Twist-2 from QCD sum rules

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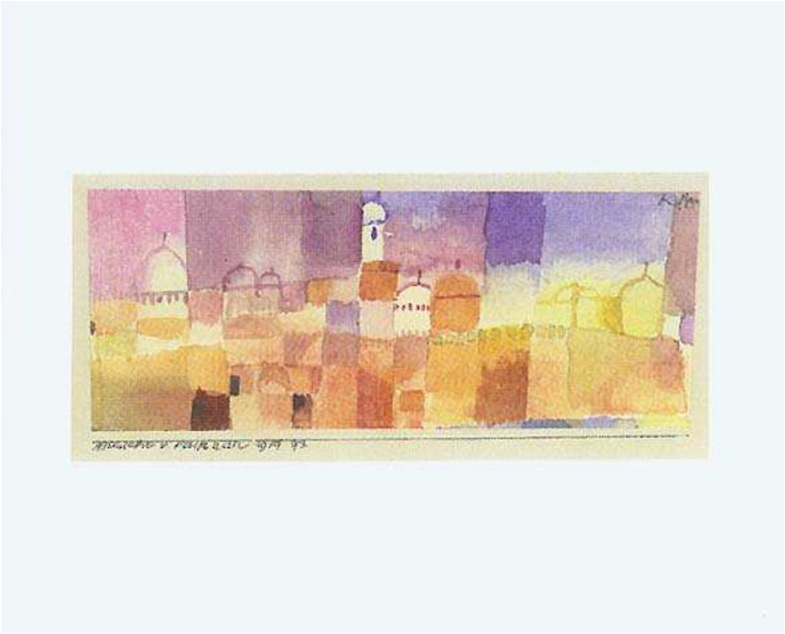

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   - conformal expansion of DA
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    - diagonal sum rules
    - operator method

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6. Summary, Conclusions – Questions

1. light meson DA, leading twist-2 (heavy e.g. B-DA (V.Braun))

- "non-local extension of decay constant matrix elements"
  "twist" = dim - spin

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\begin{aligned}
\langle 0|\bar{q}(z)\not{z}\gamma_5[z,-z]s(-z)|K(q)\rangle_\mu &= i f_K qz \int_0^1 du\, e^{i\xi qz}\phi_K(u,\mu) + O(m_K^2, z^2) \\
\langle 0|\bar{q}(z)\not{z}[z,-z]s(-z)|K^*(q)\rangle_\mu &= (ez) f_K^{\parallel} m_{K^*} \int_0^1 du\, e^{i\xi qz}\phi_K^{\parallel}(u,\mu), \\
\langle 0|\bar{q}(z)\sigma_{\mu\nu}[z,-z]s(-z)|K^*(q)\rangle_\mu &= i(e_\mu q_\nu - e_\nu^{(\lambda)} q_\mu) f_K^{\perp}(\mu) \int_0^1 du\, e^{i\xi qz}\phi_K^{\perp}(u,\mu),
\end{aligned}
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  - $[z,-z]$ being the Wilson line (gauge invariance)
  - z_μ close light-like separation
  - $\xi = u-(1-u)$. $u,(1-u)$ interpretation collinear momentum fraction of quark

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  - $[z,-z]$ being the Wilson line (gauge invariance)
  - z_μ close light-like separation
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- Corrections higher twist: (classific. e.o.m. state art (P.Ball I))
  - higher Fock states $(\langle 0|\bar{q}\sigma\cdot Gs|K\rangle)$
  - deviation from the light-cone $(O(x^2, m_\pi^2))$
  - other comb. of “good” and “bad” LC-states $(\langle 0|\bar{q}\gamma_5 s|K\rangle)$

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$$\phi_K(u,\mu) = 6u\bar{u}(1 + \sum_{n\geq 1} a_n(\mu, K) C_n^{3/2}(2u-1))$$

a. a_n Gegenbauer moments (its det. main topic of this talk)

b. $a_{2n+1} = 0$ particles def. G-parity (e.g. $a_1(\pi) = 0$ not $a_1(K) \neq 0$)

c. no mixing at LO ($C_n(2u-1)$ eigen-fct LO evolution kernel)

d. anomalous dimension $\gamma_{n+1} > \gamma_n > 0$ “conformal hierarchy”
(Asymptotic DA $\phi_K(u) \stackrel{\mu\to\infty}{\to} 6u(1-u)$ known from P-QCD)

e. Alternative reasoning $SL(2,R)$ collinear subgroup of conformal group $SO(4,2)$
C_n are representations with conformal spin $j = 2+n$ (good q-number LO)
Strong analogy partial wave exp. $[S0(3), Y_{lm}] \sim [SL(2,R), C_n]$
C_n n-nodes higher n washed out upon convolution smooth kernel

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- Point d.& e. a priori arguments justify truncation
A posteriori justification ... smoothness kernels & numerical values of moments

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$$M_0 = 1 \qquad M_2 = \tfrac{1}{5} + \tfrac{12}{35} a_2$$
$$M_1 = a_1 \qquad M_4 = \tfrac{3}{35} + \tfrac{8}{35} a_2 + \tfrac{8}{77} a_4$$

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-
  - QCD sum rules (this talk)
  - Lattice-QCD (L.Del Debbio & A.Juettner)
  - non-local condensates (N.Stefanis II)
  - Instanton vacuum model (not covered)
  - Dyson-Schwinger (yet unexplored)

2. QCD sum rule ... tool estimating low hadr. param. low lying states

- Start suitable correlation function ($\Gamma_1 = \Gamma_2 = \gamma_5 \Rightarrow$ extraction $a_1(K)$)

$$\Pi(q) = i \int_x \langle 0 | T[\bar{q}(i \overleftrightarrow{D}_\mu)\Gamma_1 s](x) \, [\bar{s}\Gamma_2 q](0) | 0 \rangle e^{iqx}$$

Hadronic world: inserting a complete set of states $1 = \sum |K\rangle\langle K| + \ldots$

$$\Pi(q) \sim a_1 \frac{f_K^2}{q^2 - m_K^2} + \text{higher states}$$

Quark-gluon world: Operator product expansion for virtualities $-q^2 << \Lambda_{QCD}^2$

$$\Pi(q) \sim c^1(q^2) + \frac{c^{\bar{q}q}}{q^4} \langle \bar{m}_s qq \rangle + \ldots$$

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- Estimate "higher states" by analytic continuation
  N.B. non-trivial bec. OPE truncated (ok smearing over interval .. Quark-Hadron Duality)
  numerics improved applying Borel transformation $\Rightarrow$ extract a_1

…continuation

- For higher n the convergence OPE breaks down parametrically: $c^1 \sim O(n^{-2})$, $\langle G^2 \rangle \sim O(1)$, $\langle \bar{q}q \rangle^2 \sim O(n)$ etc
  - only calculate lowest moments up to $n = 2$
  - intuitively understood, higher a_n more and more non-local objects and therefore OPE diff. converge
  - also problematic lattice ... derivatives CPU-cons., moments M_n background e.g. $M_4 = \frac{3}{35} + \frac{8}{35} a_2 + \frac{8}{77} a_4$
  - other methods ? non-local condensates

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- effective method calculating a_n in one go via non local operator (Ball & Boglione 03)

3. The decay constants or the a_0

- pseudoscalars O^-: f_π,f_K well known experiment ($f_\eta, f_{\eta'}$ (T.Feldmann I))
- vectors 1^-: $f_\rho^{||}$ etc experiment (mixing ...)
- $f_\rho^\perp(\mu)$ etc rely on theory

  Penguin transition $b \to (s,d)$ ($B \to K^*, \rho(\omega)$) formfactors described LCSR (DA)

  Bulk part extracting $|V_{\rm td}/V_{\rm ts}|$ (Ball & RZ JHEP 06)

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- Longitudinal decay constants from experiment
  - tau-decays $\tau^- \to V^- \nu_\tau$

$$f_{\rho^-} = (210 \pm 2_{\rm Br} \pm 1_\Gamma) \cdot {\rm MeV} \qquad f_{K^{*-}} = (220 \pm 4_{\rm Br} \pm 2_\Gamma \pm 1_{|V_{\rm us}|}) \cdot {\rm MeV}\,,$$

  - electromagnetic annihilation $V^0 \to e^+e^-$ Mixing !

$$\begin{aligned} f_{\rho^0} &= (222 \pm 2_{\rm Br} \pm 1_\Gamma) \cdot {\rm MeV} \\ f_\omega &= (187 \pm 2_{\rm Br} \pm 1_\Gamma \pm 4_{\omega\phi} \pm 1_{\rho\omega}) \cdot {\rm MeV} \\ f_\phi &= (215 \pm 2_{\rm Br} \pm 1_\Gamma \pm 4_{\omega\phi}) \cdot {\rm MeV} \quad , \end{aligned}$$

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.
1. literature sometimes mixing neglected, uncertainty estimate
2. slight tension (Ball & Braun 96) $f_{\rho^-} = 195 \pm 7 \cdot {\rm MeV}$, $f_{\rho^0} = 216 \pm 5{\rm MeV}$, isospin ?

... continued

- QCD sum rules (Ball & RZ 05/06)

$$f_\rho \;= (206 \pm 7) \cdot \mathrm{MeV} \qquad f_\rho^\perp(1\,\mathrm{GeV}) \;= (165 \pm 9) \cdot \mathrm{MeV}$$

$$f_{K^*} = (222 \pm 8) \cdot \mathrm{MeV} \qquad f_{K^*}^\perp(1\,\mathrm{GeV}) = (185 \pm 10) \cdot \mathrm{MeV}$$

($f_{K^*}^\perp$ non-trivial second resonance K_1 needs added for stability

$$\left(\frac{f_\rho^\perp}{f_\rho^\parallel}\right)_{\mathrm{SR}} (2\,\mathrm{GeV}) = 0.69 \pm 0.04 \qquad \left(\frac{f_{K^*}^\perp}{f_{K^*}^\parallel}\right)_{\mathrm{SR}} (2\,\mathrm{GeV}) = 0.73 \pm 0.04$$

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- Encouraging extra effort would be desirable

Results a_1

Recall: variable u mom. fraction of quarks. $a_1 > 0$ s-quark average mom. higher light q-quark as suggested by intuition from constituent quark model

4. a_1 SU(3) breaking – Sum Rule history

Sum rule: diagonal $\Gamma_{1,2}$ same chirality, non-diagonal opposite ch. $\mu_0 = 1\text{GeV}$

Type	$a_1(K)(\mu_0)$	$a_1^{\parallel}(K^*)(\mu_0)$	$a_1^{\perp}(K^*)(\mu_0)$	Authors	Remarks
ND	0.17	0.19	0.2	Chernyak & Zhit. 84	sign mistake
ND	-0.18	-0.4	-0.34	Ball Boglione 03	NLO,unstable
D	0.05 ± 0.02	-	-	Khodjamiran et al 04	-
OPR	0.1 ± 0.12	0.1 ± 0.07	-	Braun Lenz 04	neglect $O(m_s^2)$
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- OPR: New method can't compete yet ...
- confirmed by Lattice-QCD calculations (low uncertainty)
  $a_1(K, 2\text{GeV}) = 0.0453(9)(29)$ QCDSF/UKQCD 06 (W.Schrors)
  $a_1(K, 2\text{GeV}) = 0.055(5)$ QCDSF 06 (A.Juettner)

Operator relations

- Operator relations (Braun & Lenz 04, Ball & RZ 06)

$$\frac{9}{5}\, a_1(K) \;= -\frac{m_s - m_q}{m_s + m_q} + 4\,\frac{m_s^2 - m_q^2}{m_K^2} - 8\kappa_4(K),$$

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- First Gegenbauer moment related to twist-4 matrix elements, N.B. $\kappa_4 \to 0$ for $m_s \to m_q$

$$\langle 0|\bar{q}(gG_{\alpha\mu})i\gamma^\mu\gamma_5 s|K(q)\rangle = iq_\alpha f_K m_K^2 \kappa_4(K)\,,$$

$$\langle 0|\bar{q}(gG_{\alpha\mu})i\gamma^\mu s|K^*(q)\rangle = e_\alpha^{(\lambda)} f_K^{\parallel} m_{K^*}^3 \kappa_4^{\parallel}(K^*)\,,$$

$$\langle 0|\bar{q}(gG_\alpha{}^\mu)\sigma_{\beta\mu} s|K^*(q)\rangle = f_K^{\perp} m_{K^*}^2 \left\{\kappa_4^{\perp}(K^*)(e_\alpha q_\beta - e_\beta q_\alpha) + \ldots\right\}$$

Estimate κ_4 then we get an estimate of a_1.

Deriving relations

- Method 1. non-flavour singlet QCD energy momentum tensor (Braun & Lenz 04)

$$O_{\mu\nu} = \frac{1}{2}\bar{q}\gamma_\mu i \overleftrightarrow{D}_\nu s + \frac{1}{2}\bar{q}\gamma_\nu i \overleftrightarrow{D}_\mu s - \frac{1}{4} g_{\mu\nu}\bar{q}\, i \overleftrightarrow{\not{D}}\, s$$

and then

$$\langle 0|\partial_\mu O^\mu_\nu|K^*\rangle \stackrel{e.o.m}{=} \dots$$

  - Leads to equation $a_1^{\|}(K^*)$, (for K $\gamma_\mu \to \gamma_\mu\gamma_5$)
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- Method 2. Derive directly from e.o.m. matrix elements (Ball & RZ 06)

$$\begin{aligned} \frac{\partial}{\partial x_\mu}\, \bar{q}(x)\gamma_\mu(\gamma_5)s(-x) &= -i\int_{-1}^{1} dv\, v\bar{q}_1(x)x_\alpha g G^{\alpha\mu}(vx)\gamma_\mu(\gamma_5)q_2(-x) \\ &+ (m_q \pm m_s)\bar{q}_1(x)i(\gamma_5)q_2(-x) \end{aligned}$$

  take matrix element $\langle 0|\ldots|K\rangle$ ..(involves other e.o.m. total transl. der.)
  allows to get $a_1^{\perp}(K^*)$ on same footing as others

...continued

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- $(\kappa_4)'s$ estimated any other non-pert approach
  m_s might spoil the numerics ?

5. Second Gegenbauer moment a_2

- No dramatic history like a_1 .. $O(m_s)$ or $O(m_s^2)$ not crucial

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a_2(\pi, 1\text{GeV}) &= 0.26^{+0.21}_{-0.09} \quad \text{Khodjamirian, Mannel\&Melcher(04)} \\
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(consistent prel. Lattice-QCD)
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- vector mesons .. update coming Ball et al (06?)

Conclusions, Summary & Questions

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  - light mesons conformal symmetry $\Rightarrow$ ordering
    coefficient calculable from local hadronic matrix elements $\Rightarrow$ non-pert. methods
  - lower moments a_n results $n = 2$, higher more diff. less interesting
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  - desirable have results for $f^\perp$ from lattice-QCD
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- *Thanks for attention and interest !*