Determining the light-by-light contributions from Dyson-Schwinger equations

Christian S. Fischer

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25. October 2007

C.F., Journal of Physics **G32**, R253-R291 (2006).

Overview

1. The lbl contributions

2. The Dyson-Schwinger equations framework
   - Yang-Mills-theory
   - Quark-Gluon Interaction
   - DχSB: Quarks and Pions
   - Quark-Photon interaction

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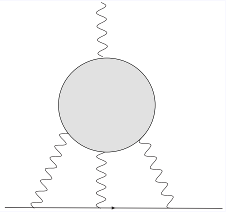
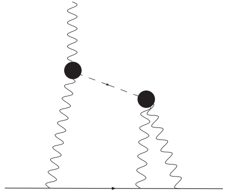
lbl - total

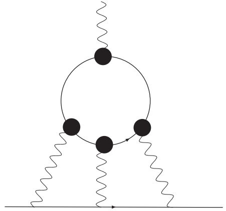


$$a_\mu^{theory} = 116591793(68) \times 10^{-11}$$
$$a_\mu^{lbl} = 100(39) \times 10^{-11}$$

F. Jegerlehner,arXiv:hep-ph/0703125.

lbl: PS-Meson-exchange and quark-loop





π_0, η, η'-pole contributions

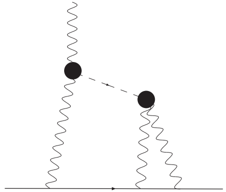
$$a_\mu^{lbl} = 100(39) \times 10^{-11}$$
$$a_\mu^{\pi} = 88(12) \times 10^{-11}$$

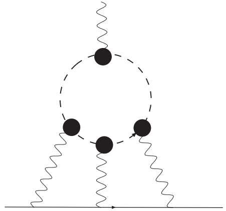
quark-loop contributions

$$a_\mu^{lbl} = 100(39) \times 10^{-11}$$
$$a_\mu^{quark} = 21(\ \ 3) \times 10^{-11}$$

F. Jegerlehner,arXiv:hep-ph/0703125.

lbl: AV-Meson exchange and meson-loop





AV-pole contributions

$$a_\mu^{lbl} = 100(39) \times 10^{-11}$$
$$a_\mu^{\pi} = 10(\ \ 4) \times 10^{-11}$$

meson-loop contributions

$$a_\mu^{lbl} = 100(39) \times 10^{-11}$$
$$a_\mu^{quark} = -19(13) \times 10^{-11}$$

F. Jegerlehner,arXiv:hep-ph/0703125.

Methodology

State of the art calculations done on basis of

- Extended Nambu-Jona-Lasinio (ENJL) model
- Vector meson dominance (VMD)
- quark-hadron duality
- short distance constraints

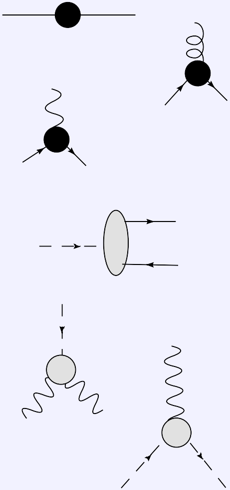
K. Melnikov and A. Vainshtein, PRD **70** (2004) 113006.
M. Knecht and A. Nyffeler, PRD **65** (2002) 073034.
M. Hayakawa and T. Kinoshita, PRD **57**, (1998) 465.
J. Bijnens, E. Pallante and J. Prades, NPB **474** (1996) 379.

- Overall: Numerical agreement between different methods
- Indiviual contributions: Disagreement
- Consistent ab initio approach desirable...

Elements of an ab initio calculation

Need to determine:

- quark propagator
- quark-gluon interaction
- quark-photon interaction
- wave function of $\pi, \eta, a_0, ...$
- $\pi - \gamma - \gamma$ form factor
- $\pi - \pi - \gamma$ form factor



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Motivation

QCD Green's functions

- are connected to confinement:
  - Gribov-Zwanziger/Kugo-Ojima scenarios
  - Positivity
  - Quark-antiquark potential
- encode DχSB
- are ingredients for hadron phenomenology
  - Bound state equations:
    Bethe–Salpeter equation / Faddeev equation

The Goal:
Ab initio description of hadrons as bound states
in terms of quark-gluon substructure

Quarks, Gluons and Ghosts:

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A, c] \; \exp\left\{ -\int d^4x \left(\bar{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \frac{(\partial A)^2}{2\xi} + \bar{c}(-\partial D)c \right) \right\}$$

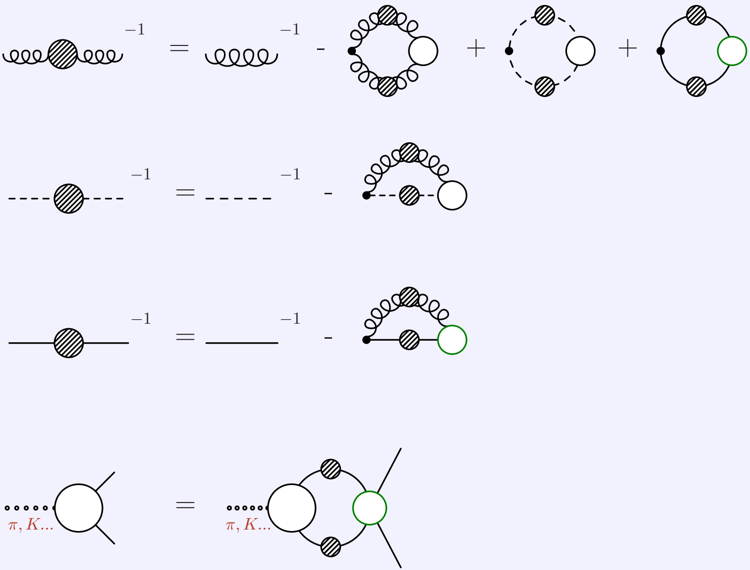
Landau gauge propagators in momentum space:

$$D^{\text{Gluon}}_{\mu\nu}(p) = \frac{\mathbf{Z}(p^2)}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

$$D^{\text{Ghost}}(p) = -\frac{\mathbf{G}(p^2)}{p^2}$$

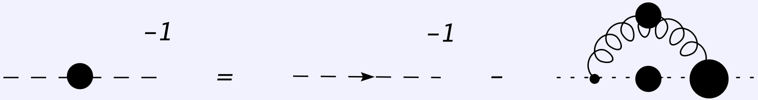
$$S^{\text{Quark}}(p) = \frac{Z_f(p^2)}{-i\not{p} + M(p^2)}$$

DSEs and BSE



Infrared Structure of YM-theory

- DSE for the ghost-propagator



- Selfconsistency in infrared:

$$Z(p^2) \sim (p^2)^{2\kappa}\ , \quad G(p^2) \sim (p^2)^{-\kappa}$$

L. v. Smekal, A. Hauck, R. Alkofer, Phys. Rev. Lett. **79** (1997) 3591

- $\kappa > 0 \quad \Rightarrow$ **Well defined global colour charge!**

P. Watson and R. Alkofer, Phys. Rev. Lett. **86** (2001) 5239

C. Lerche, L. v. Smekal, Phys. Rev. D **65** (2002) 125006.

General Infrared Structure

- $2n$ external ghost legs and m external gluon legs (one external scale p^2; **solves DSEs and STIs**):

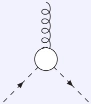
$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n-m)\kappa}$$

R. Alkofer, C. F., F. Llanes-Estrada, Phys. Lett. B **611** (2005)

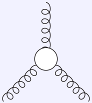
- Solution is unique!

C.F. and J. M. Pawlowski, Phys. Rev. D **75** (2007) 025012.

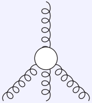
Nonperturbative Running Coupling



$$\alpha^{gh-gl}(p^2) = \alpha_\mu \, G^2(p^2) \, Z(p^2)$$

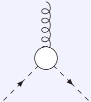


$$\alpha^{3g}(p^2) = \alpha_\mu \, [\Gamma^{3g}(p^2)]^2 \, Z^3(p^2)$$

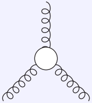


$$\alpha^{4g}(p^2) = \alpha_\mu \, \Gamma^{4g}(p^2) \, Z^2(p^2)$$

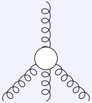
Running Coupling: IR-Universality



$$\alpha^{gh-gl}(p^2) \;=\; \alpha_\mu\, G^2(p^2)\, Z(p^2) \;\sim\; \mathbf{const}/N_c$$



$$\alpha^{3g}(p^2) \;=\; \alpha_\mu\, [\Gamma^{3g}(p^2)]^2\, Z^3(p^2) \;\sim\; \mathbf{const}/N_c$$

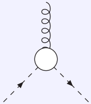


$$\alpha^{4g}(p^2) \;=\; \alpha_\mu\, \Gamma^{4g}(p^2)\, Z^2(p^2) \;\sim\; \mathbf{const}/N_c$$

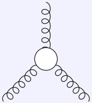
with

$$\Gamma^{3g}(p^2) \sim (p^2)^{-3\kappa}\,, \quad \Gamma^{4g}(p^2) \sim (p^2)^{-4\kappa}$$
$$G(p^2) \sim (p^2)^{-\kappa}\,, \quad Z(p^2) \sim (p^2)^{2\kappa}$$

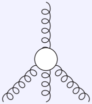
Running Coupling: IR-Universality



$$\alpha^{gh-gl}(p^2) = \alpha_\mu \, G^2(p^2) \, Z(p^2) \sim \mathbf{8.92}/N_c$$



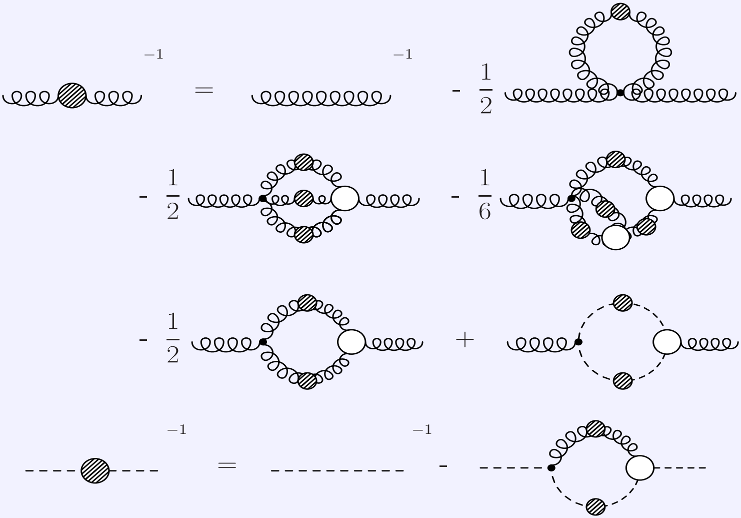
$$\alpha^{3g}(p^2) = \alpha_\mu \, [\Gamma^{3g}(p^2)]^2 \, Z^3(p^2) \sim \mathbf{const}/N_c$$



$$\alpha^{4g}(p^2) = \alpha_\mu \, \Gamma^{4g}(p^2) \, Z^2(p^2) \sim \mathbf{0.0086}/N_c$$

C. Lerche and L. v. Smekal, PRD **65** (2002) 125006

C. Kellermann and C.F., in preparation


−1
=
−1
- 1/2
- 1/2
- 1/6
- 1/2
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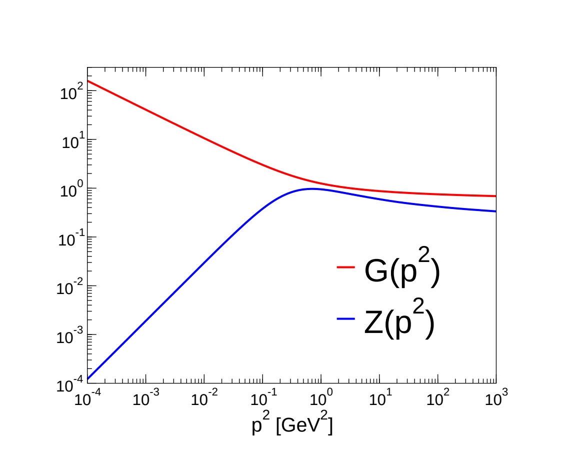
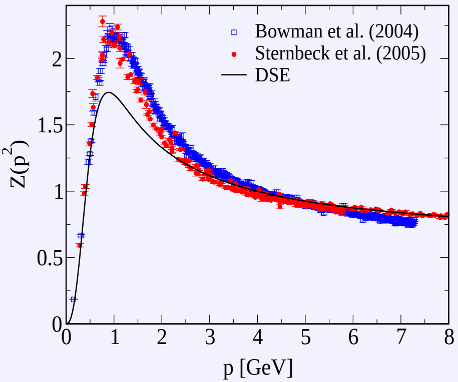
Ghost and Glue

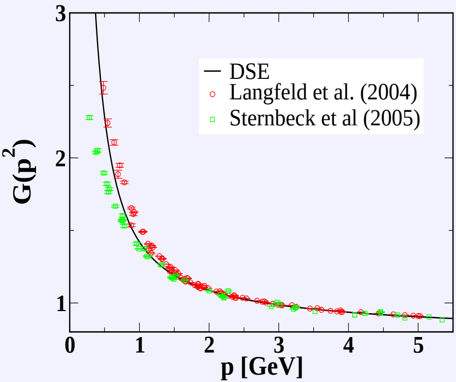


CF and Alkofer, PLB 536 (2002) 177.

- IR: $G(p^2) \sim (p^2)^{-\kappa}$ $\quad Z(p^2) \sim (p^2)^{2\kappa}$ $\qquad \kappa \approx 0.595$

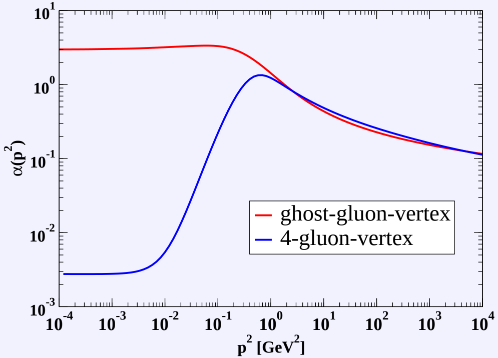
Ghost and Glue vs lattice





- Kugo-Ojima criterion satisfied: $G(p^2 = 0) \to \infty$

Running coupling



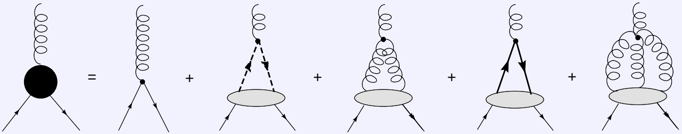
CF and Alkofer, PLB 536 (2002) 177.

Kellermann and CF, in preparation

- Ghost sector dominates deep IR

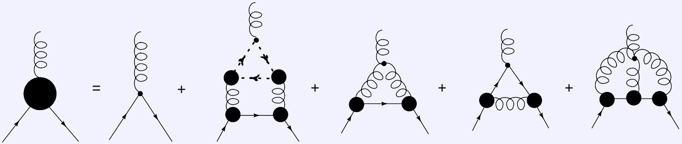
Infrared Structure of QCD

- Quark-gluon vertex:



Infrared Structure of QCD

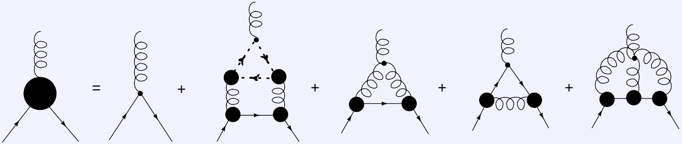
- Quark-gluon vertex: **lowest order** in skeleton expansion



$$S(p) = i\not{p}\,\frac{Z_f}{p^2+M^2} + \frac{Z_f M}{p^2+M^2}$$

Infrared Structure of QCD

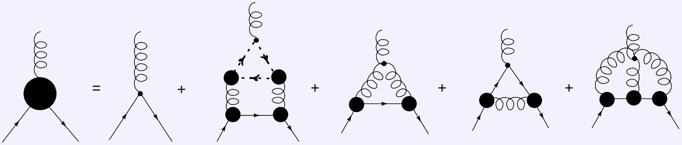
- Quark-gluon vertex: **lowest order** in skeleton expansion



$$S(p) = i\not{p}\,\frac{Z_f}{p^2+M^2} + \frac{Z_f M}{p^2+M^2}$$

$$\Gamma_\mu = ig\sum_{i=1}^{4}\lambda_i G^i_\mu \;:\quad G^1_\mu = \gamma_\mu\,,\;\; G^2_\mu = \hat{p}_\mu\,,\;\; G^3_\mu = \hat{\not{p}}\,\hat{p}_\mu\,,\;\; G^4_\mu = \hat{\not{p}}\,\gamma_\mu$$

- Quark-gluon vertex: **lowest order** in skeleton expansion



$$S(p) = i\not{p}\,\frac{Z_f}{p^2+M^2} + \frac{Z_f M}{p^2+M^2}$$

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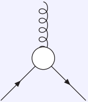
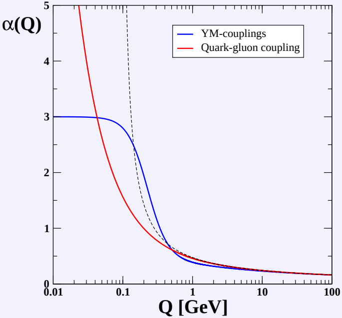
$$\lambda_{1,2,3,4} \sim (p^2)^{-1/2-\kappa} \qquad \leftrightarrow \qquad \lambda_{1,3} \sim (p^2)^{-\kappa}$$

Running Coupling: IR-slavery



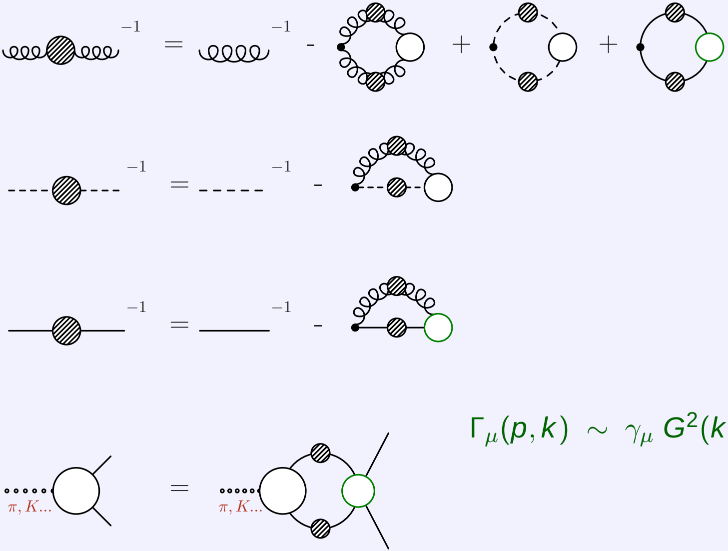
$$\alpha^{qg}(p^2) = \alpha_\mu \, [\Gamma^{qg}(p^2)]^2 \, [Z_f(p^2)]^2 \, Z(p^2) \sim \begin{cases} \frac{1}{p^2} : D\chi SB \\ const : \chi S \end{cases}$$



R. Alkofer, C. F., F. Llanes-Estrada, hep-ph/0607293.

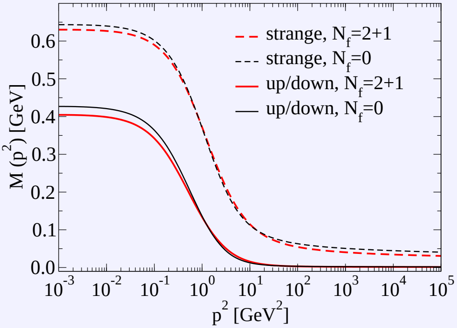
C. F., F. Llanes-Estrada, R. Alkofer, K. Schwenzer, in preparation.

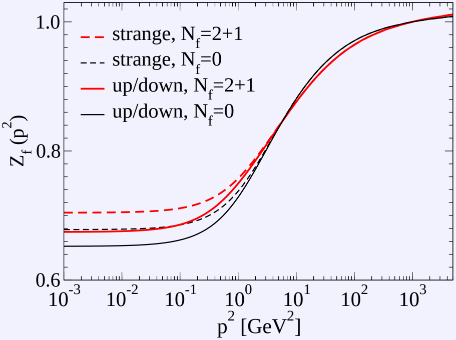
DSEs and BSE



$$\Gamma_\mu(p,k) \sim \gamma_\mu \, G^2(k)\, A(k)$$

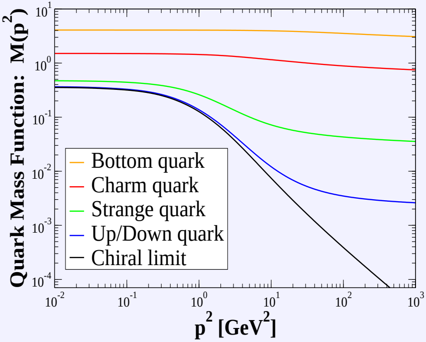
Partially unquenched quark

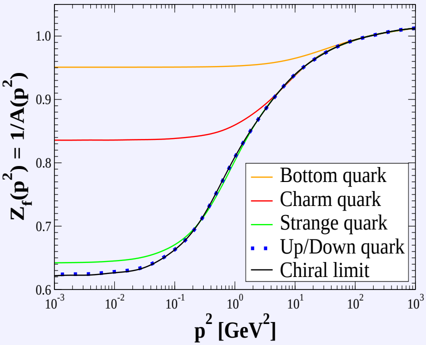




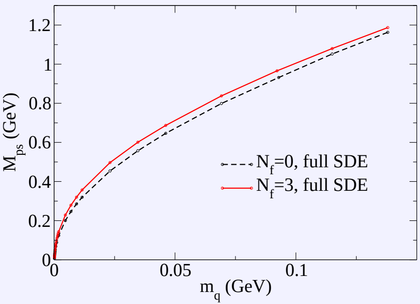
	$-(<\bar{q}q>^0)^{1/3}$ (MeV)	$M_{ch}(p^2=0)$ (MeV)
$N_f = 0$	266	416
$N_f = 2+1$	271	388

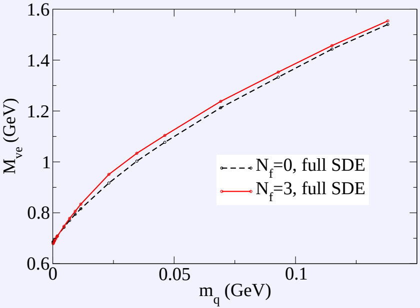
Massive Quarks





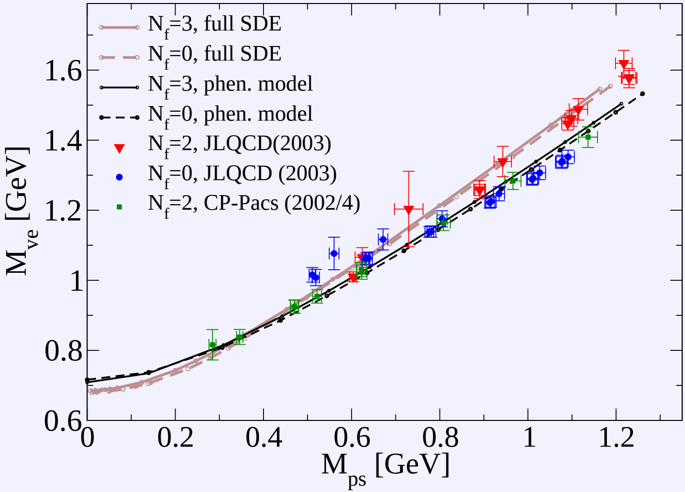
Partially unquenched light mesons I





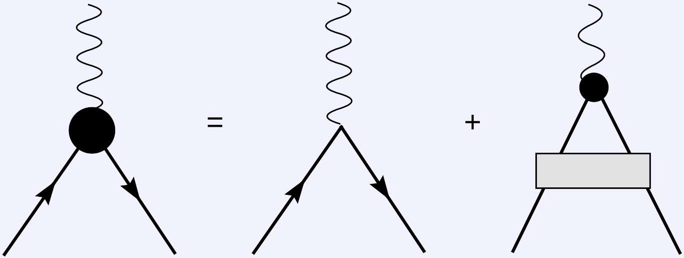
	m_u	m_s	m_π	f_π	m_K	f_K	m_ρ
$N_f = 0$	4.17	88.2	139.7	130.9	494.5	165.6	708.0
$N_f = 2+1$	4.06	86.0	140.0	131.1	493.3	169.5	695.2
$N_f = 3$	4.06		139.7	130.8			690.0
PDG	3.0-5.5	80-130	139.6	130.7	493.7	160.0	770

Partially unquenched light mesons II

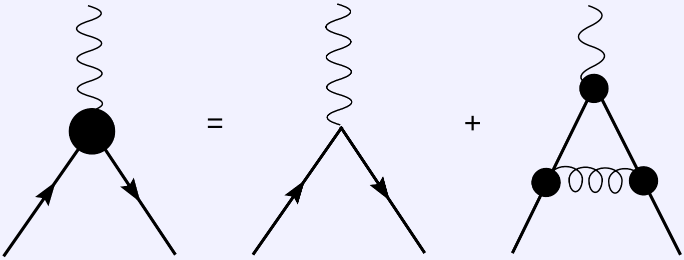


C. F., P. Watson and W. Cassing, Phys. Rev. D **72** (2005) 094025

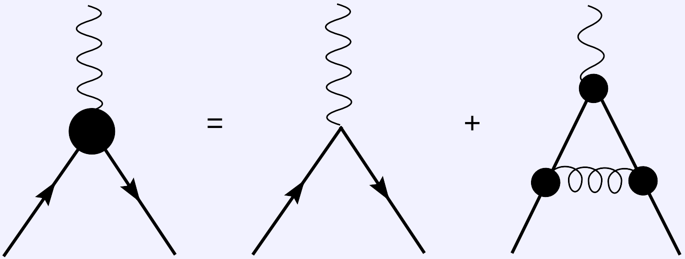
Quark-photon vertex



Quark-photon vertex



Quark-photon vertex

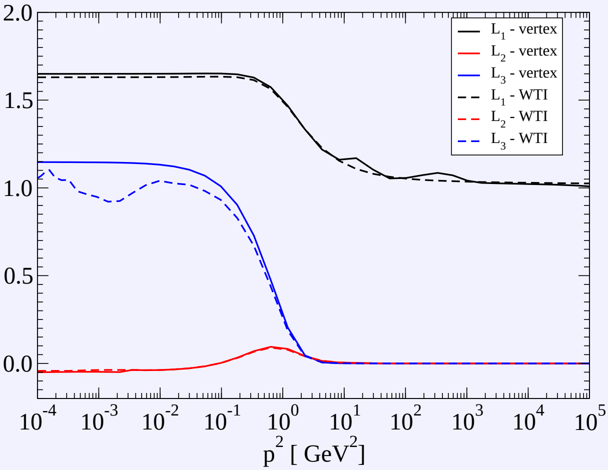


- Up to twelve independent tensor structures
- Ward-Takahashi identity (WTI)

$$k_\mu \Gamma_\mu(p,q,k) = S^{-1}(p) - S^{-1}(q)$$

C.F. and Andreas Krassnigg, work in progress

Quark-photon vertex



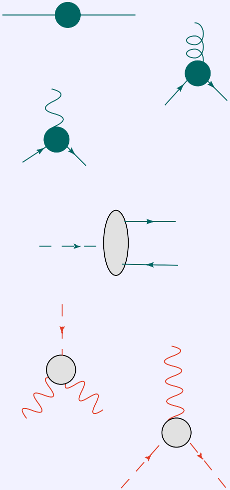
- Ward-Takahashi identity (WTI) satisfied!

$$k_\mu \Gamma_\mu(p,q,k) = S^{-1}(p) - S^{-1}(q)$$

Elements of an ab initio calculation

Need to determine:

- quark propagator
- quark-gluon interaction
- quark-photon interaction
- wave function of $\pi, \eta, a_0, ...$
- $\pi - \gamma - \gamma$ form factor
- $\pi - \pi - \gamma$ form factor



Summary

- Yang-Mills sector well under control
  - Infrared solution for any 1PI Green's function
  - Fixed point of running coupling
- Progress in Quark sector
  - Quark-Gluon-Vertex: Confinement $\leftrightarrow$ DχSB
  - Quark-Photon-Vertex: WTI satisfied
- Meson sector
  - Goldstone bosons under control
  - Missing pieces: axialvector mesons, $\pi - \gamma - \gamma$, $\pi - \pi - \gamma$