a_μ and its implications for physics beyond the SM, particularly in view of the questions of grand unification, origin of flavour and CP violation!

G.Ross, Glasgow, October 2007

a_μ and its implications for physics beyond the SM, particularly in view of the questions of grand unification, origin of flavour and CP violation!

G.Ross, Glasgow, October 2007

- Supersymmetry
- Extra dimensions
- Additional gauge bosons
- Additional (vectorlike) fermions
- Additional scalars
- Exotic flavour-changing interactions
- Nonperturbative effects
- Muon substructure
- Anomalous gauge boson couplings
- ….

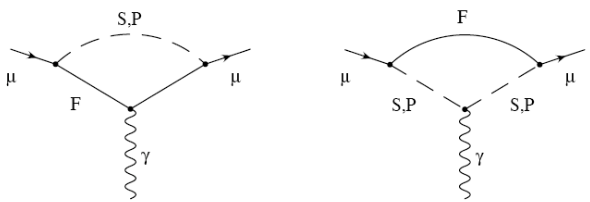
a_μ and its implications for physics beyond the SM, particularly in view of the questions of grand unification, origin of flavour and CP violation!

G.Ross, Glasgow, October 2007

- Supersymmetry   Hierarchy problem, GUT
- Extra dimensions   Hierarchy problem, GUT
- Additional gauge bosons   L-R symmetry, Family group, ..?
- Additional (vectorlike) fermions   Froggatt-Nielsen mass generation, string compact…
- Additional scalars   L-R symmetry, Familons, little Higgs…
- Exotic flavour-changing interactions   Family symmetries, technicolour…
- Nonperturbative effects   Composite structure...
- Muon substructure   × ? $1.7TeV < \Lambda_\mu < 2.3TeV \quad c.f. \quad \Lambda \geq 4-5TeV$
- Anomalous gauge boson couplings   × ?
- ….

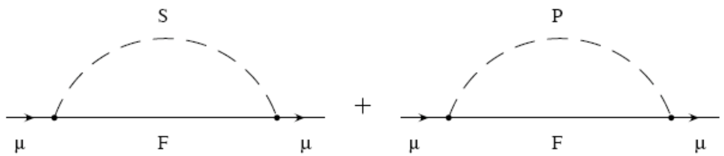
$a_\mu \Leftrightarrow m_\mu$

a_μ



$$\frac{g^2}{8\pi^2}\frac{m_\mu M_F}{M_P^2}\left(1-\frac{M_P^2}{M_S^2}\right)$$

$m_\mu \simeq$



$$\frac{g^2}{16\pi^2}M_F \ln\left(\frac{M_S^2}{M_P^2}\right)$$

$$a_\mu(\text{New Physics}) \simeq C\frac{m_\mu^2}{M_P^2},$$

$$C = \left(1-\frac{M_P^2}{M_S^2}\right) / \ln\frac{M_S^2}{M_P^2}$$

In general

$$a_\mu = C\frac{m_\mu^2}{M^2}$$

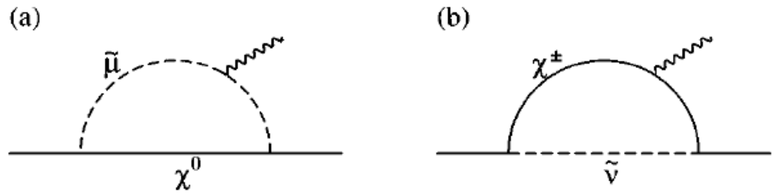
$$C \leq O(1)$$

Czarnecki,Marciano

Application to SUSY

$$\Delta a_\mu \propto g_{1,2}^2\, m_\mu^2 \frac{m_{G_{1,2}}\mu_H}{\widetilde{M}^2} \tan\beta$$

$$\Delta m_\mu \propto g_{1,2}^2\, m_\mu \frac{m_{G_{1,2}}\mu_H}{\widetilde{M}^2} \tan\beta$$



$$a_\mu = C \frac{m_\mu^2}{\widetilde{M}^2}$$

Δa_μ large $\Rightarrow$ Δm_μ large

Test of large Δm_μ?

$$\psi_\alpha = \begin{pmatrix} d \\ d \\ d \\ l \end{pmatrix}$$

Yukawa Unification

$$\frac{m_s}{m_\mu}(M_X) = \frac{1}{3}$$

$$\frac{m_d}{m_e}(M_X) = 3$$

Georgi Jarlskog

$$|V_{us}| = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta}\sqrt{\frac{m_u}{m_c}} \right|$$

$$\frac{M^{d,l}}{m_3} = \begin{pmatrix} < \varepsilon^4 & \varepsilon^3 & \\ \varepsilon^3 & a^{d,l}\varepsilon^2 & \\ & & 1 \end{pmatrix}$$

$\varepsilon \simeq 0.15$

$a^d = 1 \quad a^l = -3$

$$\bar{\psi}^\alpha \psi_\alpha$$

$$\frac{m_b}{m_\tau}(M_X) = 1$$

$$\bar{\psi}^\alpha \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix} \psi_\alpha$$

$$Det(M^l) = Det(M^d)|_{M_X}$$

Effect of Δm_μ^{SUSY}, Δm_b^{SUSY}

Parameters	Input SUSY Parameters					
$\tan\beta$	1.3	10	38	50	38	38
γ_b	0	0	0	0	-0.22	$+0.22$
γ_d	0	0	0	0	-0.21	$+0.21$
γ_t	0	0	0	0	0	-0.44
Parameters	**Comparison with GUT Mass Ratios**					
$(m_b/m_\tau)(M_X)$	$1.00^{+0.04}_{-0.4}$	0.73(3)	0.73(3)	0.73(4)	1.00(4)	1.00(4)
$(3m_s/m_\mu)(M_X)$	$0.70^{+0.8}_{-0.05}$	0.69(8)	0.69(8)	0.69(8)	0.9(1)	0.6(1)
$(m_d/3\,m_e)(M_X)$	0.82(7)	0.83(7)	0.83(7)	0.83(7)	1.05(8)	0.68(6)
$(\frac{\det Y^d}{\det Y^e})(M_X)$	$0.57^{+0.08}_{-0.26}$	0.42(7)	0.42(7)	0.42(7)	0.92(14)	0.39(7)

Serna, GGR

$$\gamma_t \approx y_t^2\,\mu A^t \frac{\tan\beta}{16\pi^2} I_3(m_{\tilde t_1}^2, m_{\tilde t_2}^2, \mu^2) \sim y_t^2 \frac{\tan\beta}{32\pi^2}\frac{\mu A^t}{m_{\tilde t}^2}$$

$$\gamma_u \approx -g_2^2 M_2 \mu \frac{\tan\beta}{16\pi^2} I_3(m_{\chi_1}^2, m_{\chi_2}^2, m_{\tilde u}^2) \sim 0$$

$$\gamma_b \approx \frac{8}{3} g_3^2 \frac{\tan\beta}{16\pi^2} M_3 \mu I_3(m_{\tilde b_1}^2, m_{\tilde b_2}^2, {M_3}^2) \sim \frac{4}{3} g_3^2 \frac{\tan\beta}{16\pi^2}\frac{\mu M_3}{m_{\tilde b}^2}$$

$$\gamma_d \approx \frac{8}{3} g_3^2 \frac{\tan\beta}{16\pi^2} M_3 \mu I_3(m_{\tilde d_1}^2, m_{\tilde d_2}^2, {M_3}^2) \sim \frac{4}{3} g_3^2 \frac{\tan\beta}{16\pi^2}\frac{\mu M_3}{m_{\tilde d}^2}$$

$$\frac{\mu M_3}{m_{\tilde b}^2} \sim -0.5, \qquad \frac{m_{\tilde b}^2}{m_{\tilde d}^2} \sim 1.0$$

$$c.f. \quad \Delta a_\mu \propto g_{1,2}^2\, m_\mu^2 \frac{m_{G_{1,2}} \mu_H}{\widetilde{M}^2} \tan\beta$$

$$\Rightarrow M_3 < 0,\ \ M_{1,2} > 0\ ?$$

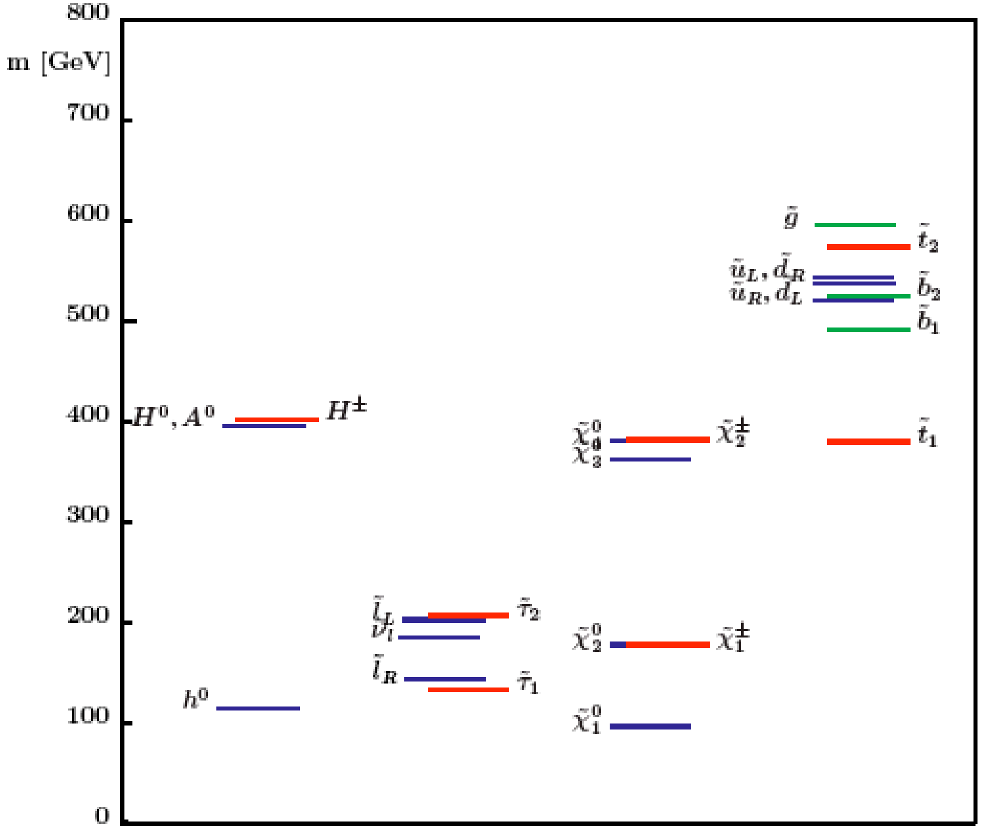
a_μ and Grand Unification

- GUT predictions for m improved
- $\widetilde{M}_{i=1,2,3}$ not universal?

a_μ and Grand Unification

- GUT predictions for m improved
- $\widetilde{M}_{i=1,2,3}$ not universal?

SPS 1



e.g. Anomaly mediation

RG invariant relations – fixed point

$$M_i = m_0 \beta_{g_i} / g_i \qquad M_3 < 0$$

$$h_{t,b,\tau} = -m_0 \beta_{Y_{t,b,\tau}}$$

$$(m^2)^i{}_j = \frac{1}{2} m_0^2 \mu \frac{d}{d\mu} \gamma^i{}_j$$

$$m_3^2 = \kappa m_0 \mu - m_0 \beta_\mu$$

$$a_\mu^{\mathrm{SUSY}} = 29.8$$

m_0	100 GeV
$m_{1/2}$	250 GeV
A_0	−100 GeV
$\tan\beta$	10
sign μ	+

a_μ and Grand Unification

- $\widetilde{M}_{i=1,2,3}$ not universal.
- Δa_μ large $\Rightarrow$ sneutrino and charginos relatively light
- $\Delta(b \to s\gamma)$ small $\Rightarrow$ heavy charged Higgs and heavy stop

$$\mathcal{BR}(b \to s\gamma)|_{\chi^\pm} \propto \mu A_t \tan\beta f(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{\chi}^+}) \frac{m_b}{v(1+\Delta m_b)}$$

Serna, GGR
Komine, Yamaguchi

Effect of ~~CP~~

New SUSY sources of ~~CP~~:

$$M_C = \begin{pmatrix} |\tilde{m}_2| e^{i\xi_2} & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & |\mu| e^{i\theta_\mu} \end{pmatrix}$$

To suppress neutron, electron EDM, $\xi_i, \theta_\mu = O(10^{-2})$ unless there are cancellations

For large phases significant corrections to a_μ

Table 1: Cases where the EDM and the g-2 experiments are satisfied

(case) ξ_2, θ_μ, ξ_3 (radian)	d_e, d_n (ecm)	a_μ^{SUSY}
(a) $-.63,.3,.37$	-4.2×10^{-27}, -5.3×10^{-26}	47.0×10^{-10}
(b)$-.85$,.4 ,.37	4.2×10^{-27}, 4.8×10^{-26}	10.8×10^{-10}
(c)$-.8$,.2 ,1.3	4.0×10^{-27}, 5.4×10^{-26}	12.2×10^{-10}
(d)$-.32$,.3 ,$-.28$	-1.2×10^{-27}, 3.3×10^{-26}	20.1×10^{-10}
(e)$-.5$,.49 ,$-.5$	1.8×10^{-27}, -6.6×10^{-27}	12.7×10^{-10}

Ibrahim, Chattopadhyay, Nath

Effect of $\not{C}P$

New SUSY sources of $\not{C}P$:

$$M_C = \begin{pmatrix} |\tilde{m}_2|e^{i\xi_2} & \sqrt{2}m_W \sin\beta \\ \sqrt{2}m_W \cos\beta & |\mu|e^{i\theta_\mu} \end{pmatrix}$$

To suppress neutron, electron EDM, $\xi_i, \theta_\mu = O(10^{-2})$ unless there are cancellations

For large phases significant corrections to a_μ

Table 1: Cases where the EDM and the g-2 experiments are satisfied

(case) ξ_2, θ_μ, ξ_3 (radian)	d_e, d_n (ecm)	a_μ^{SUSY}
(a) $-.63,.3,.37$	-4.2×10^{-27}, -5.3×10^{-26}	47.0×10^{-10}
(b)$-.85$,.4 ,.37	4.2×10^{-27}, 4.8×10^{-26}	10.8×10^{-10}
(c)$-.8$,.2 ,1.3	4.0×10^{-27}, 5.4×10^{-26}	12.2×10^{-10}
(d)$-.32$,.3 ,$-.28$	-1.2×10^{-27}, 3.3×10^{-26}	20.1×10^{-10}
(e)$-.5$,.49 ,$-.5$	1.8×10^{-27}, -6.6×10^{-27}	12.7×10^{-10}

Ibrahim, Chattopadhyay, Nath

But spontaneously broken family symmetries can naturally explain why $\xi_i, \theta_\mu = O(10^{-2})$

Vives, GGR

Origin of flavour

$$\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \simeq \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

FAMILY SYMMETRY?

Abelian, Non-Abelian $\subset (U(3))^6$

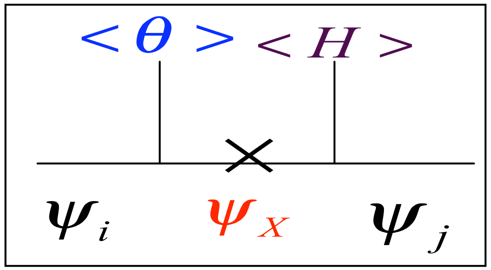
spontaneously broken:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{<\theta> \neq 0} \begin{pmatrix} 0 & 0 & 0 \\ 0 & a\varepsilon^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$a = O(1), \quad \varepsilon^2 = \frac{<\theta>}{M}$$

Messenger sector (heavy fermions, heavy Higgs)

$$m_{ij} = \frac{<\theta><H>}{M_X}$$



+family gauge group or familons

Froggatt, Nielsen

- Additional gauge bosons   L-R symmetry, Family group, ..?

$$a_\mu = C \frac{m_\mu^2}{M^2}$$

$$C \le O(1)$$

$C \ll 1$ family diagonal couplings (m_μ small)

$C \sim 1$ coupling to heavy fermions....M=1-2TeV

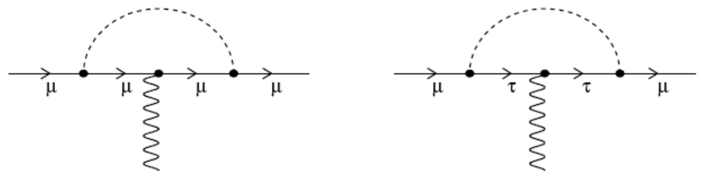
Strong correlation with FCNC

$e.g.\, Z'$ with large $\tau - \mu$ *mixing*: $\Delta a_\mu = 221 \times 10^{-11} \Rightarrow BR(\tau \to \mu\gamma) = 1 \times 10^{-6}$

Huang, Lin, Shan, Zhang

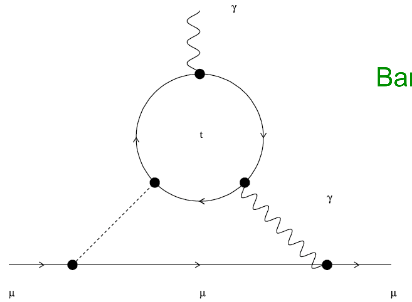
- Additional scalars   Extended Higgs structure, familons…

$$\Delta a_\mu = \pm \frac{1}{8\pi^2}\frac{m_\mu^2}{m_\phi^2}\ln\left(\frac{m_\phi^2}{m_\mu^2}\right)Y_{ii}^2$$



(a) (b)

$$\Delta a_\mu^h = \mp \frac{N_c q_t^2}{\pi^2}\frac{m_\mu m_t}{m_\phi^2}F\left(\frac{m_t^2}{m_\phi^2}\right)Y_{tt}Y_{\mu\mu}$$



Barr Zee

For $m_A \simeq 150 GeV$ two loop term can dominate.

Significant contribution if large μ-τ mixing

$$Y_{\mu\tau} = 0.04\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$$

$\tau \to \mu\gamma$ close to present bound

Summary

- Many ways to generate Δa_μ $\quad a_\mu = C \frac{m_\mu^2}{M^2}$ $\quad C \leq O(1)$

Summary

- Many ways to generate Δa_μ $\qquad a_\mu = C\frac{m_\mu^2}{M^2}$ $\qquad C \leq O(1)$

- Provides strong constraint on details of new physics

SUSY: M_i not universal, spectrum constrained

Family symmetry: Associated FCNC e.g. $\tau \to \mu\gamma$.

Constrained Yukawa structure