QCD Sum Rules with Nonlocal Condensates and the Pion Distribution Amplitude

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Contents of Report

- Physical reasons for condensate **nonlocality** (**NLC**)
  and
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- QCD **NLC SRs** for pion DA
- **Bunch** of **pion distribution amplitudes** from **NLC SRs**
- **Pion DA** from **“non-diagonal”** **NLC SRs**

Physical reasons for NLC and problems of standard QCD SR approach to pion DA

Physical reasons for NLC QCD SRs

M.S.&Radyushkin, [SJNP (1989, 1990); PRD (1992)]
A.B.& M.S., [PLB 436(1998)351]

- Condensate nonlocality is inevitable property of QCD vacuum, **confirmed** later by lattice simulations.
- Correlation distance $1/\lambda$ in QCD vacuum provides crude measure of nonlocality.

Scales $\lambda^2 = \lambda^2_{\rm quark}(\lambda^2_{\rm gluon})$ enter into hadron
dynamic quantities like DAs, Form Factors, ...
in such a way so that **singularities** appear at $\lambda^2 \to 0$

★ What is the character of these singularities?

Physical reasons for NLC QCD SRs

- the **integrable** singularities **destroy applicability** of QCD SRs for DA moments $\langle\xi^N\rangle$ at $N \geq 2$, generating terms **nondecreasing** with N:

$$\langle\xi^N\rangle \sim \left(N^0, N^1, \ldots\right); \quad \varphi(x) \sim \Big(\delta(x), \delta^{\{1\}}(x), \ldots\Big)$$

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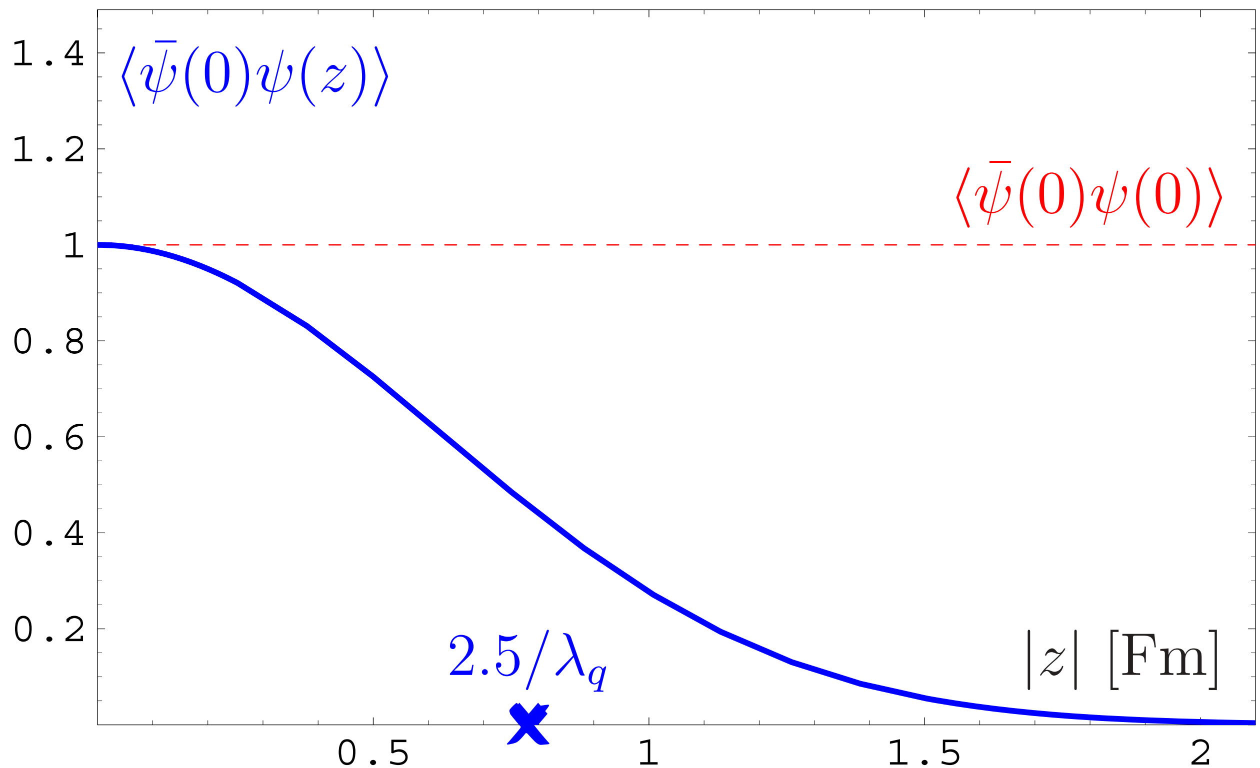
To avoid these problems, take into account nonlocality of vacuum condensate to obtain DAs and Form Factors with QCD SRs.

Quantitative measure of nonlocality given by $\Delta = \dfrac{\lambda^2}{M^2}$, where M^2 is average scale of Borel parameter.

Lattice data of Pisa group



Nonlocality of quark condensates from Pisa data **[BM, PRD 65,114511(2002)]**. **Dotted line** is local limit. Even at $|z| \simeq 0.5$ Fm, nonlocality still quite important.

Introducing NLC in QCD calculations

$$T\left(\bar{\psi}\psi\right) = \overline{\bar{\psi}\psi} + :\bar{\psi}\psi: \quad \textbf{(Wick theorem)}$$

$$\langle T\left(\bar{\psi}\psi\right)\rangle = i^{-1}\hat{S}_0(x) + \quad ?$$

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QCD PT

$$\langle :\bar{\psi}\psi: \rangle \overset{\textbf{def}}{=} 0$$

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$$\langle :\bar{\psi}(0)\psi(0): \rangle$$

$$\text{CONST} \neq 0$$

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[**SVZ'79**]

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NLC QCD SR

$$\langle:\bar{\psi}(0)\psi(z):\rangle$$

$$F_S(z^2) + \hat{z}F_V(z^2)$$

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QCD PT	**QCD SR**	**NLC QCD SR**
$\langle:\bar{\psi}\psi:\rangle \overset{\textbf{def}}{=} 0$	$\langle:\bar{\psi}(0)\psi(0):\rangle$	$\langle:\bar{\psi}(0)\psi(z):\rangle$
	CONST $\neq 0$	$F_S(z^2) + \hat{z}F_V(z^2)$
	[**SVZ'79**]	**M&R'86**
	Condensat	**Nonlocal condensat**

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Nonlocal condensat

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Distribution Amplitudes, Form Factors

Non-Local Condensates in QCD SR

- Illustration of
  NLC-model: $\langle \bar{q}(0)q(z) \rangle = \langle \bar{q}(0)q(0) \rangle e^{-|z^2|\lambda_q^2/8}$

Non-Local Condensates in QCD SR

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- A **single scale** parameter $\lambda_q^2 = \langle k^2 \rangle$ characterizing the average momentum of quarks in QCD vacuum:

$$\lambda_q^2 = \begin{cases} 0.4 \pm 0.1 \text{ GeV}^2 & [\text{ QCD SRs, 1987 }] \\ 0.5 \pm 0.05 \text{ GeV}^2 & [\text{ QCD SRs, 1991 }] \\ 0.4 - 0.5 \text{ GeV}^2 & [\text{ Lattice, 1998-2002 }] \end{cases}$$

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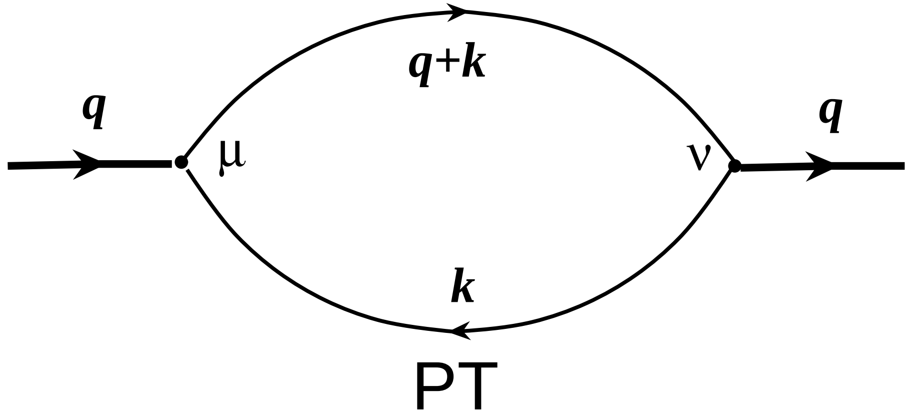
- Correlation length $\lambda_q^{-1} \sim \rho$-meson size
- Possible to include second ($\Lambda \simeq 450$ **MeV**) scale with $\langle \bar{q}(0)q(z) \rangle \Big|_{|z| \gg 1 \text{ Fm}} \sim \langle \bar{q}q \rangle e^{-|z|\Lambda}$ (not included here)

Introducing NLC in QCD calculations, example



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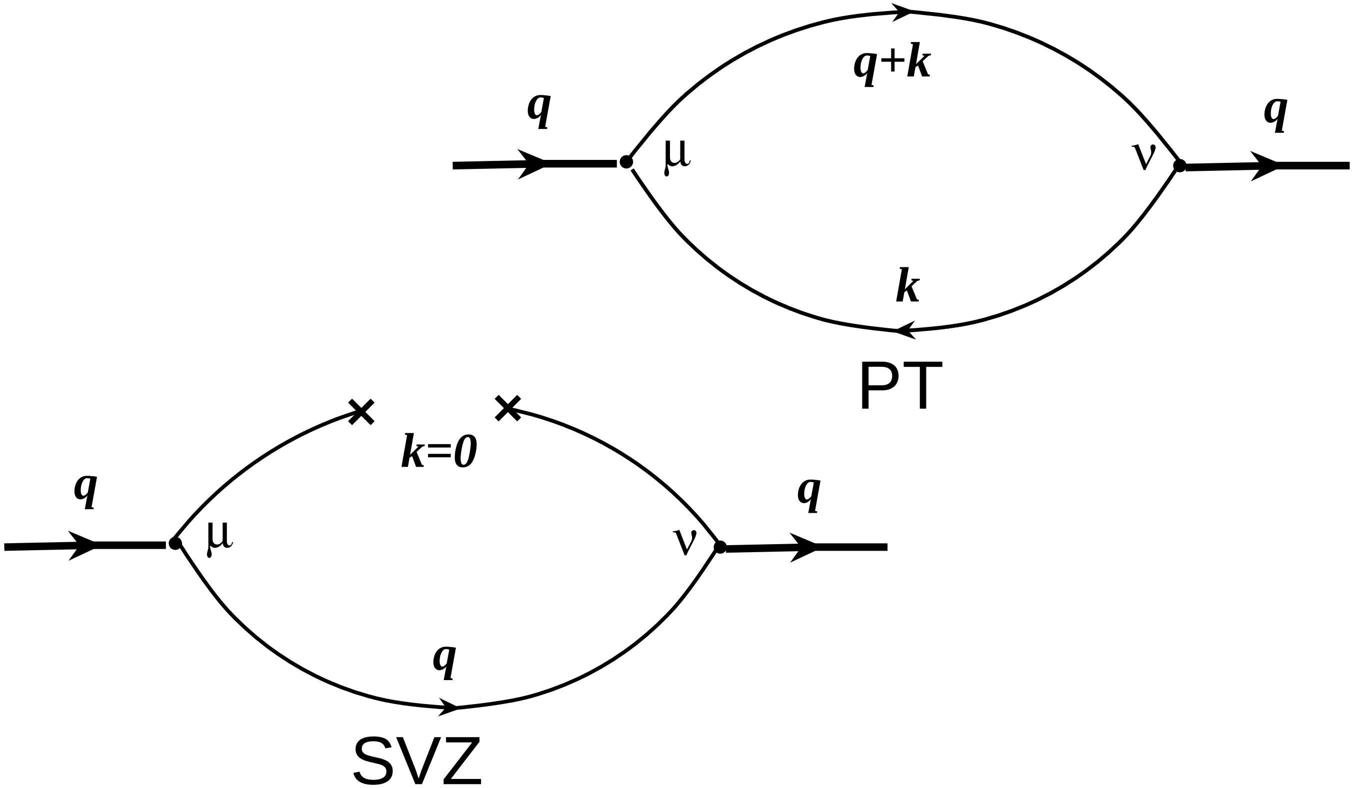

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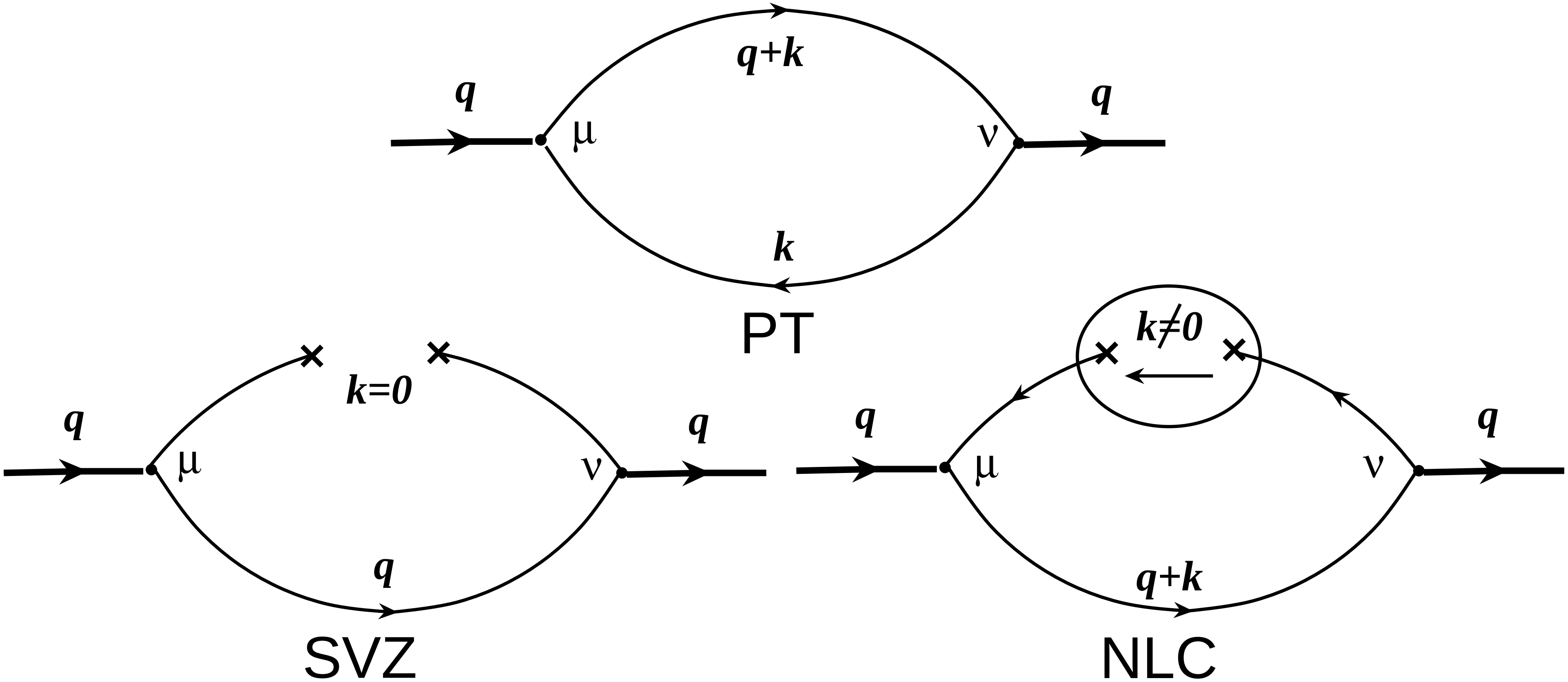

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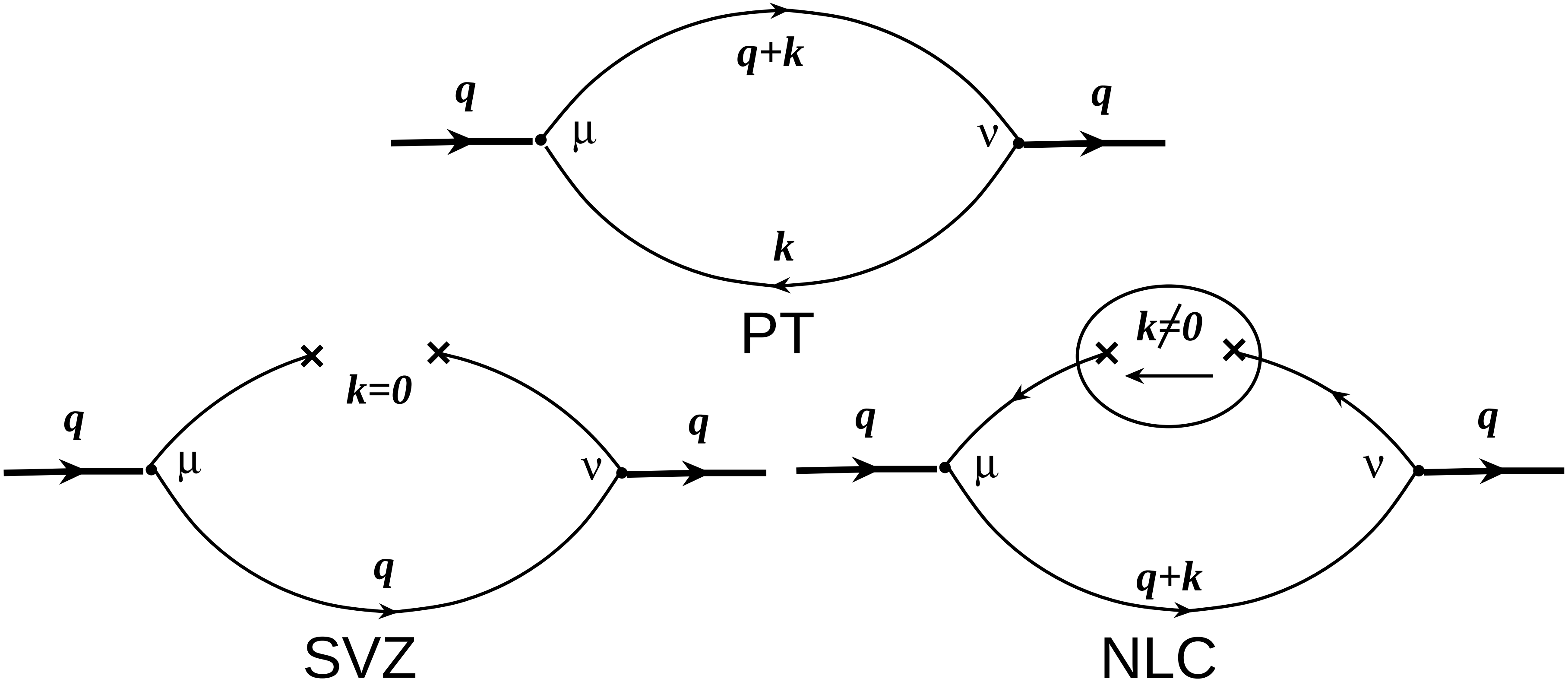


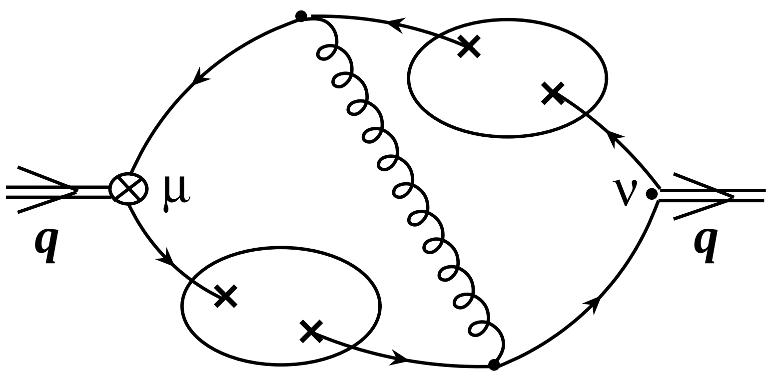
Quarks **run** through QCD vacuum with

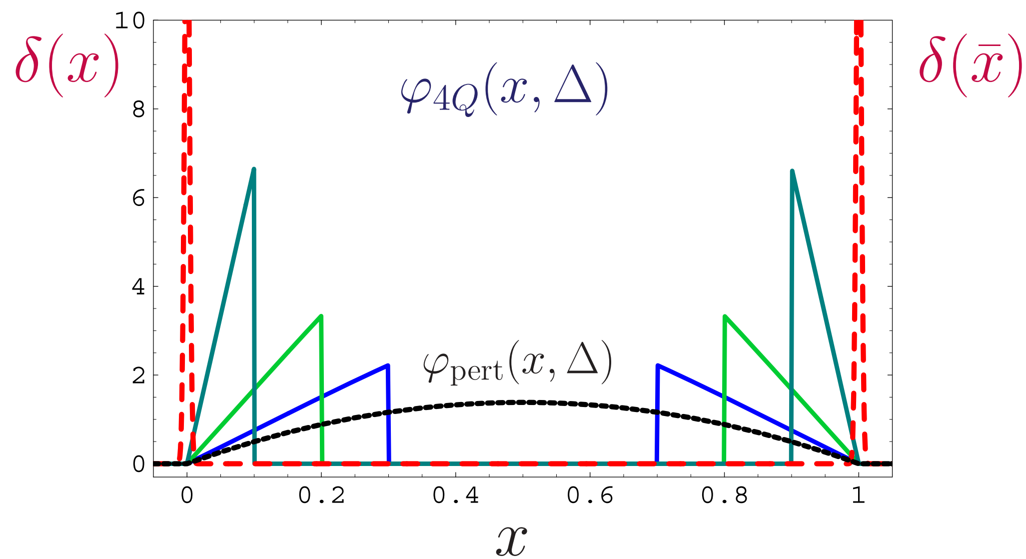
nonzero momentum $k \neq 0$:

$$\langle k^2 \rangle = \frac{\langle \bar{\psi} D^2 \psi \rangle}{\langle \bar{\psi} \psi \rangle} = \lambda_q^2 = 0.35 - 0.55\,\text{GeV}^2$$

NLC contributions to QCD SR, example

Examples for Gaussian NLC with a single parameter λ_q^2





Local limit: $\lambda_q^2/M^2 \equiv \Delta \to 0$,

$$\varphi_{4Q}^{\text{loc}}(x) \equiv \lim_{\Delta\to 0} \varphi_{4Q}^{\text{NLC}}(x;\Delta) = 9[\delta(x) + \delta(1-x)]$$

QCD NLC SRs for

Pion Distribution Amplitude

QCD NLC SR for Pion DA

Example of QCD SR with Non-Local Condensates for twist-2 pion DA $\varphi_\pi(x)$:

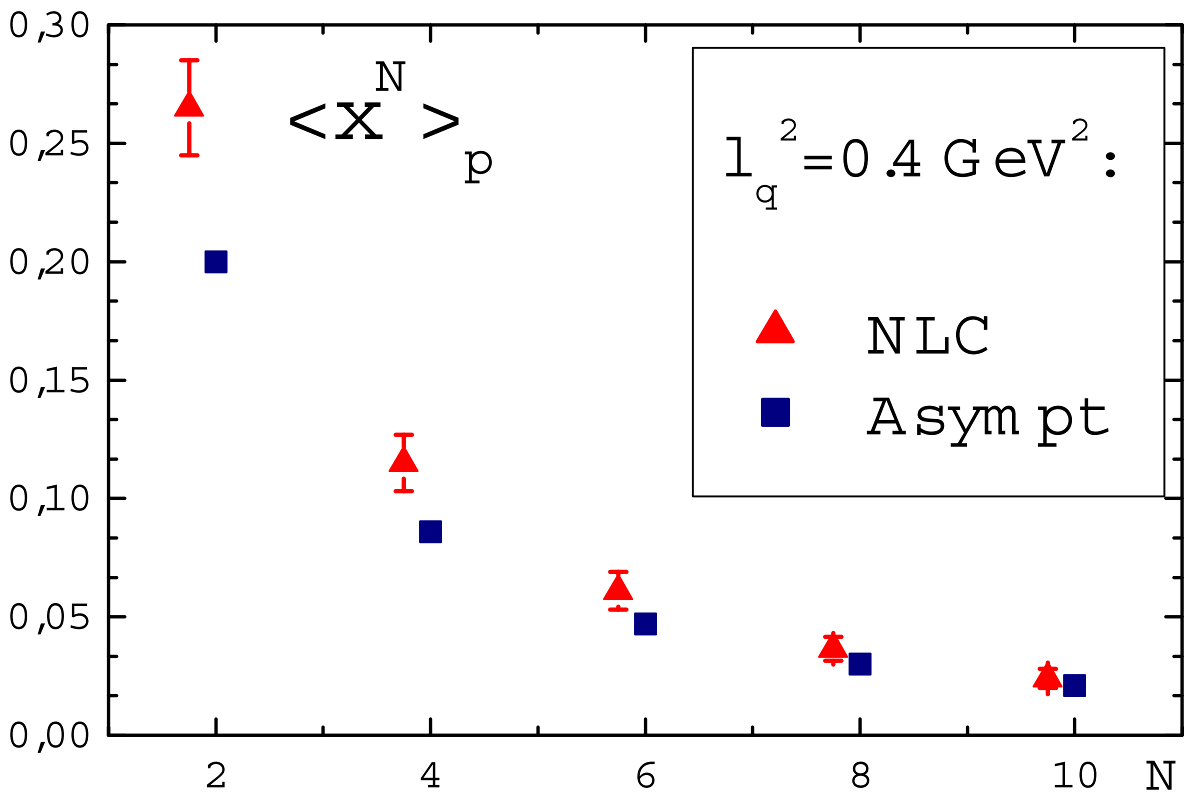
$$f_\pi^2\,\varphi_\pi(x) = \int_0^{s_0} \rho^{\text{pert}}(x;s)\,e^{-s/M^2}ds + \frac{\alpha_s\langle GG\rangle}{24\pi M^2}\,\varphi_G(x;\Delta) + \frac{16\pi\alpha_s\langle\bar{q}q\rangle^2}{81M^4}\sum_{i=2V,3L,4Q}\varphi_i(x;\Delta)$$

Local limit: $\lambda_q^2/M^2 \equiv \Delta \to 0$,

$$\begin{aligned}
\varphi_G(x;\Delta=0) &= [\delta(x)+\delta(1-x)] \\
\varphi_{2V}(x;\Delta=0) &= [x\delta'(1-x)+(1-x)\delta'(x)] \\
\varphi_{4Q}(x;\Delta=0) &= 9[\delta(x)+\delta(1-x)]
\end{aligned}$$

QCD NLC SRs for pion DA

Moments $\langle\xi^N\rangle_\pi = \int_0^1 \varphi_\pi(x)\,(2x-1)^N\,dx$ at $\mu^2 \approx 1$ GeV^2

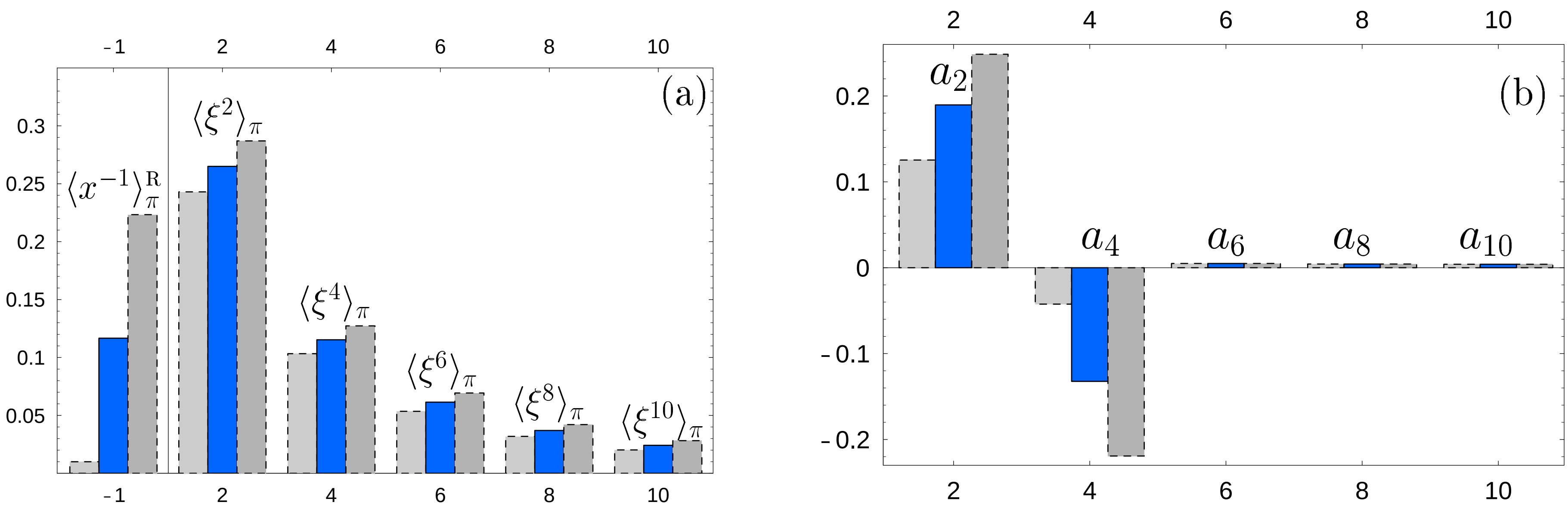


from NLC SRs

▲ **BMS, PLB 508 (2001) 279 Err., PLB 590 (2004)309**

These $\langle\xi^N\rangle_\pi$ values allow to **construct** Gegenbauer coefficients a_n for $\varphi_\pi(x)$

NLC SRs for pion DA



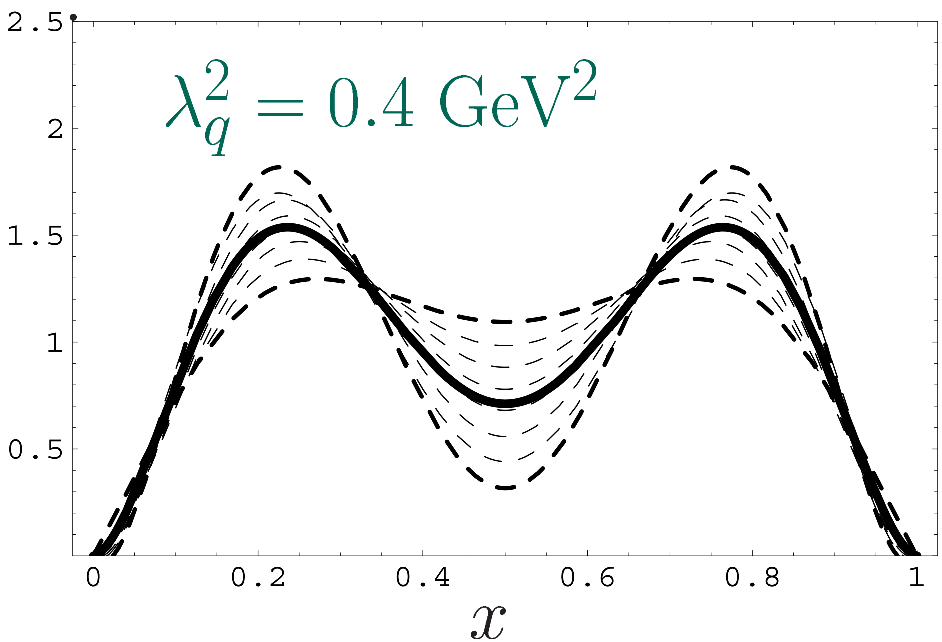
BMS [Ann. Phys. (Leipzig) 13 (2004) 629]

- These $a_{2,4,\ldots}$ values enable **reconstruction** of twist-2 DA $\varphi_\pi(x)$
- Independently, inverse moment $\langle x^{-1}\rangle_\pi^{\textbf{SR}}$ can be estimated

NLC SRs for Pion DA

produce **bunch** of self-consistent 2-parameter models $\varphi_\pi(x)$ at $\mu^2 \simeq 1$ **GeV**^{2}:

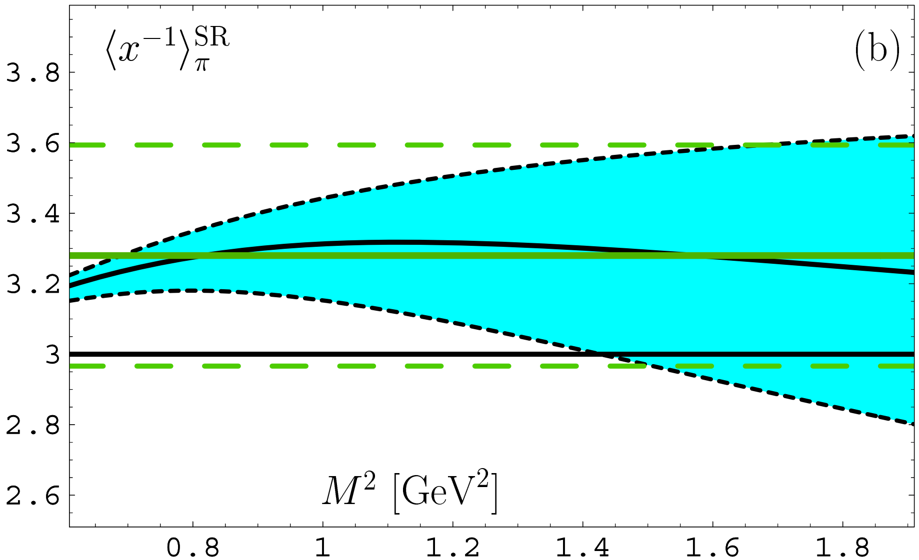
$$\varphi_\pi(x) = \varphi^{\text{as}}(x)\left[1 + a_2\, C_2^{3/2}(2x-1) + a_4\, C_4^{3/2}(2x-1)\right]$$



$$
\begin{aligned}
a_2^{\text{b.f.}} &= +0.188\\
a_4^{\text{b.f.}} &= -0.130\\
\langle x^{-1}\rangle_\pi^{\text{b.f.}} &= 3.17\\
\chi^2 &\approx 0.001\\
\langle x^{-1}\rangle_\pi^{\text{SR}} &= 3.30(30)
\end{aligned}
$$

NLC SR estimate of $\langle x^{-1}\rangle_{\pi}^{SR}$

BMS [PLB (2001)]: at $\mu^2 \simeq 1$ **GeV**^{2}



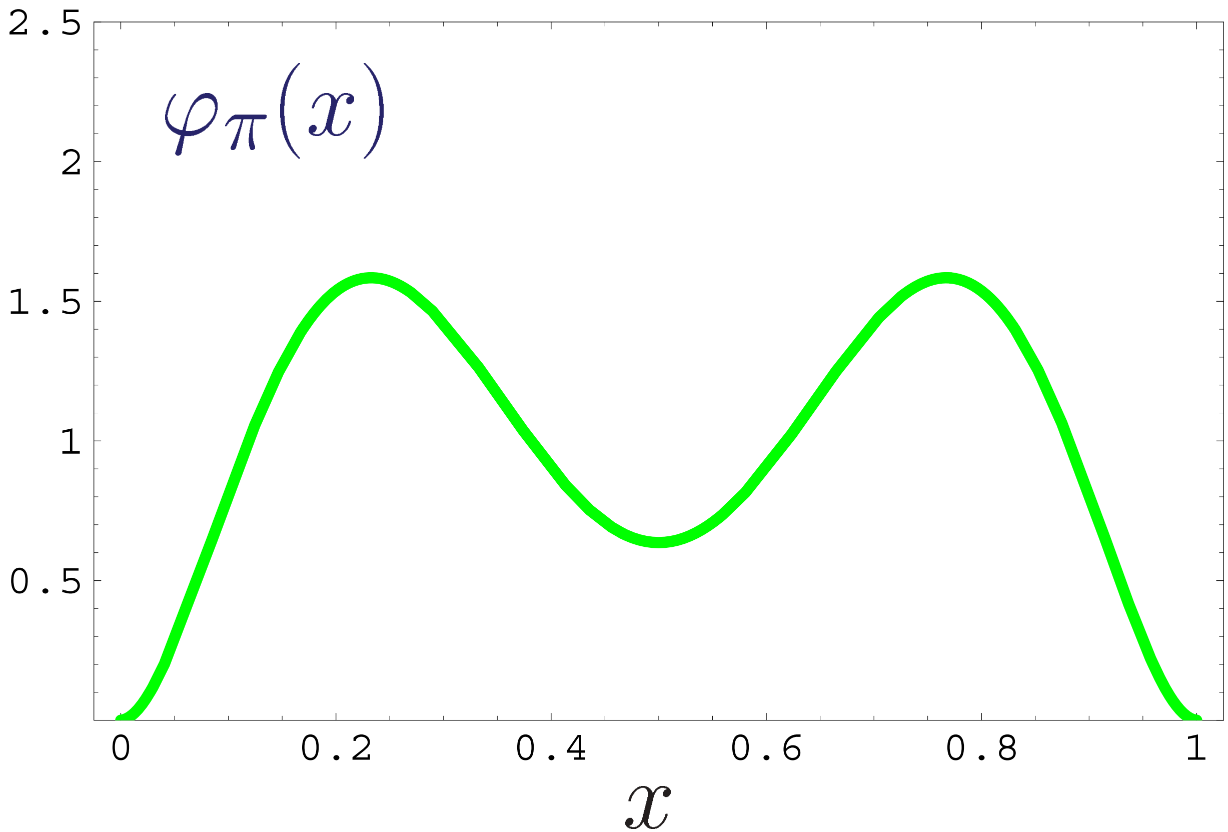
$\lambda_q^2 = 0.4$ **GeV**^{2},

$\langle x^{-1}\rangle_{\pi}^{\textbf{SR}} = 3.3 \pm 0.3,$

$\langle x^{-1}\rangle_{\pi}^{\textbf{b.f.}} = 3.17$

The moment $\langle x^{-1}\rangle_{\pi}^{\textbf{SR}}$ could be determined **only in NLC SRs** because end-point singularities absent!

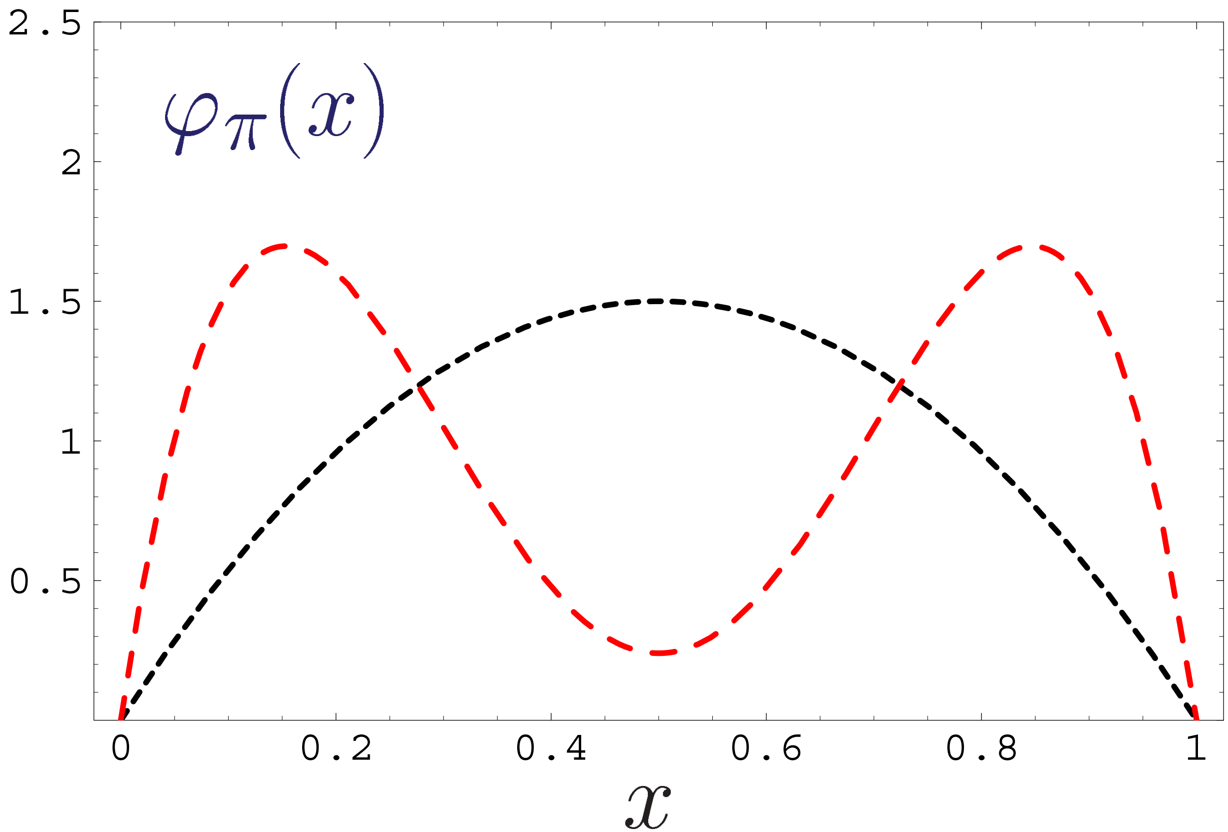
BMS vs CZ distribution amplitude



Curves	DAs
—	**BMS**

BMS DA is end-point suppressed!

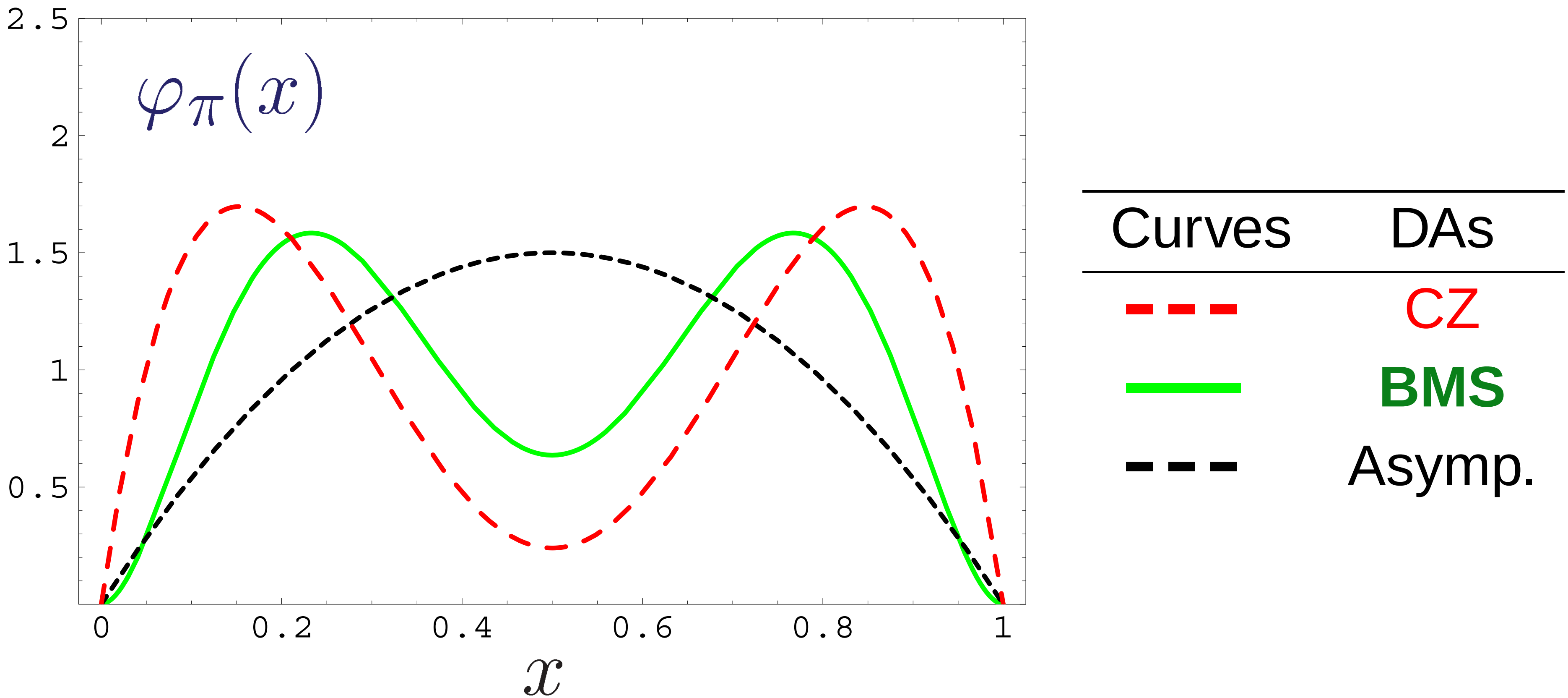
BMS vs CZ distribution amplitude



Curves	DAs
red dashed line	CZ
black dashed line	Asymp.

CZ DA: end-point enhancement

BMS vs CZ distribution amplitude

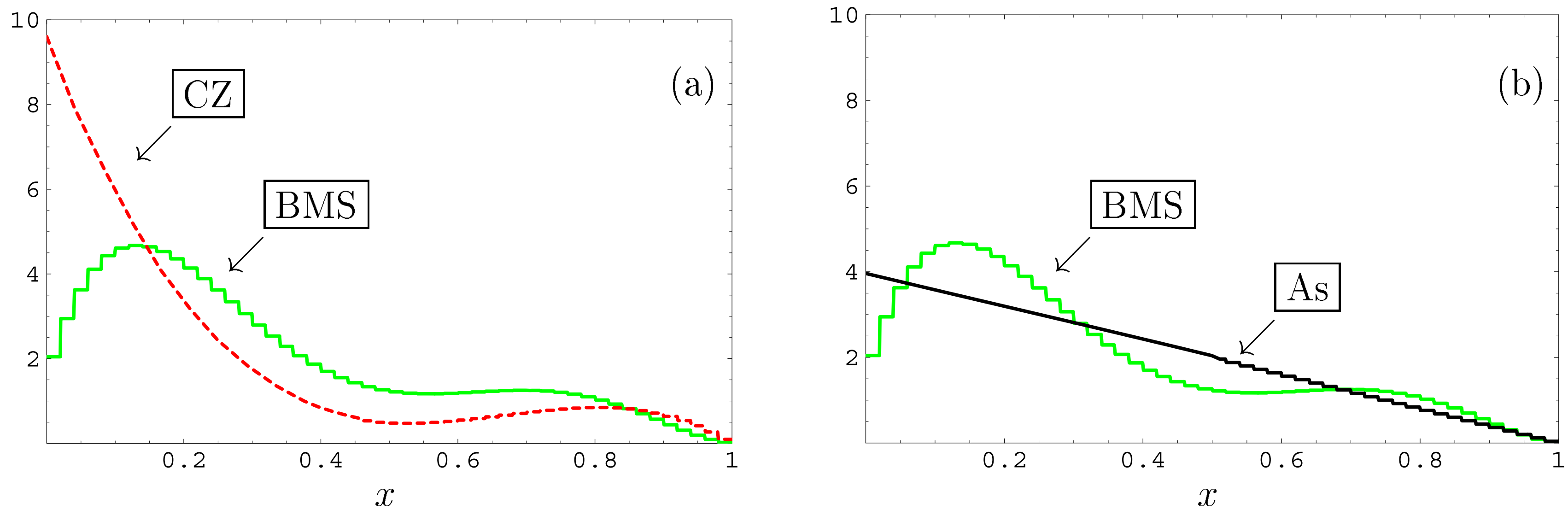


BMS bunch is 2-humped, but end-point suppressed!

Histograms for inverse moment $\langle x^{-1} \rangle_\pi$

Contributions of different DAs to inverse moment $\langle x^{-1} \rangle_\pi$, calculated as $\int_x^{x+0.02} \phi(x) dx$ and normalized to 100%, for:

(a) **CZ** and **BMS** DAs; (b) Asympt. and **BMS** DAs.



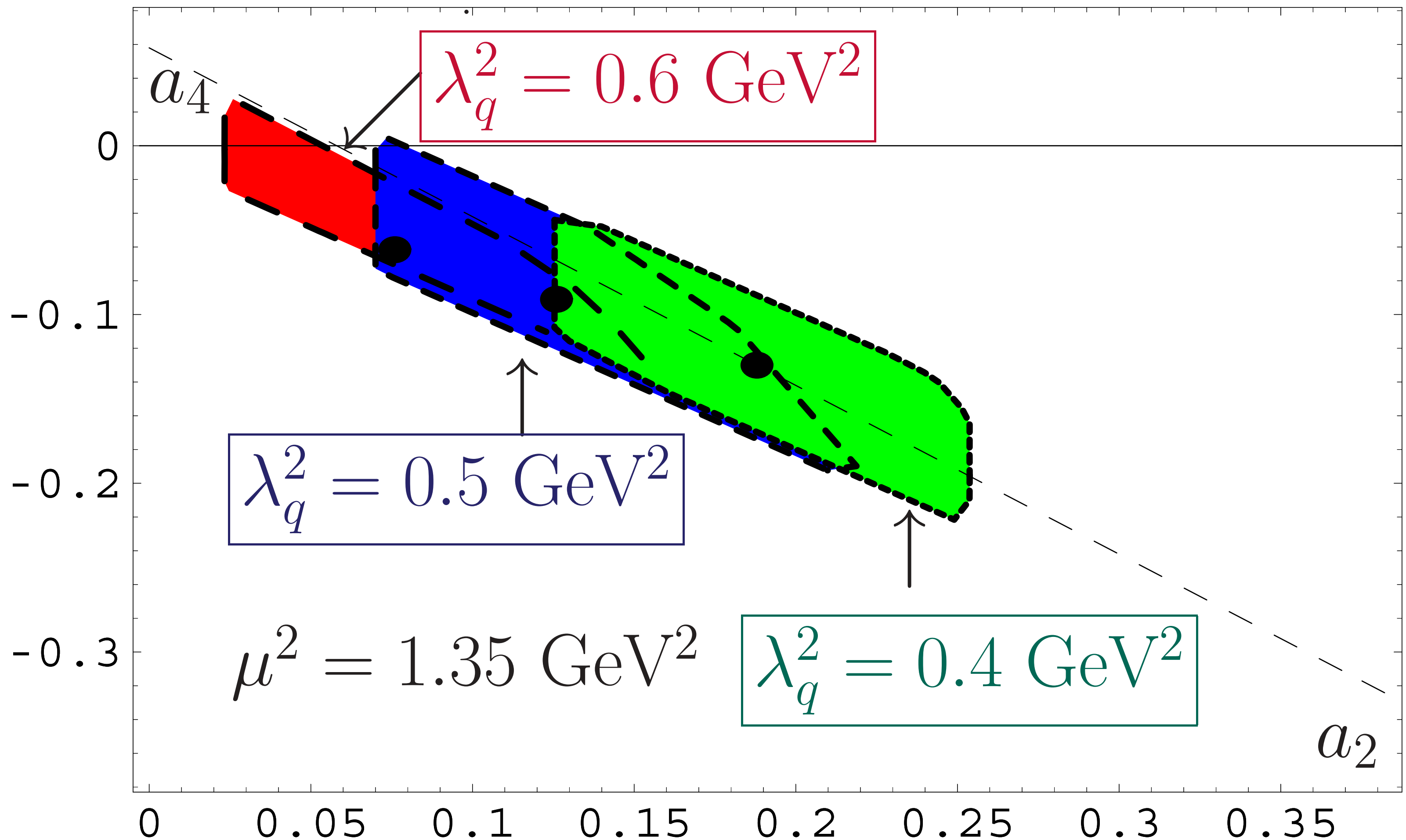
In **BMS** case region $x \leq 0.1$ contributes even less than in Asymptotic DA case.

Pion Distribution Amplitudes

Bunch from

Nonlocal Condensates SRs

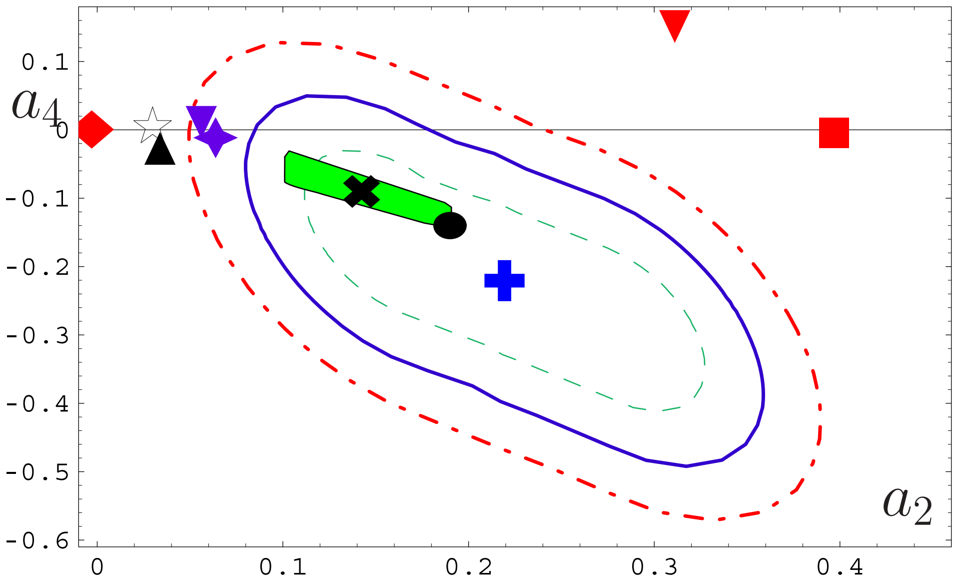
NLC SR Constraints on a_2, a_4 of Pion DA



Estimated **bunches of pion DAs** for different values of $\boldsymbol{\lambda_q^2}$.

BMS bunch vs CLEO-data Constraints

NLO Light-Cone SR $\oplus$ Twist-4 $\oplus(\mu^2 = Q^2)$
with 20% uncertainty of $\delta^2_{\text{Tw-4}}$ value
BMS [PLB 578 (2004) 91]: $\boldsymbol{\lambda_q^2 = 0.4}$ **GeV^2**, $\boldsymbol{\delta^2_{\text{Tw-4}} = 0.19(4)}$ **GeV**



- ✚= best-fit BMS, ●=SY point
- ◆ = **Asymptotic** DA
- ■ = **CZ** DA, ▼= **BF** DA
- ✖ = BMS model
- ☆, ▲ and ✦ = instantons
- ▼ = transverse lattice

BMS DA and major part of **BMS bunch** **inside** 1σ-domain (**green dashed contour**).

Pion Distribution Amplitude

from

“non-diagonal” NLC SRs

Direct determination of $\varphi_\pi(x)$ profile

Radyushkin [hep-ph/9406237];
BM [Z.Phys.C68(1995)451, MPLA11(1996)1611, PRD65]
Approach based on axial-vector correlator and PCAC :

$$\varphi_\pi(x) + \varphi_{\pi'}(x) e^{-m^2_{\pi'}/M^2} + \varphi_{\pi''}(x) e^{-m^2_{\pi''}/M^2} + \ldots$$

$$= \frac{M^2}{2}\left(1 - x + \frac{\lambda_q^2}{2M^2}\right) \cdot f_s(xM^2) + (x \to 1 - x)$$

$\varphi_\pi(x)$ – distribution of partons **in xP in pion**

directly related with

$f_s(\nu)$ – distribution of quarks **in virtuality ν** in vacuum

$$\langle \bar{q}(0) E(0,z) q(z) \rangle / \langle \bar{q} q \rangle = F = \int_0^\infty \exp\left(-z^2/4\,\nu\right) f_s(\nu) d\nu$$

Ansatze for NLC

$f(\nu)$ – quark distribution in virtuality ν.

- Gaussian ansatz, $F^G(z^2)$, takes into account **only one** NLC scale – distance of short correlations $1/\lambda_q$

$F^G(z^2) = \exp\left(-\lambda_q^2 z^2/8\right)$	$f(\nu) = \delta\left(\nu - \lambda_q^2/2\right)$

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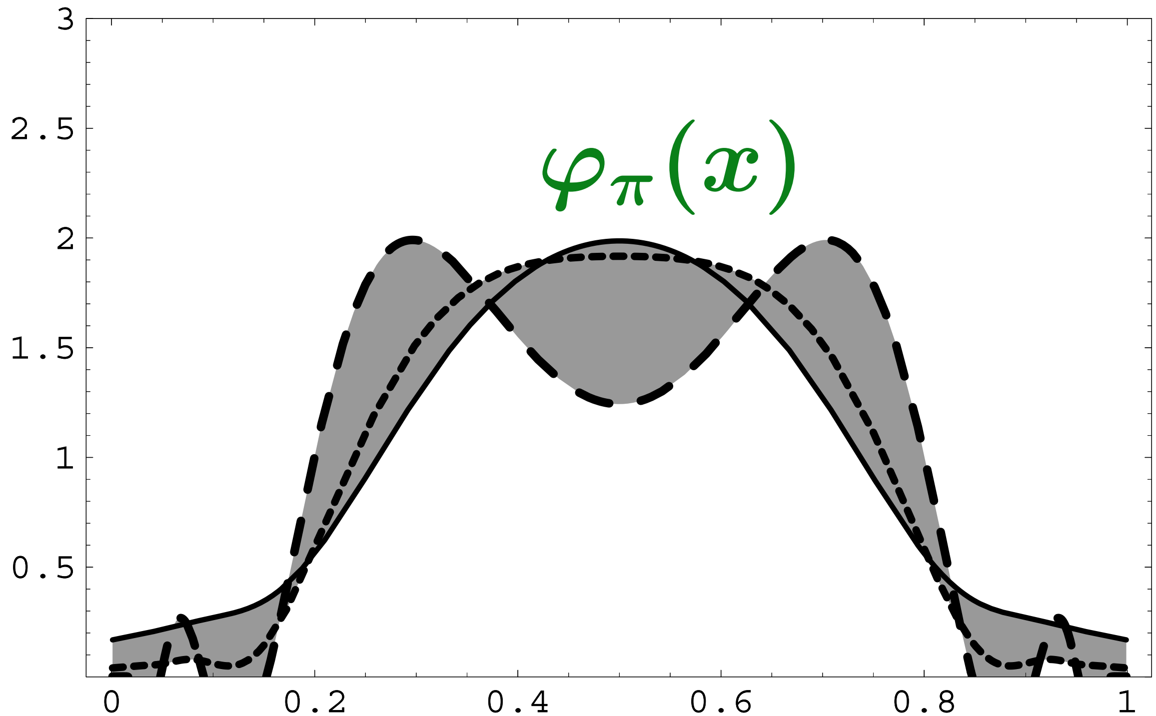
- Improved ansatz – **also** long distance correlation $1/\Lambda$ $F(z^2)\Big|_{z^2\to\infty} \sim e^{-|z|\Lambda}$:

$F(z^2) \sim K_1\left(\Lambda\sqrt{4\sigma^2 + \lvert z^2\rvert}\right)$	$f(\nu) = e^{-\Lambda^2/\nu - \sigma^2\,\nu}$

Improved ansatz allows one to predict the masses of the resonances $m_{\pi'} \simeq \mathbf{1.34}\ (1.3)_{\text{exp}}$, $m_{\pi''} \simeq \mathbf{1.86}\ (1.8)_{\text{exp}}$

Direct determination of $\varphi_\pi(x)$ profile

$$f_s(\nu) \sim \exp\left(-\Lambda^2/\nu - \sigma^2\nu\right)$$



Determined shape of $\varphi_\pi(x)$ inside **gray strip** for few Gev^2.

★ **Advantage**: endpoint suppressed; close in shape to **bunch** from the moment **NLC SRs**

★ **Disadvantage**: result sensitive to $f_s(\nu)$-ansatz.

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- based on **QCD NLC SRs** we construct admissible sets of **self-consistent pion DAs** —**bunches**—for pion, and reliable models for **DA** of polarized ρ^L and ρ^T meson.

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- Comparing **NLC SRs** with new CLEO-data **constraints** **allows to fix** value of QCD vacuum nonlocality to $\lambda_q^2 = 0.4$ GeV^2.

CONCLUSIONS

- **QCD SRs** with **NLCs** provide one the higher moments (up to $\langle \xi^{10} \rangle$) of **meson DA**
- **QCD NLC SRs** give a possibility to estimate functionals of these **DAs** containing singularities, like $\langle x^{-1} \rangle$.
- based on **QCD NLC SRs** we construct admissible sets of **self-consistent pion DAs** —**bunches**—for pion, and reliable models for **DA** of polarized ρ^L and ρ^T meson.
- Comparing **NLC SRs** with new CLEO-data **constraints** **allows to fix** value of QCD vacuum nonlocality to $\lambda_q^2 = 0.4$ GeV^2.
- **This bunch** of pion DAs **agrees well** with
  (i) **CELLO data**, (ii) **E791** data on dijet πA-production,
  (iii) **JLab F(pi)** data on pion form factor, and
  (iv) recent **lattice** data.

QCD SRs for ρ-meson

Distribution Amplitudes

ρ-Meson Distribution Amplitude

- Matrix element of nonlocal tensor current on light cone

$$\langle 0 \mid \bar{u}(z)\sigma_{\mu\nu}E(z,0)d(0) \mid \rho_{\perp}(P,\lambda)\rangle\Big|_{z^2=0} =$$

$$i f_{\rho}^{T} \left(\varepsilon_{\mu}(P,\lambda)P_{\nu} - \varepsilon_{\nu}(P,\lambda)P_{\mu}\right) \int_{0}^{1} dx\; e^{ix(zP)}\; \varphi_{\rho}^{T}(x,\mu^2) + \ldots$$

- **Gauge-invariance** due to Fock–Schwinger string:

$$E(z,0) = \mathcal{P} e^{ig\int_0^z A_{\mu}(\tau)d\tau^{\mu}}$$

- Physical meaning of $\varphi_{\rho}^{T}(x;\mu^2)$ — amplitude for transition $\rho^{T} \longrightarrow$ valence $u + d$ quarks.

QCD Sum Rules Approach

Correlator:

$$\Pi^{\mu\nu;\alpha\beta}_{(N)}(q) = i\int d^4x\; e^{iqx}\langle 0|T\Big[J^{\mu\nu +}_{(0)}(x) J^{\alpha\beta}_{(N)}(0)\Big]|0\rangle$$

with ($n^2 = 0$)

$$J^{\mu\nu}_{(N)}(x) = \bar{u}(x)\sigma^{\mu\nu}\left(n\nabla\right)^N d(x)\,.$$

Decomposition:

$$\hat{\Pi}_{(0)}(q) = \Pi_-(q^2)\,\hat{P}_1 + \Pi_+(q^2)\,\hat{P}_2$$

with

$$\hat{P}_i \cdot \hat{P}_j = \delta_{ij}\hat{P}_i\,.$$

QCD Sum Rules Approach

For the general case $N \neq 0$, a similar decomposition involves 4 new independent tensors $\hat{Q}_i$:

$$\hat{\Pi}_{(N)}(q) = \Pi_{-}(q)\,\hat{P}_1 + \Pi_{+}(q)\,\hat{P}_2 + K_1(q)\,\hat{Q}_1$$

$$+K_3(q)\,\hat{Q}_3 + K_z(q)\,\hat{Q}_z + K_q(q)\,\hat{Q}_q\,.$$

Here ρ- and b_1-terms of twist 2, 3, and 4 **are mixed**, see **Bakulev&Mikhailov, EJPC 19 (2001) 361** for more detail.

QCD Sum Rules Approach

For the general case $N \neq 0$, ρ- and b_1-terms of twist 2, 3, and 4 **are mixed**,

see **Bakulev&Mikhailov, EJPC 19 (2001) 361** for more detail.

As a result we obtain 2 types of QCD SRs with this correlator:

- Mixed-Parity SR: One SR for both $\rho \oplus b_1$
  Advantages: 4Q-condensate term is cancelled exactly.
  Disadvantages: high sensitivity to gluon NLC model.

QCD Sum Rules Approach

For the general case $N \neq 0$, ρ- and b_1-terms of twist 2, 3, and 4 **are mixed**,

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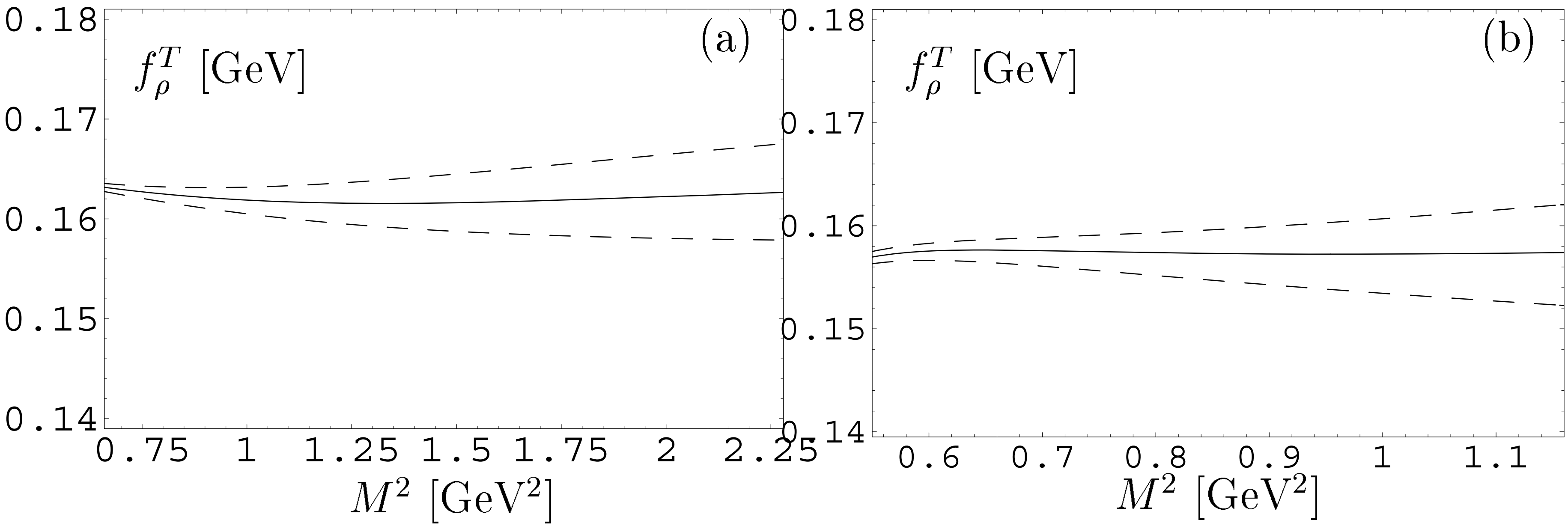
As a result we obtain 2 types of QCD SRs with this correlator:

- Mixed-Parity SR: One SR for both $\rho \oplus b_1$
  Advantages: 4Q-condensate term is cancelled exactly.
  Disadvantages: high sensitivity to gluon NLC model.
- Pure-Parity SRs: SR for $\rho \oplus$ SR for b_1
  Advantages: low sensitivity to gluon NLC model.
  Disadvantages: 4Q-condensate term contributes to both.

Quality of QCD SRs

Results for $f_\rho^T(M^2)$ from:
(a) the "mixed parity" NLC SR; (b) the "pure parity" NLC SR.
The fidelity windows = the whole range of M^2.

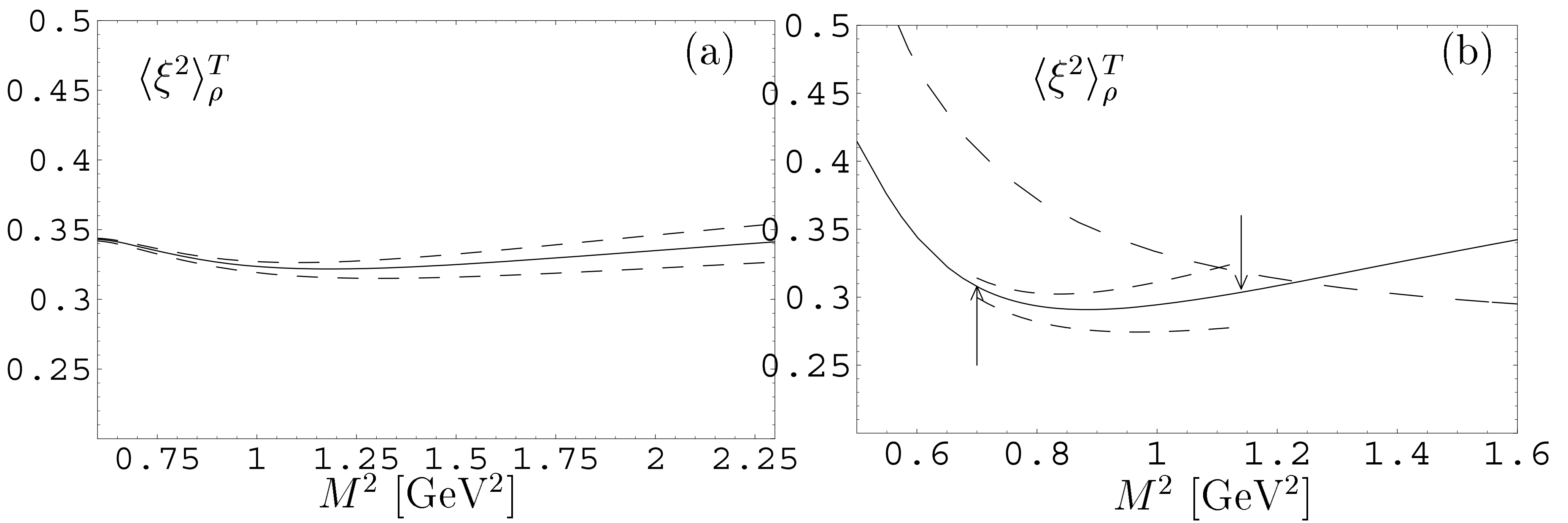


Numbers: (a) $f_\rho^T(\mu^2) = 0.162(5)$; (b) $f_\rho^T(\mu^2) = 0.157(5)$
at normalization scale $\mu^2 \approx 1$ **GeV^2**.

Quality of QCD SRs

Results for $\langle\xi^2\rangle_\rho^T(\mu^2)$ from:

(a) the “mixed parity” NLC SR; (b) the “pure parity” NLC SR. The fidelity windows = the whole range of M^2 or shown by arrows.



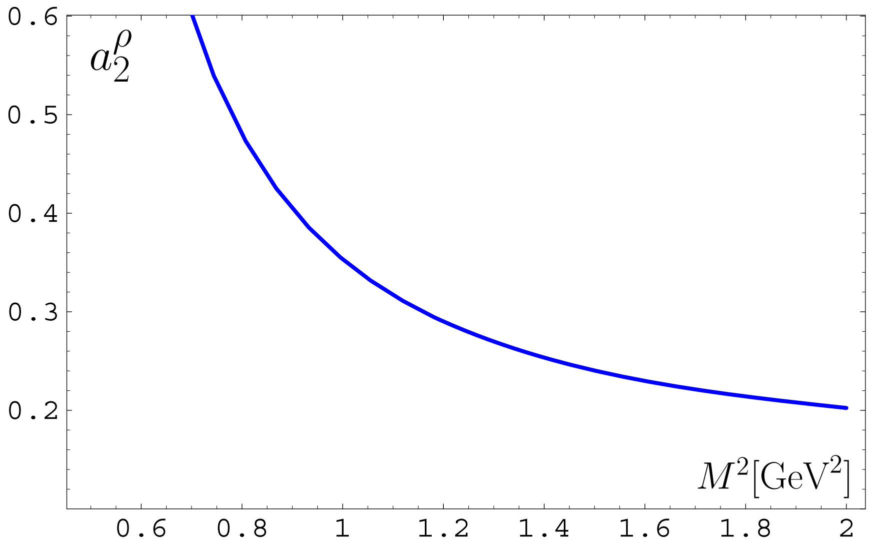
Numbers: (a) $\langle\xi^2\rangle_\rho^T = 0.33(1)$; (b) $\langle\xi^2\rangle_\rho^T = 0.30(2)$ at normalization scale $\mu^2 \approx 1$ **GeV^2**.

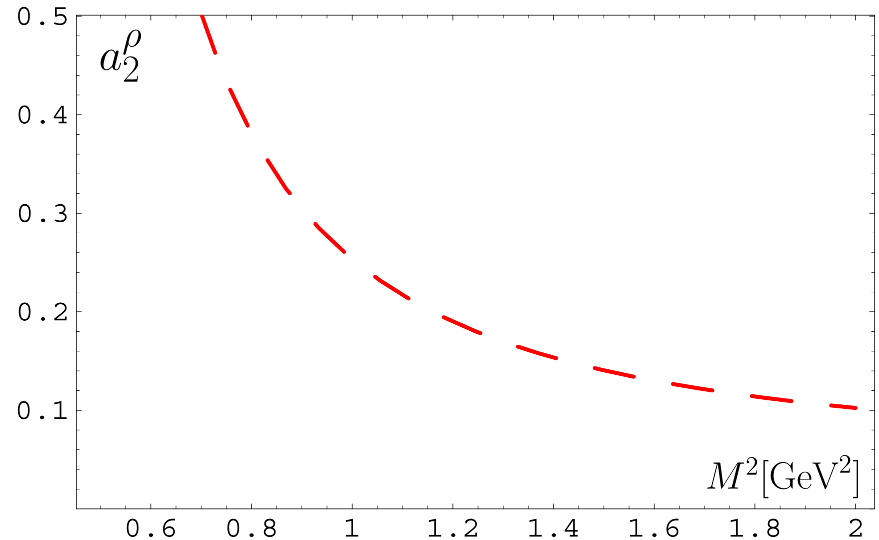
Question to Ball&Braun about $a_2^{\rho,T}$

Results for $a_2^{\rho,T} = \frac{35}{12}\left(< \xi^2 >_\rho^T - 0.2\right)$ from:

(a) the “mixed parity” SR of Ball&Braun (Mathematica);

(b) the “mixed parity” SR of Ball&Braun (published in **PRD 54 (1996) 2182**). Note different ordinates in graphics!





Mathematica: $a_2^{\rho,T} = 0.3 \pm 0.1$; **Publ.:** $a_2^{\rho,T} = 0.2 \pm 0.1$.

Question to the authors, published in **[EJPC 19(2001)361]**:

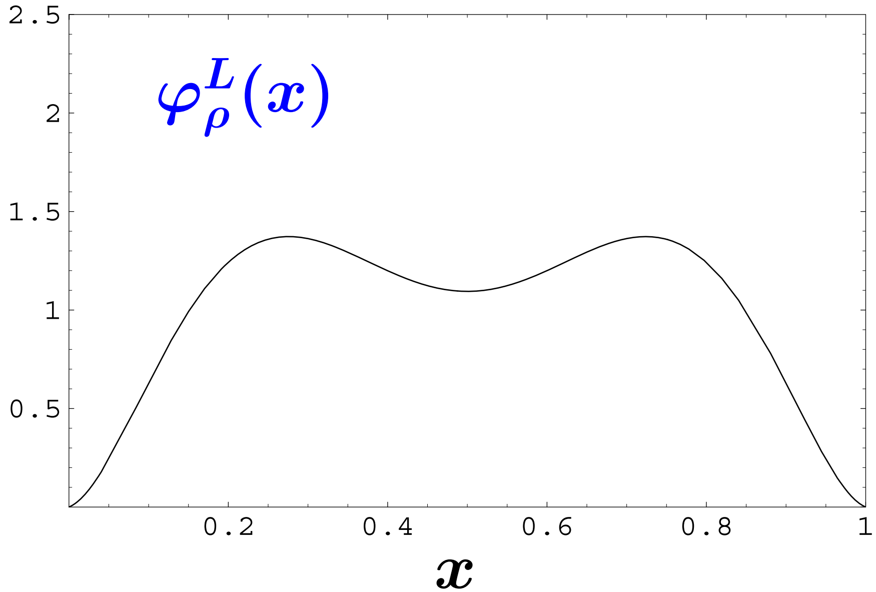
Why to declare $a_2^{\rho,T} = 0.2 \pm 0.1$?

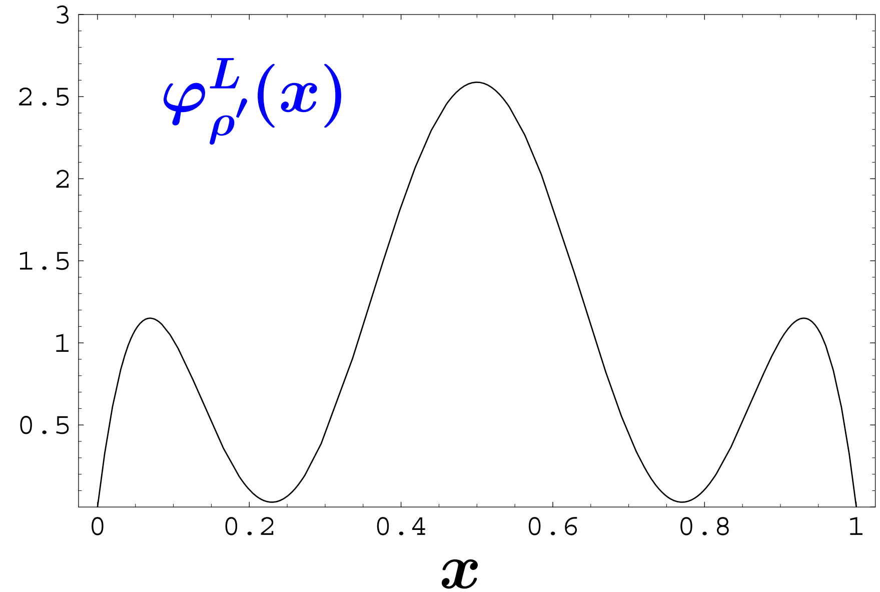
Distribution Amplitudes of ρ^L *and* ρ'^L

Moments $\langle \xi^N \rangle^L_{\rho,\rho'} = \int_0^1 \varphi^L_{\rho,\rho'}(x)\,(2x-1)^N\,dx$

from NLC QCD SRs [**PLB98**].

Then $\Rightarrow$ restore DAs φ^L_ρ and $\varphi^L_{\rho'}$:





$$\langle x^{-1} \rangle^{\text{DA},L}_\rho = 3.0$$

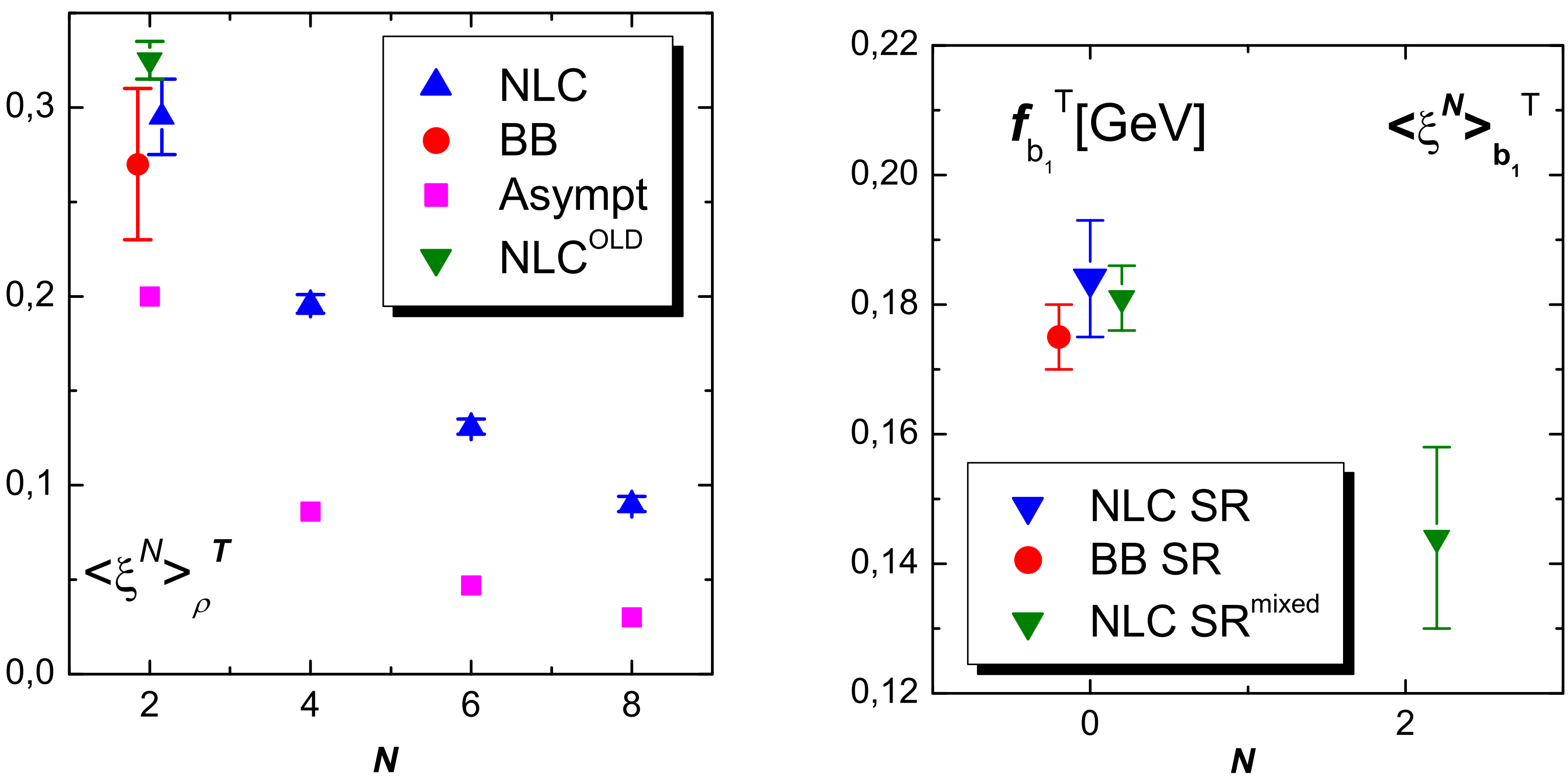
$$\langle x^{-1} \rangle^{\text{SR},L}_\rho = 3.1 \pm 0.3$$

$$\langle x^{-1} \rangle^{\text{DA},L}_{\rho'} = 4.3$$

$$\langle x^{-1} \rangle^{\text{SR},L}_{\rho'} = 4.7 \pm 0.4$$

Moments of ρ^T and b_1^T

Moments $\langle \xi^N \rangle_\rho^T = \int_0^1 \varphi_\rho^T(x)\,(2x-1)^N\,dx$ and b_1^T from NLC QCD SRs [**EJPC2000, EJPC2001**]:

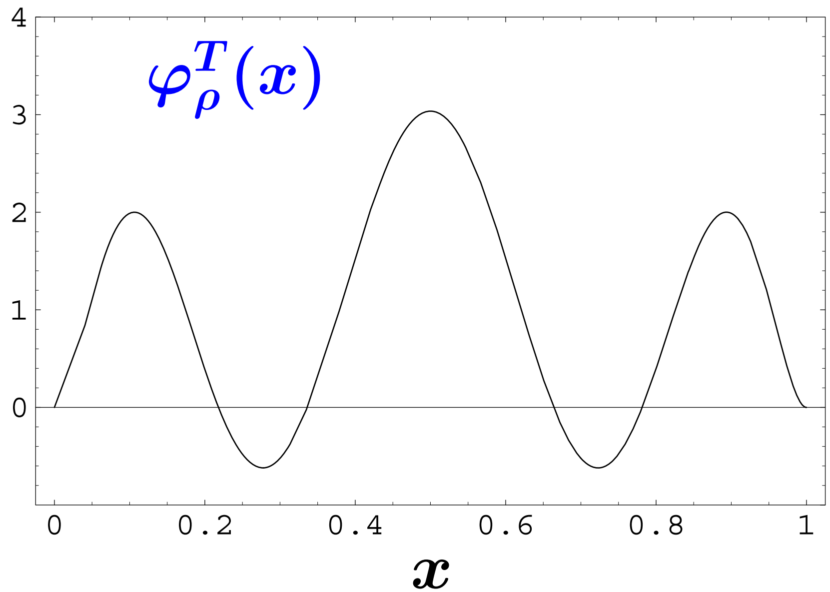


Distribution Amplitudes of ρ^T

Moments $\langle \xi^N \rangle_\rho^T = \int_0^1 \varphi_\rho^T(x)\,(2x-1)^N\,dx$

from NLC QCD SRs [**EJPC2000, EJPC2001**].

Then $\Rightarrow$ restore DA φ_ρ^T:



$$\langle x^{-1} \rangle_\rho^{\text{DA},T} = 4.15^{+0.4}_{-0.1}$$

$$\langle x^{-1} \rangle_\rho^{\text{SR},T} = 4.35 \pm 0.2$$

Conclusion: DAs of ρ^L, ρ'^L and ρ^T mesons, **significantly differ** from asymptotic distributions.