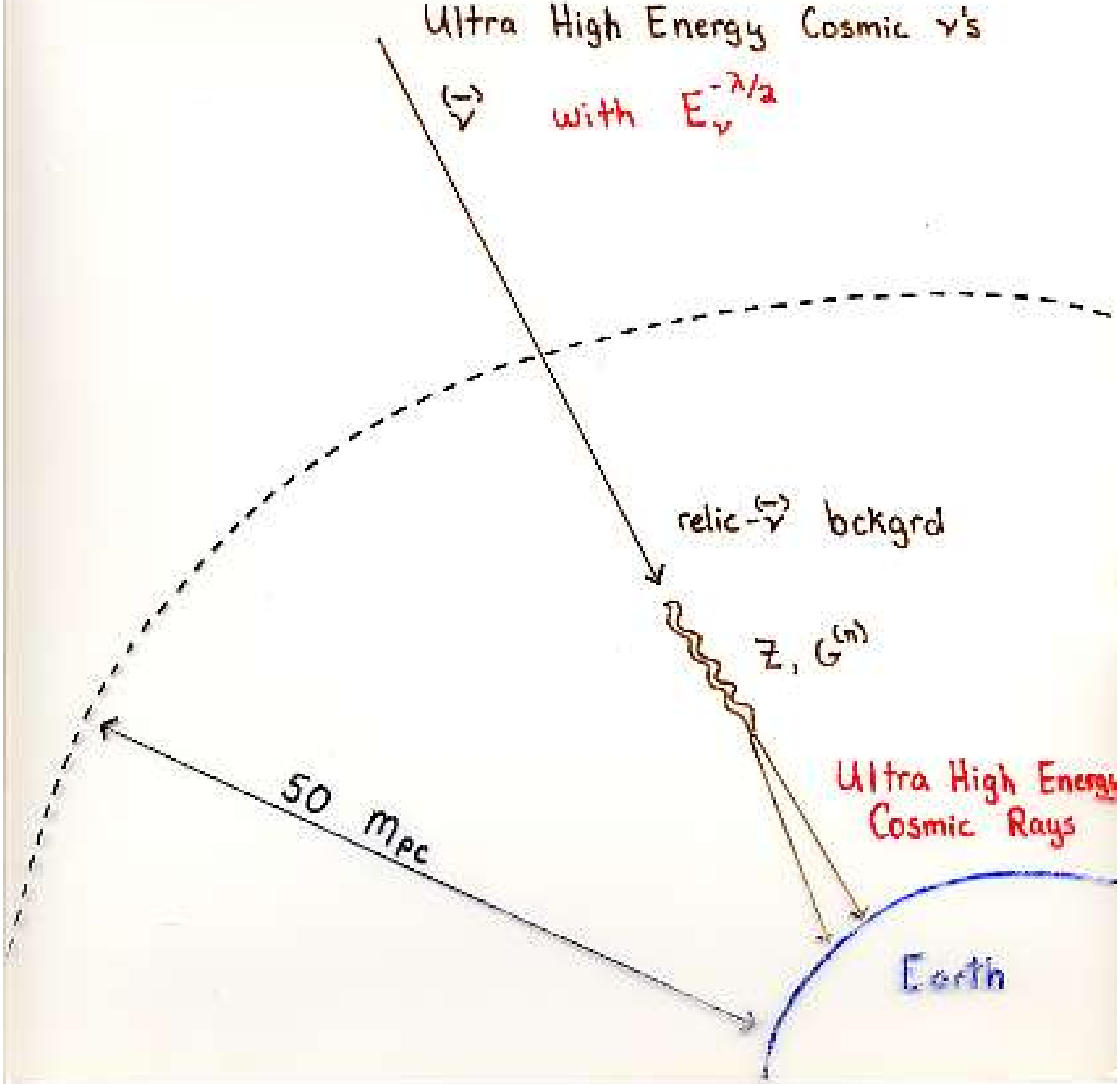
$i = q, g, DY$ for the processes $q\bar{q} \to G$, $gg \to G$, and $q\bar{q} \to Z/\gamma^*$ respectively. Only the shape of the distribution is used in the statistical tests, and the coefficients x are constrained such that

$$x_q + x_g + x_{DY} = 1 \tag{4.3}$$

In order to evaluate the discovery reach of the experiment, in terms of its ability to reveal the spin 2 nature of the resonance, the following procedure was followed, intended to mimic an ensemble of possible experimental runs:

Gravi-burst: Source of Super High Energy Cosmic Rays

JLH, Davoudi, Rizzo
hep-ph/0010354



Cosmic Rays: $\nu\bar{\nu} \to G_1 \to$ hadrons

Assume incident ν flux: $E^{-\lambda/2}$

Davoudiasl, JLH, Rizzo PRD02

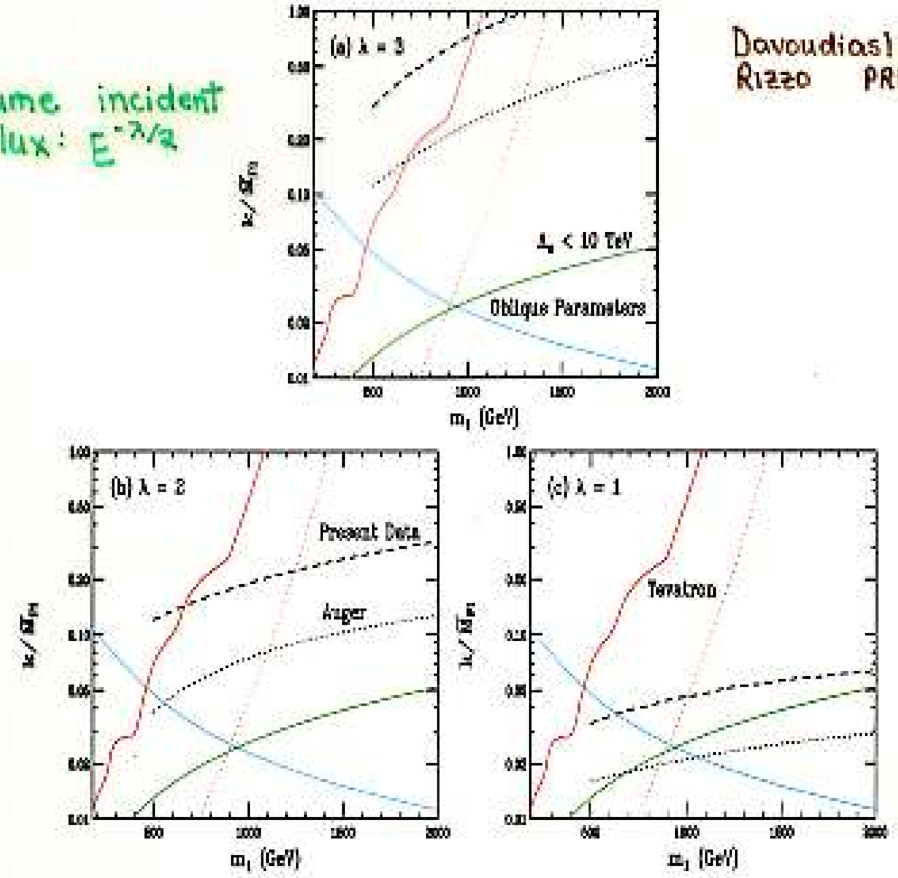


Figure 2: Allowed region in the $(k/\overline{M}_{Pl})$-m_1 plane. The solid(dotted) diagonal red curve excludes the region above and to the left from direct searches for graviton resonances at the Run I(II, 30 fb^{-1}) Tevatron. The light blue(green) curve is an indirect bound from the oblique parameter analysis (based on the hierarchy requirement that $\Lambda_\pi < 10$ TeV) and excludes the region below it. The black dashed(dotted) curves excluding the regions above them at 95% CL based on present (anticipated future Auger) cosmic ray data. The top(bottom left, bottom right) panel corresponds to $\lambda = 3(2,1)$ which describes the fall with energy of the neutrino flux as $E^{-\lambda/2}$.

Fine-tuned Summary

Large Extra-Dimensions: Large, Flat

Tower of weakly interacting, evenly spaced, almost continuous KK graviton states

Numerous at collider energies ⇒ observable in emission + exchange

LHC + NLC probe $M_D \sim 6$-10 TeV

NLC determines M_D, δ, spin-2 nature of KK states

Localized Gravity: Warped

Graviton KK tower: $m_1 \sim$ TeV, TeV^{-1} couplings
spacing given by $J_1(x_n) = 0$

Direct spin-2 resonance production

LHC probes parameter space

NLC becomes graviton factory

SM in the bulk is interesting, but problematic