LCSR with B-Meson DAs

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table of contents

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- get independent results from traditional LCSR for heavy-light form factors

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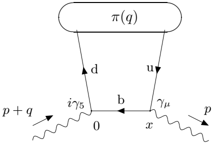
- Advantages:
  - use only B-Meson DAs instead of many different
  - SU(3)-breaking realized via threshold parameter, decay constant and strange quark mass
- Drawbacks:
  - shape of B-Meson DAs not very well known
  - even worse for three particle DAs

Sum Rules

- correlation function from traditional LCSR

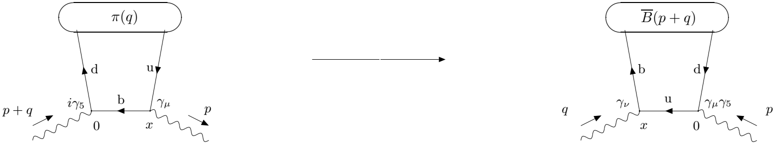
$$F_\mu(p,q) = i\int d^4x e^{-ipx}\langle\pi(q)|\mathrm{T}\{\overline{u}(x)\gamma_\mu b(x),\overline{b}(0)i\gamma_5 d(0)\}|0\rangle$$



Sum Rules

- 'inverted' correlation function

$$F_{\mu\nu}(p_B,q) = i\int d^4x e^{iqx}\langle 0|T\{\overline{d}(0)\gamma_\mu\gamma_5 u(0),\overline{u}(x)\gamma_\nu b(x)\}|\overline{B}(p_B)\rangle$$



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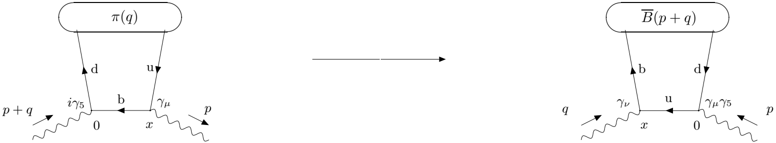

- check light cone dominance,
  $\Lambda \ll |p^2| \ll m_b^2,\ q^2 \le m_b^2 - 2*\lambda*m_b$

Two-particle contributions

- contract light quarks to free propagator

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- contract light quarks to free propagator
- use definition of B-Meson DA

Definition (B-Meson DA)

$$\langle 0|\overline{q}_\beta(z)\,b_\alpha(0)|\overline{B}(p)\rangle\Big|_{z^2=0} =: -\frac{if_B m_B}{4}\left[\frac{1+\not{v}}{2}\left\{2\tilde{\phi}_+^B(t) + \frac{\tilde{\phi}_-^B(t)-\tilde{\phi}_+^B(t)}{t}\not{z}\right\}\gamma_5\right]_{\alpha\beta}$$

Grozin/Neubert 96

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- sum rules in leading order

leading order sum rule

$f^+_{B\pi}(0)$-sum rule

$$f^+_{B\pi}(0) = \frac{f_B}{f_\pi} \int_0^{s_0^\pi/m_B} d\omega \, e^{-m_B \omega/M^2} \phi_-^B(\omega)$$

Comments

- distribution amplitude ϕ_-^B only for low ω needed
- WW-relation $\phi_-^B(0) = \lambda_B^{-1}$ can be used to constrain λ_B
- independent calculation by de Fazio, Feldmann and Hurth

leading order sum rule

approximation for $\frac{f_B}{f^+_{B\pi}\,\lambda_B}$

$$\frac{f^+_{B\pi}\,\lambda_B}{f_B} \approx \frac{1}{f_\pi}\int_0^{s_0^\pi/m_B} d\omega\, e^{m_B\,\omega/M^2}$$

Comments

- distribution amplitude ϕ^B_- only for low ω needed
- WW-relation $\phi^B_-(0) = \lambda_B^{-1}$ can be used to constrain λ_B
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leading order sum rule

$T_1^{B\to\rho}(0)$-sum rule

$$T_1^{B\to\rho}(0) = \frac{f_B}{2\, f_\rho\, m_\rho} e^{m_\rho^2/M^2} \int_0^{s_0^\rho/m_B} d\omega e^{-m_B\omega/M^2} \phi_+^B(\omega)$$

Comments

- distribution amplitude ϕ_-^B only for low ω needed
- WW-relation $\phi_-^B(0) = \lambda_B^{-1}$ can be used to constrain λ_B
- independent calculation by de Fazio, Feldmann and Hurth
- for vector form factors ϕ_+^B at low ω needed

Results to leading order

- universal form factors

Universal Form Factors

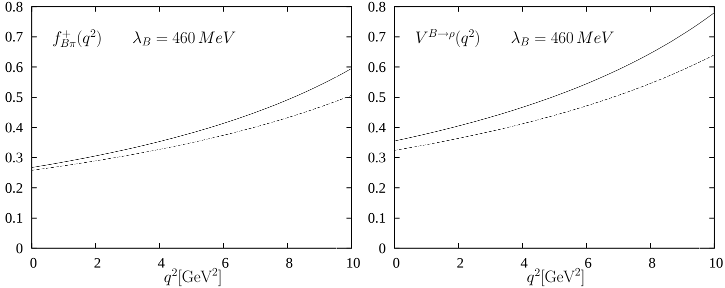
$$\xi(0) = \frac{f_B}{f_P m_B} e^{m_P^2/M^2} \int_0^{s_0^P} ds\, e^{-s/M^2} \phi_-^B(0)$$

$$\xi_\perp(0) = \frac{f_B}{2 m_B f_V m_V} e^{m_V^2/M^2} \int_0^{s_0^V} ds\, s\, e^{-s/M^2} \frac{d}{d\omega} \phi_+^B(\omega)\Big|_{\omega=0}$$

$$\xi_\parallel(0) = \frac{f_B}{m_B f_V m_V^2} e^{m_V^2/M^2} \int_0^{s_0^V} ds\, s\, e^{-s/M^2} \phi_-^B(0)$$

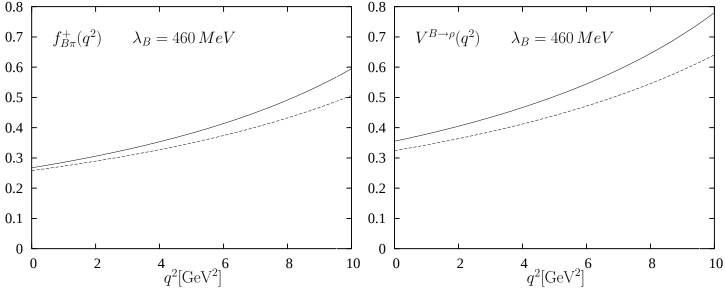
- for vector form factors there exist universal three particle contributions

Numerics in leading order



- slope of new sum rules is steeper

Numerics in leading order



- slope of new sum rules is steeper
- vector form factors in general larger than traditional results

Numerics in leading order(2)

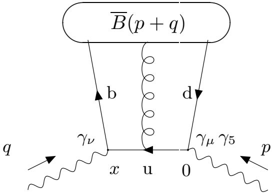
- our numerical results compared to traditional approach

form factor	this work	LCSR with light-meson DA's	ref.
$f^+_{B\pi}(0)$	0.25±0.05	0.258± 0.03	[BZ]
$f^+_{BK}(0)$	0.32 ±0.08	0.301± 0.041	[BZ]
$f^T_{B\pi}(0)$	0.23 ±0.04	0.253± 0.028	[BZ]
$f^T_{BK}(0)$	0.30 ±0.08	0.328± 0.04	[BZ]
$V^{B\rho}(0)$	0.37 ±0.10	0.323± 0.029	[BZ]
$V^{BK^*}(0)$	0.43 ±0.12	0.411± 0.033	[BZ]
$A_1^{B\rho}(0)$	0.27 ±0.08	0.242± 0.024	[BZ]
$A_1^{BK^*}(0)$	0.32 ±0.09	0.292± 0.028	[BZ]
$A_2^{B\rho}(0)$	0.24 ±0.08	0.221±0.023	[BZ]
$A_2^{BK^*}(0)$	0.30 ±0.10	0.259 ± 0.027	[BZ]
$T_1^{B\rho}(0)$	0.31 ±0.09	0.267 ± 0.021	[BZ]
$T_1^{BK^*}(0)$	0.37 ±0.10	0.333±0.028	[BZ]

- uncertainties in λ_B, f_π, f_k, M^2, m_s, f_B, λ_E included

Soft gluon corrections

- expand propagator in background field



Soft gluon corrections

- expand propagator in background field
- use definition of three-particle distribution amplitudes

Definition (Three Particle DAs)

$$\langle 0|\bar{q}_{2\alpha}(x)G_{\lambda\rho}(ux)x^\rho b_\beta(0)|\bar{B}^0(v)\rangle = \frac{f_B m_B}{4}\int_0^\infty d\omega \int_0^\infty d\xi\, e^{-i(\omega+u\xi)v\cdot x}$$

$$\times\Bigg[(1+\not{v})\Bigg\{(v_\lambda\gamma_\rho - v_\rho\gamma_\lambda)\Big(\Psi_A(\omega,\xi)-\Psi_V(\omega,\xi)\Big) - i\sigma_{\lambda\rho}\Psi_V(\omega,\xi)$$

$$-\left(\frac{x_\lambda v_\rho - x_\rho v_\lambda}{v\cdot x}\right)X_A(\omega,\xi) + \left(\frac{x_\lambda\gamma_\rho - x_\rho\gamma_\lambda}{v\cdot x}\right)Y_A(\omega,\xi)\Bigg\}\gamma_5\Bigg]_{\beta\alpha}$$

Kawamura et al.

Generic result(1)

- generic formula for three-particle contributions

Three-particle contributions

$$\Delta F(q^2, s_0, M^2) = \int_0^{\sigma_0(q^2,s_0)} d\sigma \exp\left(\frac{-\sigma m_B^2 - (m^2 - \sigma q^2)/\bar{\sigma} + m_{P(V)}^2}{M^2}\right)$$
$$\times \left(-I_1^{(F)}(\sigma) + \frac{I_2^{(F)}(\sigma)}{M^2} - \frac{I_3^{(F)}(\sigma)}{2M^4}\right)$$
$$+ \frac{e^{-(s_0 - m_{P(V)}^2)/M^2}}{m_B^2}\left\{\eta(\sigma)\left[I_2^{(F)}(\sigma) - \frac{1}{2}\left(\frac{1}{M^2} + \frac{1}{m_B^2}\frac{d\eta(\sigma)}{d\sigma}\right) I_3^{(F)}(\sigma)\right.\right.$$
$$\left.\left.\left. - \frac{\eta(\sigma)}{2m_B^2}\frac{dI_3^{(F)}(\sigma)}{d\sigma}\right]\right\}\right|_{\sigma=\sigma_0}$$

Generic result(2)

- integrals I_n for different powers of propagator denominators

Integrals I_n

$$I_n^{(F)}(\sigma) = \frac{1}{\bar{\sigma}^n} \int_0^{m_B\sigma} d\omega \int_{m_B\sigma-\omega}^{\infty} \frac{d\xi}{\xi} \Big[C_n^{(F,\Psi A)}(\sigma,u,q^2)\Psi_A^B(\omega,\xi) + C_n^{(F,\Psi V)}(\sigma,u,q^2)\Psi_V^B(\omega,\xi) + C_n^{(F,XA)}(\sigma,u,q^2)\overline{X}_A^B(\omega,\xi) + C_n^{(F,YA)}(\sigma,u,q^2)\overline{Y}_A^B(\omega,\xi) \Big]\Big|_{u=\frac{(m_B\sigma-\omega)}{\xi}}$$

Three-particle distribution amplitudes(1)

Known information

- equation of motion constraints

Three-particle distribution amplitudes(1)

Known information

- equation of motion constraints
- normalization of three-particle distribution amplitudes

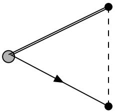
Normalization

$$
\begin{aligned}
\int_0^\infty d\omega \int_0^\infty d\xi\, \Psi_A(\omega,\xi) &= \frac{\lambda_E^2}{3} \\
\int_0^\infty d\omega \int_0^\infty d\xi\, \Psi_V(\omega,\xi) &= \frac{\lambda_H^2}{3} \\
\int_0^\infty d\omega \int_0^\infty d\xi\, X_A(\omega,\xi) &= \int_0^\infty d\omega \int_0^\infty d\xi\, Y_A(\omega,\xi) = 0
\end{aligned}
$$

Three-particle distribution amplitudes(2)

What to do

- shape of three-particle distribution amplitudes unknown
- following Grozin/Neubert and Braun/Ivanov/Korchemsky



two-particle distribution amplitudes:

$$\phi_+^B(\omega) \sim \omega\Theta(s_0 - \omega)$$
$$\phi_-^B(\omega) \sim \Theta(s_0 - \omega)$$

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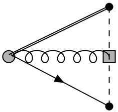


three-particle distribution amplitudes:

$$
\begin{aligned}
\Psi_A(\omega,\xi) &\sim C\,\xi^2\Theta(2s_0 - (\omega+\xi)) \\
\Psi_V(\omega,\xi) &\sim C\,\xi^2\Theta(2s_0 - (\omega+\xi)) \\
X_A(\omega,\xi) &\sim C\,\xi(2\omega-\xi)\Theta(2s_0 - (\omega+\xi)) \\
Y_A(\omega,\xi) &\sim -C/4\,\xi\Theta(2s_0 - (\omega+\xi))
\end{aligned}
$$

Models

- two different models

Model 1 (Local Duality)

$$\begin{aligned}
\Psi_A^{LD}(\omega,\xi) &= \Psi_V^{LD}(\omega,\xi) = \\
&\left(\frac{35\lambda_E^2}{4\tilde{s}_0^4}\right)\xi^2\left(1-\frac{\omega+\xi}{2\tilde{s}_0}\right)^3\Theta(2\tilde{s}_0-\omega-\xi)\,, \\
X_A^{LD}(\omega,\xi) &= \left(\frac{35\lambda_E^2}{4\tilde{s}_0^4}\right)\xi(2\omega-\xi)\left(1-\frac{\omega+\xi}{2\tilde{s}_0}\right)^3\Theta(2\tilde{s}_0-\omega-\xi)\,, \\
Y_A^{LD}(\omega,\xi) &= -\left(\frac{35\lambda_E^2}{16\tilde{s}_0^4}\right)\xi\left(1-\frac{\omega+\xi}{2\tilde{s}_0}\right)^3 \\
&(2\tilde{s}_0-13\omega+3\xi)\Theta(2\tilde{s}_0-\omega-\xi)\,.
\end{aligned}$$

Models

- two different models

Model 2 (Exponential)

$$\Psi_A(\omega,\xi) = \Psi_V(\omega,\xi) = \frac{\lambda_E^2}{6\omega_0^4}\,\xi^2 e^{-(\omega+\xi)/\omega_0}\,,$$

$$X_A(\omega,\xi) = \frac{\lambda_E^2}{6\omega_0^4}\,\xi(2\omega-\xi)\,e^{-(\omega+\xi)/\omega_0}\,,$$

$$Y_A(\omega,\xi) = -\frac{\lambda_E^2}{24\omega_0^4}\,\xi(7\omega_0-13\omega+3\xi)e^{-(\omega+\xi)/\omega_0}$$

Equations of motion

- three-particle states affect two-particle distribution amplitudes via equation of motions

Equations of motion

$$\begin{aligned} \omega \frac{d\phi_-^B(\omega)}{d\omega} + \phi_+^B(\omega) &= I(\omega)\,, \\ \left(\omega - 2\bar{\Lambda}\right) \phi_+^B(\omega) + \omega\phi_-^B(\omega) &= J(\omega)\,, \end{aligned}$$

Kawamura et al.

- $I(\omega)$, $J(\omega)$: three-particle 'source terms'

Equations of motion

- solutions were obtained by Kawamura et al. in 2001

Solutions

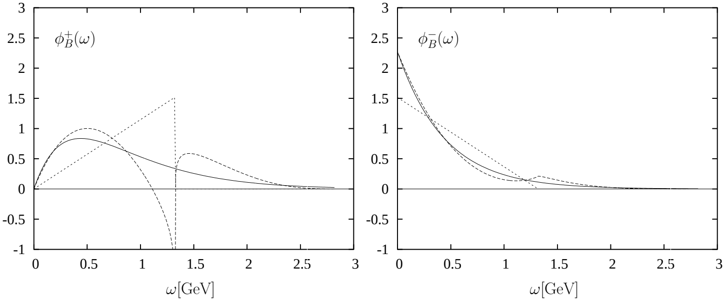
$$\phi_+^B(\omega) = \phi_+^{B,WW}(\omega) + \frac{\omega}{2\bar{\Lambda}}\Phi(\omega)\,,$$
$$\phi_-^B(\omega) = \phi_-^{B,WW}(\omega) + \frac{2\bar{\Lambda} - \omega}{2\bar{\Lambda}}\Phi(\omega) + \frac{J(\omega)}{\omega}\,,$$

WW-Part

$$\phi_+^{(B,WW)}(\omega) = \frac{\omega}{2\bar{\Lambda}^2}\Theta(2\bar{\Lambda} - \omega) \qquad \phi_-^{(B,WW)}(\omega) = \frac{2\bar{\Lambda} - \omega}{2\bar{\Lambda}^2}\Theta(2\bar{\Lambda} - \omega)$$

Equations of motion

- using our models to calculate corrections



- our two models nearly indistinguishable at small ω for same moments

Some further comments

- for our models $I(\omega) = 0$, $J(\omega = 0) = 0$

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- old model by Grozin/Neubert

Grozin/Neubert model

$$\phi_+^B(\omega) = \frac{\omega}{\omega_0} e^{\frac{\omega}{\omega_0}}, \qquad \phi_-^B(\omega) = \frac{1}{\omega_0} e^{\frac{\omega}{\omega_0}}$$

sets $\lambda_E^2 = \lambda_H^2 = \frac{2}{3}\bar{\Lambda}^2 = \frac{3}{2}\lambda_B^2$

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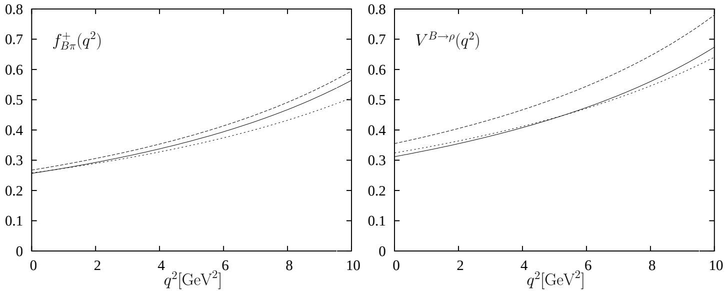
- fixing moments of our exponential model to this values $\longrightarrow$ Grozin/Neubert model
- these two models give consistent set of DAs

Numerics

- all numerics calculated with the exponential model



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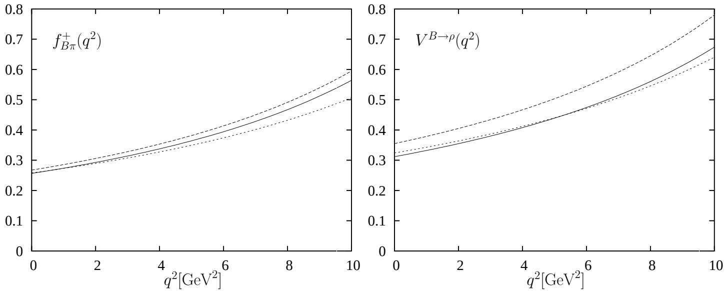

Comparison

- good agreement with Ball/Zwicky fit for low q^2
- agreement lost at $q^2 \sim 6 - 8\,Gev^2$

Numerics(2)

- our preliminary results

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- full analysis until now only for $f^+_{B\pi}$ and $T_1^{B\to V}$

SU(3)-breaking

SU(3)-breaking-relations

$$\frac{T_1^{B\to K^*}(0)}{T_1^{B\to\rho}(0)} = 1.22 \pm 0.13$$

$$\frac{f^+_{B\to K}(0)}{f^+_{B\to\pi}(0)} = 1.29 \pm 0.12$$

- uncertainties cancel to a large amount
- results agree with recent calculations by Ball/Zwicky and depend only weak on λ_B

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- SU(3) breaking effects far less sensitive to λ_B and f_B, already $\sim 10\%$ accuracy
- inclusion of m_s brings one additional three particle DA we neglected into the game (Geyer, Witzel 06)

Backupslide(1)

$\xi_\perp$ with 3-particle DAs

$$\xi_\perp(0) = \frac{f_B}{2m_B f_V m_V} e^{m_V^2/M^2} \int_0^{s_0^V} ds \left(s\, e^{-s/M^2} \frac{d}{d\omega}\phi_+^B(\omega)\Big|_{\omega=0} - \int_0^\infty \frac{d\xi}{\xi}\left(\Psi_A(0,\xi) + \Psi_V(0,\xi) + X_A(0,\xi)\right)\right)$$

In our models

$$-\int_0^\infty \frac{d\xi}{\xi}\left(\Psi_A(0,\xi) + X_A(0,\xi)\right) = \frac{1}{2}J(0) = 0$$

- in our model, only Ψ_V gives contribution

Sum rule example

Sum rule for f^+_{BP}

$$f^+_{BP}(q^2) = \frac{f_B m_B}{\text{fP}} \left\{ \int\limits_0^{\sigma_0(q^2,\text{s0})} d\sigma \exp\left(\frac{-\sigma m_B^2 - (m^2 - \sigma q^2)/(1-\sigma) + m_P^2}{M^2} \right) \right.$$
$$\times \left[\frac{(1-\sigma)^2 m_B^2}{(1-\sigma)^2 m_B^2 + m^2 - q^2} \phi_-^B(m_B\sigma) + \left(1 - \frac{(1-\sigma)^2 m_B^2}{(1-\sigma)^2 m_B^2 + m^2 - q^2} \right) \phi_+^B(m_B\sigma) + \right.$$
$$\left. \left. + \frac{2(1-\sigma)\left(m^2 - q^2\right) m_B}{\left((1-\sigma)^2 m_B^2 + m^2 - q^2\right)^2} \Phi_\pm^B(m_B\sigma) \right] + \Delta f^+_{BP}(q^2, s_0, M^2) \right\},$$

$$\Phi^B_{pm}(m_B\sigma) = \int_0^{m_B\sigma} d\rho (\phi_+^B(\rho) - \phi_-^B(\rho))$$

Δf^+_{BP}: Three-particle contributions