Cosener’s House 7th June 2003

Inflationary density perturbations

David Wands

Institute of Cosmology and Gravitation
University of Portsmouth

outline

- some motivation
- Primordial Density Perturbation
  (and conserved quantities on large scales)
- constraints on single-field models
- predictions from multi-field models
- conclusions

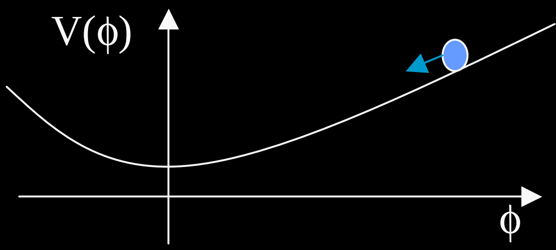
Cosmological inflation:

Starobinsky (1980)

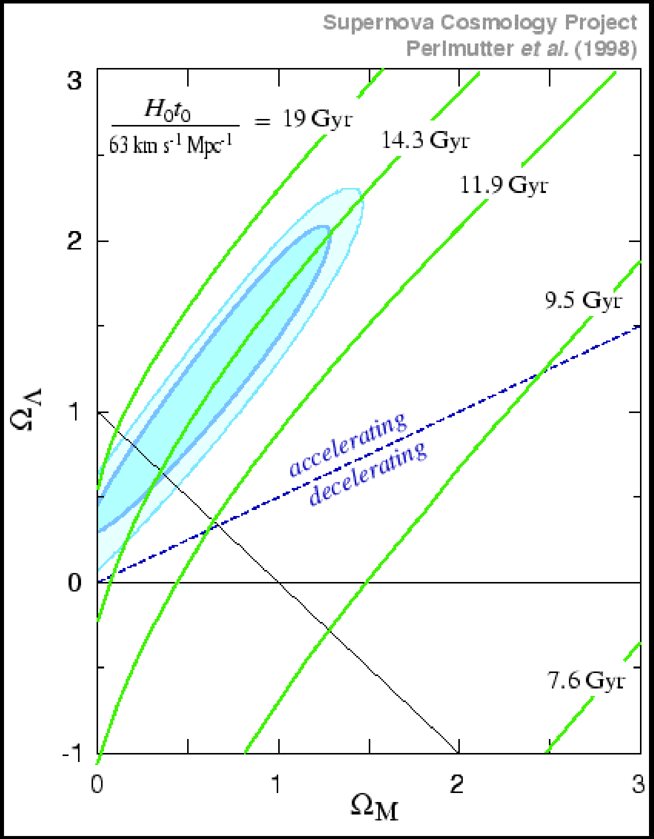
Guth (1981)

- period of accelerated expansion
  in the very early universe
- requires negative pressure

  e.g. self-interacting scalar field



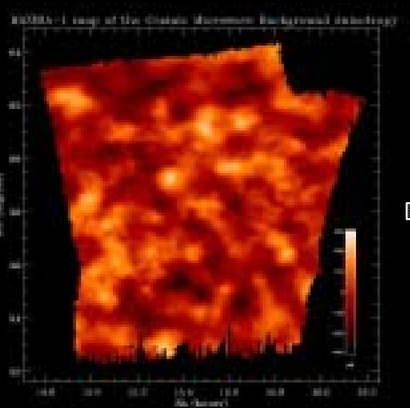
- speculative and uncertain physics
- just the kind of peculiar cosmological
  behaviour we observe today



Motivation:

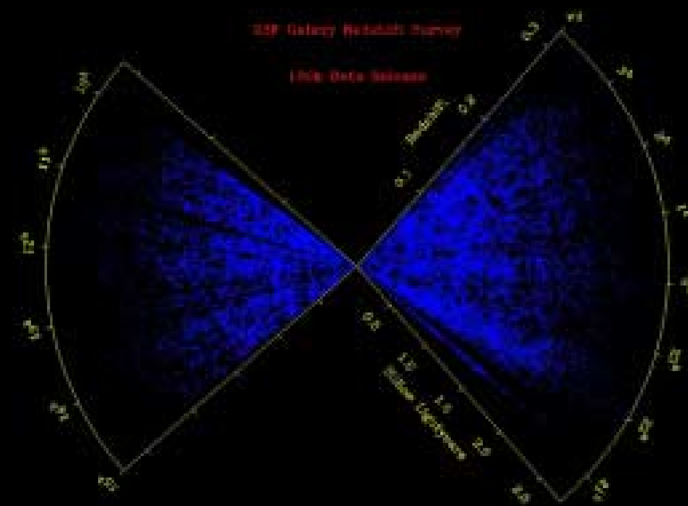
inflation in very early universe testable through *primordial perturbation spectra*

- radiation/matter density perturbations
- \+ gravitational waves *(we hope)*



gravitational

instability



new observational data offers precision tests of cosmological parameters and the ***P****rimordial* ***D****ensity* ***P****erturbation*

Primordial Density Perturbation

e.g., epoch of primordial nucleosynthesis

cosmic fluid consists of

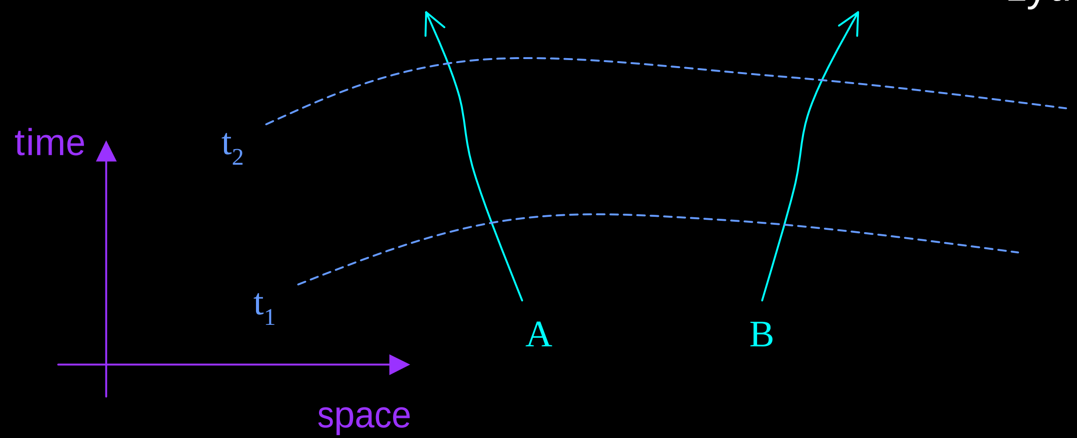
- photons, γ, neutrinos, ν, baryons, B, cold dark matter, CDM, (+quintessence?)

- total density perturbation, or
  curvature perturbation $\mathbf{R} \approx \delta\rho \,/\, \rho$
- relative density perturbations, or
  isocurvature pertbns $\mathbf{S_i} = \delta(n_i / n_\gamma) / (n_i / n_\gamma)$

- large-angle CMB: $(\Delta T/T)_{lss} \approx [\, \mathbf{R} - 2\mathbf{S}_m \,] \,/\, 5$

Conserved cosmological perturbations

Lyth & Wands in preparation



For every quantity, x, that obeys a **local conservation equation**

$$\frac{dx}{dN} = y(x) \quad , \qquad e.g. \quad \dot{\rho}_m = -3H\rho_m$$

where $dN = Hdt$ is the locally-defined expansion along comoving worldlines

there is a **conserved perturbation**

$$\zeta_x \equiv \delta N = \frac{\delta x}{y(x)}$$

where perturbation $\delta x = x_A - x_B$ is a evaluated on hypersurfaces separated by uniform expansion $\Delta N = \Delta \ln a$

examples:

(i) total energy conservation: $$\frac{d\rho}{dN} = H^{-1}\dot{\rho} = -3(\rho + P)$$

for perfect fluid / adiabatic perturbations, $P=P(\rho)$

$$\Rightarrow \quad R \equiv \zeta_\rho = \frac{\delta\rho}{3(\rho + P)} \quad \text{conserved}$$

(ii) energy conservation for non-interacting perfect fluids:

$$H^{-1}\dot{\rho}_i = -3(\rho_i + P_i) \quad \text{where } P_i = P_i(\rho_i) \quad \Rightarrow \quad \zeta_i = \frac{\delta\rho_i}{3(\rho_i + P_i)}$$

(iii) conserved particle/quantum numbers (e.g., B, B-L,...)

$$H^{-1}\dot{n}_i = -3n_i \quad \Rightarrow \quad \zeta_i = \frac{\delta n_i}{3n_i}$$

Primordial Density Perturbation (II)

epoch of primordial nucleosynthesis

perturbed cosmic fluid consists of

– photons, ζ_γ, neutrinos, ζ_ν, baryons, ζ_B, cold dark matter, ζ_{CDM}, (+quintessence, ζ_Q)

- total density perturbation, or curvature perturbation

$$R = \sum_i \left(\frac{\dot{\rho}_i}{\dot{\rho}} \right) \zeta_i$$

- relative density perturbations, or isocurvature perturbtns

$$S_i = 3\left(\zeta_i - \zeta_\gamma\right)$$

where do these perturbations come from?

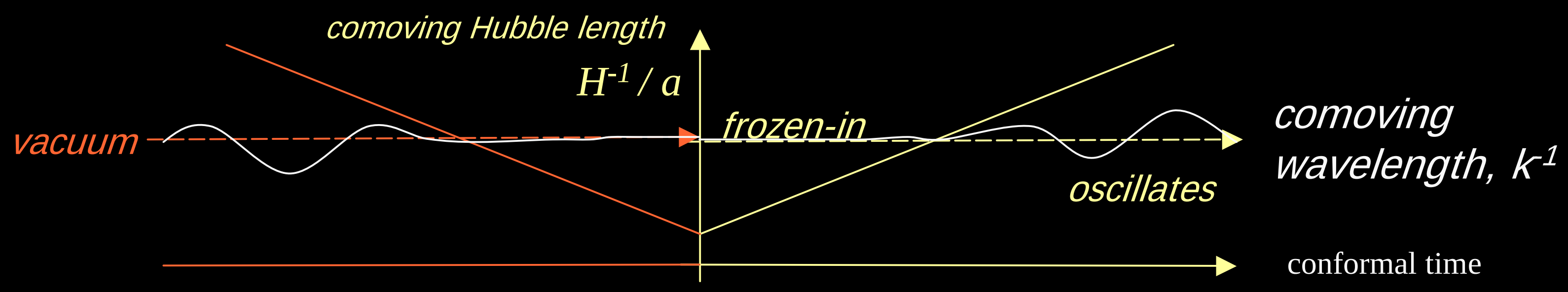
perturbations in an FRW universe:

wave equation $\delta\ddot{\phi} + 3H\delta\dot{\phi} - \nabla^2 \delta\phi = 0$

Characteristic timescales for comoving wavemode k

- *oscillation period/wavelength* *a / k*
- *Hubble damping time-scale* H^{-1}

- *small-scales k > aH* *under-damped oscillator*
- *large-scales k < aH* *over-damped oscillator ("frozen-in")*



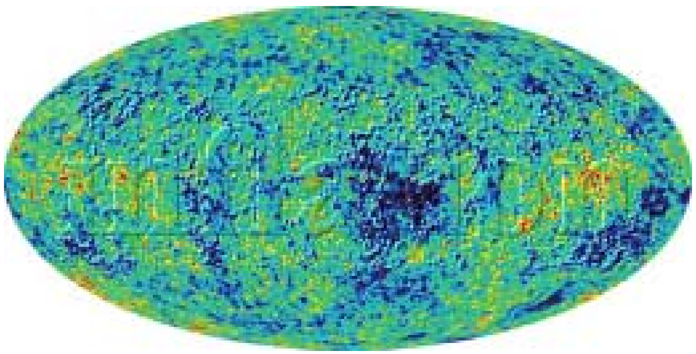
inflation

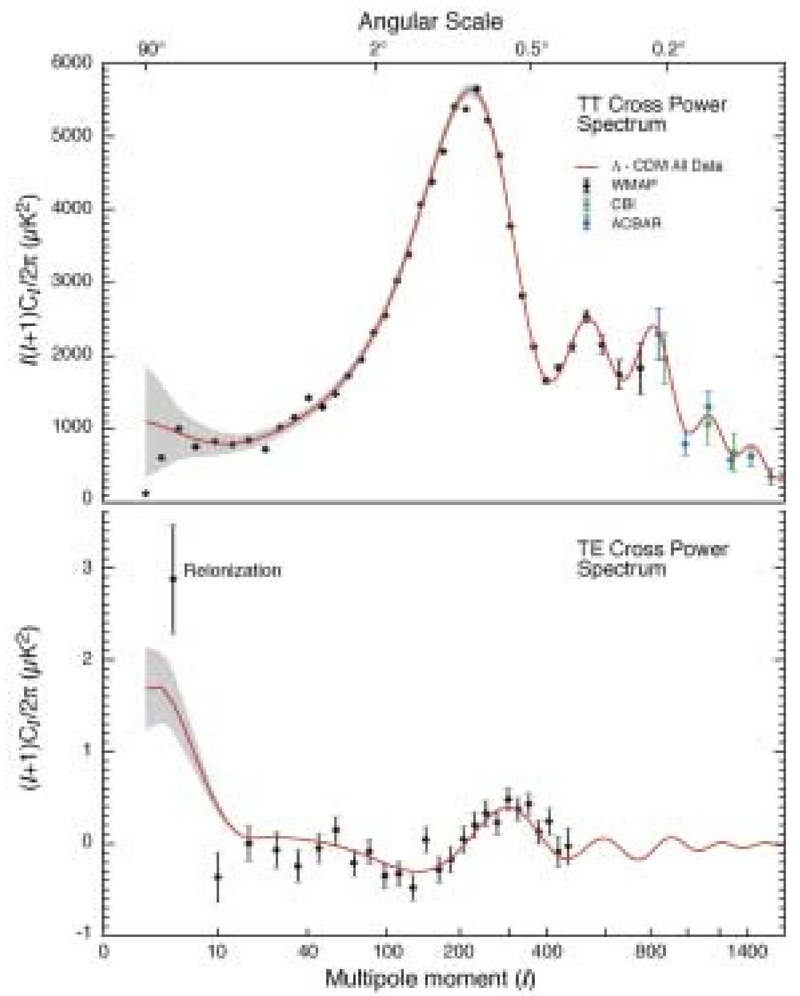
accelerated expansion
(or contraction)

radiation or matter era

decelerated expansion

Wilkinson Microwave Anisotropy Probe
February 2003





coherent oscillations

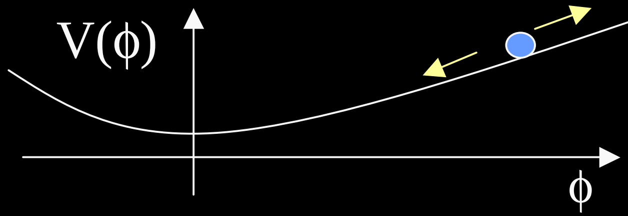
in photon-baryon plasma

from primordial density perturbations

on super-horizon scales

Vacuum fluctuations

Hawking '82, Starobinsky '82, Guth & Pi '82



- *small-scale/underdamped zero-point fluctuations* $\delta\phi_k \approx \frac{e^{-ik\eta}}{\sqrt{2k}}$
- *large-scale/overdamped perturbations in growing mode*

  linear evolution $\Rightarrow$ *Gaussian random field*

$$\left\langle \delta\phi^2 \right\rangle_{k=aH} \approx \frac{4\pi k^3 \left|\delta\phi_k\right|^2}{(2\pi)^3} = \left(\frac{H}{2\pi}\right)^2$$

fluctuations of any light fields (m<3H/2) `frozen-in' on large scales

*** assumes Bunch-Davies vacuum on small scales ***

all modes start sub-Planck length for $k/a > M_{Pl}$ Niemeyer; Brandenberger & Martin (2000)

effect likely to be small for $H << M_{Pl}$ Starobinsky; Niemeyer; Easther et al ; Kaloper et al (2002)

Inflaton -> matter perturbations

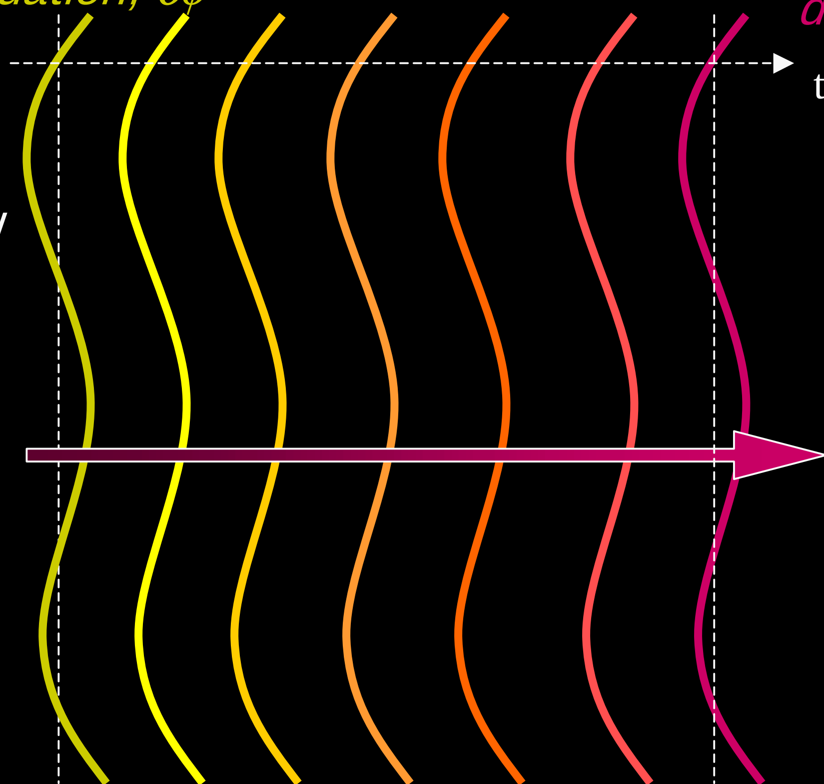
for adiabatic perturbations on super-horizon scales $\dot{R} = 0$

during inflation

scalar field fluctuation, $\delta\phi$

during matter+radiation era

density perturbation, $\delta\rho$



scalar curvature on uniform-field hypersurfaces

$$R = \frac{H\delta\sigma}{\dot{\sigma}}$$

scalar curvature on uniform-density hypersurfaces

$$R = \frac{H\delta\rho}{\dot{\rho}}$$

necessarily adiabatic primordial perturbations $\left(\zeta_\gamma = \zeta_B = \zeta_\nu = \zeta_{cdm}\right) = \zeta_\sigma$

Inflaton scenario:

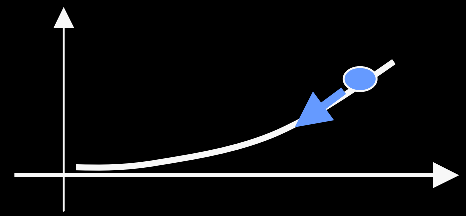
see, e.g., Lyth & Riotto

Kinney, Melchiorri & Riotto (2001)

high energy / not-so-slow roll

1. **large field** ($\Delta\varphi < M_{Pl}$)

   e.g. chaotic inflation

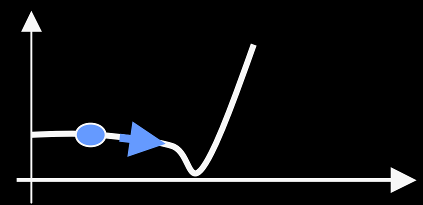


$$0 < \eta < \varepsilon$$

not-so-high energy / very slow roll

2. **small field**

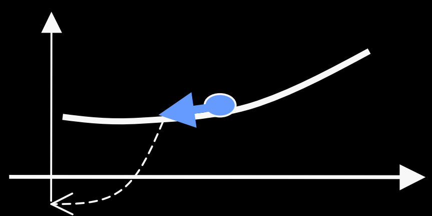
   e.g. new or natural inflation



$$\eta < 0$$

3. **hybrid inflation**

   e.g., susy or sugra models



$$0 < \varepsilon < \eta$$

slow-roll solution for potential-dominated, over-damped evolution gives useful approximation to growing mode for *{ ε , |η| } << 1*

$$\varepsilon \equiv \frac{M_P{}^2}{16\pi}\left(\frac{V_\phi}{V}\right)^2 \approx -\frac{\dot{H}}{H^2} \qquad \eta \equiv \frac{M_P{}^2}{8\pi}\left(\frac{V_{\phi\phi}}{V}\right) = \frac{m^2}{H^2}$$

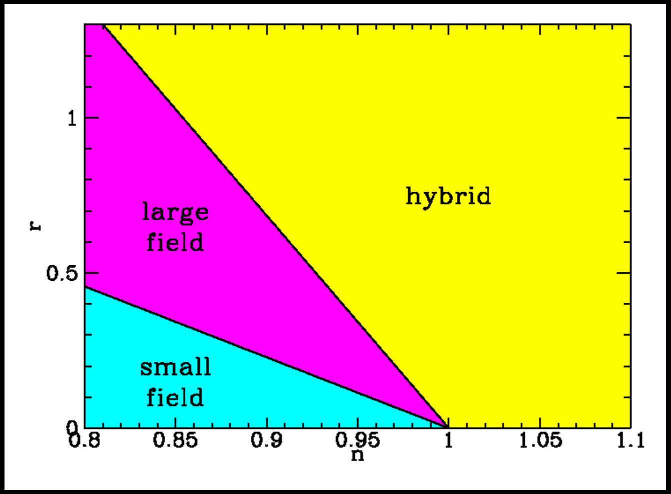
can be distinguished by observations

- slow time-dependence during inflation
  -> weak scale-dependence of spectra

$$n = 1 - 6\varepsilon + 2\eta$$

- tensor/scalar ratio suppressed at low energies/slow-roll

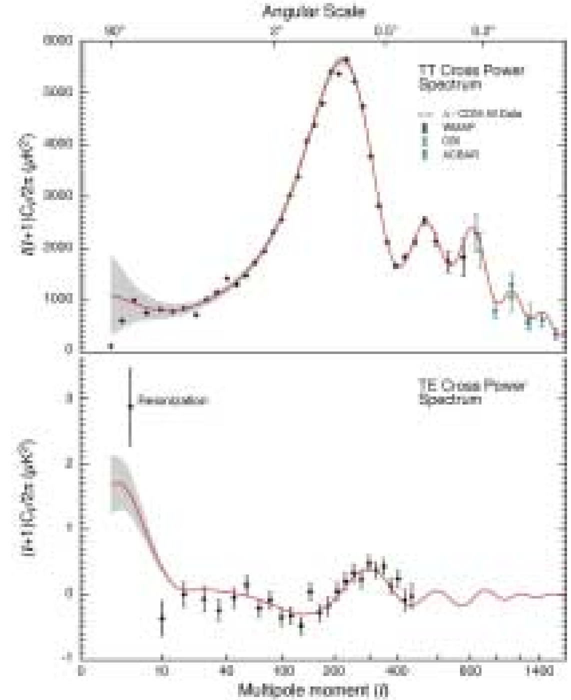
$$\frac{\langle T^2 \rangle}{\langle R^2 \rangle} = 16\varepsilon$$



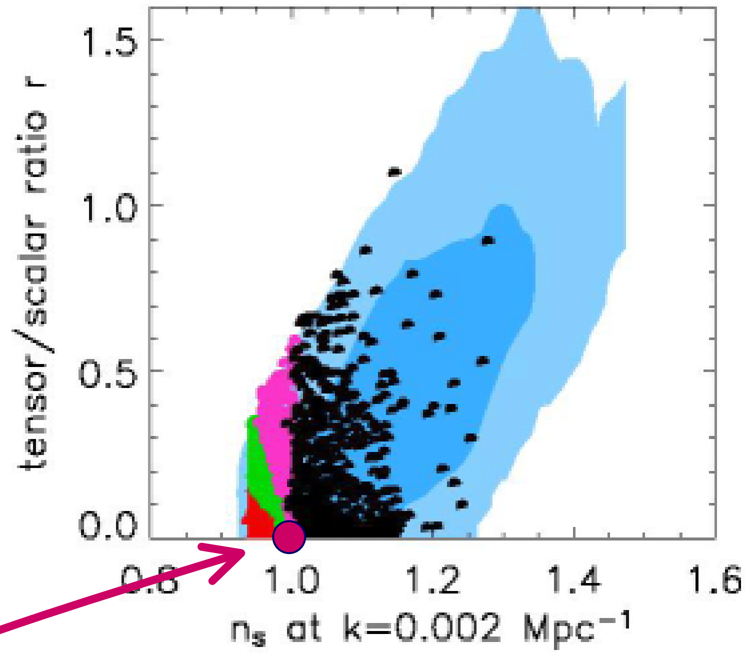
WMAP constraints (I)

- Microwave background only (WMAPext)

  Peiris et al (2003)



$r < 1.28$



spectral index

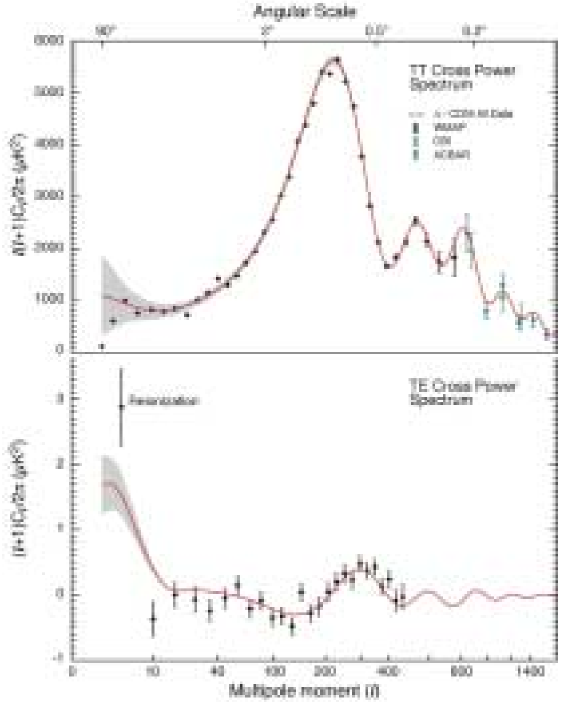
$$n_R \approx 1 - 6\varepsilon + 2\eta$$

Harrison-Zel'dovich

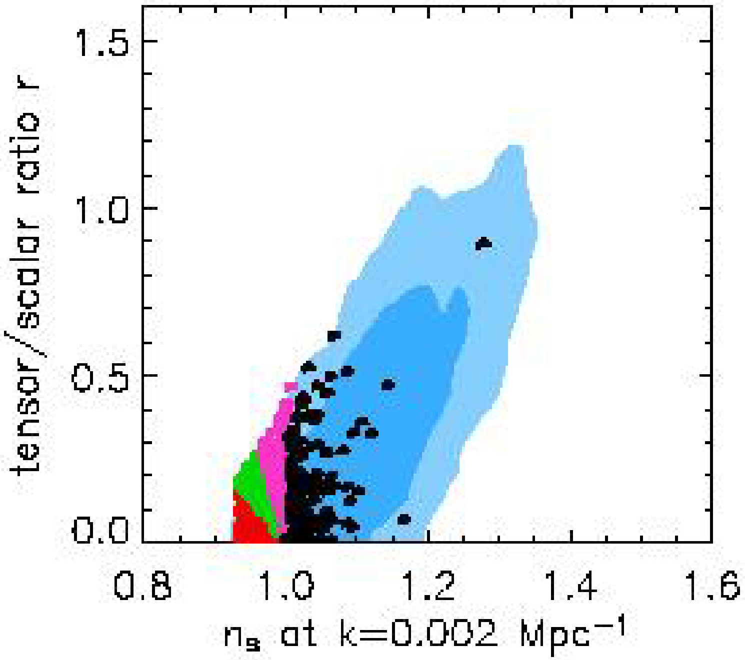
$$\boldsymbol{n \to 1, \quad r \to 0}$$

WMAP constraints (II)

- Microwave background + 2dF + Ly-alpha
  Peiris et al (2003)



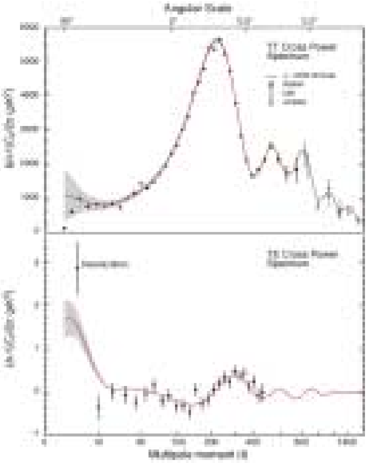
$r < 1.28$



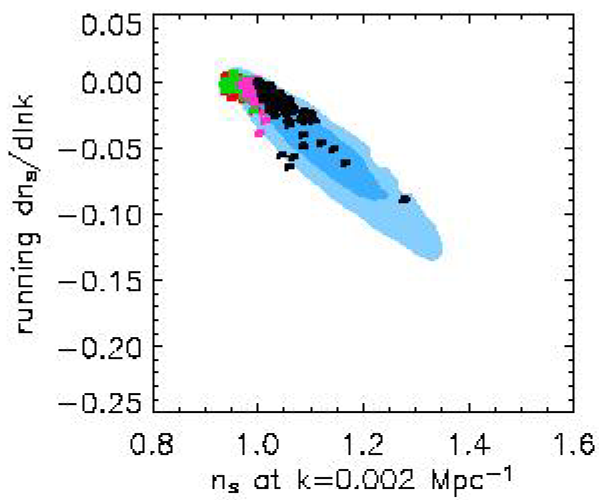
spectral index $n_R \approx 1 - 6\varepsilon + 2\eta$

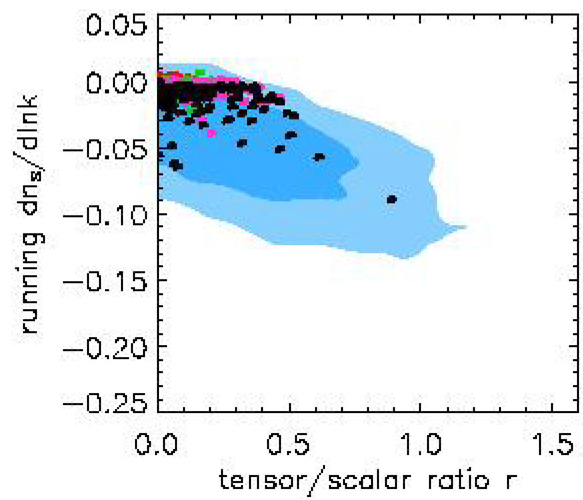
WMAP constraints (III)

- Microwave background + 2dF + Ly-alpha
  Peiris et al (2003)



$$dn_R / d \ln k = -0.055^{+0.028}_{-0.029}$$





$$n_R = 1.13 \pm 0.08$$

$$r < 1.28$$

scale-dependent tilt?

$$\frac{dn_R}{d\ln k} \approx -2\xi - 8\varepsilon(3\varepsilon - 2\eta)$$

- third slow-roll parameter

$$\xi \equiv \frac{M_{Pl}^{\ 4}}{64\pi^2}\frac{V_{,\phi}V_{,\phi\phi\phi}}{V^2}$$

  - *involving four derivatives of the potential, not two*
  - *the beginning of the end for slow-roll?*

- inflaton effective mass is not constant

$$\frac{d\eta}{dN} \approx -2\xi + 2\varepsilon\eta$$

slow-roll inflation could be just a passing phase!

digging deeper:

- additional fields may play an important role
  - initial state — new inflation
  - ending inflation — hybrid inflation
  - enhancing inflation — assisted inflation

    warm inflation

    brane-world inflation
  - ***producing density perturbations***
- may yield additional information
  - **non-gaussianity**
  - **isocurvature (non-adiabatic) modes**

"Notice all the computations, theoretical scribblings, and lab equipment, Norm. … Yes, curiosity killed these cats."

Inflation -> primordial perturbations (II)

scalar field fluctuations

two fields (σ,χ)

curvature of uniform-field slices

$$R_* = \frac{H\delta\sigma}{\dot{\sigma}}$$

isocurvature

$$S_* = \frac{H\delta\chi}{\dot{\sigma}}$$

density perturbations

matter and radiation (m,γ)

curvature of uniform-density slices

$$R = \frac{H\delta\rho}{\dot{\rho}}$$

$$S = \frac{\delta n_m}{n_m} - \frac{\delta n_\gamma}{n_\gamma}$$

$$\dot{R} = \alpha H S,$$

$$\dot{S} = \beta H S$$

$$\begin{pmatrix} R \\ S \end{pmatrix}_{\substack{\text{primordial} \\ k \ll aH}} = \begin{pmatrix} 1 & T_{RS} \\ 0 & T_{SS} \end{pmatrix} \begin{pmatrix} R_* \\ S_* \end{pmatrix}_{\substack{\text{inflation} \\ k = aH}}$$

model-dependent transfer functions

Amendola, Gordon, Wands & Sasaki (2002)

Wands, Bartolo, Matarrese & Riotto (2002)

examples:

- field dynamics during inflation

  Polarski & Starobinsky; Sasaki & Stewart; Garcia-Bellido & Wands; Steinhardt & Mukhanov; Adams, Ross & Sarkar; Langlois... (1996+)

- variable couplings during/after reheating

  Dvali, Gruzinov & Zaldariaga; Kofman (2003)

- late-decaying scalar : ***the curvaton scenario***

  Enqvist & Sloth; Lyth & Wands; Moroi & Takahashi (2001+)

curvaton scenario:

Lyth & Wands, Moroi & Takahashi, Enqvist & Sloth (2002)

assume negligible curvature perturbation during inflation $\left\langle R_*{}^2 \right\rangle = 0$

light during inflation, hence acquires isocurvature spectrum $\left\langle S_*{}^2 \right\rangle \approx \frac{\delta\rho_\chi}{\rho_\chi}$

late-decay, hence energy density non-negligible at decay $T_{RS} \approx \Omega_{\chi,decay}$

large-scale density perturbation generated entirely by non-adiabatic modes after inflation

$$\left\langle R^2 \right\rangle = T_{RS}{}^2 \left\langle S_*{}^2 \right\rangle \approx \Omega^2_{\chi,decay} \left\langle (\delta\rho_\chi / \rho_\chi)^2 \right\rangle$$

- negligible gravitational waves
- 100%correlated residual isocurvature modes
- detectable non-Gaussianity if $\Omega_{\chi,decay} << 1$

primordial isocurvature perturbations from curvaton?

Moroi & Takahashi; Lyth, Ungarelli & Wands '03

- cdm, neutrinos, baryon asymmetry all created *after* curvaton decays

$$\zeta = \left(\zeta_\gamma = \zeta_\nu = \zeta_{cdm} = \zeta_B\right) \approx \Omega_{\chi,\mathrm{decay}}\,\zeta_\chi \qquad \Rightarrow \qquad S_i = 0$$

- cdm/baryon asymmetry created at high energies *before* curvaton decay

$$\zeta = \zeta_\gamma \approx \Omega_{\chi,\mathrm{decay}}\zeta_\chi \quad , \quad \zeta_m = 0 \qquad \Rightarrow \qquad S_m = -3\zeta$$

  - 100% correlation between curvature and "**residual" isocurvature mode**
  - naturally of same magnitude

- neutrino asymmetry ($\xi<0.1$) created at high energies before curvaton decay

$$\Rightarrow \quad S_\upsilon = -\frac{135}{7}\left(\frac{\xi}{\pi}\right)^2 \zeta$$

Observational constraints

Gordon & Lewis, astro-ph/0212248v2

using CMB + 2dF + HST + BBN

isocurvature / curvature ratio

$$B = S_B / \zeta$$

c.f. Peiris et al

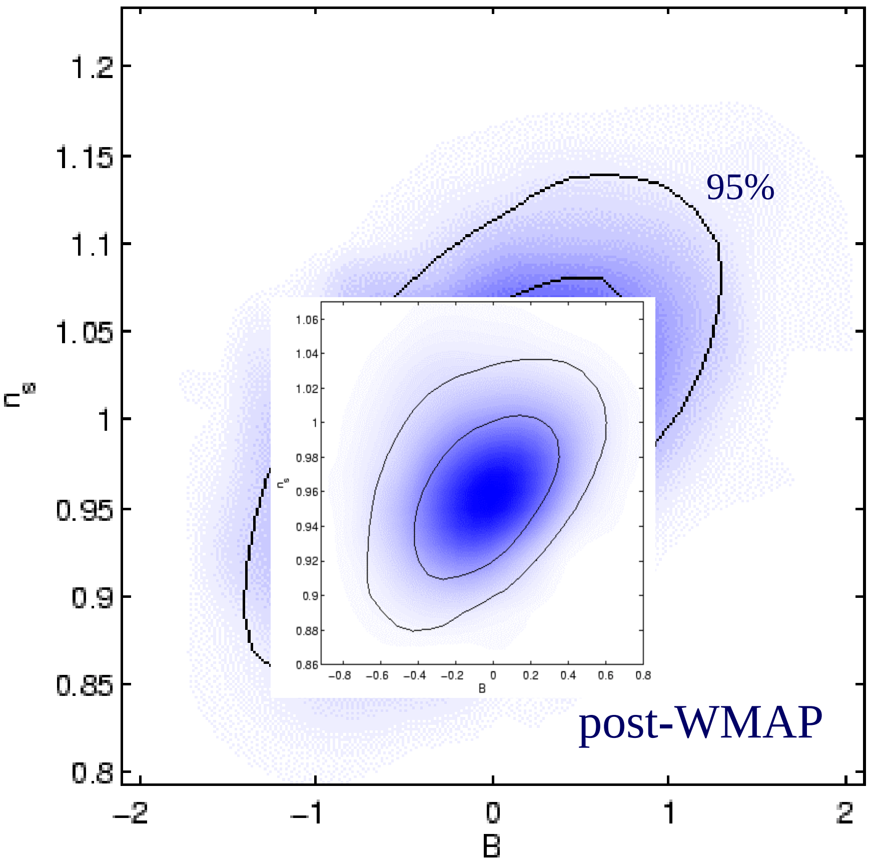
$$f_{iso} = S_{cdm}/\zeta$$

$$\approx 0.1\ B$$

and marginalised over correlation angle

$$\text{->}\ f_{iso} < 0.33$$

pre-WMAP



isocurvature perturbations from curvaton (II)

Lyth, Ungarelli & Wands '02

Gupta, Malik & Wands in preparation

- cdm/baryon asymmetry created *by* curvaton decay

$$\zeta = \zeta_\gamma \approx \Omega_{\chi,\mathrm{decay}} \zeta_\chi \quad , \quad \zeta_m = \zeta_\chi$$

$$\Rightarrow \quad S_m = 3\left(1-\Omega_{\chi,decay}\right)\zeta_\chi = 3\left(\frac{1-\Omega_{\chi,decay}}{\Omega_{\chi,decay}}\right)\zeta$$

  - curvature and isocuravture perturbations naturally of same magnitude
  - *relative magnitude related to non-Gaussianity*

non-Gaussianity

simplest kind of non-Gaussianity:

Komatsu & Spergel (2001)

Wang & Kamiokowski (2000)

$$\frac{\delta\rho}{\rho} \approx \left(\frac{\delta_1\rho}{\rho}\right) + f_{NL}\left(\frac{\delta_1\rho}{\rho}\right)^2$$

recall that for curvaton

$$\frac{\delta\rho}{\rho} \approx \Omega_{\chi,\mathrm{decay}}\frac{\delta\rho_\chi}{\rho_\chi} \approx \Omega_{\chi,\mathrm{decay}}\left(\frac{\delta\chi}{\chi} + \left(\frac{\delta\chi}{\chi}\right)^2\right)$$

corresponds to

$$\frac{\delta_1\rho}{\rho} \approx \Omega_{\chi,\mathrm{decay}}\left(\frac{\delta\chi}{\chi}\right) \quad , \quad f_{NL} \approx \frac{1}{\Omega_{\chi,\mathrm{decay}}}$$

Lyth, Ungarelli & Wands '02

significant constraints on f_{NL} from WMAP $f_{NL} < 134$

hence $\Omega_{\chi,\mathrm{decay}} > 0.01$ and $10^{-5} < \delta\chi/\chi < 10^{-3}$

observable parameters

- **inflaton regime**
  - curvature & tensor perturbations
  - n_s & tensor/scalar ratio = n_t
- **curvaton regime**
  - curvature + isocurvature perturbations
  - $n_s = n_{iso}$ & isocurvature/curvature ratio
- **intermediate regime**
  - n_s , n_{iso} , n_{corr} , n_t , tensor/scalar, iso/curvature, correlation

Wands, Bartolo, Matarrese & Riotto, '02

Conclusions:

1. ***Observations*** of ***tilt*** of density perturbations *($n \neq 1$)* and ***gravitational waves*** *($\varepsilon > 0$)* can distinguish between slow-roll models
2. ***Isocurvature*** perturbations and/or ***non-Gaussianity*** may provide valuable info
3. ***Non-adiabatic*** perturbations in multi-field models are an additional source of curvature perturbations on large scales
4. ***Consistency relations*** remain an important test in multi-field models - can falsify slow-roll inflation
5. ***More precise data*** allows/requires us to study more detailed models!