Ideas how to calculate the 1st inverse moment of the B-DA on the Lattice

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Why I am giving this talk?

This year there have been two papers (eg. hep-lat/0607018) on computing the first moment of the kaons distribution function ($\langle\xi\rangle$) using lattice QCD.

$$\langle\, 0 \,|\, \bar{q}(0)\, \gamma_\rho \gamma_5\, D_\mu\, s(0) \,|\, K(p) \,\rangle = \langle\xi\rangle\, f_K\, p_\rho\, p_\mu$$

I am not aware of any work on the distribution amplitude of B mesons from the lattice. (see remarks at the end of hep-ph/0407221, Hashimoto and Onogi). Computing something similar to the above equation for a B meson should be possible ....

Just for completeness....

From hep-ph/0407221, for $B \to \pi\pi$

$$\langle \ \pi(p')\pi(q) \mid Q_i \mid B(p)\rangle = f^{B\to\pi}(q^2) \int_0^1 du T_i^I(u)\Phi$$

$$+ \int_0^1 d\xi du dv T_i^{II}(\xi,u,v)\Phi_B(\xi)\Phi_\pi(u)\Phi_\pi(v)$$

where Q_i is a four fermion operator and T^I and T^{II} are perturbative. This type of analysis is a major industry in B physics.
The $\Phi_B(\xi)$ is the light cone distribution of the B meson, that lattice QCD could (perhaps) compute.

What is required?

From the review article hep-ph/0606226 by Grozin. The first moments are related to Λ (HQET parameter, that loosely is the binding energy).

$$\langle 0 \mid \overline{Q} E \alpha q \mid B \rangle = -iF\lambda_E^2 u$$

where F is the decay constant and $E = i[D_0, D]$.

$$\langle 0 \mid \overline{Q} H.\sigma q \mid B \rangle = -iF\lambda_H^2 u$$

$H = iD \times D$, (D are covariant derivatives).
May be able to calculate λ_H and λ_E. Note, I don't see these parameters in parameterisations of B meson wave-function.

NRQCD

This formalism has been used with Asqtad sea quarks in the B and Upsilon systems.

$$S_{NRQCD} = \sum_x [\phi_t^\dagger \phi_t - \phi_t^\dagger (1 - \frac{a\delta H}{2})_t (1 - \frac{aH_0}{2n})_t^n$$

$$U_t^\dagger (t-1)(1 - \frac{aH_0}{2n})_{t-1}^n (1 - \frac{a\delta H}{2})_{t-1} \phi_{t-}$$

$$aH_0 = \frac{\Delta^2}{2aM_0}$$

These are other possible formalisms (FNAL, Sommer et al., blocking) for the b quark, but they would require a lot more development, and validation.

B meson moments are not needed

Unfortunately, as reviewed by Vladimir Braun at this meeting, the moments of B meson wave function can not simply be related to the light cone distribution of the B meson that is required in B meson studies. This means that computing multiple derivatives in B meson matrix elements is not useful for phenomenology. What the continuum people want is an estimate of the inverse moment of the B meson light cone wave function.

The first inverse moment

From Lee and Neubert, hep-ph/0509350

$$\lambda_B^{-1} = \int_0^\infty d\omega \frac{\phi_+^B(\omega,\mu)}{\omega}$$

$$\sigma_B \lambda_B^{-1} = \int_0^\infty d\omega \frac{\phi_+^B(\omega,\mu)}{\omega} \log\frac{\omega}{\mu}$$

where $\phi_+^B(\omega,\mu)$ is light cone wave function.
The parameters λ_B and σ_B are required in the analysis of B decays. For example λ_B is used by Ball and Zwicky (hep-ph/0603232) to estimate 4 fermion contribution to $B \to \rho\gamma$.

λ_B

Ball and Kou (hep-ph/0301135) computed

$$\lambda_B = 0.6 \ \text{GeV}$$

from form factors for $B \to \gamma e \nu$. After the discussions yesterday, perhaps lattice can compute the form factors for $B \to \gamma e \nu$:

$$F^{hard}_{A,HQL}(E_\gamma) = \frac{f_B m_B Q_u}{2E_\gamma} \frac{1}{\lambda_B}$$

(+ radiative corrections ?) and then extract λ_B. There have been some recent papers on computing meson decays to two photons using lattice QCD (eg $\eta_c \to \gamma\gamma$) by Dudek and Edwards (hep-ph/0607140).

A formalism is proposed

Aglietti et al., hep-ph/9806277 proposed (but didn't test) a formalism for extracting meson light-cone wave function from lattice QCD.
Based on lattice correlators to extract $F^\mu(p_\pi, q) \equiv$

$$\int d^4x e^{iqx} \langle \pi^-(p_\pi) \mid \overline{d}(x)\gamma^\mu\gamma_5 S(x;0)u(0) \mid 0\rangle$$

where $S(x;0)$ is a scalar Feynman propagator

$$-D^2 S(x;0) = \delta^4(x-0)$$

$$F^\mu(p_\pi, q) = f_\pi p_\pi^\mu \int_0^1 du \frac{\Phi_\pi(u)}{(q+p_\pi)^2 + 2(q+p_\pi).k + i\epsilon}$$

Unfold the integral to get the $\Phi_\pi(u)$ pion light cone

The method is tested...

The method proposed by Aglietti et al., was tested by Orsay lattice group. Abade et al. PRD64, 074511, 2001 Title:

`Preliminaries on a lattice analysis of the pion light cone wave function A partonic signal`

They didn't actually obtain the light cone wave-function of the pion. There were constraints from $p = \frac{2\pi}{L}$, so worked at $u = 1/2$, where u is fraction of momentum carried by quark.

$$\Phi_\pi^{CZ} = 120u(1-u)(u-0.5)^2$$

Some things we can calculate ...

Lattice QCD can be used to study other distributions inside the B meson, probably not useful because they are not light cone distributions, but just in case....

B meson wave functions

In the past people have extracted "wave functions" in coulomb gauge from lattice QCD.

$$\sum_{\underline{x}} \langle 0 \mid \bar{b}(\underline{x}+\underline{r})\Gamma q(\underline{x}) \mid B \rangle$$

From hep-lat/9411087, Draper and McNeile.

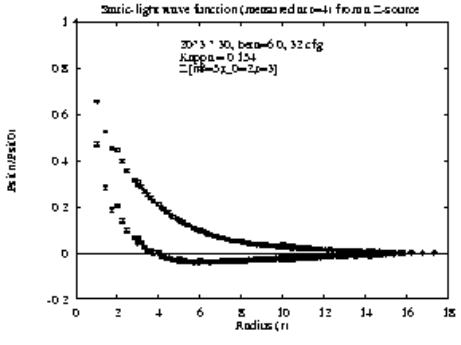
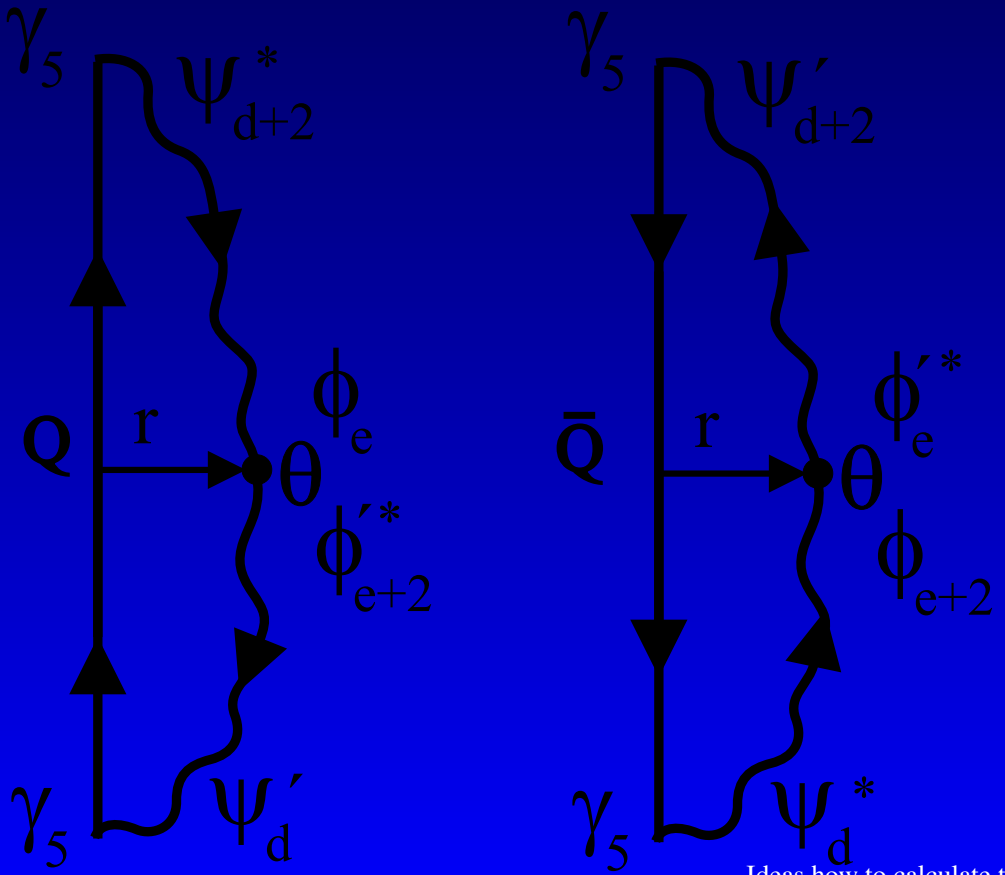


Figure 4. 1S and 2S wave functions normalized to unity at the origin for a static-light meson.

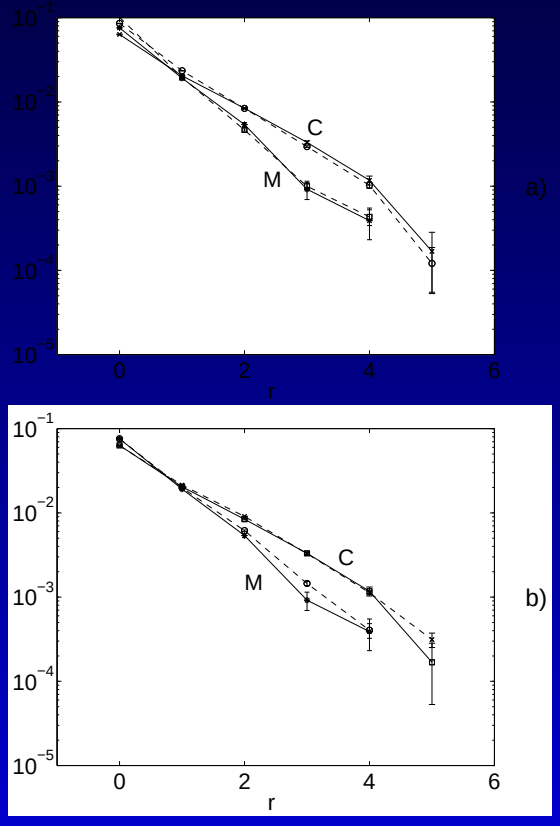
Charge and matter radial distributions

Green, Koponen, Pennanen, C. Michael, hep-lat/0105027, hep-lat/0206015 Calculation of charge matter distributions of heavy-light mesons from lattice QCD. The heavy quark is static.



Radial distribution of charge and matter of static-light meson

From hep-lat/0105027, Green et al.



Used to test relativistic potential models.

In place of conclusions

The continuum HQET community would like a non-perturbative calculation of the first inverse moment of the B meson distribution amplitude.

- The formalism of Aglietti (hep-ph/9806277) may perhaps be used to compute the entire distribution amplitude. This looks challenging to implement numerically.
- Perhaps $B \to \gamma e \nu$ can be computed on the lattice, and then λ_B extracted, in a similar manner to Ball and Kou (hep-ph/0301135). There were worries about soft corrections,