Meson wave functions from the lattice

Wolfram Schroers

DESY NIC

QCDSF/UKQCD Collaboration

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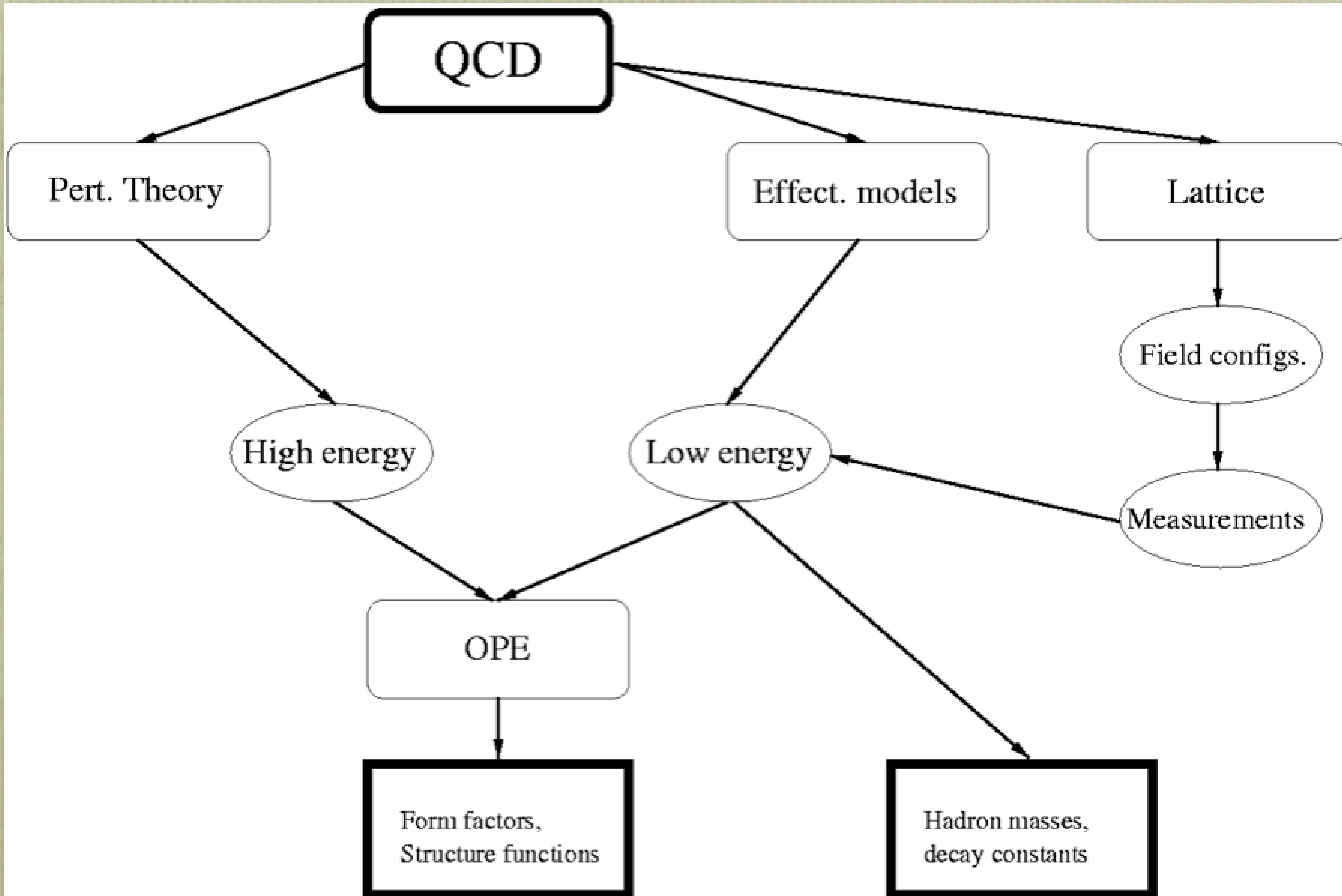
Outline

- Basics of lattice calculations
- Renormalization
- Chiral perturbation theory
- QCDSF results (hep-ph/0606012)
- Summary & Outlook


QCD
Pert. Theory
Effect. models
Lattice
Field configs.
High energy
Low energy
Measurements
OPE
Form factors,
Structure functions
Hadron masses,
decay constants

Lattice QCD

- **Goal**: Qualitative & quantitative insight into hadronic functions from first principles
  - Comparison experiment ↔ theory

    ⇒ Credibility for predictions
- **Advantage**: Vary parameters, e.g., m_q, N_C, N_F

Quark masses

- **Heavy quark regime:**

  confinement, flux tubes, adiabatic potential

- **Light quark regime:**

  chiral symmetry breaking, instantons, chiral perturbation theory

Getting observables

- In principle, three extrapolations
  - Infinite-volume extrapolation
  - Continuum extrapolation
  - Chiral extra- (or inter-) polation

Fermion discretizations

- Unimproved Wilson fermions
- Improved Wilson
- Improved staggered fermions
- Ginsparg-Wilson fermions
  - Domain-wall
  - Overlap

Practically **very** important question!

Fermion discretizations

Simple,
well understood

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Simple, well understood

Cheap

Chiral, $O(a^2)$, **but** expensive

The challenge

- Light valence fermions are possible today
- Light sea quarks remain major issue

Possible solutions:

- Hybrid calculations
- Full GW: Either full DWF or Overlap
- Full Wilson-type fermions

Renormalization

- Matching between lattice regularization at scale a^{-1} and continuum MS-bar scheme
    - Perturbative
    - Non-perturbative: RI-MOM scheme
    - Non-perturbative: Schrödinger functional

Perturbative

- Analytical calculation possible
- No lattice-systematic errors, no statistical uncertainties
- Lattice pert.-theory restricted to leading order
- Uncontrolled error from higher orders
- Only feasible if Z is $O(1)$

NP-renorm: RI-MOM

- Possible for any lattice action
- Requires gauge-fixing ⇒ Lattice Gribov copies
- May fail to yield result, but can be diagnozed
- Applicable to arbitrary Z-factors
- Does not require resampling of gauge fields

  But needs chiral extrapolation

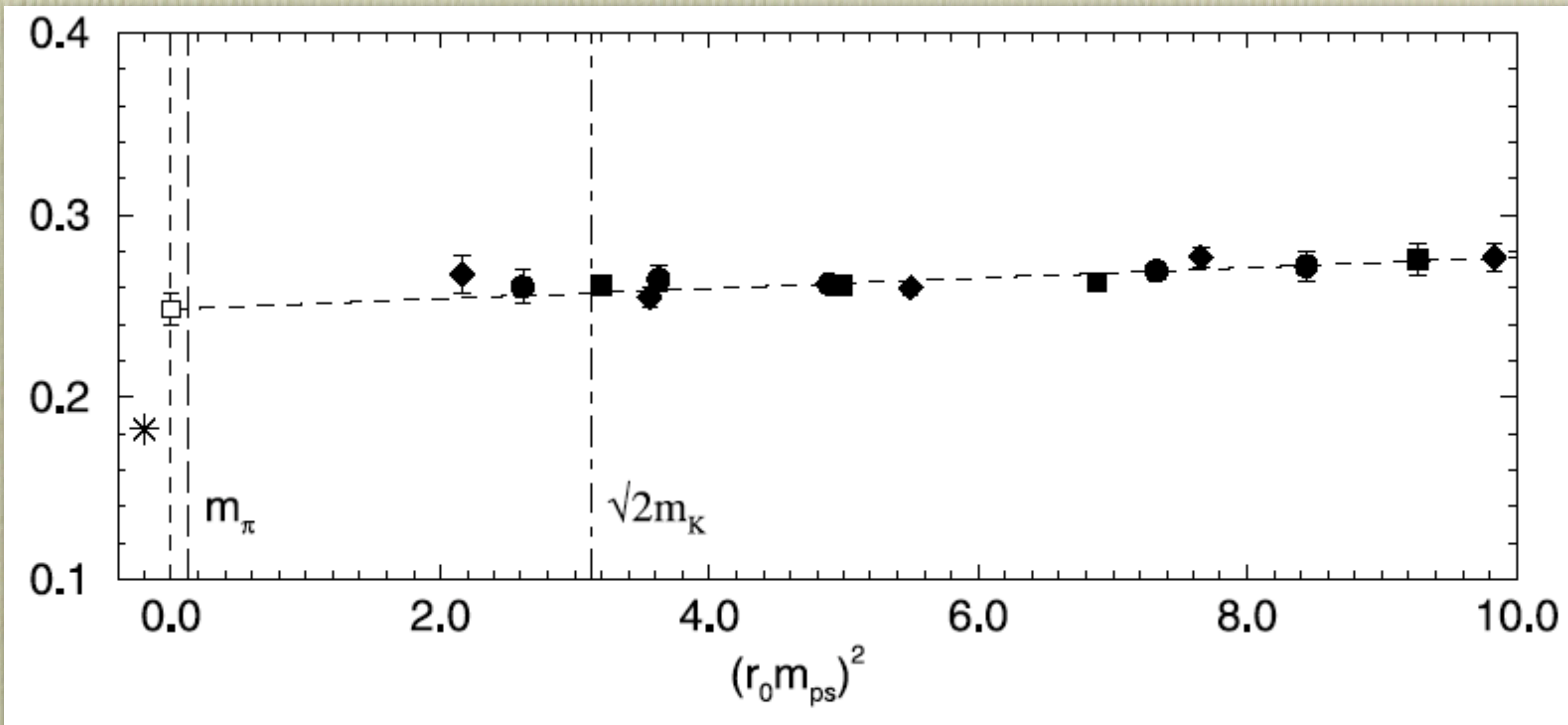
NP-renorm: SF

- Applicable to any lattice action
- No gauge fixing and no chiral extrapolation required (exactly chiral)
- Requires resampling of gauge fields at several volumes (operator-dependend)
- Matching to MS-bar must be done on your own
- Very complicated to be used

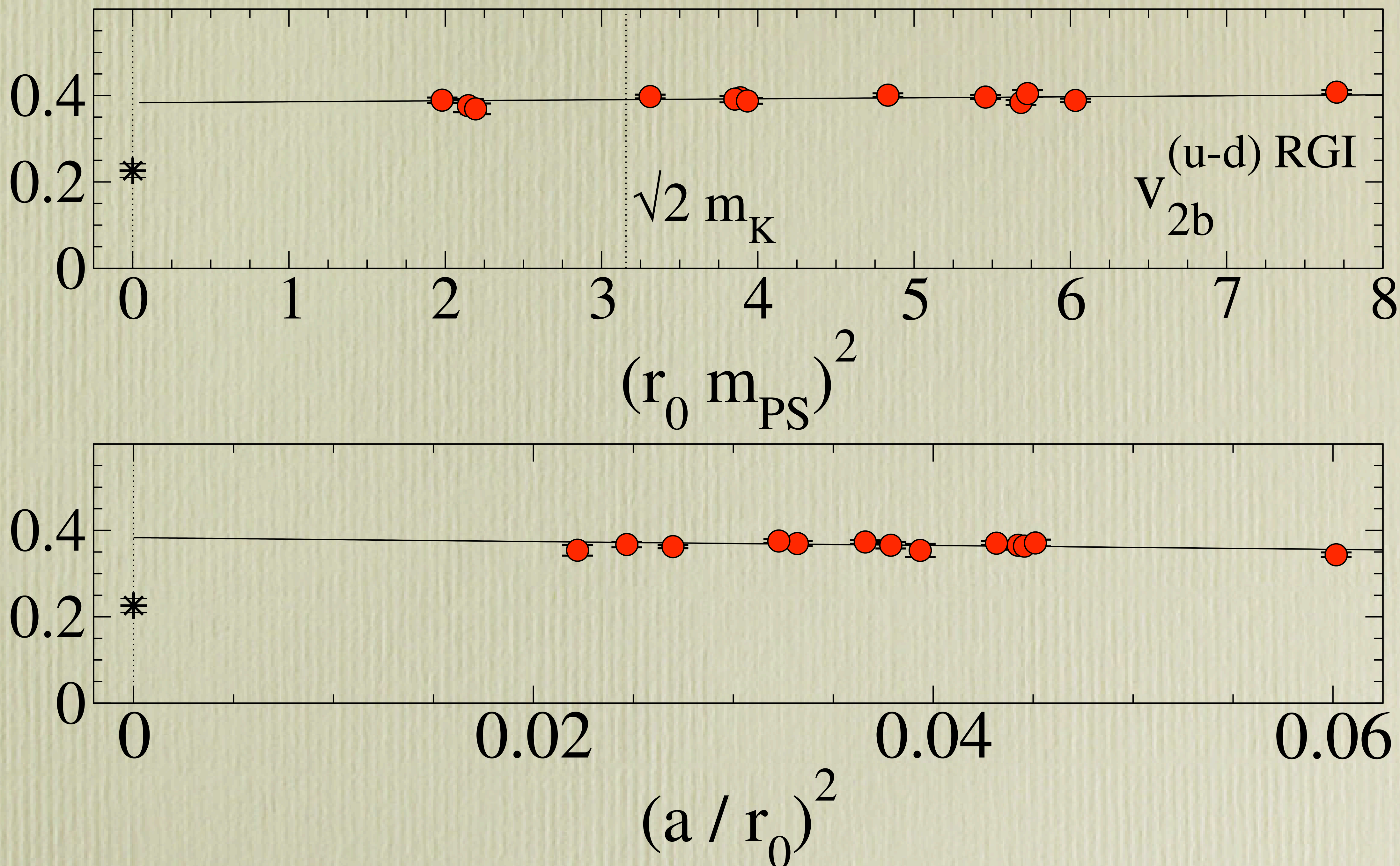
Chiral extrapolations

- Unresolved problem: $\langle x \rangle_{u-d}$
- Success story: g_A

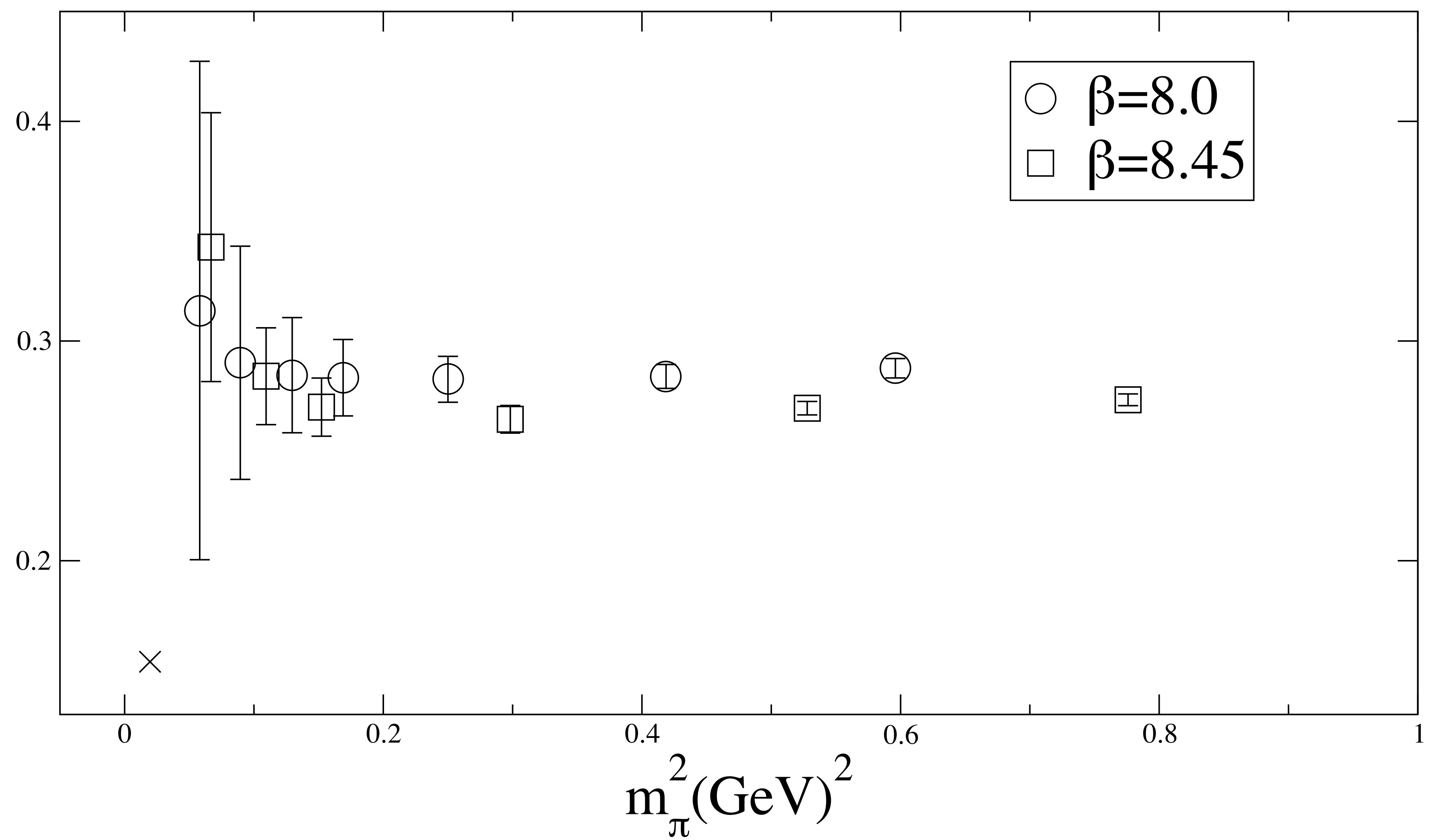
Open issue: $\langle x \rangle_{u-d}$



Phys.Rev. D71:114511 (2005) (M. Göckeler et al)



Nucl.Phys.Proc.Suppl. 140:399-404 (2005)
(J. Zanotti et al)



PoS LAT2005:363 (2005)
(T. Streuer et al)

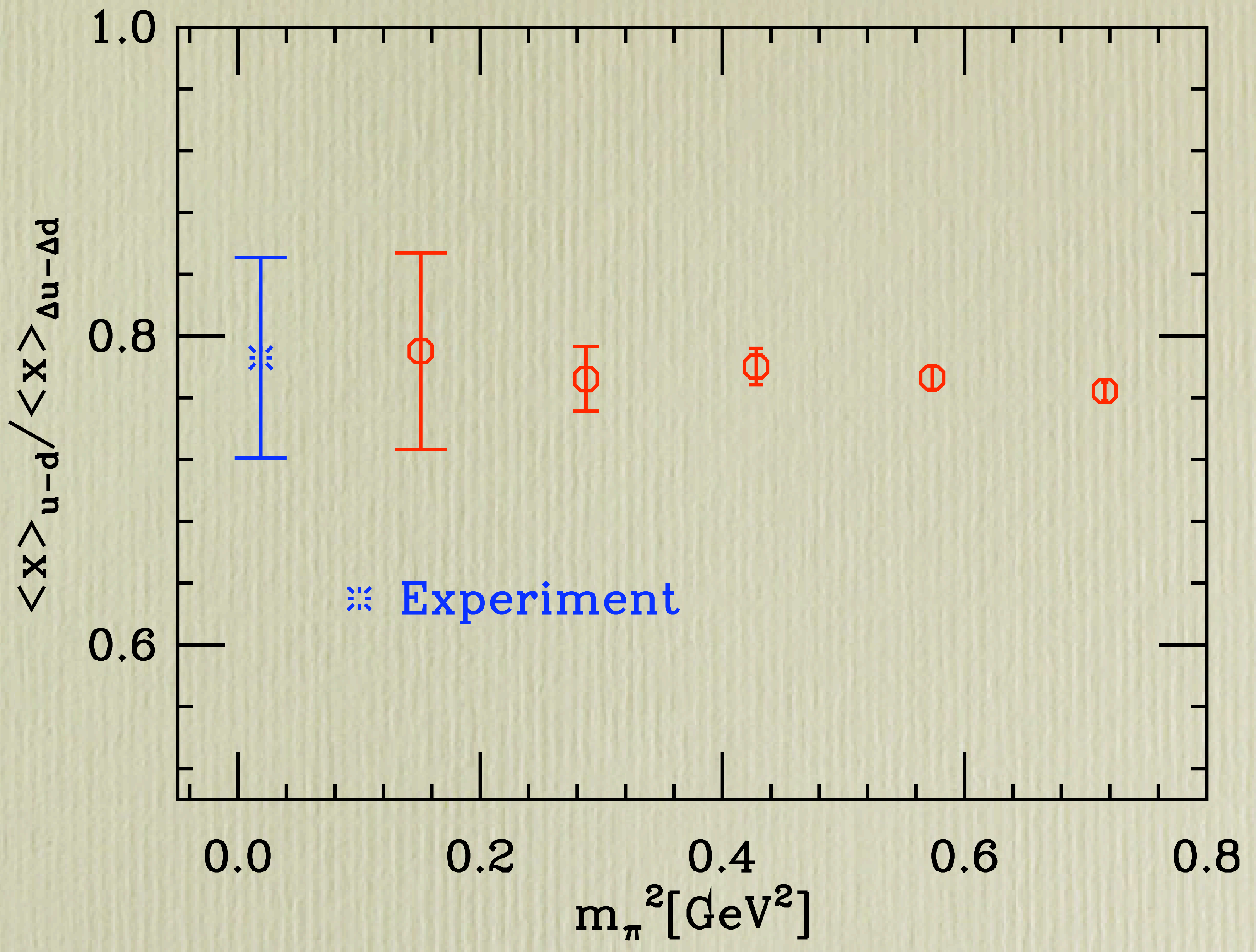
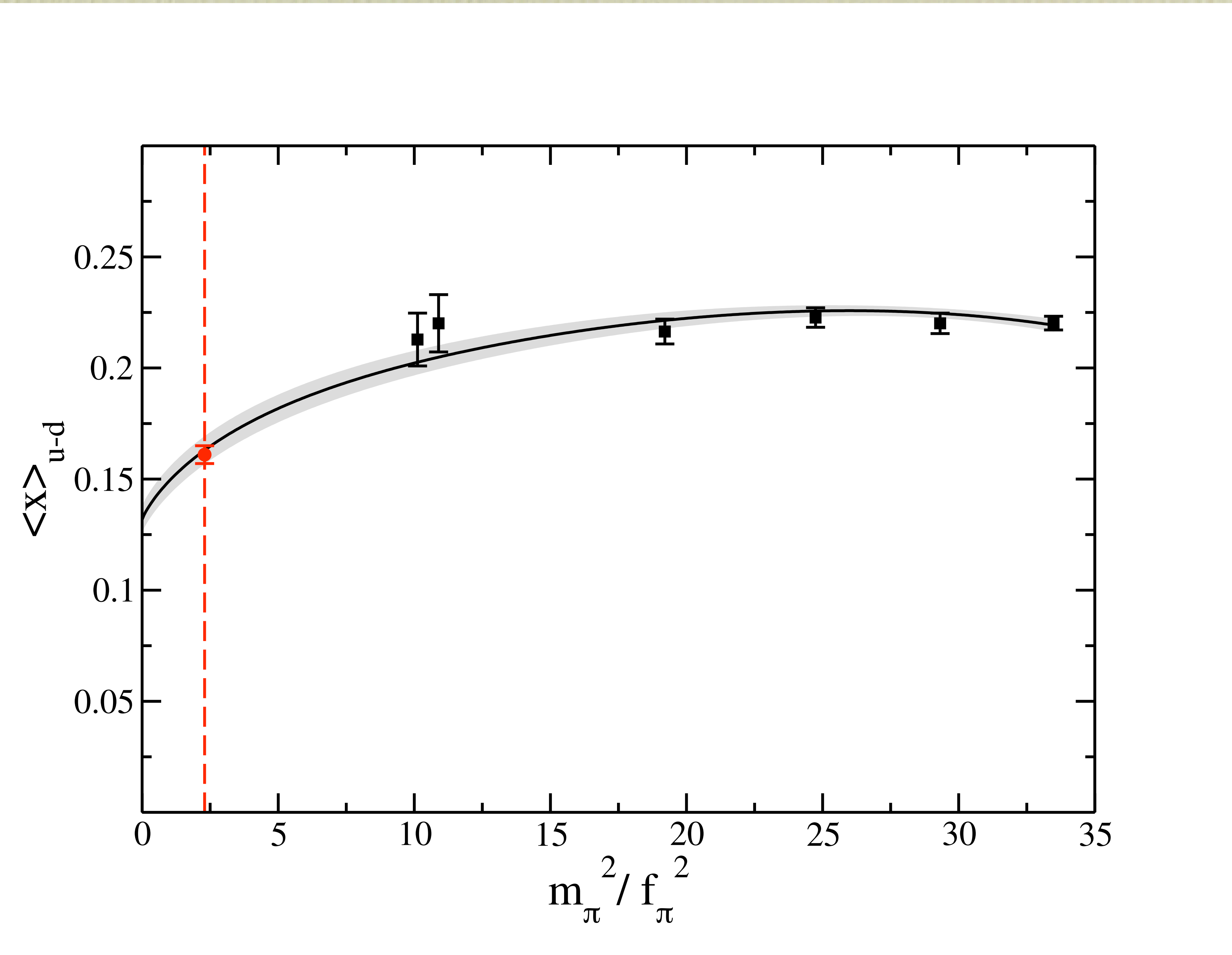


FIG. 9: The ratio of the flavor non-singlet momentum fraction to the helicity distribution (octagons). The experimental expectation is marked by the burst symbol.

hep-lat/0505024 (K. Orginos et al)

Momentum Fraction: $\langle x \rangle_{u-d}$

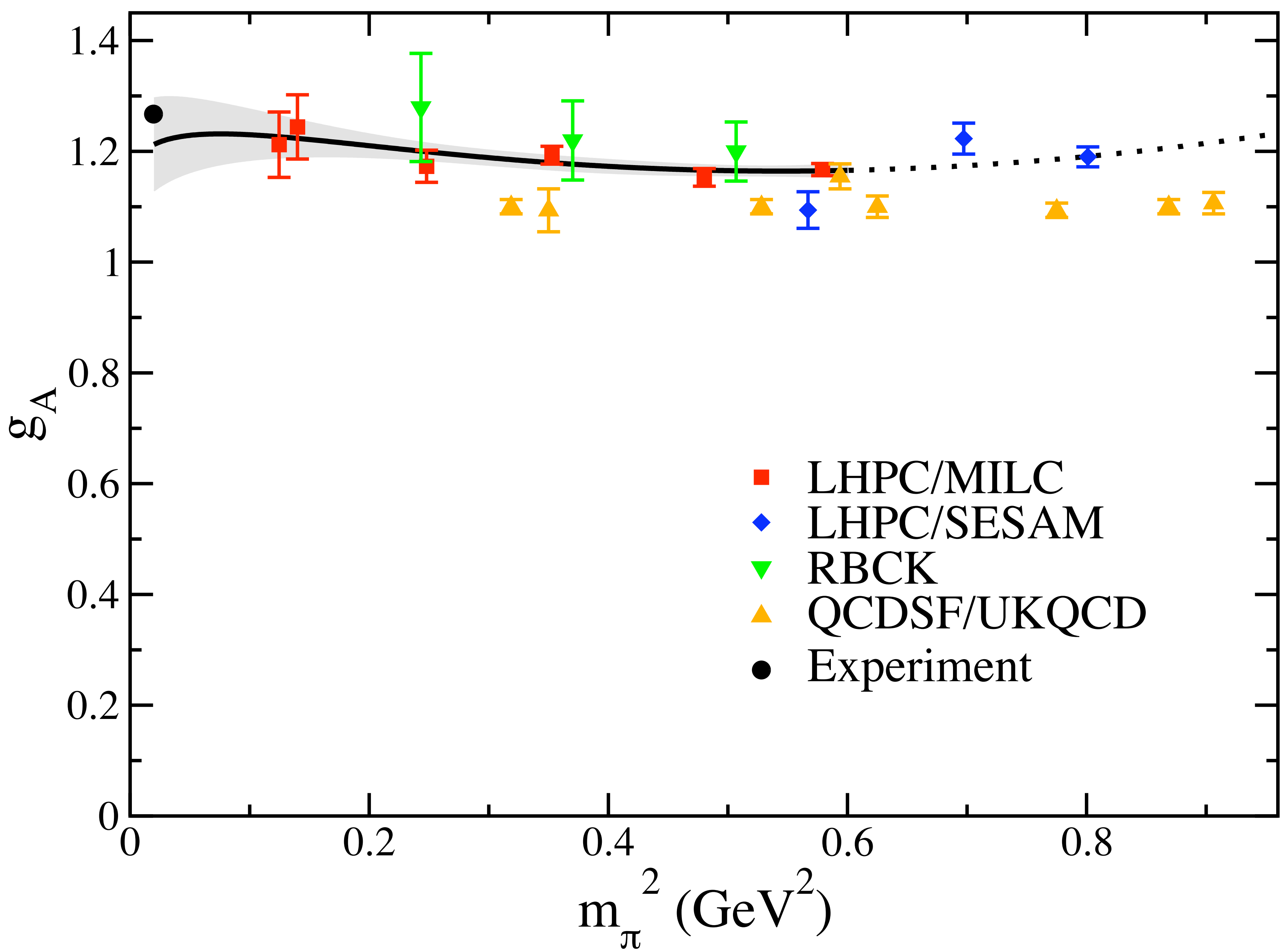


D.B. Renner, talk at Lattice 2006

LHPC: Hybrid calculations

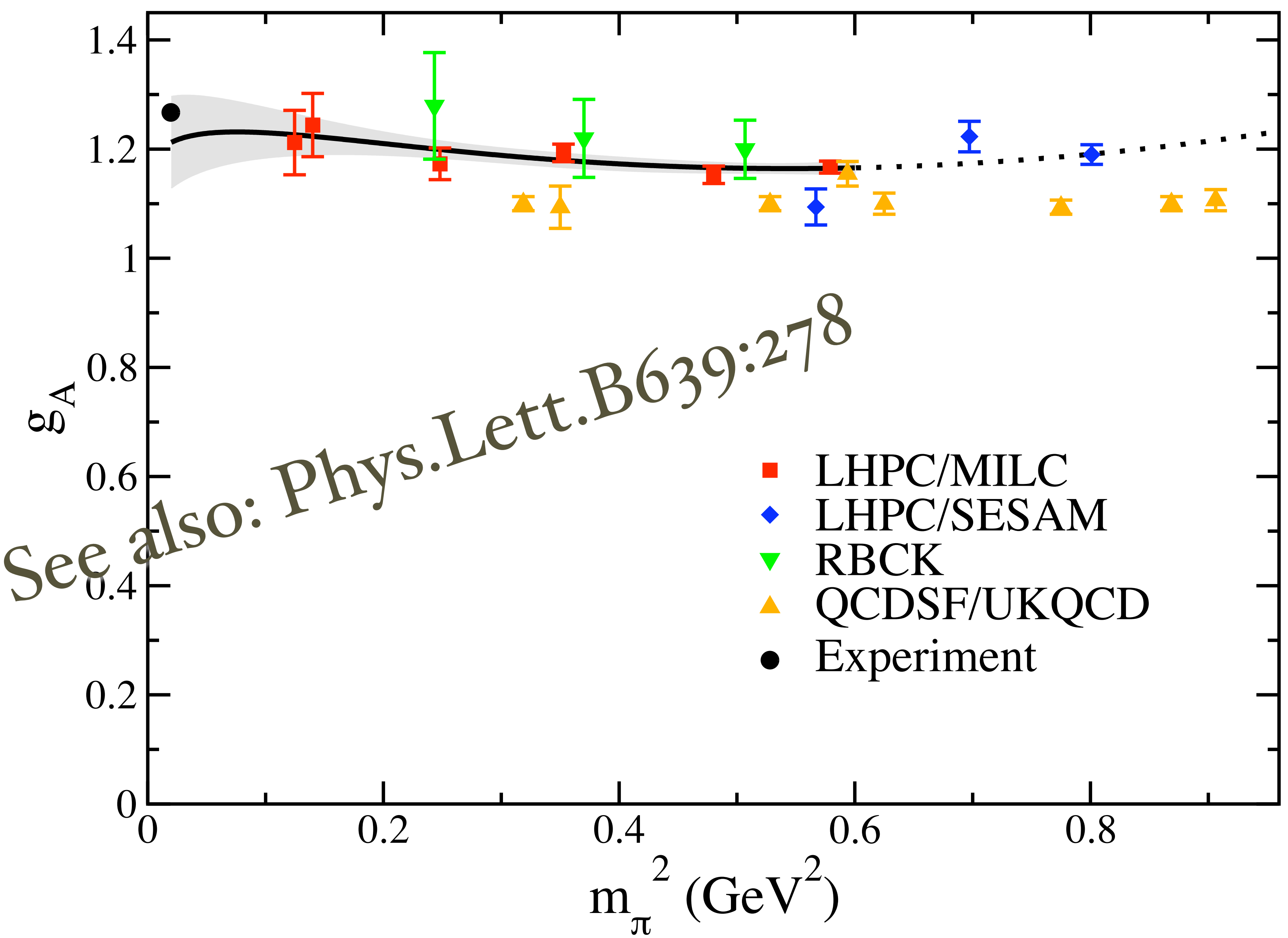
- Hybrid approach: Asqtad & DWF
- Achievement: 5% acc. at m_π=354 MeV
- Lattice sizes $(2.5\text{fm})^3$ and $(3.5\text{fm})^3$
- Six constants: f_π, m_Δ-m_N, $g_{N\Delta}$, g_A, $g_{\Delta\Delta}$, C
- First three: physical values, others are fit
- Total error from constr. parameters: <1%

Combined results full QCD



Phys.Rev.Lett. 96:052001 (2006)

Combined results full QCD



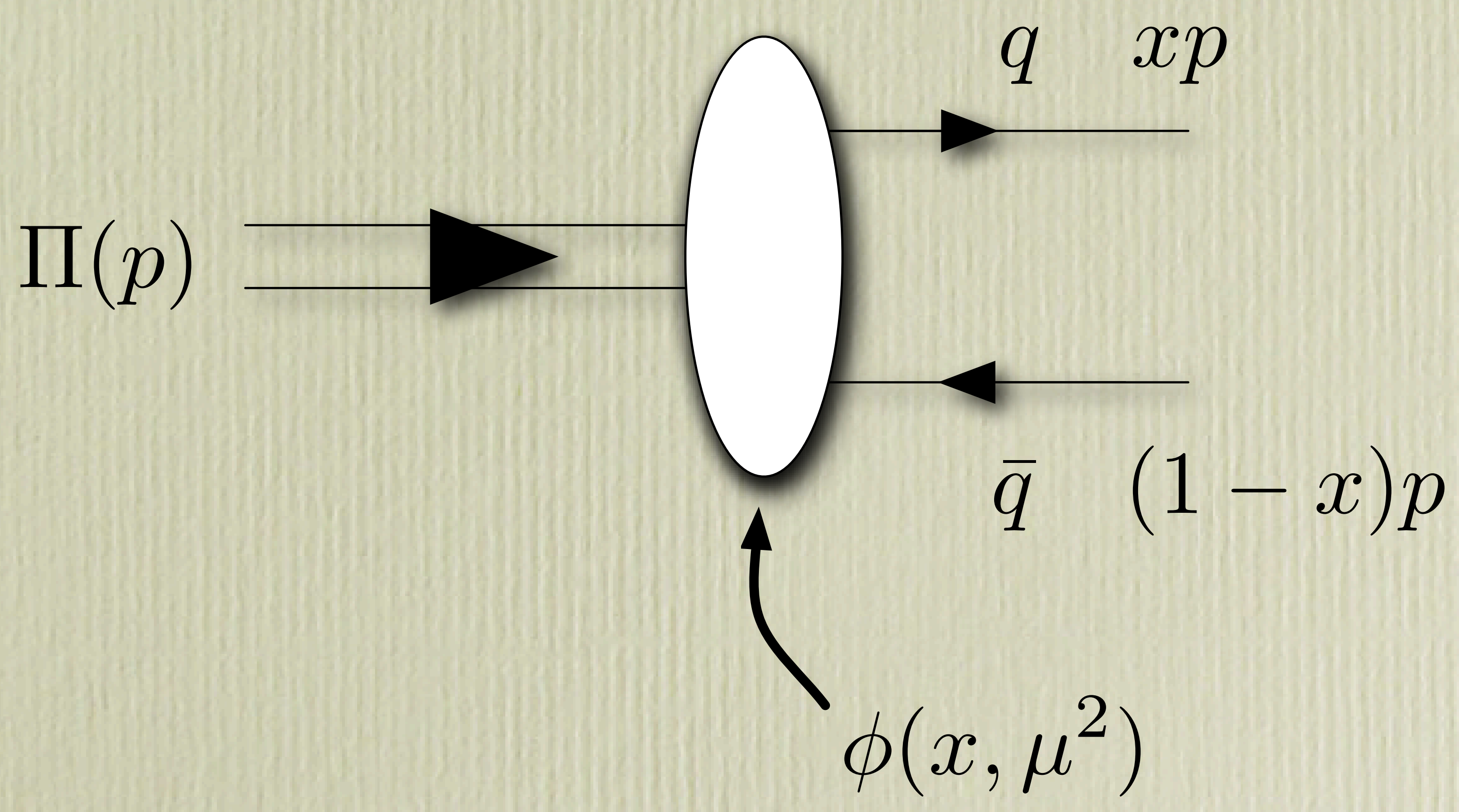
See also: Phys.Lett.B639:278

Phys.Rev.Lett. 96:052001 (2006)

Results for g_A

Experimental (neutron β decay)	$g_A = 1.2695(29)$
PRL 96:052001 (2006)	$g_A = 1.226(84)$
hep-lat/0603028	$g_A = 1.31(9)(7)$

Leading twist meson DAs



Scale dependence

$$\phi(x,\mu^2) \sim \int_{k_\perp^2<\mu^2} d^2k_\perp \phi(x,k_\perp)$$

- Process-independent, carries info on meson
- Scale dependence: ERBL-evolution

  Bethe-Salpeter equation

  ⇒ Eigenfunctions are Gegenbauer polynomials
  $$C_n^{3/2}(2x-1)$$

$$\phi(x,\mu^2) = 6x(1-x)\sum_{n=0}^{\infty} a_n(\mu^2) C_n^{3/2}(2x-1)$$

- For π, ρ, η, η' and Φ:

  G parity => $a_{odd} = 0$

  => mirror symmetry

- K, K*: $a_{odd} \neq 0$

$$\phi(x,\mu^2) = 6x(1-x)\sum^{\infty} a_n(\mu^2) C_n^{3/2}(2x-1)$$

- For π,

  G pari

  => mir

Goal:
Compute lowest moments
$a_1(\mu^2), a_2(\mu^2)$
for kaon and
$a_2(\mu^2)$
for pion

- K, K*: $a_{odd} \neq 0$

Lattice calculations

$$\langle \Omega | \mathcal{O}_{\{\mu_0 \ldots \mu_n\}} | \Pi(\vec{p}) \rangle = \mathrm{i}\, f_\Pi \, p_{\mu_0} \ldots p_{\mu_n} \langle \xi^n \rangle$$

$$x = \frac{1}{2}(1 + \xi)$$

- Local operator via light-cone OPE
- Yields moments of DAs w.r.t. *x*
- Matrix elements from ratios of two-point functions

Choice of operators

$$\mathcal{O}^a_{41} = \mathcal{O}_{\{41\}}, \qquad \vec{p} = (2\pi/L, 0, 0)$$
$$\mathcal{O}^b_{44} = (\mathcal{O}_{44} - 1/3(\mathcal{O}_{ii}))\,, \qquad \vec{p} = \vec{0}$$

$$\mathcal{O}^a_{412} = \mathcal{O}_{\{412\}}, \qquad \vec{p} = (2\pi/L, 2\pi/L, 0)$$
$$\mathcal{O}^b_{411} = (\mathcal{O}_{411} - 1/2(\mathcal{O}_{422} + \mathcal{O}_{433}))\,, \qquad \vec{p} = (2\pi/L, 0, 0)$$

Renormalization

$$\langle \xi^2 \rangle = \frac{Z^{\mathcal{S}}_{412}}{Z_{\mathcal{O}_4}} \langle \xi^2 \rangle^{\rm bare} + \frac{Z^{\mathcal{S}}_{\rm mix}}{Z_{\mathcal{O}_4}}$$

β	$Z^{\rm RGI}_{1a}$	$Z^{\rm RGI}_{1b}$	$Z^{\rm RGI}_{2a}$	$Z_{\mathcal{O}_4}$
5.20	1.52(4)	1.55(5)	2.4(1)	0.765(5)
5.25	1.52(4)	1.55(5)	2.4(1)	0.769(4)
5.29	1.54(4)	1.56(5)	2.45(10)	0.772(4)
5.40	1.57(3)	1.60(4)	2.5(1)	0.783(4)

β	$\mu^2 = 1/a^2$ [GeV^2]	$Z^{\overline{\rm MS}}_{\rm mix}$
5.20	5.3361	−0.00258
5.25	6.2001	−0.00253
5.29	6.9696	−0.00250
5.40	9.7344	−0.00240

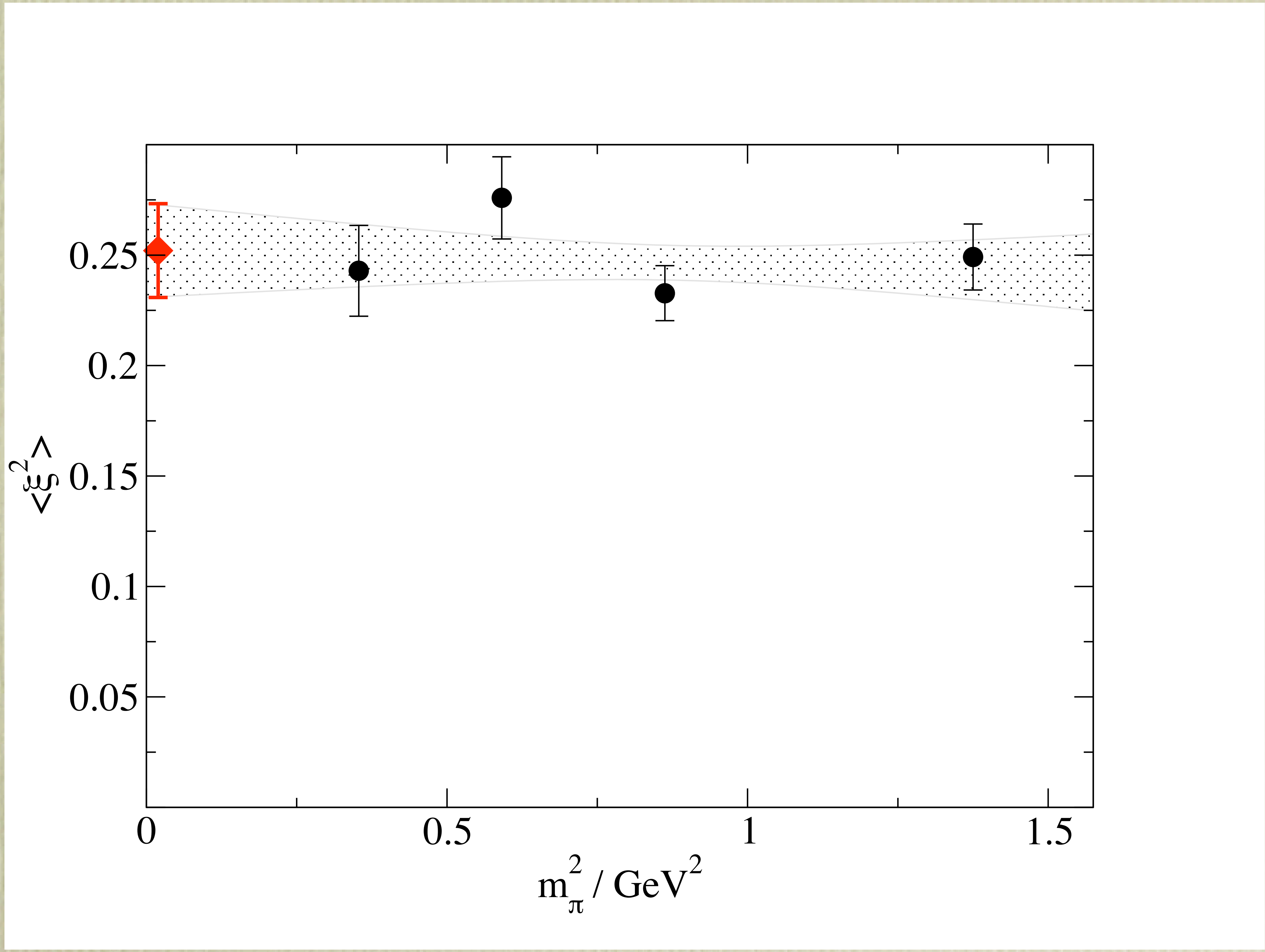
Changing schemes

β	$\mu^2 = (1/a)^2$ [GeV^2]	$Z_{2a}^{\overline{\rm MS}}/Z_{2a}^{\rm RGI}$
5.20	5.3361	0.5650
5.25	6.2001	0.5545
5.29	6.9696	0.5465
5.40	9.7344	0.5262

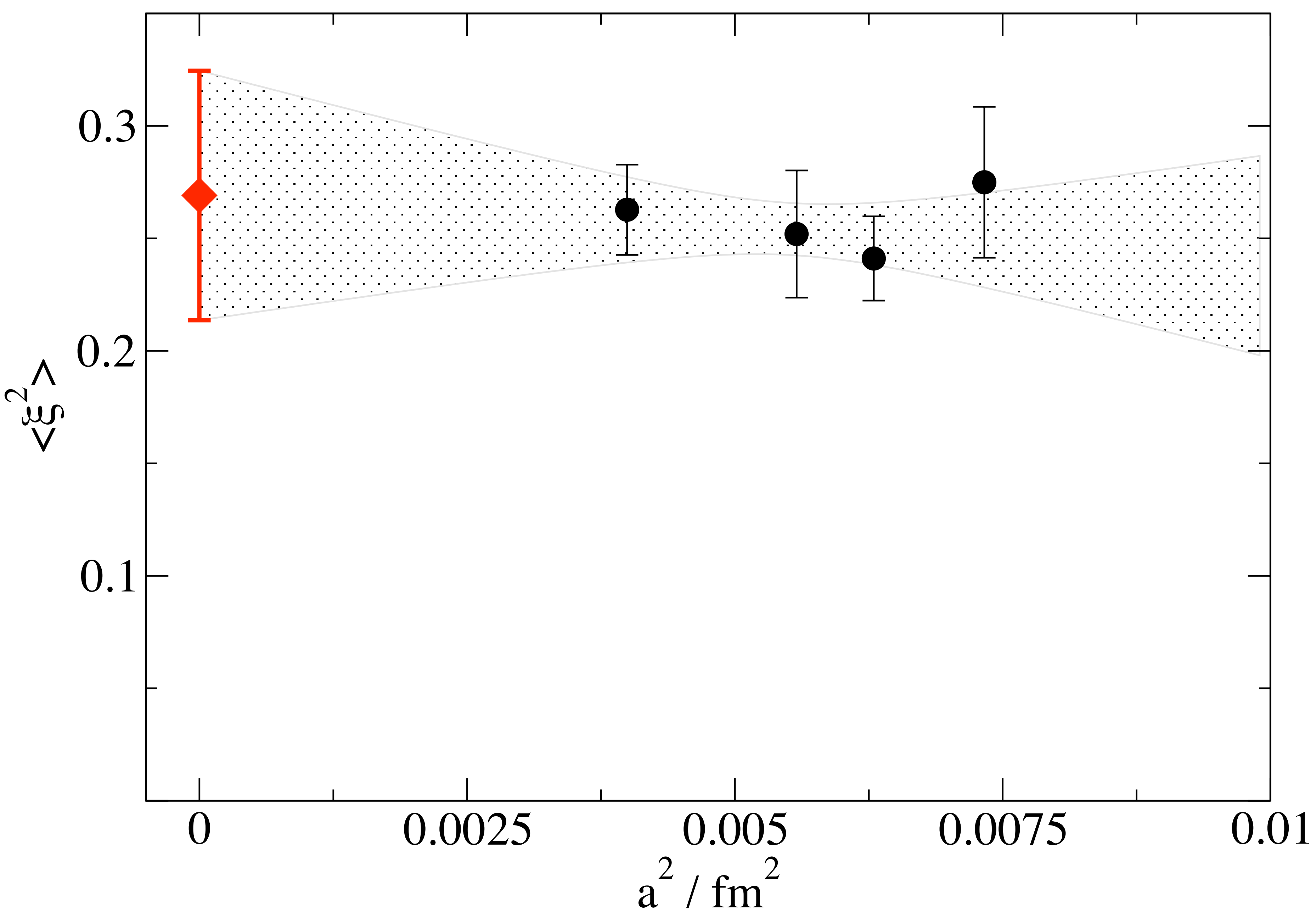
Working points

β	κ_{sea}	Volume	r_0/a	am_π
5.20	0.13420	$16^3 \times 32$	4.077(70)	0.5847(12)
5.20	0.13500	$16^3 \times 32$	4.754(45)	0.4148(13)
5.20	0.13550	$16^3 \times 32$	5.041(53)	0.2907(15)
5.25	0.13460	$16^3 \times 32$	4.737(50)	0.4932(10)
5.25	0.13520	$16^3 \times 32$	5.138(55)	0.3821(13)
5.25	0.13575	$24^3 \times 48$	5.532(40)	0.25556(55)
5.29	0.13400	$16^3 \times 32$	4.813(82)	0.5767(11)
5.29	0.13500	$16^3 \times 32$	5.227(75)	0.42057(92)
5.29	0.13550	$24^3 \times 48$	5.566(64)	0.32696(64)
5.29	0.13590	$24^3 \times 48$	5.840(70)	0.23956(71)
5.40	0.13500	$24^3 \times 48$	6.092(67)	0.40301(43)
5.40	0.13560	$24^3 \times 48$	6.381(53)	0.31232(67)
5.40	0.13610	$24^3 \times 48$	6.714(64)	0.22081(72)

Dynamical Clover, $n_f=2$



χPT: Chen et.al., PRL92:202001(2004)



0.3
0.2
0.1
$\langle \xi^2 \rangle$
0
0.0025
0.005
0.0075
0.01
a^2 / fm^2

Our result:

$$\langle \xi^2 \rangle_\pi (\mu^2 = 4\,\mathrm{GeV}^2) = 0.269(39)$$

$$a_2^\pi (4\,\mathrm{GeV}^2) = 0.201(114)$$

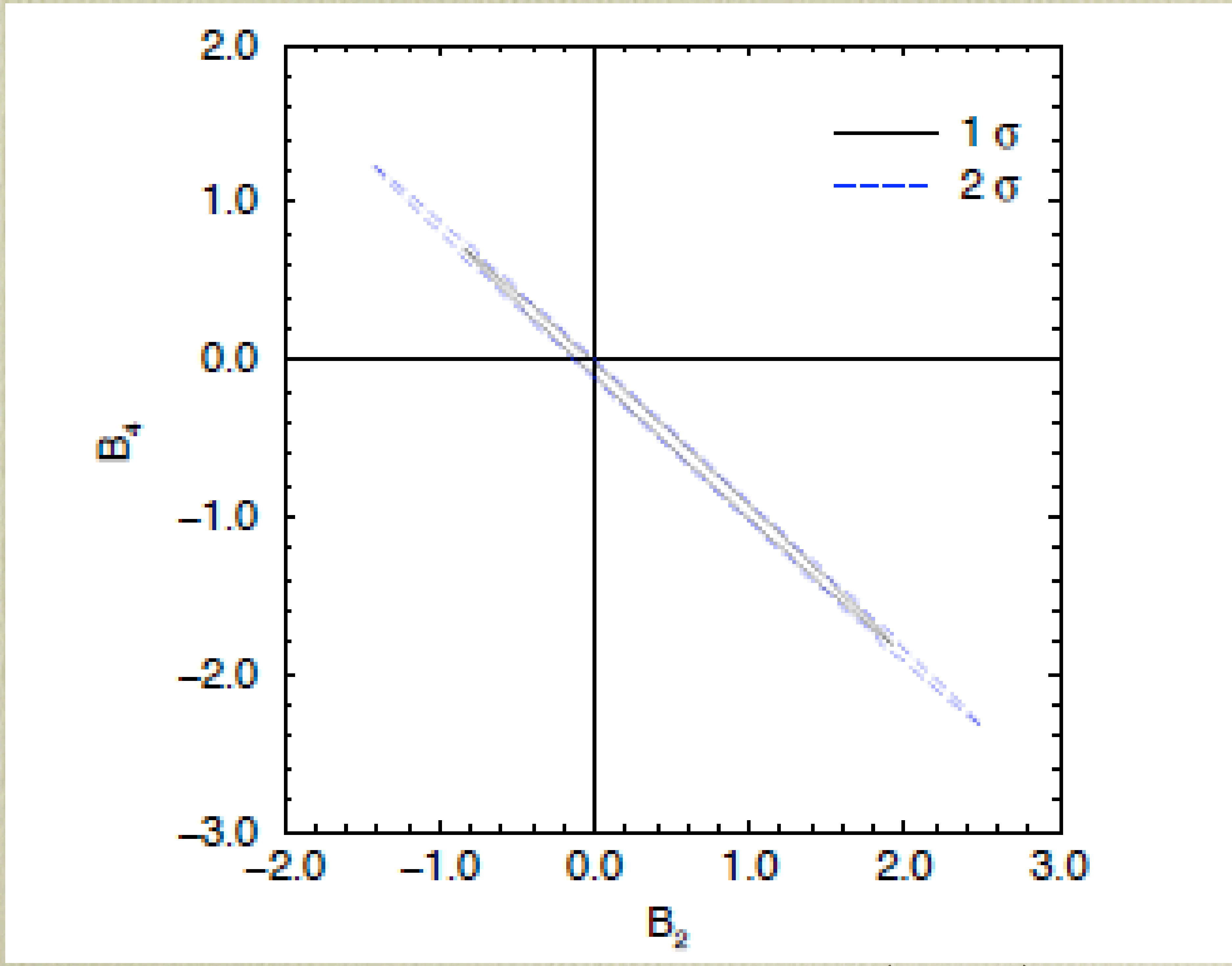
Compare to Del Debbio et.al., NPPS119:416(2003):

$$\langle \xi^2 \rangle_\pi (\mu^2 = 4\,\mathrm{GeV}^2) = 0.286(49)^{+0.030}_{-0.013}$$

Larger than asymptotic value:

$$\langle \xi^2 \rangle_\pi (\mu^2 \to \infty) = 0.2$$

$F_{\pi^0 \to \gamma^* \gamma}$ at leading twist



Diehl et.al., EPJC22:439(2001)

$F_{\pi^0 \to \gamma^* \gamma}$ various models

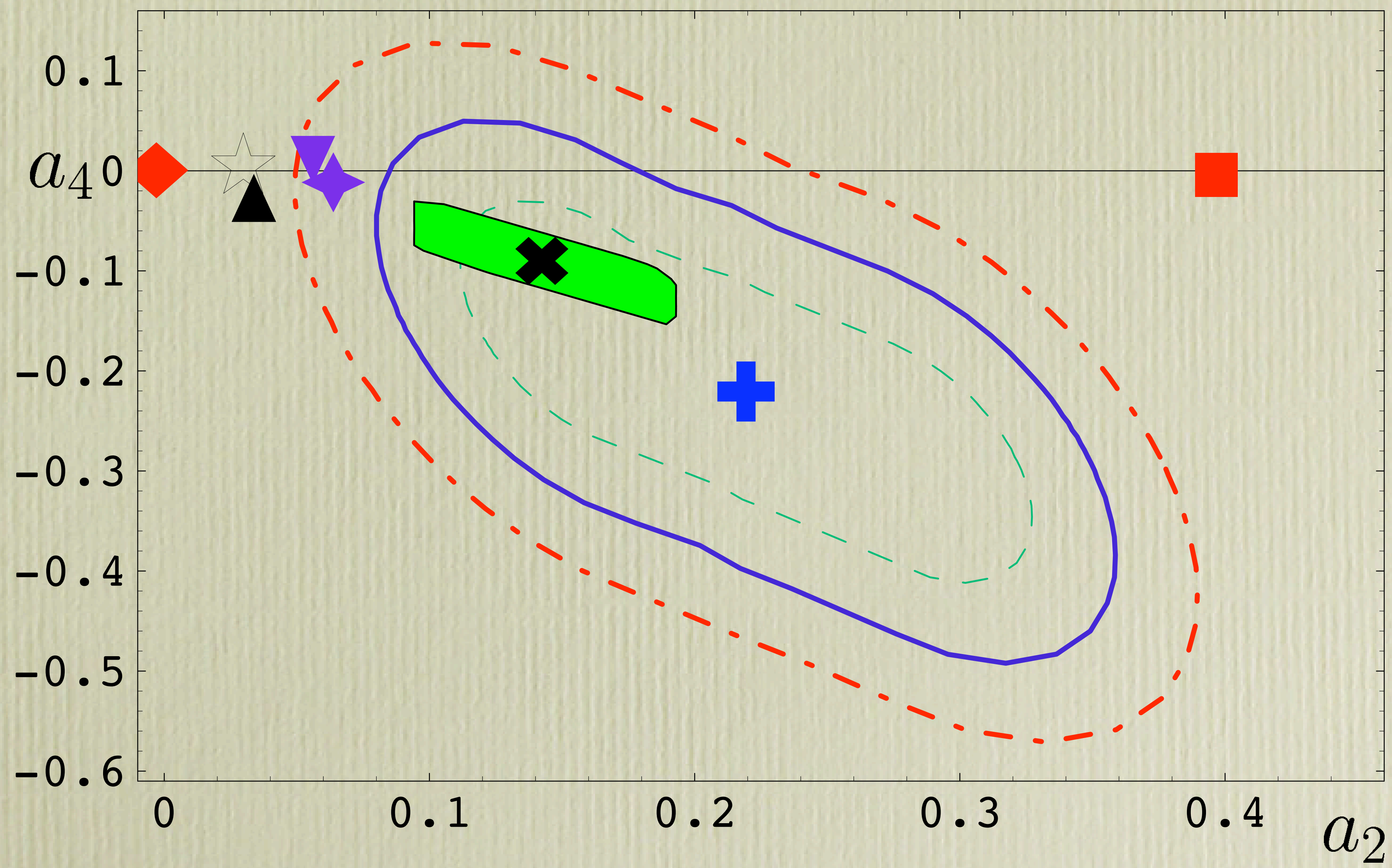


Figure from Bakulev et.al., PLB578:91(2004)

$F_{\pi 0 \to \gamma^* \gamma}$ various models

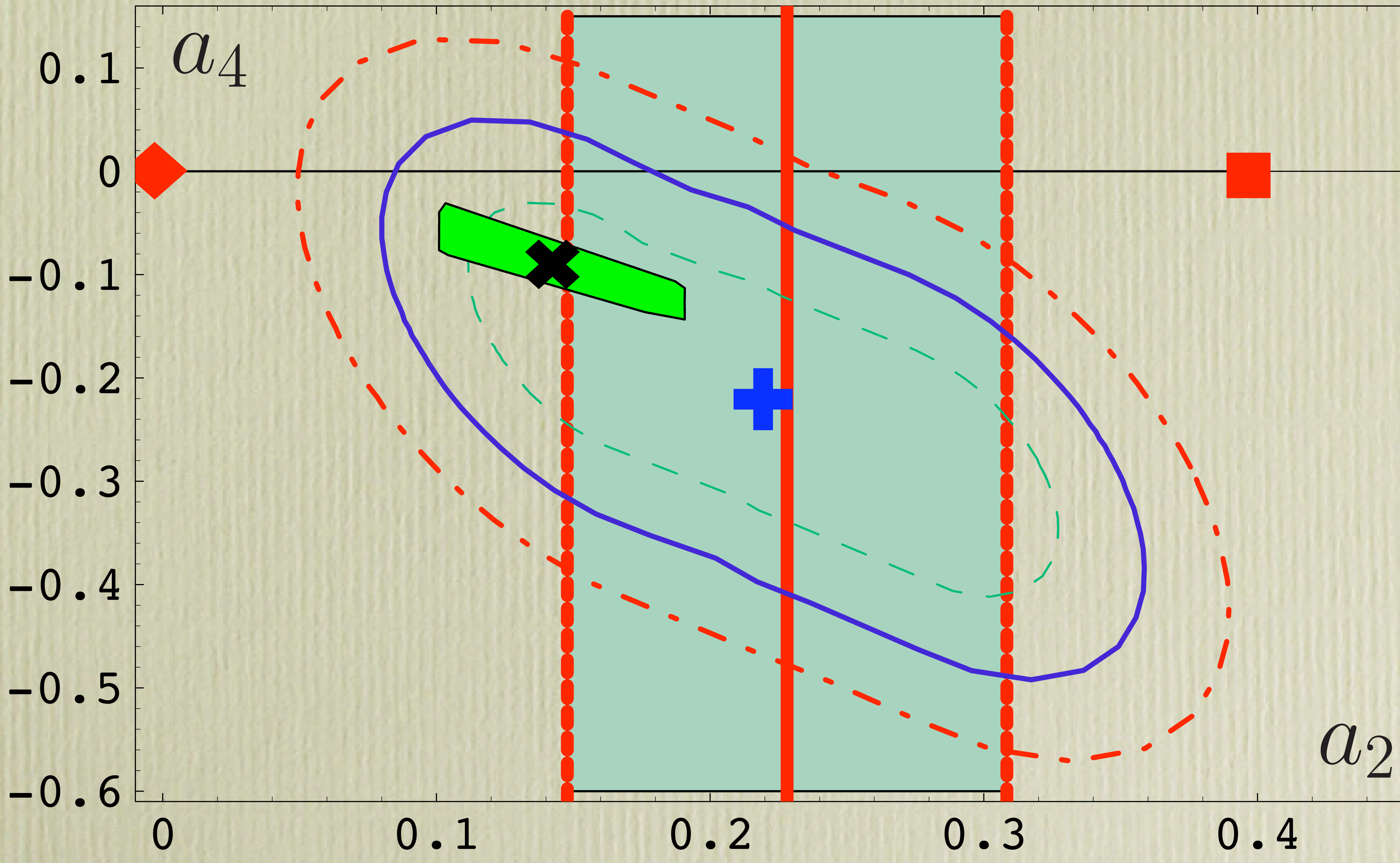


Figure from Bakulev et.al., PLB578:91(2004)

$F_{\pi 0 \to \gamma^* \gamma}$ various models

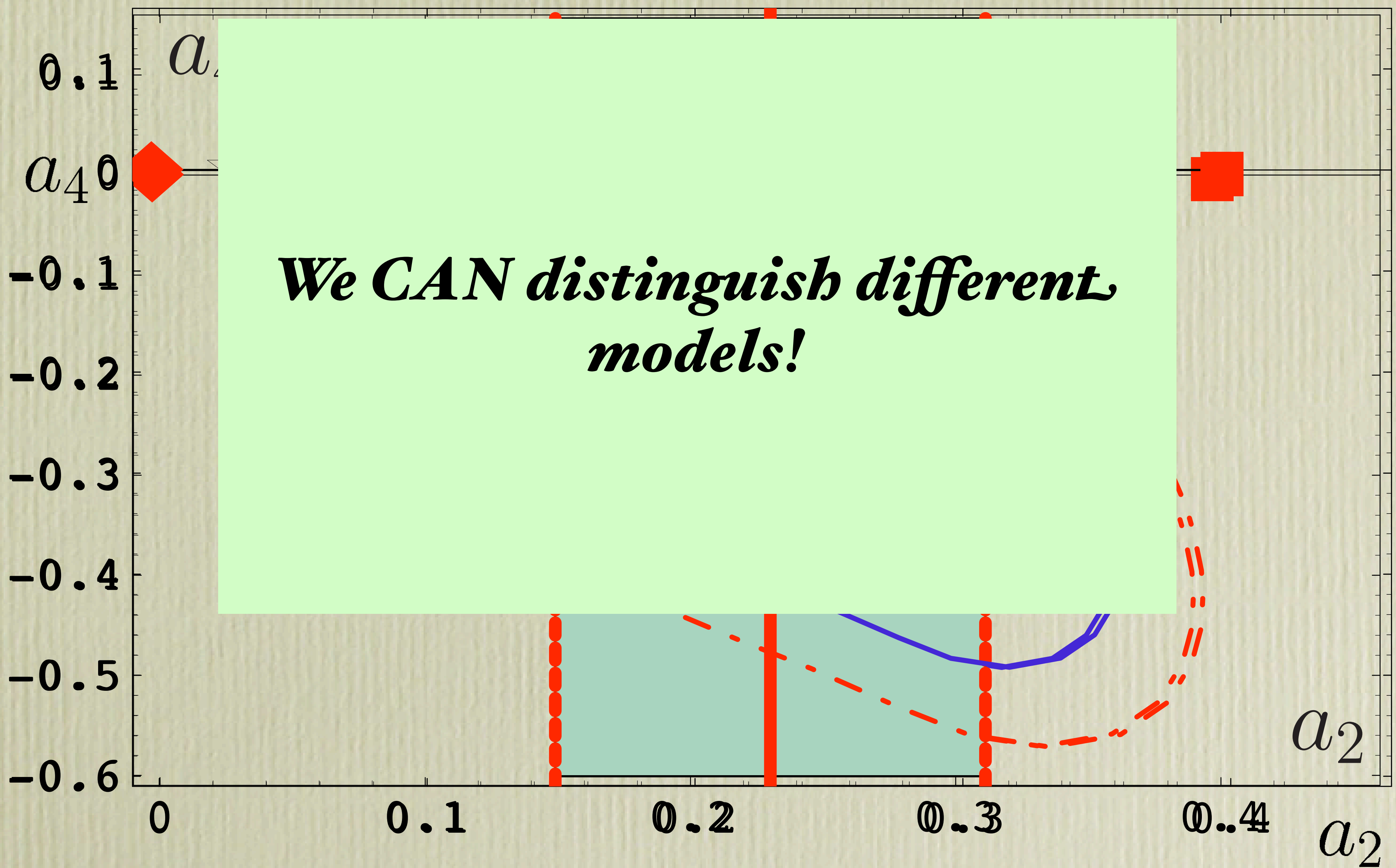
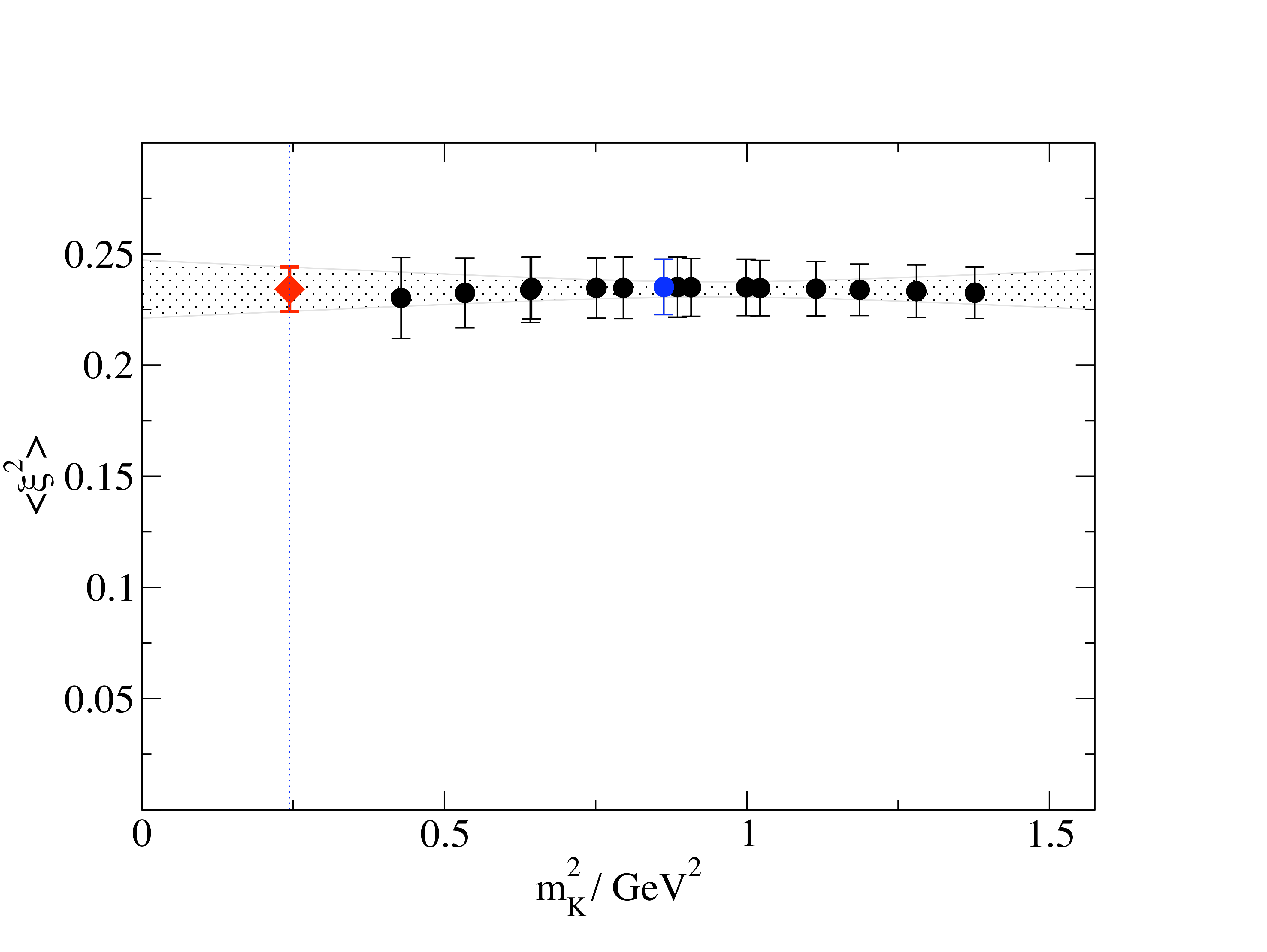


Figure from Bakulev et.al., PLB578:91(2004)

Mass non-deg. quarks

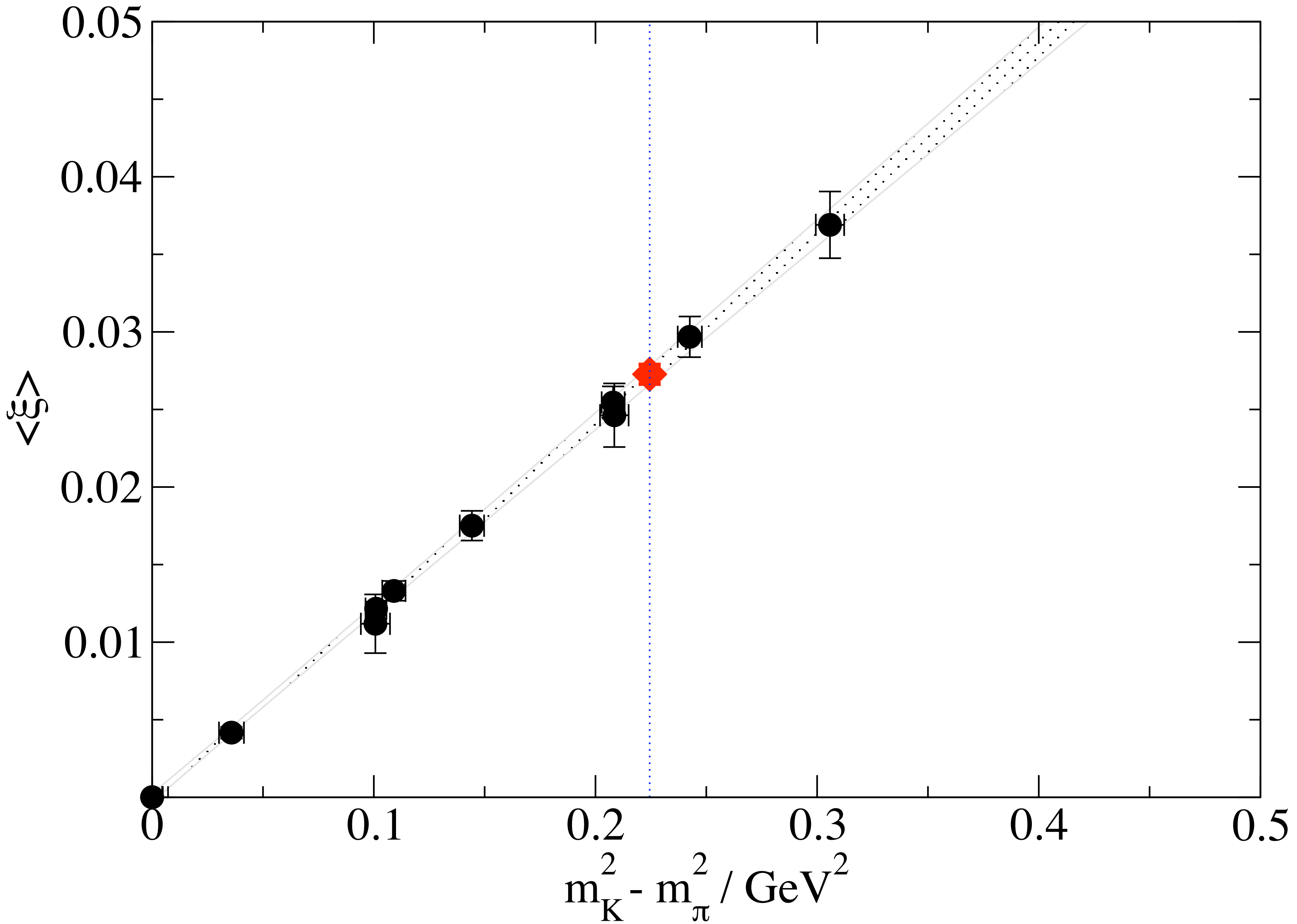


Averaging over 4 values of κ_{sea}:

$$\langle \xi^2 \rangle_K (\mu^2 = 4\,\mathrm{GeV}^2) = 0.260(6)$$

$$\langle \xi^2 \rangle_K / \langle \xi^2 \rangle_\pi \simeq 1$$

Chernyak&Zhitnisky:		Ball et.al. Khodjamirian et.al.:
$0.59(4)$	⟷	$\simeq 1$



0.05
0.04
0.03
0.02
0.01
⟨ξ⟩
0
0.1
0.2
0.3
0.4
0.5
$m_K^2 - m_\pi^2$ / GeV^2

Averaging over 4 values of κ_{sea}:

$$\langle \xi \rangle_K(\mu^2 = 4\,\mathrm{GeV}^2) = 0.0272(5)$$

$$a_1^K(4\,\mathrm{GeV}^2) = 0.0453(9)(29)$$

Recent controversy in literature, see Ball et.al., hep-lat/0603063:

$$a_1^K(4\,\mathrm{GeV}^2) = 0.05(25)$$

Compatible with hep-lat/0607018 (Next talk of A. Jüttner)

$$a_1^K(4\,\mathrm{GeV}^2) = 0.055(5)$$

Summary

- $a_2^\pi(4\,\mathrm{GeV}^2) = 0.201(114)$: larger than asymptotic values, can distinguish models
- $a_2^K(4\,\mathrm{GeV}^2) = 0.175(18)(47)$: about the same as a_2^π, also distinguishes models
- $a_1^K(4\,\mathrm{GeV}^2) = 0.0453(9)(29)$: compatible with sum-rule estimate

Outlook

- Lower pion masses (300 MeV and below)
- Improved chiral perturbation theory (J.W. Chen, private communication)
- Higher twist contributions
- Other mesons
- Nucleon - N. Warkentin @ Regensburg U.