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# *Constraints on Pion DA from experiments*

N. G. Stefanis

Institut für Theoretische Physik II, Ruhr-Universität Bochum,  
D-44780 Bochum, Germany

September 28, 2006

# Contents of Report

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- CLEO data on  $F_{\gamma\gamma^*\pi}(Q^2) \Rightarrow$  pion DA  
[ PRD 67 (2003) 074012; 73 (2006) 056002]  
[PLB 578 (2004) 91]

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- What can E791 data add on pion DA?  
[LC03 Proc.; Ann. Phys.(Leipzig) 13(2004) 629]  
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- Recent Lattice data vs.  
CLEO analysis and pion DA  
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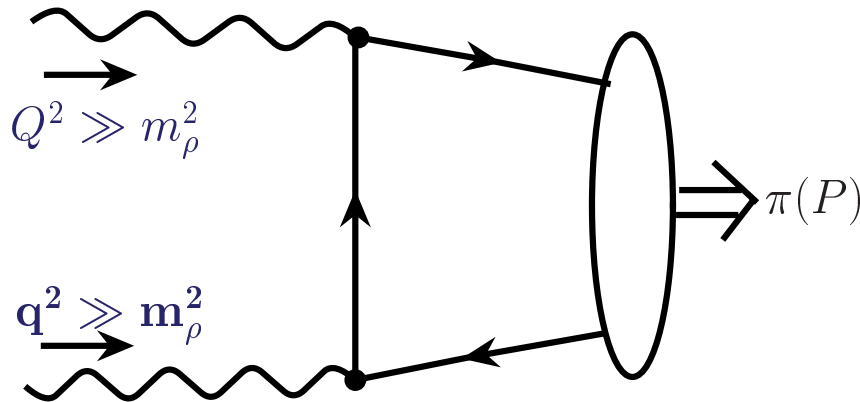
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**NLO Light-Cone SRs  $\Rightarrow$**   
**CLEO data on  $F_{\gamma\gamma^*\pi}(Q^2) \Rightarrow$**   
**Constraints on Pion DA**

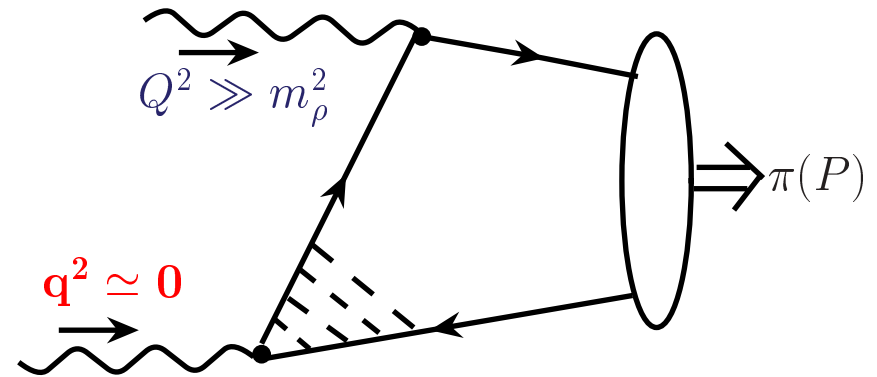
# $\gamma^* \gamma \rightarrow \pi$ : Why Light-Cone Sum Rules?

For  $Q^2 \gg m_\rho^2$ ,  $q^2 \ll m_\rho^2$  pQCD factorization valid only in leading twist and higher twists are of importance  
**[Radyushkin–Ruskov, NPB (1996)].**

Reason: if  $q^2 \rightarrow 0$  one needs to take into account interaction of real photon at long distances of order of  $O(1/\sqrt{q^2})$



**pQCD is OK**

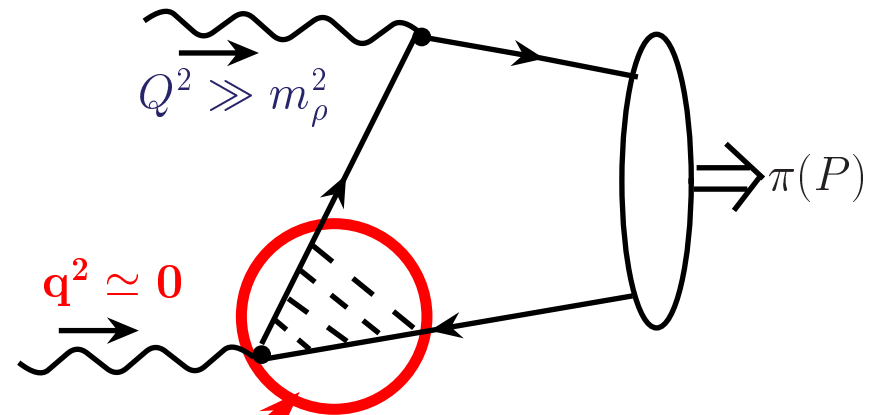


**LCSRs should be applied**

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Reason: if  $q^2 \rightarrow 0$  one needs to take into account interaction of real photon at long distances of order of  $O(1/\sqrt{q^2})$



To account for long-distance effects in pQCD, one needs to introduce light-cone **DA** of real photon



# $\gamma^* \gamma \rightarrow \pi$ : *Light-Cone Sum Rules*

---

**[Khodjamirian, EJPC (1999)]**: LCSR effectively accounts for long-distances effects of real photon using quark-hadron duality in vector channel and dispersion relation in  $q^2$

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[**Khodjamirian, EJPC (1999)**]: LCSR effectively accounts for long-distances effects of real photon using quark-hadron duality in vector channel and dispersion relation in  $q^2$

$$F_{\gamma\gamma^*\pi}(Q^2, q^2 \rightarrow 0) = \frac{1}{\pi} \int_0^{s_0} \frac{\text{Im}F_{\gamma^*\gamma^*\pi}^{\text{PT}}(Q^2, s)}{m_\rho^2} e^{(m_\rho^2 - s)/M^2} ds + \frac{1}{\pi} \int_{s_0}^{\infty} \frac{\text{Im}F_{\gamma^*\gamma^*\pi}^{\text{PT}}(Q^2, s)}{s} ds$$

$s_0 \simeq 1.5 \text{ GeV}^2$  – effective threshold in vector channel,  
 $M^2$  – Borel parameter (0.5 – 0.9  $\text{GeV}^2$ ).

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$s_0 \simeq 1.5 \text{ GeV}^2$  – effective threshold in vector channel,  
 $M^2$  – Borel parameter (0.5 – 0.9  $\text{GeV}^2$ ).

**Real-photon limit  $q^2 \rightarrow 0$  can be easily done!**

# CLEO-data analysis

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- Accurate NLO evolution for both  $\varphi(x, Q_{\text{exp}}^2)$  and  $\alpha_s(Q_{\text{exp}}^2)$ , taking into account quark thresholds;

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- The relation between “**nonlocality**” scale and **twist-4** magnitude  $\delta_{\text{Tw-4}}^2 \approx \lambda_q^2/2$  was used to re-estimate  $\delta_{\text{Tw-4}}^2 = 0.19 \pm 0.02$  at  $\lambda_q^2 = 0.4 \text{ GeV}^2$

# CLEO-data analysis

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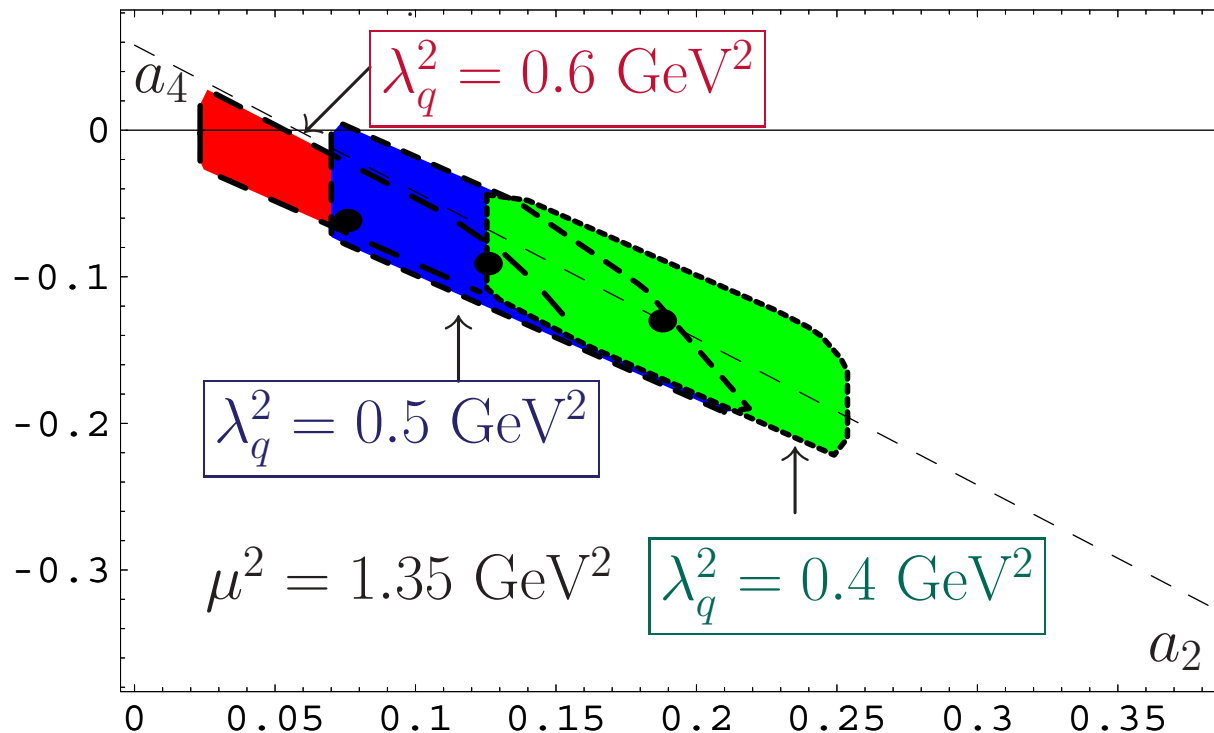
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- New procedure of data processing to disentangle the statistical and theoretical uncertainties
- Constraints on  $\langle x^{-1} \rangle_\pi$  from CLEO data.

# NLC SR Constraints on $a_2, a_4$ of Pion DA



$\lambda_q^2$  – average virtuality of vacuum quarks – the single parameter of NLC approach [PLB 508(2001)279]

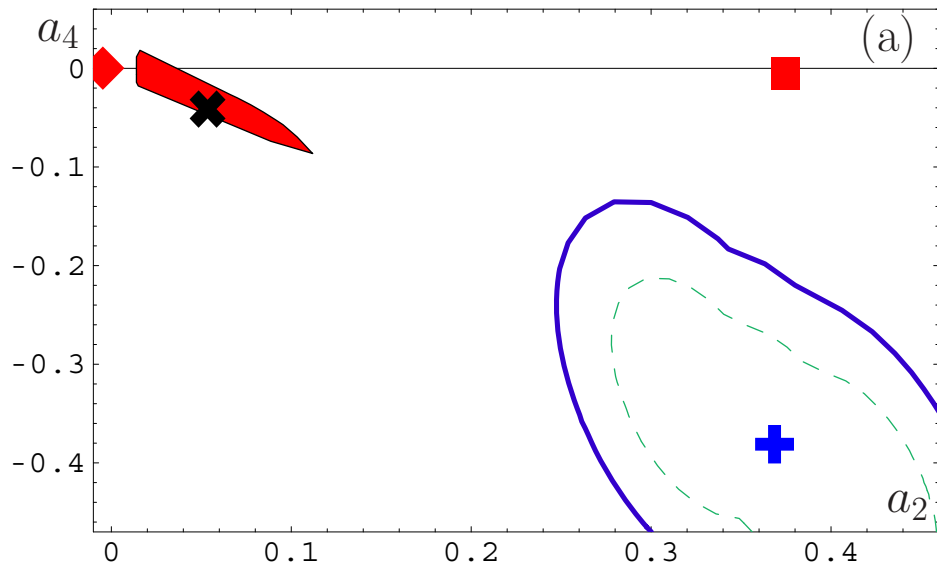
Main purpose of analysis: to use CLEO data to restrict  $\lambda_q^2$





# NLC SR Results vs NLO CLEO Constraints

[BMS, PRD 67 (2003) 074012]

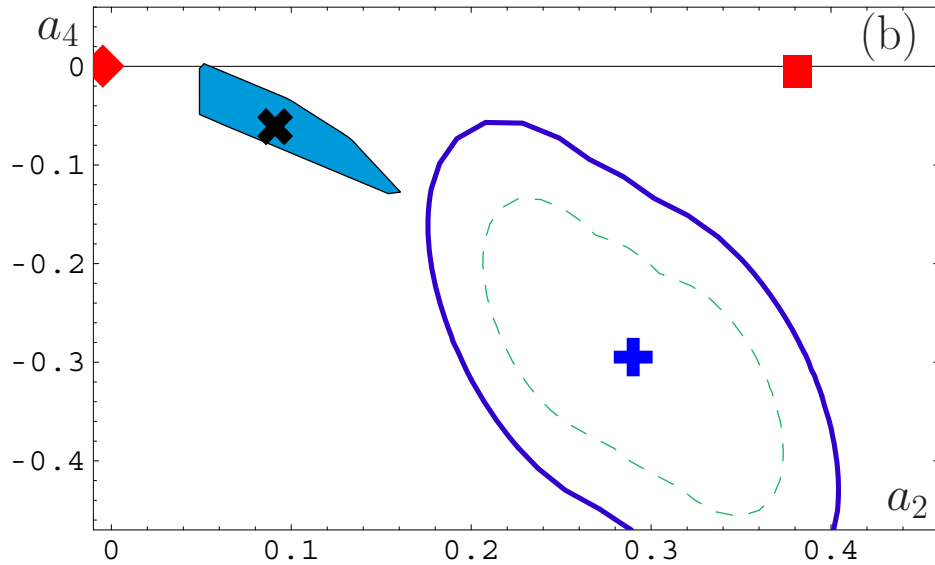


$$\blacksquare \Leftrightarrow \lambda_q^2 = 0.6 \text{ GeV}^2, \\ \delta_{\text{TW-4}}^2 = 0.28(3) \text{ GeV}^2$$

No agreement with CLEO data for  $\lambda_q^2 = 0.6 \text{ GeV}^2$

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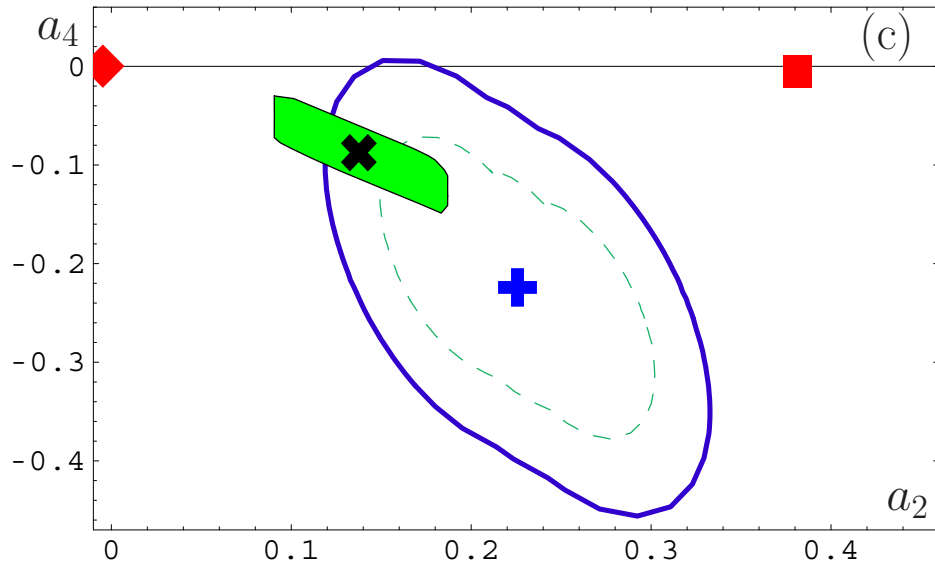


**—**  $\Leftrightarrow \lambda_q^2 = 0.5 \text{ GeV}^2,$   
 $\delta_{\text{TW-4}}^2 = 0.23(2) \text{ GeV}^2$

Bad agreement with CLEO data for  $\lambda_q^2 = 0.5 \text{ GeV}^2$

# NLC SR Results vs NLO CLEO Constraints

[BMS, PRD 67 (2003) 074012]

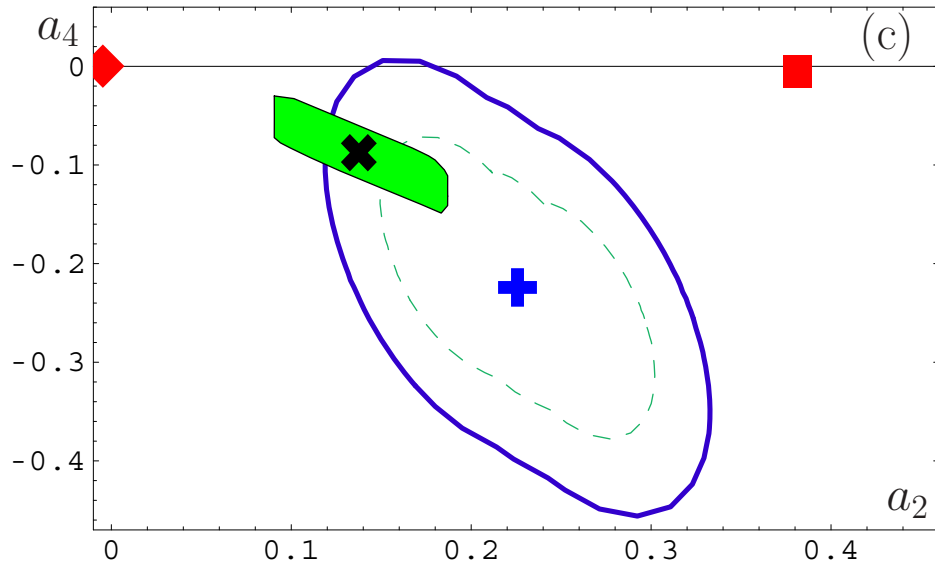


$$\text{Green region} \Leftrightarrow \lambda_q^2 = 0.4 \text{ GeV}^2, \\ \delta_{\text{TW-4}}^2 = 0.19(2) \text{ GeV}^2$$

Good agreement with CLEO data for  $\lambda_q^2 = 0.4 \text{ GeV}^2$  

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$$\text{Green region} \Leftrightarrow \lambda_q^2 = 0.4 \text{ GeV}^2, \\ \delta_{TW-4}^2 = 0.19(2) \text{ GeV}^2$$

Good agreement with CLEO data for  $\lambda_q^2 = 0.4 \text{ GeV}^2$  

For  $\lambda_q^2 \lesssim 0.35 \text{ GeV}^2$  agreement may be better, but:

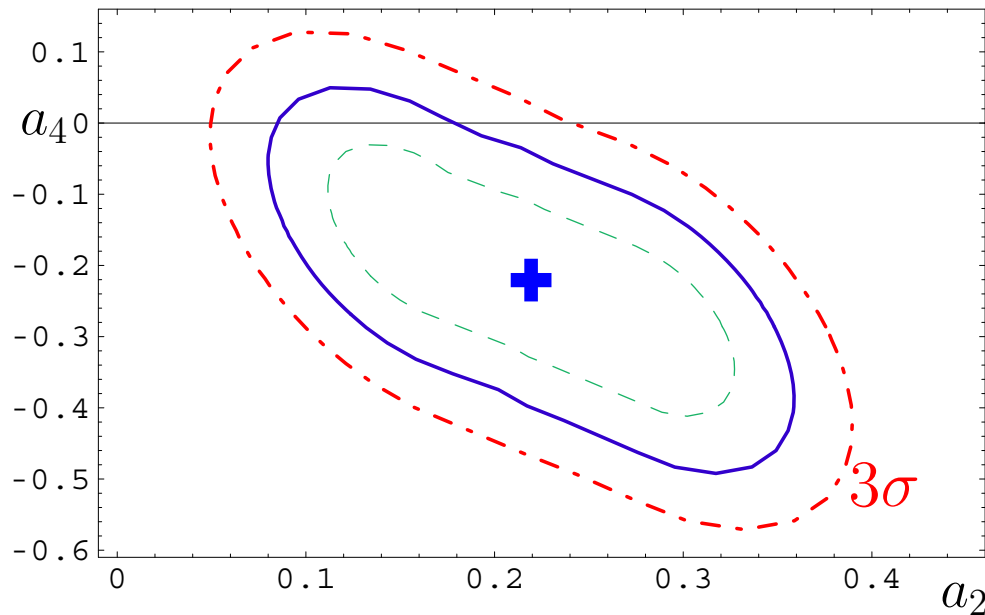
- QCD SRs become **unstable**
- **disagreement** with lattice estimates

[Bakulev&Mikhailov, PRD 65 (2002) 114511]

# NLC SRs vs CLEO Constraints

**NLO Light-Cone SR  $\oplus$  Twist-4  $\oplus$  ( $\mu^2 = Q^2$ )**  
with **20%** uncertainty of  $\delta_{\text{Tw-4}}^2$ :  $\delta_{\text{Tw-4}}^2 = 0.19(4) \text{ GeV}^2$

**BMS [PLB 578 (2004) 91]:  $\lambda_q^2 = 0.4 \text{ GeV}^2$**

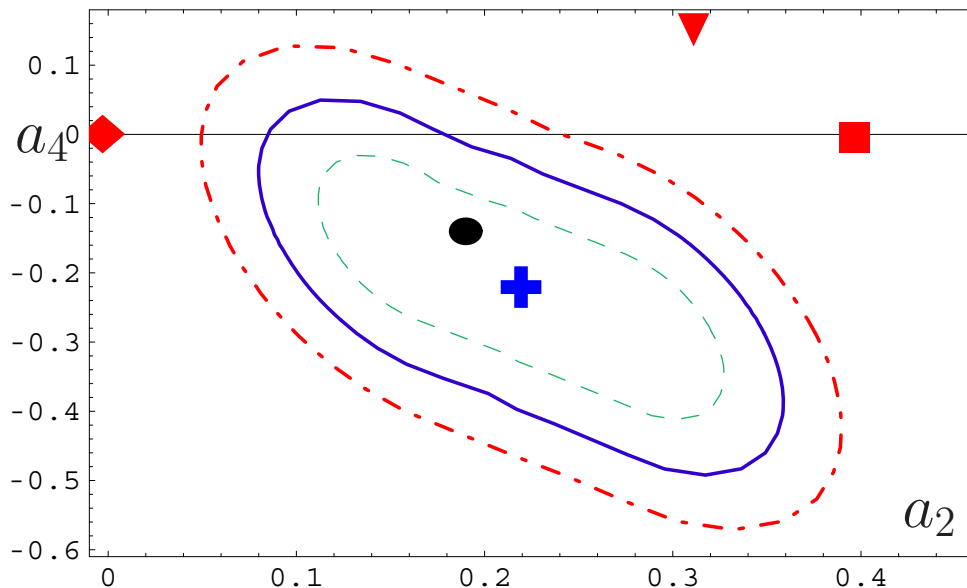


**+** = best-fit **BMS** point

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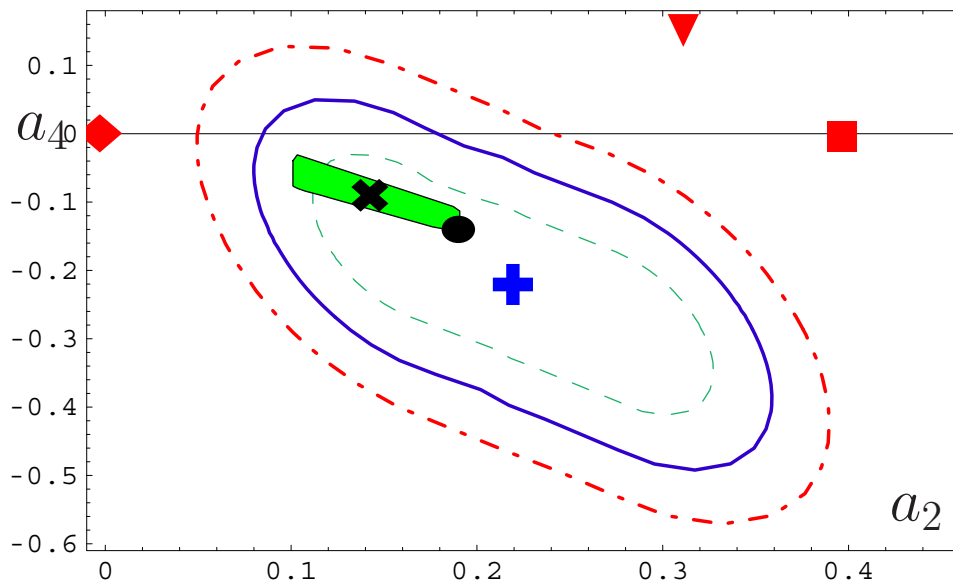
**+** = best-fit BMS, **●** = SY points  
**◆** = Asymptotic DA  
**■** = CZ DA, **▼** = BF DA

Even with 20% uncertainty in twist-4  
**CZ, BF DA excluded at least at  $4\sigma$ -level!**  
**As DA — at  $3\sigma$ -level.**

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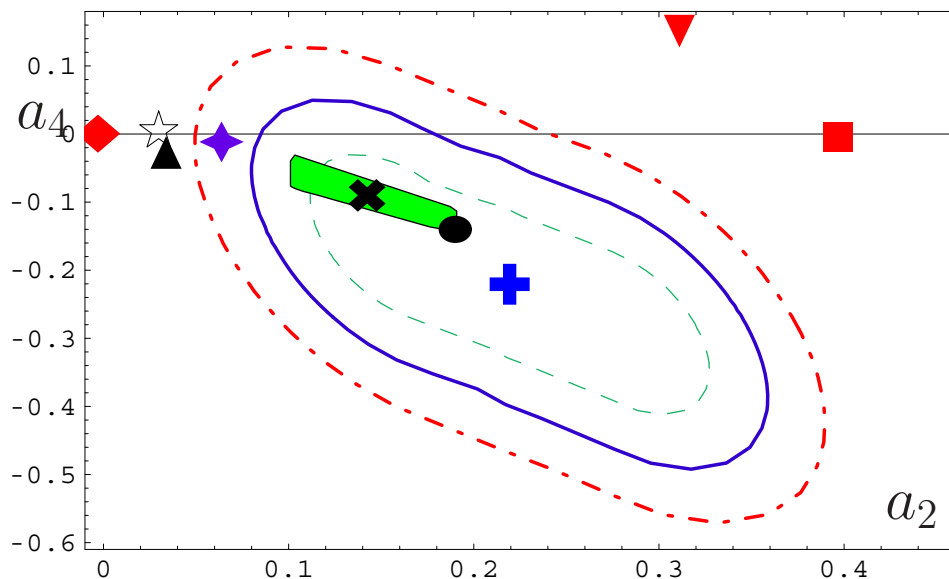
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- x** = BMS model

**CZ DA excluded at least** at  $4\sigma$ -level! **As DA** — at  $3\sigma$ -level.  
BMS DA and most of **BMS bunch** — inside  $1\sigma$ -domain.

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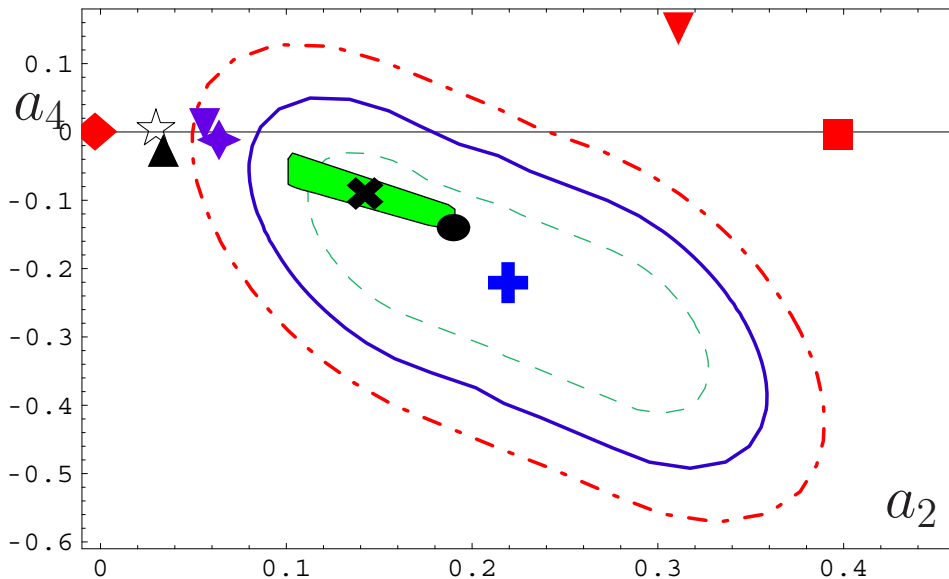
BMS DA and most of **BMS bunch** — inside  $1\sigma$ -domain.  
Instanton-based models — near  $3\sigma$ -boundary  
(**Praszalowicz** et al -model is close to  $2\sigma$ -boundary).



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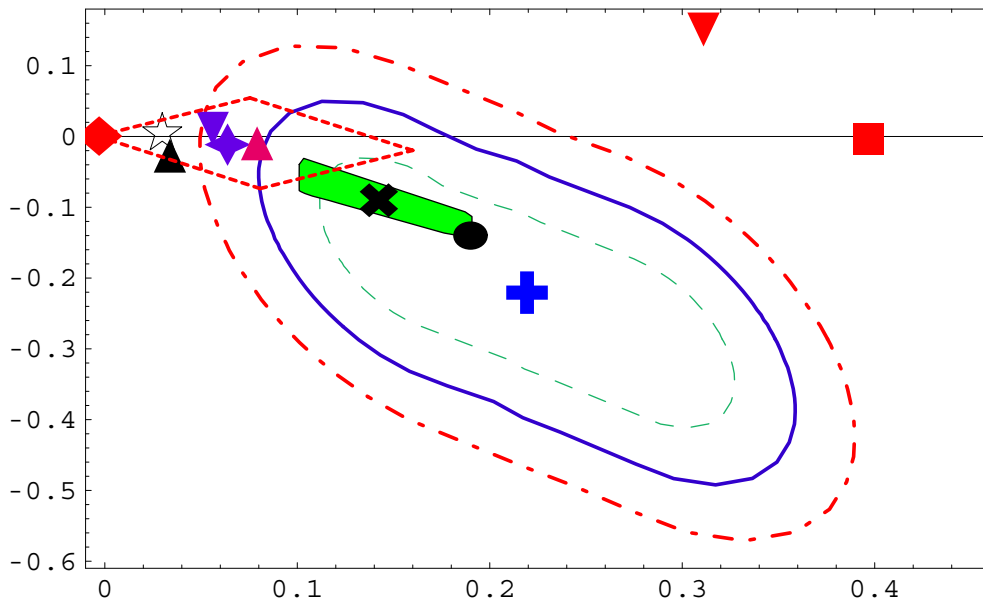
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BMS DA and most of **BMS bunch**—inside  $1\sigma$ -domain.   
**Transverse lattice** model — near  $3\sigma$ -boundary.

# NLC SRs vs CLEO Constraints

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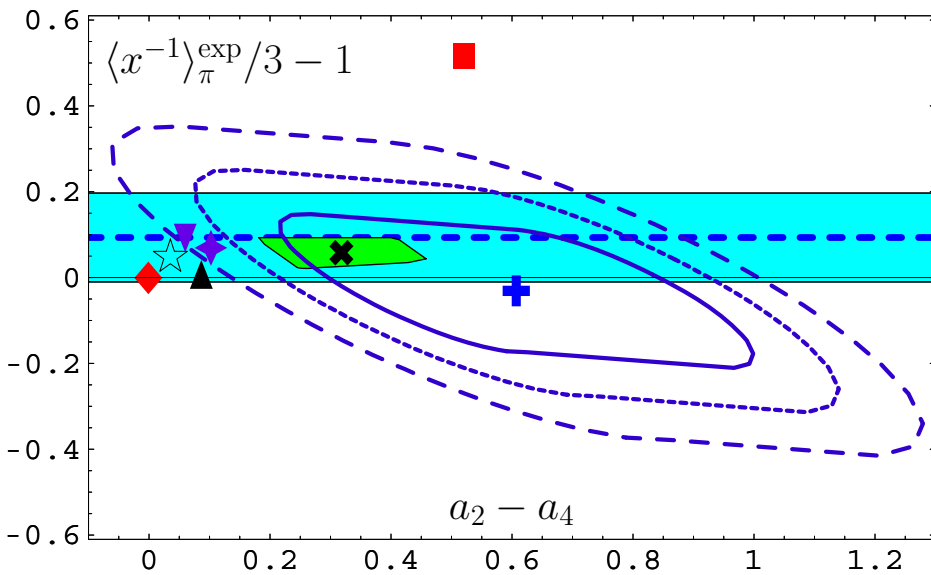


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- ▼** = transverse lattice
- ▲** = BZ model

**▲** in diamond = **[Ball and Zwicky (2005)]** constraint at  $2\sigma$ -boundary.

# New CLEO data constraints for $\langle x^{-1} \rangle_\pi$

**BMS [PLB 578 (2004) 91]:** evolution to  $\mu^2 = 1 \text{ GeV}^2$



$$\lambda_q^2 = 0.4 \text{ GeV}^2,$$

$$\frac{1}{3} \langle x^{-1} \rangle_\pi^{\text{SR}} - 1 = 0.1 \pm 0.1$$



See also **Bijnens&Khodjamirian [EPJC (2002)]:**

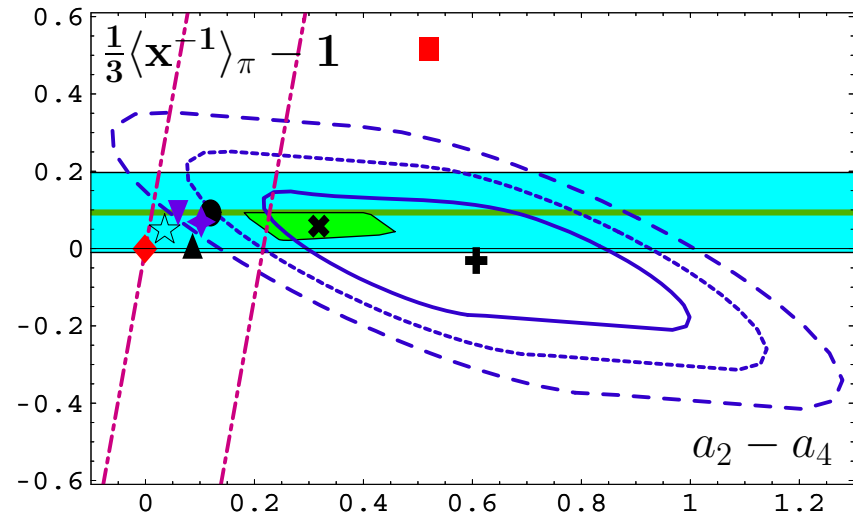
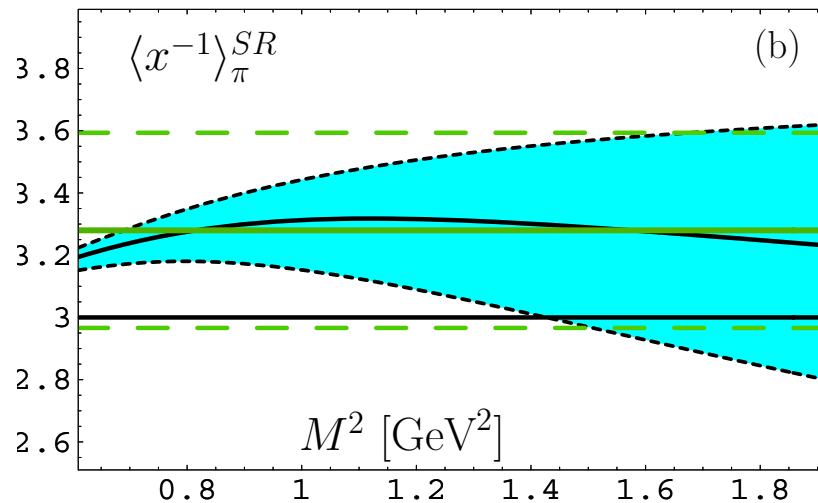
$$\frac{1}{3} \langle x^{-1} \rangle_\pi - 1 = 0.24 \pm 0.16$$

**Again:**

**Good agreement of a theoretical “tool” of different origin with CLEO data**

# New CLEO data constraints for $\langle x^{-1} \rangle_\pi$

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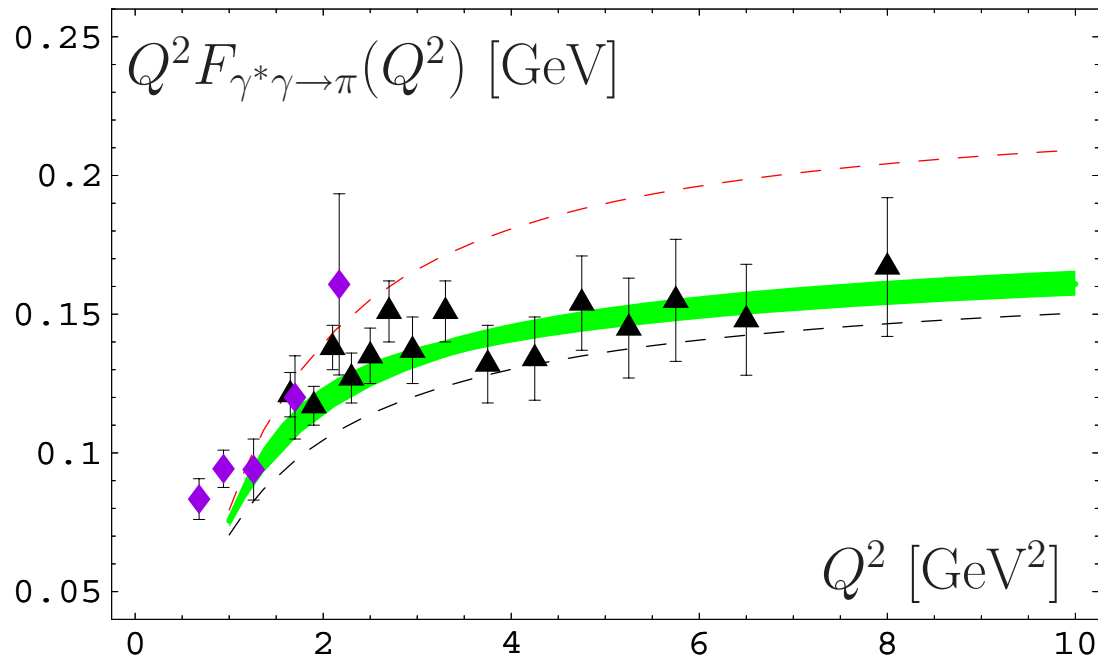


$$\lambda_q^2 = 0.4 \text{ GeV}^2, \quad \frac{1}{3}\langle x^{-1} \rangle_\pi^{SR} - 1 = 0.1 \pm 0.1$$

**Again:**

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# LCSR vs. CELLO (◆) & CLEO (▲) data



curve	DA
---	CZ
—	BMS bunch
- - -	Asymp.

- **BMS bunch** describes rather well all data above  $Q^2 \gtrsim 1.5 \text{ GeV}^2$ ;
- Low- $Q^2$  CELLO data (only statistical errors shown) **excludes As DA** and high- $Q^2$  CLEO data **excludes CZ DA**.

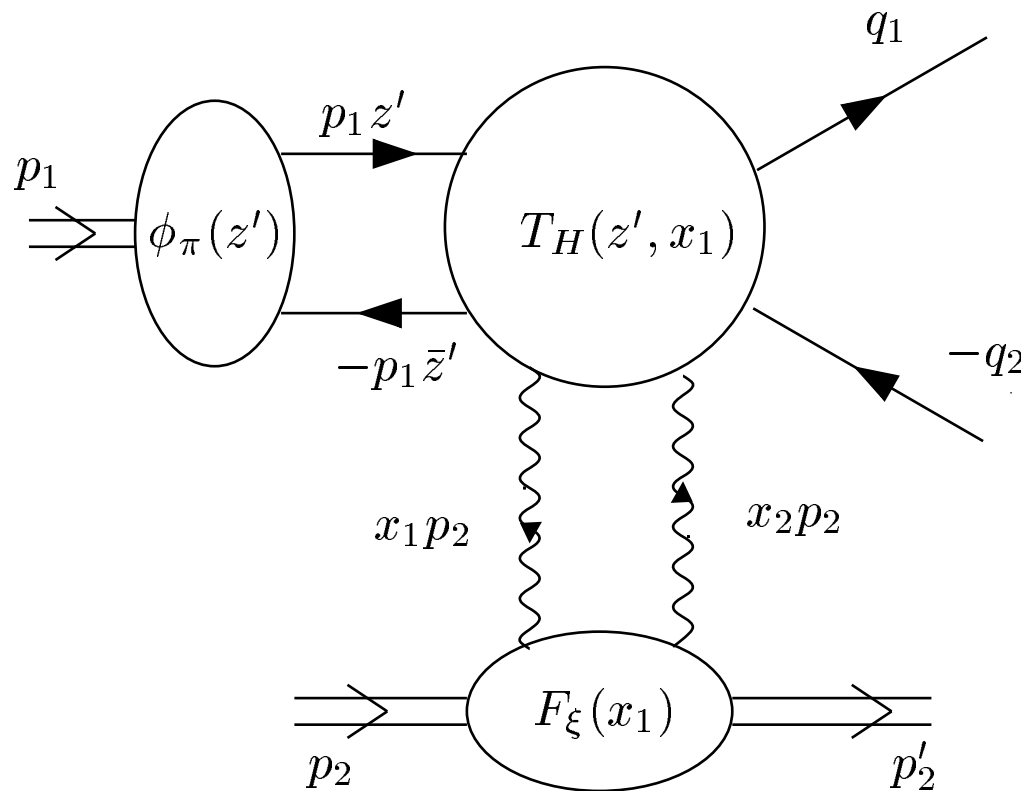
# *Diffraction Dijet Production*

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**What can  
E791 data add?**

# E791: Diffractive dijet production

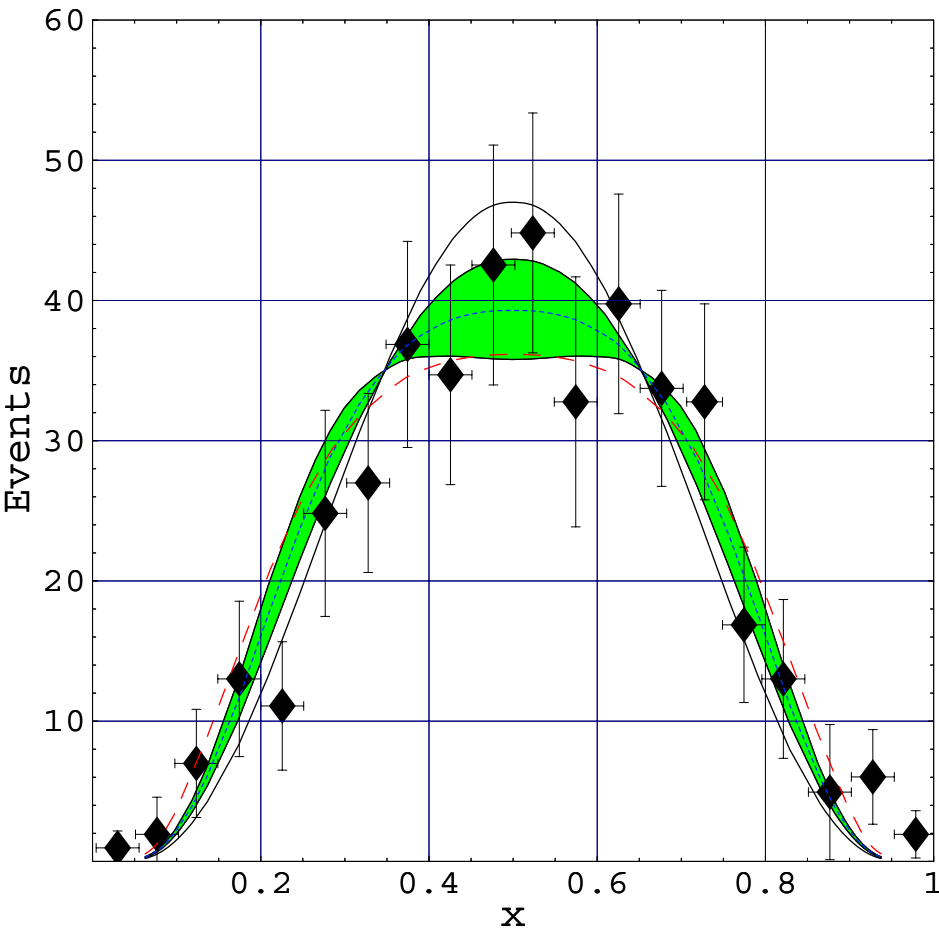
Frankfurt et al. [PLB (1993)]: Rough estimations  
Braun et al. [NPB (2002)]: Account for hard GEXs



$$q_\perp^2 \simeq 4 \text{ GeV}^2$$
$$s \simeq 1000 \text{ GeV}^2$$

# E791: Good agreement with BMS bunch

Following convolution procedure of **Braun et al.**, we found



[PLB 578 (2004) 91]

	DA	$\chi^2$
—	Asymp.	12.56
—	BMS bunch	10.96
- - -	CZ	14.15

(accounting for 18 data points)

**Our bunch** of pion DAs has maximum uncertainty in the central region, but **agrees well** with E791 data!



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**JLab data for  $F_\pi(Q^2)$**   
**in**  
**Analytic NLO pQCD**

# Analytic Perturbation Theory

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**Analyticization** means procedure to obtain analyticity of hadronic observables in whole  $Q^2$  region via dispersion relations (**Radyushkin, Krasnikov&Pivovarov, Dokshitzer, Beneke&Braun, Shirkov&Solovtsov**):

Analytization combines

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Analytization combines

- **RG invariance**  $\implies$  resummation of UV logs and correct QCD asymptotics
- **Causality**  $\implies$  spectral representation  
 $\implies$  **no Landau singularity**

# Analytic Perturbation Theory

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**Analytic Perturbation Theory** expresses QCD observables over **non-power sequences**  $\{\mathcal{A}_k^{(L)}(Q^2)\}$  in  $L$ -loop order [**Shirkov, NPB Proc. 64 (1998) 106**].  
At 1-loop:

$$\mathcal{A}_k^{(1)}(Q^2) = \frac{1}{\pi} \int_0^{\infty} \frac{\rho_k^{(1)}(\sigma) d\sigma}{\sigma + Q^2 - i\epsilon} ; \rho_k^{(1)}(\sigma) = \text{Im} \left( \frac{4\pi}{b_0 \ln(-\sigma/\Lambda^2)} \right)^k$$

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with 1-loop explicit expressions

- $\mathcal{A}_1^{(1)}(Q^2) = \frac{4\pi}{b_0} \left[ \frac{1}{\ln(Q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 - Q^2} \right]$

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- $\mathcal{A}_1^{(1)}(Q^2) = \frac{4\pi}{b_0} \left[ \frac{1}{\ln(Q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 - Q^2} \right]$
- $\mathcal{A}_2^{(1)}(Q^2) = \left( \frac{4\pi}{b_0} \right)^2 \left[ \frac{1}{\ln^2(Q^2/\Lambda^2)} + \frac{Q^2 \Lambda^2}{(\Lambda^2 - Q^2)^2} \right]$

# Analytic Perturbation Theory

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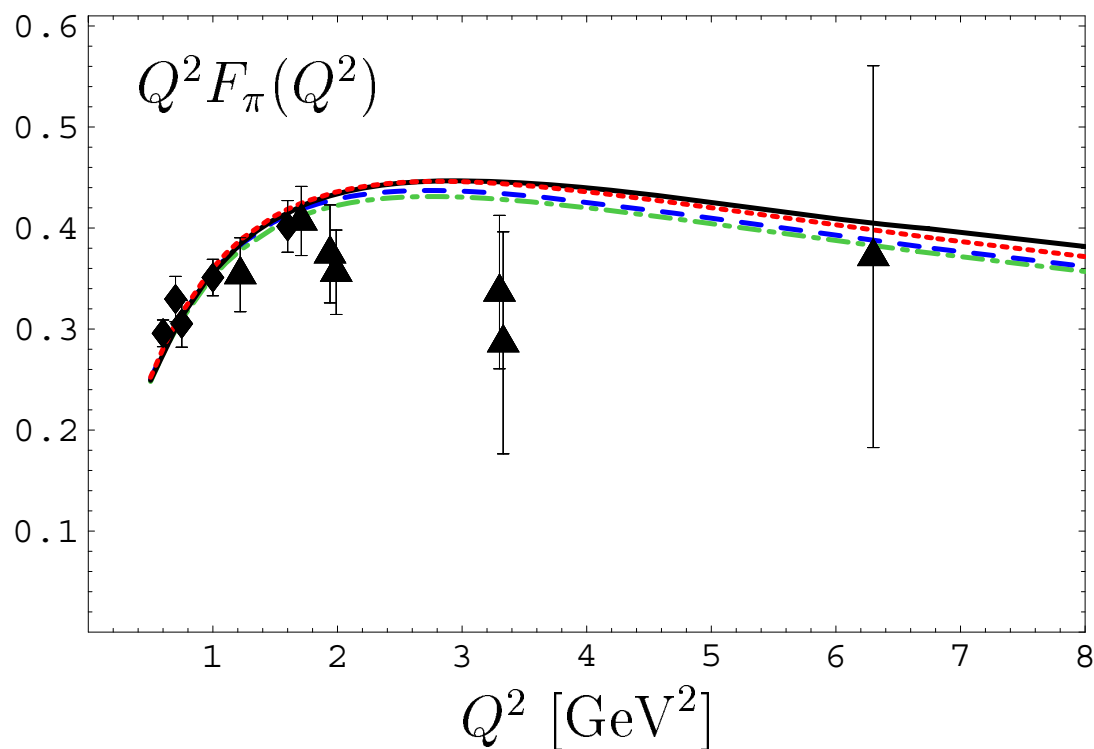
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**Important:**  $\mathcal{A}_2(Q^2) \neq [\mathcal{A}_1(Q^2)]^2$

# Pion form factor in analytic NLO pQCD

[Bakulev-Passek-Schroers-Stefanis,  
PRD 70 (2004) 033014]



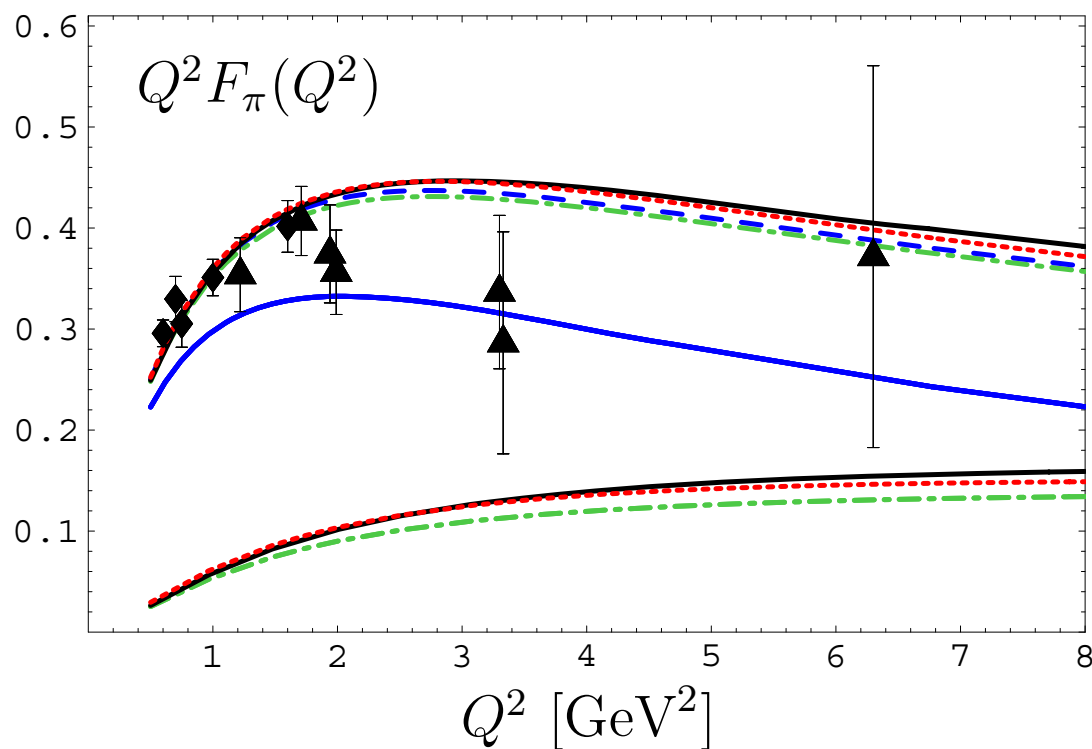
Curves	Schemes
—	$\mu_R^2 = 1 \text{ GeV}^2$
- - -	$\mu_R^2 = Q^2$
⋯	<b>BLM scale</b>
- · - ·	<b><math>\alpha_V</math>-scheme</b>

★ De facto insensitive to scheme/scale setting



# Pion form factor in analytic NLO pQCD

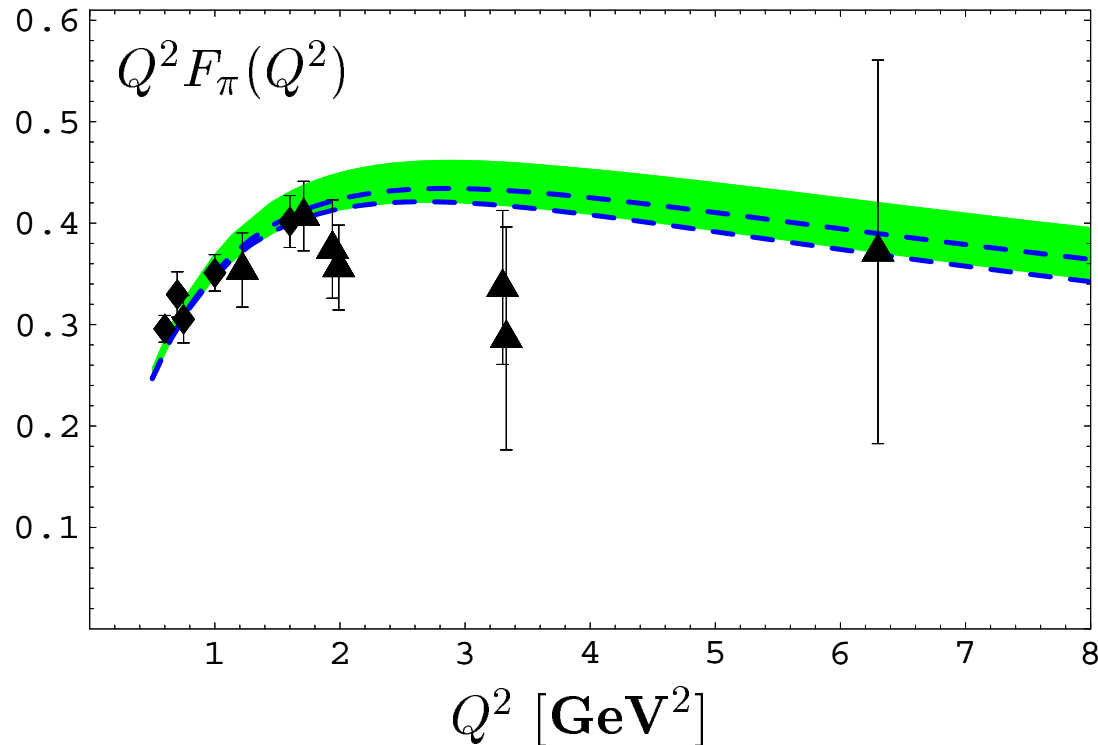
[Bakulev-Passek-Schroers-Stefanis,  
PRD 70 (2004) 033014]



Curves	Schemes
—	$\mu_R^2 = 1 \text{ GeV}^2$
- - -	$\mu_R^2 = Q^2$
⋯	<b>BLM scale</b>
- · - ·	<b><math>\alpha_V</math>-scheme</b>
—	<b>soft part</b>

★ De facto insensitive to scheme/scale setting

# Pion FF in analytic NLO pQCD



**Green strip** includes

- **NLC QCD SRs uncertainties (pion DA bunch);**
- **scale-setting ambiguities** at NLO level.

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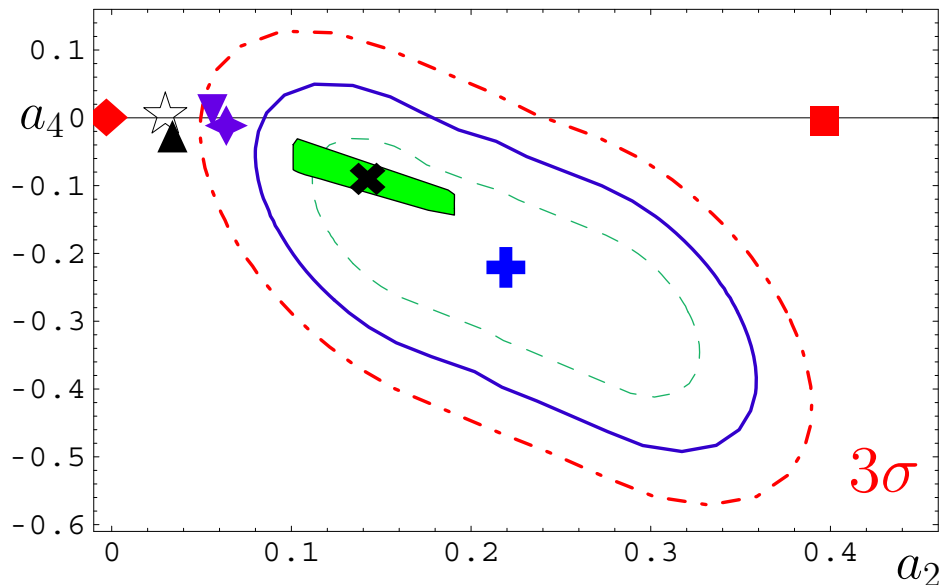
# New Lattice Data for pion DA

# CLEO Constraints and Lattice Data

**NLO Light-Cone SR  $\oplus$  Twist-4  $\oplus$  ( $\mu^2 = Q^2$ )**

with 20% uncertainty of  $\delta_{\text{Tw-4}}^2$ :  $\delta_{\text{Tw-4}}^2 = 0.19 \pm 0.04 \text{ GeV}^2$

**BMS [PLB 578 (2004) 91]:  $\lambda_q^2 = 0.4 \text{ GeV}^2$**



- +** = best-fit point
- ◆** = **Asymptotic** DA
- = **CZ** DA
- ×** = BMS model
- ☆**, **▲** and **◆** = instantons
- ▼** = transverse lattice

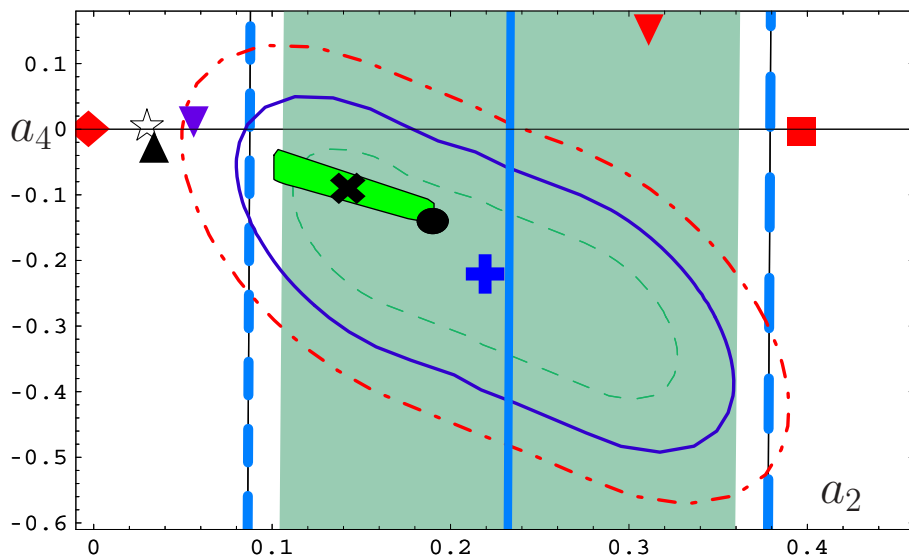
BMS DA and most of **BMS bunch** — inside  $1\sigma$ -domain.  
**Transverse lattice** model — near  $3\sigma$ -boundary.

# CLEO Constraints and Lattice Data

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**BMS [PLB 578 (2004) 91]:  $\lambda_q^2 = 0.4 \text{ GeV}^2$**



◆ = Asymptotic DA

■ = CZ DA

▼ = BF DA

+ = best-fit point, ● = SY point

✕ = BMS model

☆, ▲ = instantons

▼ = transverse lattice

gray strip = L. Del Debbio'04

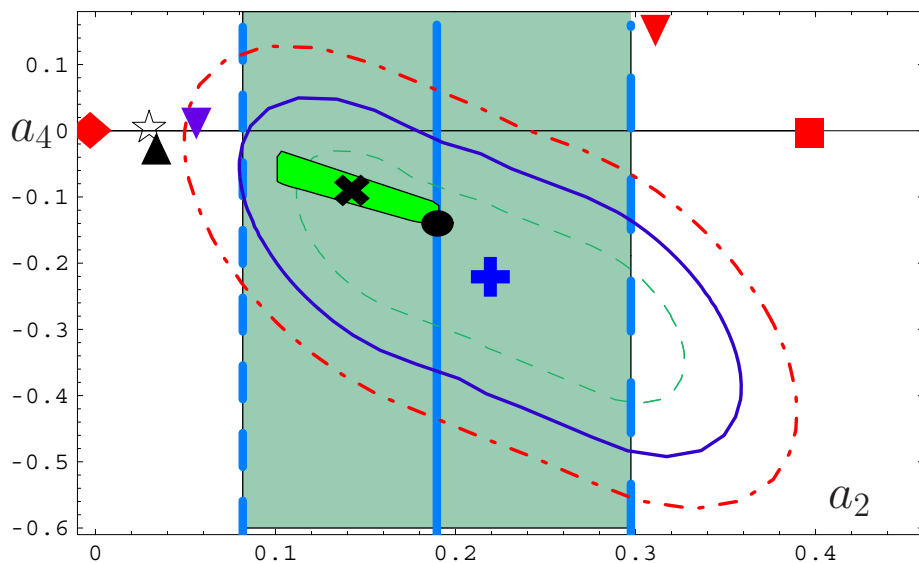
BMS DA and most of **BMS bunch** — inside  $1\sigma$ -domain  
and inside **2004 lattice strip [PRD 73 (2006) 056002]**.

# CLEO Constraints and Lattice Data

**NLO Light-Cone SR  $\oplus$  Twist-4  $\oplus$  ( $\mu^2 = Q^2$ )**

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**BMS [PLB 578 (2004) 91]:  $\lambda_q^2 = 0.4 \text{ GeV}^2$**



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- = CZ DA, ▼ = BF DA
- + = best-fit point, ● = SY point
- ✕ = BMS model
- ☆, ▲ = instantons
- ▼ = transverse lattice
- gray strip = QCD SF/UK'06

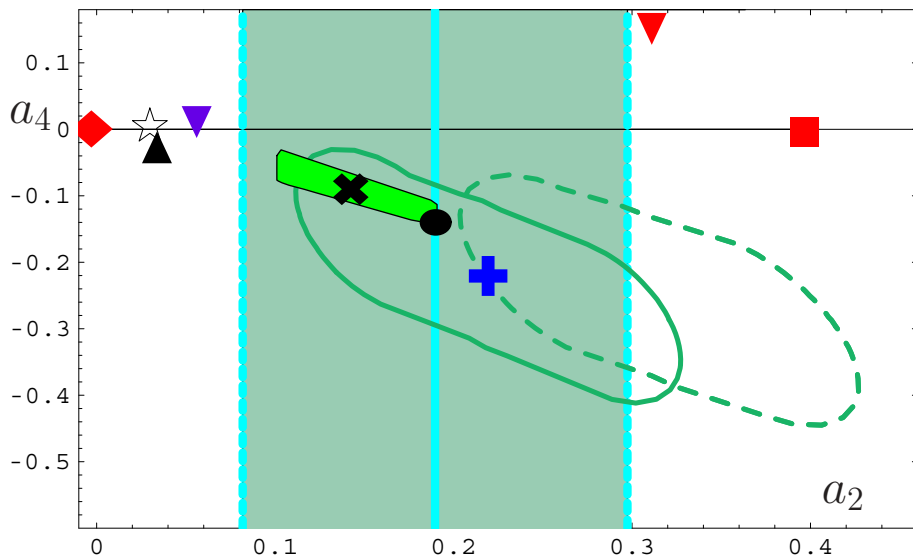
BMS DA and most of **BMS bunch** — inside  $1\sigma$ -domain and **inside new 2006 lattice strip [hep-lat/0606012]**. 

# CLEO Constraints and Lattice Data

**NLO Light-Cone SR  $\oplus$  Twist-4  $\oplus$  ( $\mu^2 = Q^2$ )**

with 10% uncertainty of  $\delta_{\text{Tw-4}}^2$ :  $\delta_{\text{Tw-4}}^2 = 0.19 \pm 0.02 \text{ GeV}^2$

**[PRD 73 (2006) 056002]:  $\lambda_q^2 = 0.4 \text{ GeV}^2$**




- ◆ = Asymptotic DA
- = CZ DA, ▼ = BF DA
- + = best-fit point, ● = SY point
- ✕ = BMS model
- ☆, ▲ = instantons
- ▼ = transverse lattice
- gray strip = QCD SF/UK'06

BMS DA and most of **BMS bunch** — inside  $1\sigma$ -domain and inside **lattice strip**. **Dashed contour** = renormalon model estimation of **Twist-4** in CLEO data analysis.

# CONCLUSIONS


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- **QCD SR** with **NLC** for pion DA **gives us** admissible sets(**bunches**) of DAs for each  $\lambda_q^2$  value. 




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- **QCD SR** with **NLC** for pion DA **gives us** admissible sets of DAs for each  $\lambda_q^2$  value.
- Comparing **NLC SRs** with new CLEO **constraints** **allows to fix** the value of QCD vacuum nonlocality  $\lambda_q^2 = 0.4 \text{ GeV}^2$ .
- NLO LCSR **produces new constraints** on pion DA parameters ( $a_2, a_4$ ) in conjunction with CLEO data.
- **This bunch** of pion DAs **agrees well** with **E791** data on diffractive dijet production, with **JLab F(pi)** data on pion EM form factor and with recent **lattice** data. 

# Collaborators & Publications

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## Collaborators

- A. P. Bakulev BLTPh, JINR, Dubna
- S. V. Mikhailov BLTPh, JINR, Dubna
- A. I. Karanikas University of Athens, Athens

## Publications

- A.P.B., S.V.M., N.G.S. PLB 508 (2001) 279
- A.P.B., S.V.M., N.G.S. PRD 67 (2003) 074012
- A.P.B., S.V.M., N.G.S. PLB 578 (2004) 91
- A.P.B. *et al.* PRD 70 (2004) 033014
- A.P.B., N.G.S. NPB 721 (2005) 50
- A.P.B., A.I.K., N.G.S. PRD 72 (2005) 074015
- A.P.B., S.V.M., N.G.S. PRD 73 (2006) 056002