R-Symmetry, Gauge Singlets and Seiberg Duality

Part 1 - Background

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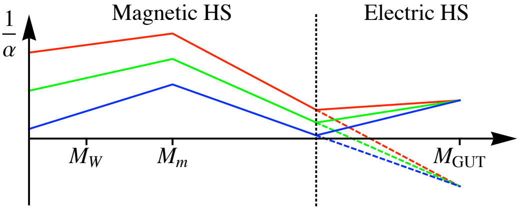
February 26, 2009

Why bother?

- Seiberg duality gives us a different angle on supersymmetric gauge theories.
- We believe it will help in understanding many aspects of BSM physics such as SUSY breaking, proton decay and unification.
- Problem: currently, dualities only exist for theories with highly constrained and unrealistic superpotentials.
- By including gauge singlets in our theories we have alleviated some of these constraints.
- Our ultimate goal is to use these tools to find a dual theory to the supersymmetric $\mathrm{SU}(5)$ Georgi-Glashow model.

Example: "Dualification"

- In models of SUSY breaking with direct mediation, the messengers deflect the gauge coupling unification.
- Extrapolating to higher scales it may appear as though unification occurs at a negative, unphysical value of $\frac{1}{\alpha}$.
- In the dual theory the unification is much more natural[1].



[1] S. Abel, V.V. Khoze - arXiv:809.5262v1[hep-ph]

What is R-symmetry?

- An R-symmetry is a global symmetry which does not commute with SUSY.
- For $\mathcal{N} = 1$ we can only have $\mathrm{U}(1)$ R-symmetries.
- Chiral supermultiplet $\Phi = (\varphi, \psi, F)$ with $R_\Phi = R$

$$\implies R_\varphi = R, \quad R_\psi = R - 1 \quad \text{and} \quad R_F = R - 2.$$

- Understood by writing Φ as a superfield

$$\Phi = \varphi + \theta\psi + \theta^2 F \quad \text{with} \quad R_\theta = 1.$$

- R-symmetries are like rotations in superspace.

R-symmetry and you

- In superspace, the Lagrangian is

$$\mathcal{L} = \int \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}\, \Phi^\dagger \Phi + \left[\int \mathrm{d}^2\theta\; W(\Phi) + \text{h.c.} \right].$$

- $\mathcal{L}$ is invariant under all R-symmetries so $R_W = 2$.
- At a conformal fixed point the R-symmetry is absorbed into the superconformal algebra giving

$$\dim O = \frac{3}{2} R_O.$$

a-maximisation

- Given one non-anomalous R-symmetry under which superfields Φ_i have R-charges R_i, we can always define another

$$R_i' = R_i + \sum p_i Y_i .$$

- Intriligator and Wecht showed[2] that the exact R-symmetry, which is absorbed into the superconformal algebra, (probably) maximises

$$a \propto 3 \operatorname*{Tr}_{\psi} \left[R^3\right] - \operatorname*{Tr}_{\psi} \left[R\right] .$$

- All relevant deformations to the superpotential then satisfy $R_{\Delta W} < \frac{2}{3}$.

[2] K. Intriligator, B. Wecht - arXiv:hep-th/0304128

SQCD - the electric theory

- SQCD is the supersymmetric generalisation of QCD, with gauge group $\mathrm{SU}(N)$ and F_Q flavours of quark/antiquark.
- It has no superpotential
- The matter content and global symmetries are

	$\mathrm{SU}(N)$	$\mathrm{SU}(F_Q)_L$	$\mathrm{SU}(F_Q)_R$	$\mathrm{U}(1)_B$	$\mathrm{U}(1)_R$
Q	$\mathbf{N}$	$\mathbf{F_Q}$	$\mathbf{1}$	$1/N$	$1-\frac{N}{F_Q}$
$\tilde{Q}$	$\overline{\mathbf{N}}$	$\mathbf{1}$	$\overline{\mathbf{F_Q}}$	$-1/N$	$1-\frac{N}{F_Q}$

SQCD+M - the magnetic theory

- SQCD+M is SQCD, but with an elementary meson field M. It has gauge group $\mathrm{SU}(n)$ and F_Q flavours of quark/antiquark.
- It has superpotential $W = M\tilde{q}q$.
- The matter content and global symmetries are

	$\mathrm{SU}(n)$	$\mathrm{SU}(F_Q)_L$	$\mathrm{SU}(F_Q)_R$	$\mathrm{U}(1)_B$	$\mathrm{U}(1)_R$
q	$\mathbf{n}$	$\overline{\mathbf{F_Q}}$	$\mathbf{1}$	$1/n$	$1-\frac{n}{F_Q}$
$\tilde{q}$	$\overline{\mathbf{n}}$	$\mathbf{1}$	$\mathbf{F_Q}$	$-1/n$	$1-\frac{n}{F_Q}$
M	$\mathbf{1}$	$\mathbf{F_Q}$	$\overline{\mathbf{F_Q}}$	0	$\frac{2n}{F_Q}$

Putting it together

- Seiberg postulated[3] that SQCD and SCQD+M with

$$n = F_Q - N$$

  describe the same infrared physics.
- The mesons are required to match up the moduli spaces of the two theories

$$\tilde{Q}Q \longrightarrow M.$$

- The magnetic superpotential is required to project out the redundant composite magnetic mesons.
- It is often a strong-weak duality.

[3] N. Seiberg - arXiv:hep-th/9411149

Testing the duality

There are four main tests for dualities in supersymmetric gauge theories.

1. The global symmetries of each theory must match.
2. The classical moduli spaces of the theories must match.
3. The 't Hooft anomaly matching conditions must be satisfied.
4. The duality must be maintained under deformations of the two theories.

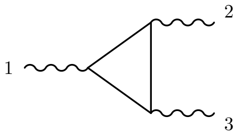
Moduli space (or baryon) matching

- Meson matching is trivial as we added in elementary mesons by hand. The only remaining, non-trivial moduli are the baryons.

	$\mathrm{SU}(F_Q)_L$	$\mathrm{U}(1)_B$	$\mathrm{U}(1)_R$
$B = \epsilon^{(N)} Q^N$	$\mathbf{asym(N)}$	1	$N - \frac{N^2}{F_Q}$
$b = \epsilon^{(n)} q^n$	$\overline{\mathbf{asym(n)}}$	1	$n - \frac{n^2}{F_Q}$

- But $\overline{\mathbf{asym(n)}} = \mathbf{asym(N)}$ and $n - \frac{n^2}{F_Q} = N - \frac{N^2}{F_Q}$ for $n = F_Q - N$ so everything matches.

't Hooft anomaly matching



- Imagine gauging each of the global symmetries. This creates many new anomalies.
- If the theories are dual, the anomalies created should have the same values in each theory.
- We have to calculate the anomalies for all global symmetries and hope they match.
- This is a fully quantum mechanical test that ensures we are using the right degrees of freedom.

Deforming the superpotential

- If the theories truly describe the same physics, the duality should survive any deformations to the superpotential.
- Consider adding a quark mass term to the electric theory $\Delta W_{\rm el} = m\tilde{Q}Q$.
- If we add the equivalent term $\Delta W_{\rm mag} = \mu m M$ to the magnetic superpotential the duality remains.
- We can check that the same global symmetries are broken in the same way for each theory. The moduli spaces are also deformed equivalently.

Adding more stuff

- We're not actually that interested in SQCD. It would be nice to find dualities with different matter content.
- Consider adding a field X, in the adjoint of the gauge group.
- The mesons are now more complicated

$$M_j = \tilde{Q} X^j Q.$$

- Unfortunately the number of independent mesons now depends on N, making it much more difficult to match electric and magnetic mesons.
- We need to do something clever!

The chiral ring

- Fortunately, Kutasov and Schwimmer did do something clever[4].
- By adding a superpotential $W = X^{k+1}$ we find the F-term equation $X^k = 0$.
- This *truncates the chiral ring*, reducing the possible mesons to

$$M_j = \tilde{Q} X^j Q, \quad j = 0, \ldots, k-1$$

which is now totally independent of N.

[4] D. Kutasov, A. Schwimmer - arXiv:hep-th/9505004

The electric theory

- The electric theory has gauge group $\mathrm{SU}(N)$, F_Q flavours of quark/antiquark and one adjoint.
- It has superpotential $W_{\mathrm{el}} = t_0 \frac{X^{k+1}}{k+1}$.
- The matter content and global symmetries are

	$\mathrm{SU}(N)$	$\mathrm{SU}(F_Q)_L$	$\mathrm{SU}(F_Q)_R$	$\mathrm{U}(1)_B$	$\mathrm{U}(1)_R$
Q	$\mathbf{N}$	$\mathbf{F_Q}$	$\mathbf{1}$	$1/N$	$1 - \frac{N}{F_Q(k+1)}$
$\tilde{Q}$	$\overline{\mathbf{N}}$	$\mathbf{1}$	$\overline{\mathbf{F_Q}}$	$-1/N$	$1 - \frac{N}{F_Q(k+1)}$
X	$\mathbf{adj}$	$\mathbf{1}$	$\mathbf{1}$	0	$\frac{2}{k+1}$

The magnetic theory

- The magnetic theory has gauge group $\mathrm{SU}(n)$ with $n = kF_Q - N$, F_Q flavours of quark/antiquark, one adjoint and k mesons.
- It has superpotential $W_{\mathrm{mag}} = -t_0 \frac{x^{k+1}}{k+1} + \frac{t_0}{\mu^2} \sum_{j=0}^{k-1} M_j \tilde{q} x^{k-1-j} q$.
- The matter content and global symmetries are

	$\mathrm{SU}(n)$	$\mathrm{SU}(F_Q)_L$	$\mathrm{SU}(F_Q)_R$	$\mathrm{U}(1)_B$	$\mathrm{U}(1)_R$
q	$\mathbf{n}$	$\overline{\mathbf{F_Q}}$	$\mathbf{1}$	$1/n$	$1 - \frac{n}{F_Q(k+1)}$
$\tilde{q}$	$\overline{\mathbf{n}}$	$\mathbf{1}$	$\mathbf{F_Q}$	$-1/n$	$1 - \frac{n}{F_Q(k+1)}$
x	$\mathbf{adj}$	$\mathbf{1}$	$\mathbf{1}$	0	$\frac{2}{k+1}$
M_j	$\mathbf{1}$	$\mathbf{F_Q}$	$\overline{\mathbf{F_Q}}$	0	$\frac{2n+2F_Q(j+1)}{F_Q(k+1)}$

Baryon matching

- The most general baryons in the KSS model are constructed out of dressed quarks $Q_j = X^j Q$ with $j = 0, \ldots, k-1$.
- Setting $\sum_j r_j = N$,

	$\mathrm{SU}(F_Q)_L$
$B_{\{r_j\}} = \epsilon^{(N)} \prod_j Q_j^{r_j}$	$\prod_j \mathbf{asym(r_j)}$
$b_{\{r_j\}} = \epsilon^{(n)} \prod_j q_j^{F_Q - r_j}$	$\prod_j \overline{\mathbf{asym(F_Q - r_j)}}$

- $\overline{\mathbf{asym(F_Q - r_j)}} = \mathbf{asym(r_j)}$ and all $\mathrm{U}(1)$ charges match.
- All other tests are also passed, (especially anomaly matching!) including...

Deformations of the KSS model

- We can consider a more general electric superpotential

$$W_{\mathrm{el}} = \sum_{i=0}^{k-1} t_i \frac{X^{k+1-i}}{k+1-i}$$

where we have chosen a basis such that $t_1 = 0$.

- This superpotential leads to a very rich vacuum structure but also explicitly breaks the R-symmetry.
- If the magnetic superpotential is simultaneously deformed to

$$W_{\mathrm{mag}} = -\sum_{i=0}^{k-1} t_i \frac{x^{k+1-i}}{k+1-i} + \frac{1}{\mu^2}\sum_{i=0}^{k-1} t_i \sum_{j=1}^{k-i} M_j \tilde{q} x^{k-1-j} q$$

the duality is maintained.

Dualities with antisymmetrics

- A similar approach can be taken[5] for models with an antisymmetric representation of the gauge group A.
- The superpotential $W_{\rm el} = (\tilde{A}A)^{k+1}$ gives the F-term equations $A(\tilde{A}A)^k = (\tilde{A}A)^k\tilde{A} = 0$, which truncate the chiral ring.
- The allowed mesons are

$$\begin{array}{lll} M_j = \tilde{Q}(\tilde{A}A)^j Q & j = 0,\ldots,k & \in (\mathbf{F_Q}, \overline{\mathbf{F_Q}}) \\ P_j = Q(\tilde{A}A)^j \tilde{A} Q & j = 0,\ldots,k-1 & \in (\mathbf{asym}, \mathbf{1}) \\ \tilde{P}_j = \tilde{Q}A(\tilde{A}A)^j \tilde{Q} & j = 0,\ldots,k-1 & \in (\mathbf{1}, \overline{\mathbf{asym}}) \end{array}$$

[5] K. Intriligator, R.G. Leigh, M.J. Strassler - arXiv:hep-th/9506148

The magnetic theory

- This theory is dual to the theory with gauge group $\mathrm{SU}(n)$, where

$$n = (2k+1)\,F_Q - 4k - n.$$

- The magnetic superpotential is

$$\begin{aligned} W_{\mathrm{mag}} &= (\tilde{a}a)^{k+1} + \sum_{j=0}^{k} M_j \tilde{q}(\tilde{a}a)^{k-j} q + \\ & \sum_{j=0}^{k-1} \left[P_j q (\tilde{a}a)^{k-1-j} \tilde{a} q + \tilde{P}_j \tilde{q} a (\tilde{a}a)^{k-1-j} \tilde{q} \right] \end{aligned}$$

- The elementary mesons are those defined earlier.

Baryon matching

- Baryon matching is now more complicated.

	$\mathrm{SU}(F_Q)_L$
$B_r = \epsilon^{(N)} A^r Q^{N-2r}$	$\mathbf{asym(N-2r)}$
$b_r = \epsilon^{(n)} a^{k(F_Q-2)-r} q^{F_Q-N+2r}$	$\overline{\mathbf{asym(F_Q-N+2r)}}$

- $\overline{\mathbf{asym(F_Q-N+2r)}} = \mathbf{asym(N-2r)}$ and all $\mathrm{U}(1)$ charges match. The anomaly matching works out too.